Inelastic Scattering of 14.0-MeV Protons by the Even Isotopes of Cadmium*

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Angular distributions of 14.0-MeV protons scattered elastically and inelastically by targets of ^{106,108,110,112,114,116}Cd have been obtained. Coupled-channel analysis has been used to extract extended optical-model parameters that generate fits to the scattering data of the isotopic sequence. The analysis has yielded deformabilities that have been used to calculate equivalent electromagnetic-transition enhancement factors and ratios of $B(E2)$ values, which have been compared with the results of Coulomb-excitation studies.

1. INTRODUCTION

IN this paper we report our studies of the scattering of 14.0-MeV protons from isotopically enriched targets of ^{106,108,110,112,114,116}Cd. The coupled-channel calculation[1] in the form of Tamura's computer code[2,3] is used to analyze the experimental results. This type of calculation has shown its usefulness in the deduction of reaction mechanisms and nuclear spectroscopic information, and it has the especially important capability of treating exactly, within the framework of the collective model, single-step and multiple-step processes. The coupled-channel calculation computer code includes spin-orbit distortion, complex form factor, and Coulomb excitation. The excited states of the cadmium nuclei are described in the collective model as vibrations of nuclei about the spherical equilibrium shape.[4,5] By studying a sequence of isotopes, we also obtain information on the behavior of the potential parameters of the extended optical model[6–8] as a function of neutron excess. In our analysis of the experimental data we sought a smoothly varying set of potential parameters that would account for the data reasonably well for the entire isotopic sequence rather than optimizing fits to each target nucleus.

The idea of vibration about a spherical shape does not apply exactly to the cadmium isotopes, because the static quadrupole moment of the first excited state of ^{114}Cd has been measured to be nonzero. Bromley and Weneser[9] have discussed this result and summarized the Coulomb-excitation studies on the cadmium isotopes. We can still apply Tamura's code because the various strength parameters that enter the calculation can be varied and need not be the values assigned by the vibrational model. Also, it is possible to describe states as mixtures of one- and two-phonon states. In other words, the present analysis employs the vibrational collective model in such a way that it provides a framework to describe the strengths and angular-momentum transfers between levels in a nucleus. These are just the quantities that can be reliably deduced from studies of inelastic scattering of particles.

Much of the previous experimental work on the cadmium isotopes has concentrated on the isotopes with mass numbers 110–116. Cookson and Darcey[10] have measured with 12-keV resolution the position of the states excited by inelastic proton scattering by ^{110,112,114,116}Cd. We have used their level assignments in the present work. Sakai *et al.*[11] have measured the angular distributions of the low-lying excited states in ^{114}Cd, and Sakai and Tamura[12] have applied coupled-channel analysis to these results. Koike[13] has studied the nucleus ^{112}Cd with proton inelastic scattering and has applied distorted-wave Born approximation (DWBA) analysis to the quadrupole, octupole, and hexadecapole states.

Recently Makofske *et al.*[14] have reported the excitation of quadrupole and octupole states in even isotopes of Sn, Cd, and Te with 16-MeV protons. In the case of Sn, a coupled-channel analysis has been used[15] to explain the diminution of the cross section for excitation of the quadrupole state as a function of neutron excess because of the accompanying increase of the imaginary part of the potential.

Coulomb-excitation studies have been carried out by McGowan *et al.*,[16] using oxygen ions. Using α particles,

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[3] T. Tamura, Oak Ridge National Laboratory Report No. ORNL-4152, 1967 (unpublished).
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[16] F. K. McGowan, R. L. Robinson, P. H. Stelson, and J. L. C. Ford, Jr., Nucl. Phys. **66**, 97 (1965).

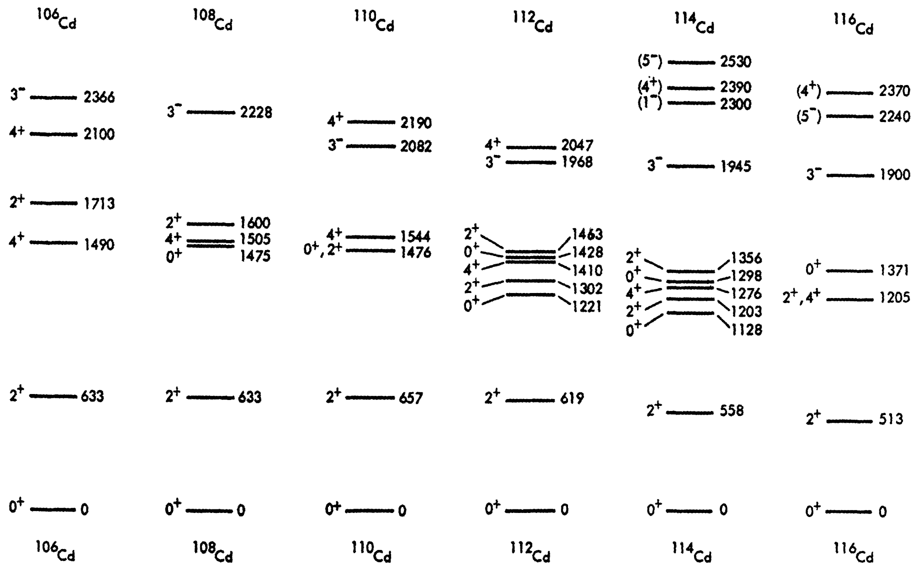


FIG. 1. Levels seen well enough in the present experiment to obtain angular distributions. Known levels separated by less than 50 keV were seen as doublets. Most of the values of the excitation energies are taken from the work of Cookson and Darcey (Ref. 10).

