Study of the C¹² (p, p_α) Be^s Reaction and Excited States of C¹²[†]

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The C¹² (p, p_{α}) Be⁸ reaction has been studied for several proton and α -particle angles using a bombarding energy of 57 MeV. It is found that the reaction proceeds primarily by sequential α decay, i.e., the proton inelastically scatters, leaving C¹² in an excited state which then decays by emitting an α particle. The properties of several of the excited states of C¹² in the giant dipole region were investigated. States at 19.7, 21.1, 22.2, and 26 MeV were found which have a significant $T=0$ component. In addition, the C¹²(p , 2p) B¹¹ reaction was measured simultaneously. These data indicate that a C^{12} state at 20.3 MeV decays to the ground state of B¹¹ by proton emission.

INTRODUCTION

THE primary objective of most of the previous **1** studies of $(p, p\alpha)$ reactions on light nuclei has been to obtain information on the α -cluster parentage of such nuclei.¹⁻⁴ Several of these studies, carried out at energies above 100 MeV, have been analyzed with reasonable success using the quasi-elastic, $p-\alpha$ scattering approximation and have yielded information on the parentage and momentum distribution of α clusters in light nuclei.

The initial objective of the present study was to investigate the nature of the reaction mechanism of the $C^{12}(p, p\alpha)$ Be⁸ reaction at energies in the region of 60 MeV. The early results clearly indicated that this reaction proceeds primarily by sequential processes instead of a quasi-elastic scattering process at such energies. This sequential process involves initial inelastic proton scattering leading to excited states of $C¹²$ which subsequently α decay by emitting an α particle. This observation of a dominant sequential process is in contrast to $(p, p\alpha)$ studies at approximately the same energy on $Li^{6,6} Li^{7,6}$ and Be⁹.⁷ These reactions all showed large contributions from quasi-elastic scattering. The study of C^{12} was then extended to investigate the properties of several of the observed excited states.

The states in the region of excitation between 20 to

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178

²⁶ MeV in C" were of greatest interest. By observing both the inelastically scattered proton and the decay α particle we were able to establish the mode of decay of these states as well as the mechanism by which they are excited. In order to obtain additional information on these excited states we also studied the proton decay mode of states in the same region of excitation by observing the $C^{12}(p, 2p)B^{11}$ reaction. This information in conjunction with the results of previous investigations of states in this region of excitation of C^{12} has enabled us to obtain certain spectroscopic information about these states.

EXPERIMENTAL METHOD

The 57-MeV proton beam from the Oak Ridge Isochronous Cyclotron was energy analyzed to provide a beam with an energy spread of about 100 keV. A 1.0 mg/cm' carbon foil target was placed at the center of a 30-in. diam scattering chamber.

The particles emitted from the target were detected in two counter telescopes which were coplanar with the beam. One of these telescopes, which we shall call the proton telescope, consisted of a $500-\mu$ totally depleted solid-state surface-barrier detector ΔE_p , and a NaI(T1) detector E_p of thickness 1.25 in. The other telescope, which we call the α -particle telescope, consisted of two solid-state surface-barrier detectors: a $100-\mu$ totally depleted detector ΔE_{α} , and a 1200- μ detector E_{α} . The solid angles were defined by a circular slit on the α particle telescope to be 8.00×10^{-3} sr, and by a rectangular slit on the proton telescope to be 5.66×10^{-3} sr. The height of the rectangular slit was twice the width.

Figure 1 shows a schematic block diagram of the electronics used throughout most of the experiment. The timing signals were derived from the ΔE_{p} detector and the E_{α} detector, using zero-crossover circuits. A time-to-amplitude converter (TAC) and a singlechannel analyzer were used as the coincidence circuit. The time resolution was about 15 nsec full width at half maximum (FWHM); thus, accidental events from consecutive rf beam bursts were well separated.

A fast multiplier circuit of the type designed by 1698

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Ridge Associated Universities.
 $\frac{1}{2}$ A. A. James and H. G. Pugh, Nucl. Phys. 42, 441, (1963).
 $\frac{1}{2}$ T. Yuasa, Phys. Lett

⁴ C. Ruhla, M. Riou, M. Gusakow, J. C. Jacmart, M. Liu, and
L. Valentin, Phys. Letters 6, 283, (1963).
⁵ M. Jain, M. Epstein, H. D. Holmgren, H. G. Pugh, and P. G.
Roos, Bull. Am. Phys. Soc. 12, 466 (1967).
⁶ H. G.