Hansen and Nathan[17] have located the positions of the octupole states in the even cadmium nuclei with masses between 110 and 116. The angular distribution for particles exciting the octupole state has a characteristic shape that has been used in the present experiment to confirm the location of these levels and, in addition, to identify the positions of the octupole state in ^{106}Cd and ^{108}Cd.

2. EXPERIMENTAL PROCEDURE

Protons were accelerated to 14.0 MeV by the Livermore variable-energy cyclotron.[18] The beam was momentum-analyzed by a 90° bending magnet with a 76.2-cm radius of curvature and focused by a quadrupole triplet to a spot at the center of a 60.9-cm-diam scattering chamber. The beam was monitored by a Faraday-cup–current-integrator combination.

The targets consisted of self-supporting metallic films having areal densities about 1 mg cm^{-2}. The isotopic composition of the various targets is presented in Table I. The minimum enrichment, in the case of ^{108}Cd, is 80.8%. The ^{106}Cd target was much smaller than the other targets. Geometric considerations made it impossible to obtain scattering data beyond 90° for this nucleus.

The detector telescope used to observe the scattered particles subtended a solid angle of 2.50×10^{-4} sr. It consisted of a transmission-type surface-barrier ΔE detector and a lithium-drifted silicon E detector. The ΔE detector was 51 μm thick, operated with a reverse bias of 25 V, and the E detector was 3000 μm thick, operated with a reverse bias of 250 V. These detectors were cooled to dry-ice temperature to minimize leakage currents. The particle identification circuit of Goulding, Landis, Cerny, and Pehl[19] was used to select the type of particle to be recorded. The spectra were stored in an 800-channel pulse-height analyzer.

The punched-paper-tape output of the pulse-height analyzer was used to generate computer cards that were used as input to the CDC 3600 computer. Several computer programs were then used to plot the spectra, to fit the spectra with sums of Gaussian-shaped peaks, obtaining peak locations and areas, and to deduce and plot angular distributions. In the present experiment we were able to obtain angular distributions for levels separated by more than 50 keV.

3. EXPERIMENTAL RESULTS AND COUPLED-CHANNEL ANALYSES

The levels that were seen well enough in the present experiment to obtain angular distributions are shown in the level diagram in Fig. 1. The possible members of

TABLE I. Isotopic compositions of cadmium targets used in the present experiment.

	Target					
at.% of	^{106}Cd	^{108}Cd	^{110}Cd	^{112}Cd	^{114}Cd	^{116}Cd
^{106}Cd	88.6	0.47	...	...	...	0.03
^{108}Cd	0.43	80.8	...	...	...	0.02
^{110}Cd	1.99	5.13	97.2	0.13	0.07	0.21
^{111}Cd	1.62	3.18	1.04	0.35	0.09	0.24
^{112}Cd	2.72	4.35	0.90	98.5	0.26	0.53
^{113}Cd	1.18	1.76	0.27	0.44	0.33	0.37
^{114}Cd	2.51	3.53	0.49	0.55	99.1	1.44
^{116}Cd	0.99	0.79	0.09	0.07	0.07	97.2

[17] O. Hansen and O. Nathan, Nucl. Phys. **42**, 197 (1963).
[18] H. P. Hernandez, J. M. Peterson, B. H. Smith, and C. J. Taylor, Nucl. Instr. Methods **9**, 287 (1960).
[19] F. S. Goulding, D. A. Landis, J. Cerny, and R. H. Pehl, Nucl. Instr. Methods **31**, 1 (1964).

the two-quadrupole-phonon triplet lie at energies that are more than double the excitation energies of the first excited states. In ^{106}Cd there is no indication of an excited 0^+ state, while in ^{112}Cd and ^{114}Cd, there are additional 0^+ and 2^+ states that are sometimes described[12] as quasiparticle excitations. The octupole states are now identified in each nucleus and their excitation energies decrease monotonically in going from ^{106}Cd to ^{116}Cd.

Some known levels separated by less than 50 keV

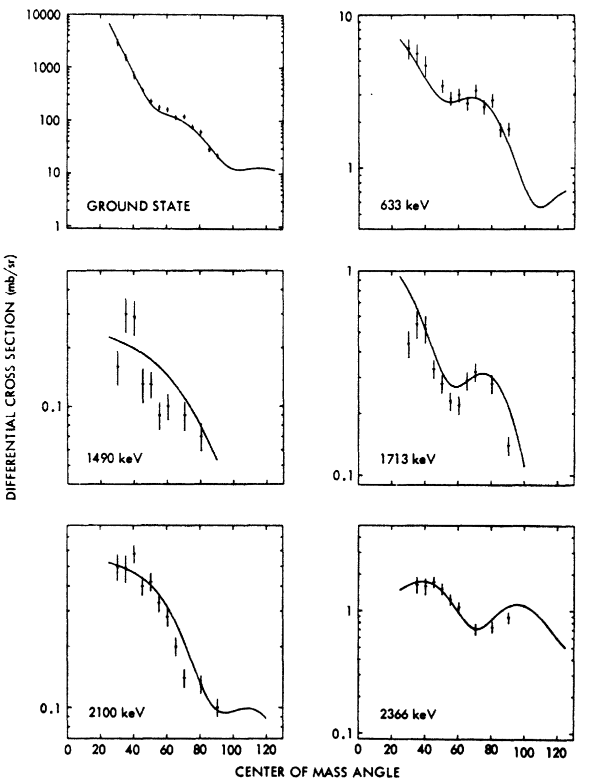


FIG. 2. Angular distributions for the scattering of 14.0-MeV protons from ^{106}Cd. The ^{106}Cd target was much smaller than the other targets used in this experiment and geometric considerations made it impossible to collect data beyond 90° for this target. The solid curves are the results of a coupled-channel calculation described in the text.

were seen as unresolved doublets. The angular distributions for the levels of each nucleus are shown in Figs. 2–7. The solid curve in each case is the result of a coupled-channel calculation performed on a CDC 6600 computer. The fits were chosen by visual inspection of the quality of fits to a number of angular distributions. A detailed least-squares procedure was not followed.

The coupled-channel calculation computer code has great capability and flexibility in terms of the number of states than can be coupled together simultaneously. To limit the time required to perform a calculation, however, one limits the number of states coupled at any one time. It is most convenient for purposes of

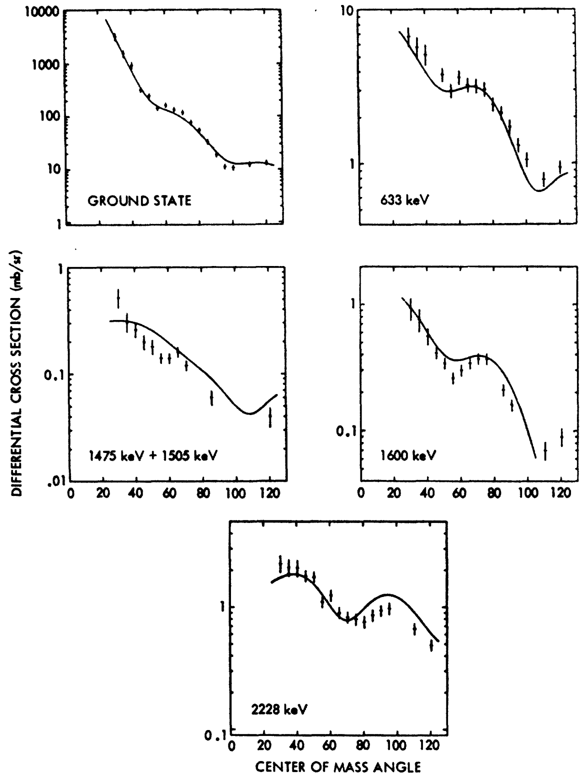


FIG. 3. Angular distributions for the scattering of 14.0-MeV protons from ^{108}Cd.

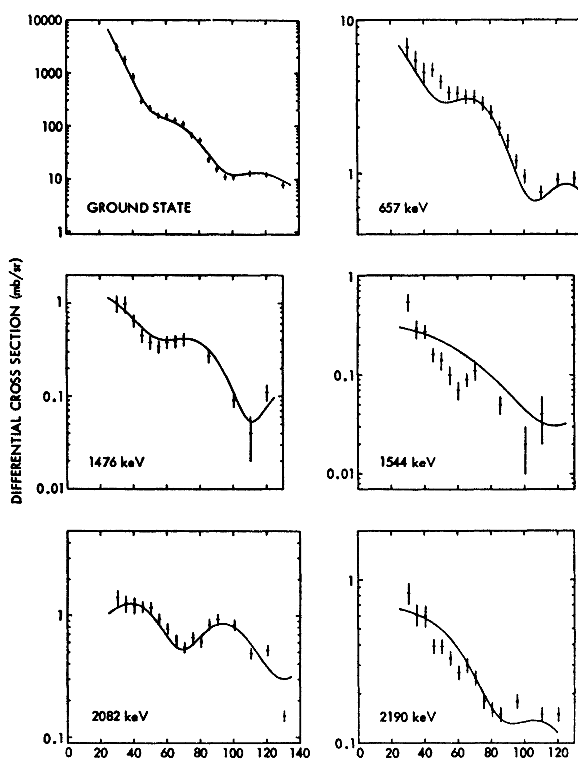


FIG. 4. Angular distributions for the scattering of 14.0-MeV protons from ^{110}Cd.

TABLE II. Parameters used in the 0_0^+–2_1^+–3_1^- calculation. $a=0.65$ fm, $\bar{a}=0.47$ fm, $V_{so}=6.5$ MeV, and $W=0.0$ MeV.

A	V (MeV)	W_D (MeV)	r_0 (fm)	$r_0A^{1/3}$ (fm)	Q (keV)	β_{02}	Q (keV)	β_{03}	β_{02} (EM)[a]
^{106}Cd	52.0	13.6	1.250	5.92	−633	0.186	−2366	0.194	0.186±5%
^{108}Cd	52.5	14.0	1.241	5.92	−633	0.199	−2228	0.207	0.195±10%
^{110}Cd	53.0	14.4	1.235	5.92	−657	0.192	−2082	0.168	0.183±4%
^{112}Cd	53.5	14.8	1.228	5.92	−619	0.195	−1968	0.173	0.186±4%
^{114}Cd	54.0	15.2	1.220	5.92	−558	0.193	−1945	0.164	0.193±4%
^{116}Cd	54.5	15.6	1.213	5.92	−513	0.201	−1900	0.160	0.201±3%

[a] Taken from compilation of Stelson and Grodzins (Ref. 20).