Fro. 1. Block diagram of electronics.

Radeka⁸ was used to identify the particles entering into the α -particle telescope. The output of this multiplier circuit was fed into two single-channel analyzers which had windows set about the peaks corresponding to α particles and to protons. A coincidence between one of these outputs and the output from the single-channel analyzer set on the TAC spectrum was used to gate the linear signals from the two telescopes. The linear signals were then sent to a 20 000 channel pulse-height analyzer operated as two 100×100 two-dimensional arrays. Coincident events in which a proton was recorded in the α -particle telescope were routed into one 100×100 array, and those resulting from an α particle were routed into the other 100×100 array.

The two signals from the α -particle telescope were added to obtain $(\Delta E_{\alpha} + E_{\alpha})$ but only the E_{p} pulse from the proton telescope was recorded. Since the ΔE_{p} pulse

was not added to E_p , the ΔE_p detector acted as an absorber and provided discrimination against He' and He4 particles with energies less than about 32 MeV in the proton telescope. The degradation of the energy resolution for the proton signal resulting from such an absorber was small compared to the energy resolution of the NaI crystal. In addition, a 1600-channel analyzer was used in order to monitor various gains and discriminator windows during data acquisition, as is shown in the block diagram.

ENERGY SPECTRA AND DATA REDUCTION

Data for the C¹²(\hat{p} , $\hat{p}\alpha$) Be⁸ reaction at $\theta_p = 101^\circ$ and $\theta_{\alpha} = -30^{\circ}$ were obtained at an incident energy of 56.5 MeV. These angles correspond to the quasifree scattering angles (angles at which it is kinematically possible for the Be⁸ nucleus to be left at rest in the laboratory system). The remaining data for the $C^{12}(p, p\alpha)$ Be⁸ reaction were taken simultaneously with the data for

⁸ V. Radeka, Brookhaven National Laboratory Report No. 7448 (unpublished) .

Fro. 2. Two-dimensional spectrum of the $C^{12}(p, p\alpha)Be^8$ reaction. The intensity in a given channel is denoted by the size of the dot. The two curves correspond to the calculated three-body kinematic curves for the $C^{12}(p, p\alpha)$ reaction to the ground and first excited (2.9-MeV) states of Be⁸.

the $C^{12}(p, 2p)B^{11}$ reaction at an energy of 57.8 MeV. The following combinations of angles were studied:
 $\theta_p = 58^\circ$, $\theta_\alpha = -37^\circ$; $\theta_p = 58^\circ$, $\theta_\alpha = -60^\circ$; and $\theta_p = 58^\circ$, $\theta_{\alpha} = -85^{\circ}$. As previously indicated, the designations θ_{α} and E_{α} refer to the all-solid-state telescope which was used to detect both protons and α particles.

A representative two-dimensional spectrum of the $(p, p\alpha)$ data as a function of E_p and E_α is shown in Fig. 2. Since the ΔE_{α} counter stopped 12-MeV α particles (3-MeV protons), the observations were limited to α -particle energies above this value. The data clearly illustrate the presence of the three-body kinematic lines for the C¹²(\hat{p} , $\hat{p}\alpha$) Be⁸ reaction proceeding to the ground state and first excited state of Be⁸.

Figure 3 illustrates a two-dimensional spectrum for the C¹²(\dot{p} , 2 \dot{p}) B¹¹ reaction obtained at $\theta_p = 58^\circ$ and $\theta_{\alpha} = -85^{\circ}$ (the second proton). The kinematic line for the C¹²(ϕ , 2 ϕ) B¹¹ reaction to the ground state of B¹¹ as well as various excited states of B¹¹ can be seen.

Other possible three-body and four-body reactions which can result from bombardment of C¹² with

FIG. 3. Two-dimensional spectrum of the C¹²(p , 2 p) B¹¹ reaction. The intensity in a given channel is denoted by the size of the dot. The two curves for the C¹²(ϕ , 2 ϕ) reaction to the ground and first excited (2.124 MeV) states of $B¹¹$.

medium-energy protons do not contribute to the kinematic regions of interest in either the $(p, p\alpha)$ or the $(p, 2p)$ spectra because their Q values are more negative or because of electronic or kinematic restrictions.