discussion to group the experimental results into the various coupling schemes employed in the coupled-channel calculation. We first consider the 0^+-2_1^+-3_1^- coupling scheme (where the subscript indicates the number of phonons) that couples together the ground state, the one-quadrupole-phonon state, and the one-octupole-phonon state. This is the basic calculation that we used to obtain the optical parameters from the fits to the data. The same parameters were used in the other types of coupling schemes. The next type of calculation that we consider is the 0_0^+-2_1^+-0_2^+-2_2^+-4_2^+, where, in this case, the subscript indicates the number of quadrupole phonons. After these calculations, we consider the 0^+-4^+, and 0^+-2^+ weak-coupling schemes and some possible quadrupole-octupole-two-phonon states suggested by Sakai and Tamura.[12]

A. Results for States Included in the 0_0^+-2_1^+-3_1^- Coupling Scheme

The 0_0^+-2_1^+-3_1^- coupling includes the most strongly excited states and is therefore best suited to determine the parameters for the potentials in the extended optical model. This calculation included spin-orbit distortion and Coulomb excitation, and allowed both real and imaginary parts of the potential to vibrate.

The magnitude of a calculated inelastic cross section depends on the combination of β and W_D. To determine W_D, we took β_{02} for the first 2^+ state to have the value assigned by Stelson and Grodzins[20] in the compilation of $B(E2)$ determinations by electromagnetic means. We then made a series of calculations using the usual set of geometric parameters for protons ($r_0=1.25$ fm, $a=0.65$ fm, $\bar{a}=0.47$ fm), varying V, W_D, and β_{03}, seeking a set of smoothly varying potential parameters that would account for the data. Since β_{02} was kept fixed, W_D was quite sensitive to the magnitude of the cross section for exciting the 2^+ state. Table II gives the values of V and W_D that we settled upon.

During the course of these calculations (all of which were carried out with $R=1.25A^{1/3}$ fm), it became

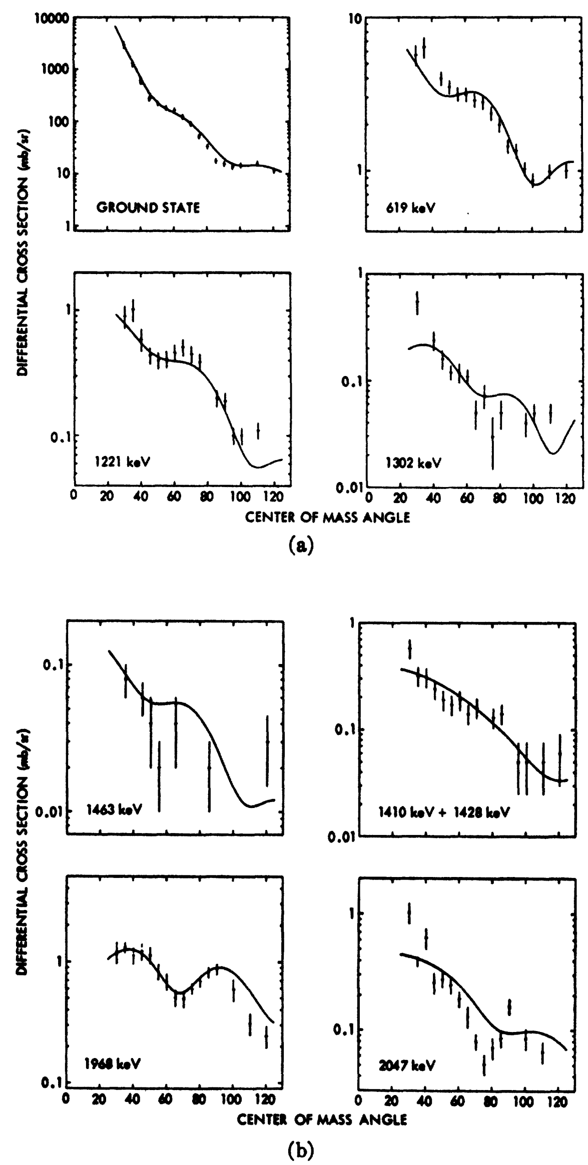
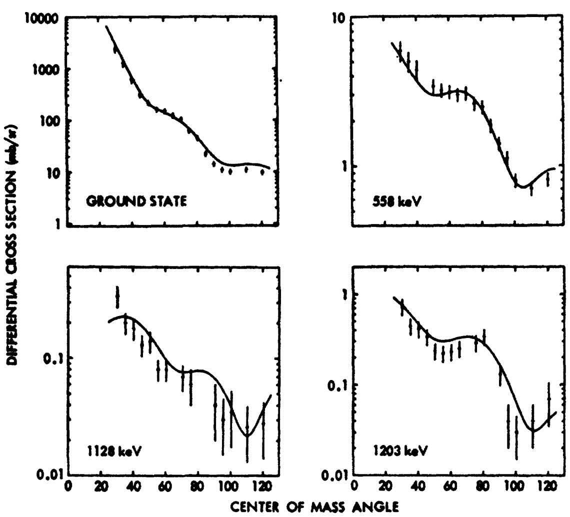
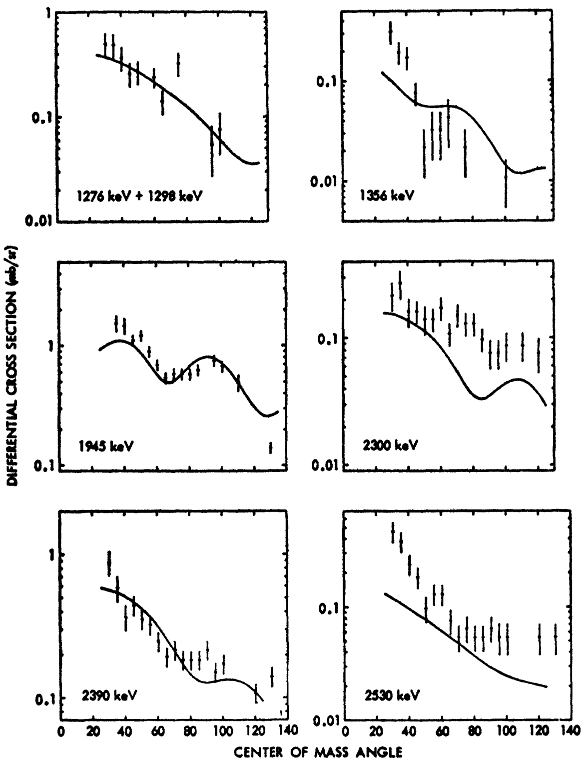


FIG. 5. Angular distributions for the scattering of 14.0-MeV protons from ^{112}Cd.

[20] P. H. Stelson and L. Grodzins, Nucl. Data A1, 1 (1965).



(a)



(b)

FIG. 6. Angular distributions for the scattering of 14.0-MeV protons from ^{114}Cd. The levels at 2300 and 2530 keV are considered to be quadrupole-octupole-two-phonon states and are shown with curves calculated with values of β_{02} and β_{03} obtained from the 558- and 1945-keV levels.

evident that the theoretical curves for the quadrupole-phonon states so generated were migrating towards 0° with increasing mass much faster than the data indicated. We found that using the same value for the nuclear radius (R=5.92 fm) markedly improved the fits to the data. Finally, the values of β_{02} were adjusted slightly to give optimum fits, but in no case did the final adjustment exceed the error incidated by Stelson and Grodzins.[20]

The potentials in Table II may be represented by

$$V=45.4+27.5[(N-Z)/A]+(0.4Z/A^{1/3}) \quad (1)$$

and

$$W_D=11.5+22(N-Z)/A. \quad (2)$$

These equations are in reasonable agreement [the constant multiplying $(N-Z)/A$ in Eq. (1) being slightly higher in our case] with the results of Satchler[21] and of Kossanyi-Demay and Swiniarski,[22] who considered only elastic scattering data.

As we have noted, the fits to the data were improved by deviating from an $A^{1/3}$ increase of the nuclear potential radius. The $A^{1/3}$ dependence has been investigated both experimentally and theoretically for a variety of nuclei. Most of the experimental results have dealt with the nuclear charge radius. Elastic electron scattering from isotopes of Ca,[23] from Sn,[24] and from Ti[25] have shown that the charge radius does not increase as $A^{1/3}$. Muonic x-ray studies of many nuclei[26] and calculations of Coulomb energy differences of the isobaric analog states of Ti,[27] as well as some other nuclei, lead to similar results. It may be said that the existence of deviations from the $A^{1/3}$ behavior for the nuclear charge radius has been well demonstrated.

Theoretically, this effect may be explained by noting that the proton binding energies of nuclei increase with neutron excess. The more tightly bound protons give rise to a smaller charge radius than would be expected from the $A^{1/3}$ dependence. Perey and Schiffer[28] show that an isospin-dependent term in the optical potential leads naturally to the increased proton binding energies and hence to the $A^{1/3}$ deviations.

The question whether the nuclear potential radius or the nuclear matter radius follows the same behavior is unanswered. Sood and Leighton[29] assume that the

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[29] P. C. Sood and H. G. Leighton, Nucl. Phys. **A111**, 209 (1968).

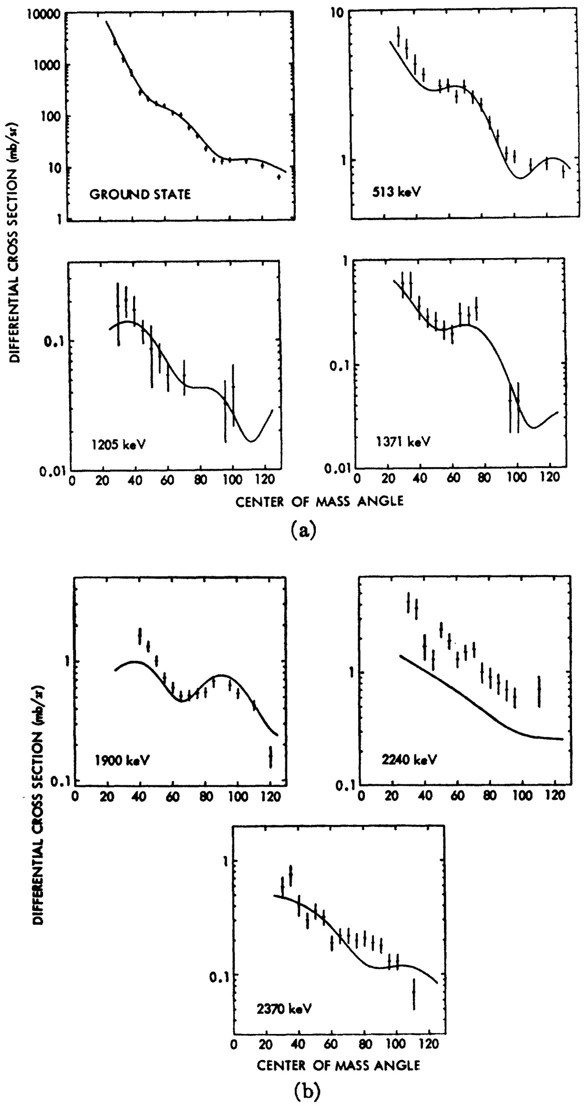

FIG. 7. Angular distributions for the scattering of 14.0-MeV protons from ^{116}Cd.