The real-to-accidental rate throughout the twodimensional energy spectrum is proportional to the product of the two singles counting rates. The computed two-dimensional array of accidental events was normalized to the actual accidental rate by comparing a section of the computed array to the corresponding section of the actual coincidence array where, because

FIG. 4. Projections of the C¹²(p , $p\alpha$) Be⁸ data onto the proton energy axis. Also shown in the figure is the momentum of the recoil Be⁸. In the plane-wave impulse approximation for quasielastic scattering this momentum is equal to that of the knocked out α particle before the collision. The arrows denote excitation energies in C¹².

of kinematic restrictions, no contributions from real events could occur. Using such a procedure we estimated that the real-to-accidental ratio for the $C^{12}(\rho, \rho \alpha)$ Be⁸ data was typically 15:1 and for the $C^{12}(p, 2p)B^{11}$ data 7:1 in the regions of interest along the kinematic lines. Since the real-to-accidental ratios were generally large, the data were not corrected for accidental events.

The distributions of events along the prominent three-body kinematic lines were projected onto the proton (E_p) axis for all of the data. In all cases the regions corresponding to the ground state and first excited state of the residual nuclei were clearly separated. Only those parts of the projected spectrum which correspond to regions in the two-dimensional array that

FIG. 5. Projections of the $C^{12}(p, p\alpha)$ Be⁸(g.s.) reaction onto the proton energy axis. The arrows denote excitation energies in C¹².

were entirely above the proton or α -particle energy cutoff were retained.

The projected spectra for the C¹²(p , $p\alpha$) Be⁸ reaction are shown in Figs. 4–6. In the case of the $C^{12}(p, 2p)B^{11}$ reaction the data obtained for all three experimental runs with $\theta_p = +58^\circ$ and $\theta_\alpha = -37^\circ$, -60° , and -85° , were very similar; hence, the three spectra projected onto the E_p axis at the fixed θ_p were summed in order to improve the statistical accuracy of the data. The summed projected spectrum for the $C^{12}(\rho, 2\rho)B^{11}$ reaction leading to the ground state of $B¹¹$ is shown in Fig. 7. The statistical uncertainty of projected spectra for excited states of $B¹¹$ was too large to obtain any significant results.

The error bars in the above figures indicate the relative uncertainties in the data. The absolute values of the cross section were uncertain to about 50%. Uncertainties in the relative cross sections were primarily due to statistical fluctuations and those in the absolute value, to nonuniformity of the targets.

DISCUSSION OF THE DATA

The C¹²(ϕ , $\phi \alpha$) Be⁸ data at 160 MeV³ is dominated by quasi-elastic scattering. Using the higher energy results as a guide, we might also expect to see a quasi-elastic peak at $q_{\alpha} = 0$ with a width of approximately 1 F^{-1} in our data for $\theta_p = 101^\circ$ and $\theta_q = -30^\circ$. None of our data in the region of quasifree scattering angles exhibit such a broad peak. The data at all angles studied exhibit a number of narrow peaks, all with a width less than 0.25 F⁻¹. In fact, the cross section for the transition to ground state of Be' at the quasifree scattering angles of $\theta_p = 101^{\circ}$ and $\theta_{\alpha} = -30^{\circ}$ has a minimum at $q = 0$. If we assume that the background in the ground-state data shown in Fig. 4 is due to quasifree scattering, the maximum quasifree cross section at $q=0$ must be less than $5 \mu b / sr^2$ MeV.

All of the results obtained at bombarding energies near 57 MeV for the C¹²(ϕ , $\phi \alpha$) Be⁸ reaction indicate that this reaction proceeds predominantly by a sequential mechanism of the form

$$
\begin{array}{c}\nC^{12} + p \rightarrow p' + C^{12*} \\
\downarrow^{\infty} \\
\rightarrow^{\infty} + Be^{8}.\n\end{array}
$$

Other types of sequential processes such as

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C^{12}+p\rightarrow Be^{8}+Li^{5}
$$

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$$
C^{12}+p\rightarrow\alpha+B^{9}
$$

\n
$$
\rightarrow p+Be^{8}
$$

do not appear to contribute significantly to the reaction in the region of phase space studied in the present experiment.