depth of the proton potential well has the form

$$V_p \simeq V_0[1+\tau(N-Z)/A],$$

with $\tau \simeq 0.5$. From the binding energies of the protons in the Ti and Xe isotopes, Sood and Leighton obtain values of V_pR^2 for each isotopic sequence as a function of $(N-Z)/A$. Assuming first that R goes as $A^{1/3}$, and then assuming R to be constant, these authors evaluate τ for both cases and find that the $A^{1/3}$ dependence leads to the wrong sign and magnitude for both Ti and Xe isotopic sequences. They quote this result as evidence for deviation from the $A^{1/3}$ behavior of the potential radius. Greenlees, Pyle, and Tang[30] have analyzed elastic proton scattering using a reformulation of the optical model. They obtain values of the nuclear matter radius from their fits to data on various isotopes of Ni, Co, Zr, Sn, and Pb that deviate from $A^{1/3}$ dependence.

In our case, the fits for the Cd isotopes were improved, particularly for the inelastic states, by maintaining a constant radius. The results of these fits, however, do not yield unique sets of parameters. It is possible that varying other parameters (say, the diffuseness) or combinations of parameters would have led to similar fits. This type of ambiguity is always present in potential model analyses. We can only say, in summary, that our fits were improved by using a constant radius for the isotopic sequence and this behavior, while not sensitively determined by our data, is not contradicted by other studies of the behavior of the nuclear matter radius.

B. Results for States Included in the 0_0^+-2_1^+-0_2^+-2_2^+-4_2^+ Coupling Scheme

The next type of calculation performed was one that calculated the excitation of the two-quadrupole-phonon state proceeding through the one-quadrupole-phonon state. It was determined that the insensitivity of the calculation and the limits on the accuracy of the experimental measurements did not merit the inclusion of spin-orbit distortions in the calculation, because their inclusion doubled the time necessary to do a calculation. The Coulomb excitation and complex form factor were maintained. The potential parameters employed in this calculation, and all that follow, were those listed in Table II and determined from the 0_0^+-2_1^+-3_1^- fits to the data.

It was found that adding a small component of single-phonon state to the second 2^+ state, and thereby directly coupling this state to the ground state, improved the fit to this angular distribution. In Fig. 8 we show how the character of the fit changes with the addition of this single-phonon admixture.

The parameters of the analyses of the two-quadrupole-phonon states are gathered in Table III. In those cases in which unresolved doublets were present, we used the same values for β_{2I} for each component. From the parameters listed in Tables II and III we can deduce reduced electromagnetic transition rates and compare to results obtained in Coulomb-excitation studies by McGowan *et al.*[16] This is done in Table IV. No errors have been assigned to our values of $B(E2)$ for the one-quadrupole-phonon state, since our methods of analysis set these to the values listed by Stelson and Grodzins.[20] The $B(E3)$ values for the one-octupole-phonon states are assigned errors of 15%. The ratios of $B(E2)$ values for two-quadrupole-phonon and one-quadrupole-phonon $[B(E2;\ I_2\rightarrow 2_1)/B(E2;\ 2_1\rightarrow 0_0)]$ deduced from (p, p') data are assigned errors of 30%.

The over-all agreement between the Coulomb-excita-

[30] G. W. Greenlees, G. J. Pyle, and Y. C. Tang, Phys. Rev. **171**, 1115 (1968).

TABLE III. Parameters of the analyses of states with one- and two-quadrupole phonons.

	One-phonon states			Two-phonon states			
Target	Q (keV)	I^π	β_{02}	Q (keV)	I^π	β_{2I}	β_{0I}
^{106}Cd	−633	2^+	0.186	−1713	2^+	0.100	0.056
				−1490	4^+	0.132	
				−1475	0^+	0.110	
^{108}Cd	−633	2^+	0.199	−1600	2^+	0.145	0.060
				−1505	4^+	0.100	
				−1476	0^+	0.130	
^{110}Cd	−657	2^+	0.192	−1476	2^+	0.130	0.060
				−1544	4^+	0.150	
				−1221	0^+	0.145	
^{112}Cd	−619	2^+	0.195	−1302	2^+	0.145	0.055
				−1410	4^+	0.163	
				−1128	0^+	0.117	
^{114}Cd	−558	2^+	0.193	−1203	2^+	0.140	0.040
				−1276	4^+	0.170	
				−1371	0^+	0.144	
^{116}Cd	−513	2^+	0.201	−1205	2^+	0.140	0.040
				−1205	4^+	0.140	

tion results and the values deduced from the (p, p') data is satisfactory. The enhancement factors for the octupole transitions are significantly higher for the (p, p') results. The ratio $B(E2; I_2\rightarrow 2_1)/B(E2; 2_1\rightarrow 0_0)$ is not 2, as predicted by the vibrational model; it is generally less than 2 and there is a tendency for the values deduced from the (p, p') data to be lower than the Coulomb-excitation results.

In ^{112}Cd and ^{114}Cd there are weakly excited 0^+ states lying close to the 4^+ state of the two-quadrupole-

TABLE IV. Ratios of reduced electromagnetic transition rates deduced from (p, p') data and compared with Coulomb-excitation studies.

		^{106}Cd	^{108}Cd	^{110}Cd	^{112}Cd	^{114}Cd	^{116}Cd
$\frac{B(E2; 2_1\rightarrow 0_0)}{B(E2)_{\mathrm{sp}}}$	CE[a]			32±2.6	34±2.4	35±2.5	36±2.5
	(p, p')[b]	32	37	34	35	36	37
$\frac{B(E2; 2_2\rightarrow 2_1)}{B(E2; 2_1\rightarrow 0_0)}$	CE[a]			1.30±0.25	1.50±0.32	1.21±0.25	0.74±0.17
	(p, p')[b]	0.58±0.17	1.06±0.32	0.92±0.28	1.10±0.33	1.05±0.32	0.97±0.29
$\frac{B(E2; 4_2\rightarrow 2_1)}{B(E2; 2_1\rightarrow 0_0)}$	CE[a]			1.42±0.19	1.82±0.23	1.80±0.23	1.63±0.36
	(p, p')[b]	1.00±0.30	0.60±0.18	1.22±0.37	1.10±0.33	1.55±0.47	0.97±0.29
$\frac{B(E2; 0_2\rightarrow 2_1)}{B(E2; 2_1\rightarrow 0_0)}$	CE[a]					0.85±0.17	0.83±0.17
	(p, p')[b]		0.60±0.18	0.92±0.28	1.40±0.42	0.73±0.22	1.03±0.31
$\frac{B(E3; 3_1\rightarrow 0_0)}{B(E3)_{\mathrm{sp}}}$	CE[a]			20±4.0	20±4.0	17±3.3	13±2.6
	(p, p')[b]	36±5.4	41±6.2	27±4.1	29±4.4	25±3.8	24±3.6

[a] Coulomb-excitation results of McGowan *et al.* (Ref. 16).

[b] Deduced from (p, p') data of the present experiment.

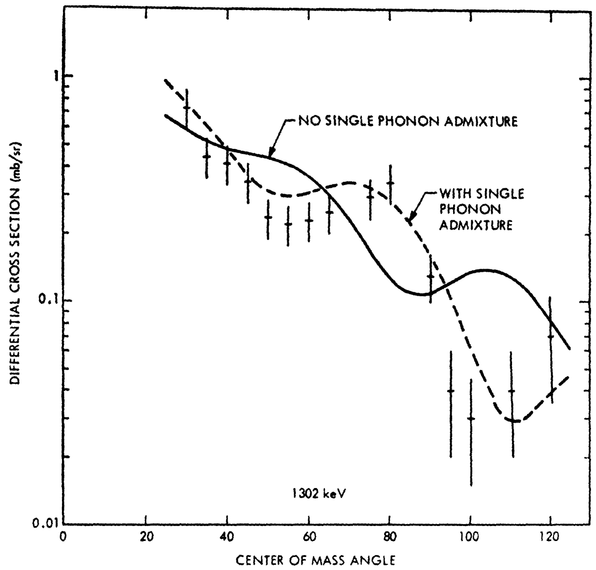

FIG. 8. Improvement brought about by adding a single-phonon admixture to the two-quadrupole-phonon state at 1302 keV in ^{112}Cd.

phonon triplet. The contribution of this state has been ignored and it is believed that it has no qualitative effect on the analyses.

C. Results for State Included in the 0^+-2^+ and 0^+-4^+ Coupling Schemes

The weak levels coupled to the ground state in the 0^+-2^+ and 0^+-4^+ coupling schemes could easily be computed with DWBA. It was, of course, more convenient for us to use the coupled-channel calculation. The parameters are shown in Table V. The weak 2^+ levels in ^{112}Cd and ^{114}Cd both have β_{02} values of 0.025, indicating that they are indeed quasiparticle excitations and not collective in nature.

We are more certain of the 4^+ assignments in ^{106}Cd, ^{110}Cd, and ^{112}Cd than we are of those in ^{114}Cd and ^{116}Cd. All the values of β_{04} fall in the neighborhood of 0.08, but the quality of the fits to the first three levels is somewhat better than it is to the latter two. Moreover, the 4^+ levels in ^{106}Cd, ^{110}Cd, and ^{112}Cd occur at 3.3 times the energy of the first 2^+ state, while the levels in ^{114}Cd and ^{116}Cd fall considerably higher in energy. The pure vibrational model predicts a 4^+ state at three times the energy of the quadrupole state.[31]

TABLE V. Parameters of the analyses of the states in 0^+-2^+ and 0^+-4^+ coupling schemes.

Target	Q (keV)	I^π	β_{0I}
^{112}Cd	−1463	2^+	0.025
^{114}Cd	−1356	2^+	0.025
^{106}Cd	−2100	4^+	0.078
^{110}Cd	−2190	4^+	0.091
^{112}Cd	−2047	4^+	0.074
^{114}Cd	−2390	(4^+)	0.090
^{116}Cd	−2390	(4^+)	0.083

D. Results for Quadrupole-Octupole-Two-Phonon States and Three-Quadrupole-Phonon Calculation

Sakai and Tamura[12] have identified the 2300- and 2530-keV states in ^{114}Cd as being quadrupole-octupole-two-phonon states. By comparison we have assigned a 5^- to the 2240-keV state of ^{116}Cd. The solid curves in graphs showing the experimental data for these states were generated using the same strength and optical parameters as were used in the $0_0{}^+$-$2_1{}^+$-$3_1{}^-$ calculations. The theoretical curves fall below the experimental data, but the shapes of the curves do follow the data.

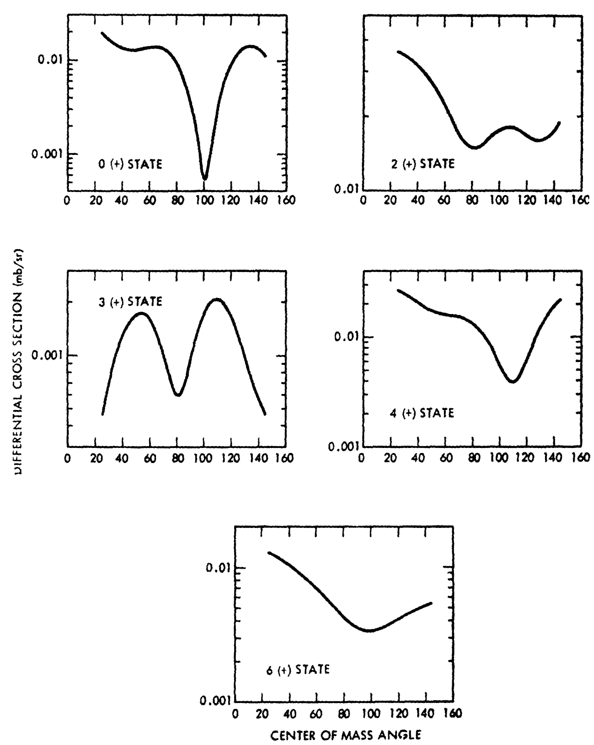

FIG. 9. Theoretical curves for the scattering to three-quadrupole-phonon states. Although the angular distributions have characteristic shapes, their magnitudes are too small to have been observed in the present experiment. All states were assumed to fall at 1800 keV for the purpose of the calculation.

States that have a three-quadrupole-phonon character should appear at roughly three times the energy of the first 2^+ state. We have calculated the cross sections for exciting these states and present them in Fig. 9. The theoretical calculations indicate that the shapes of

[31] L. J. Tassié, Australian J. Phys. 15, 135 (1962).

the various curves are distinguishable, but their magnitudes are so small that the present experiment could not see them. It is possible, however, that an experiment with highly enriched targets, good resolution, and very low background could observe states having a three-quadrupole-phonon character.

4. SUMMARY

We have measured the scattering of 14.0-MeV protons from the even isotopes of cadmium. Coupled-channel analysis, using optical-model parameters that vary in a reasonable way as a function of neutron excess, provides good fits to the experimental angular distributions. By using values for β_{02} from the compilation of Stelson and Grodzins,[20] we have determined W_D, since the cross section for exciting the first 2^+ state is sensitive to the combination of β_{02} and W_D. The analysis thus yields a value for the imaginary part of the isospin-dependent part of the extended optical model. The analysis indicates that the nuclear radius does not vary as $A^{1/3}$ in the isotopic sequence; appreciable improvement in the quality of the fits, especially those to the one-quadrupole-phonon and one-octupole-phonon states, is achieved when a constant value for the nuclear radius is employed. The octupole state has been identified in each nucleus, and enhancement factors 29 to 40 times single-particle estimates are deduced for these transitions. The second 2^+ state, sometimes thought of as a member of a two-quadrupole-phonon triplet, has some direct coupling to the ground state. Finally, the β values deduced indicate substantial reductions below the vibrational-model prediction for the ratios of the strength of the electric quadrupole transition between a member of the two-quadrupole-phonon triplet and the first 2^+ state to that of the 2^+ state to the ground state.

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Nuclear Levels in Re^{186}†

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Nuclear levels in Re^{186} have been investigated by combining the following experimental techniques: the study of (d, p) and (d, t) reactions with 12-MeV deuterons and a broad-range magnetic spectrograph; the investigation of γ transitions between 3.6 and 6.2 MeV from neutron capture using a Ge(Li) spectrometer; the measurement of (n,γ) radiation between 28 and 770 keV with a bent-crystal spectrometer; and the measurement of the (n, e^-) spectrum in the energy range 0–350 keV with a magnetic β spectrometer. With the aid of (d, p) angular distributions and transition multipolarities, the energy levels up to an excitation energy of approximately 500 keV have been interpreted as arising from the coupling of the proton orbital [402↑] to the neutron orbitals [512↓], [510↑], and [503↑]. A possible proton excited state at 314 keV has been classified as resulting from the configuration p[514↑]; n[512↓]. Many higher-lying levels have also been observed. A Coriolis band-mixing calculation has been performed in an attempt to understand further the Re^{186} level structure below 500 keV. The (d, p) intensity pattern of the ground-state rotational band is in considerable disagreement with theoretical prediction, and it is suggested that residual interaction effects may be severely distorting the simple two-particle wave functions of the low-lying configurations.

I. INTRODUCTION

THE odd-odd nucleus Re^{186} presents an interesting spectroscopic problem. Although this nucleus is characterized by a moderately strong deformation[1] $\beta \simeq 0.22$, it occupies a mass region where the deformation is beginning to change relatively rapidly as a

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[1] L. Armstrong and R. Marrus, Phys. Rev. **138**, B310 (1965).