Peaks corresponding to four excited states of C^{12} in the region of excitation energies between 18 and 32 MeV were observed in the C¹²(p , $p\alpha$)Be⁸ reaction. States at 22.2 ± 0.5 and 26.3 ± 0.5 MeV were observed in the reaction leading to the ground state of Be^8 and at 19.7 ± 0.5 , 21.1 ± 0.3 , and 26 ± 0.8 MeV for the reaction leading to the 2.9 MeV state. The width of all of the states observed appeared to be greater than the energy resolution of the proton detector (about 1.5%).

Several of the states observed lie in the region of the $T=1$, $J^{\pi}=1^-$ giant dipole resonance of C¹². It should be noted, however, that a pure $T=1$ state is forbidden by isospin conservation to decay via α -particle emis-

FIG. 6. Projections of the
C¹²(\dot{p} , $\dot{p}\alpha$) Be⁸ (2.9-MeV) reaction onto the proton energy axis. The arrows denote excitation energies in C¹².

sion to either the ground state or the first excited state of Be8. Furthermore, as a result of the hypothesized particle-hole character of the giant dipole states⁹ such states might be expected to decay primarily by the nucleon decay mode which is isospin allowed.

The summed projection of the three kinematic curves onto the E_p (protons inelastically scattered at a fixed $\theta_p = 58^\circ$) axis for the C¹²(p, 2p)B¹¹ (g.s.) reaction shown in Fig. 7 exhibits only one pronounced peak corresponding to an excitation energy of 20.3 ± 0.5 MeV ⁹ V. Gillet and N. Vinh Mau, Nucl. Phys. 54, 321 (1964).

in $C¹²$. This peak does not correspond to any of the peaks seen in the $C^{12}(p, p\alpha)$ Be⁸ leading to either the ground state or 2.9-MeV state of Be⁸. Furthermore, we do not see a strong contribution from the main part of the giant dipole resonance at 22.5 MeV in the C¹²(ϕ , 2 ϕ) B¹¹ reaction leading to either the ground state or any other region of excitation of B¹¹. Based on the particle-hole picture of the giant resonance, this is somewhat surprising. However, the absence of a peak in this reaction may only indicate that the excitation of the giant dipole state by inelastic proton scattering is very weak at the

relatively large angle of 58'. DWBA calculations for $L=1$ show that the cross section peaks near 10[°] and falls off rather rapidly with increasing angle. The absence of any strong states in the region of the giant dipole resonance in recent 60-MeV inelastic proton scattering data¹⁰ on C¹² near 60° also appears to support this hypothesis.

SPECTROSCOPIC PROPERTIES OF THE STATES OF C¹²

A large number of experiments involving studies of the $B^{11}(\rho, \gamma) C^{12}$,¹¹ $B^{11}(\rho, n) C^{11}$,¹² $B^{11}(\rho, \alpha) Be^{8}$,^{13,14} the $B^{11}(p, \gamma) C^{12}$,¹¹ $B^{11}(p, n) C^{11}$,¹² $B^{11}(p, \alpha) B^{13}$,¹³
 $C^{12}(\gamma, p) B^{11}$,¹⁵ and $C^{12}(\gamma, n) C^{11*}$ ¹⁶ reactions, and other reactions such as $C^{12}(p, p')C^{12*}$ at 60 MeV¹⁰ and $C^{12}(\alpha, \alpha') C^{12*}$ at 62 MeV¹⁷ have been performed which yield information about states in the giant dipole region of C^{12} . The accuracy of the energy measurements and the large widths of many of the levels makes it somewhat difficult to compare different experiments. However, in this section we have attempted to compare the results of the various experiments in order to obtain additional information about the spectroscopic properties of the states in C¹² observed. The results of all the above studies are summarized in Table I. γ_0 and α_0 refer to transitions to the ground state of the residual nucleus and γ_1 and α_1 refer to transitions to the first excited state.

1. 19.7-MeV State. The 19.7-MeV state is only seen in the C¹²(p , $p\alpha$)Be⁸ (2.9) reaction at $\theta_p=101^\circ$ and $\theta_{\alpha} = -30^{\circ}$. The data for this reaction at other angles does not extend to sufficiently low excitation energies. Due to statistics and the excitation of the 22.2-MeV state, a weak peak would be difficult to identify in the data for the $\text{C}^{12}(\rho, \, p\alpha) \text{Be}^8$ (g.s.) reaction at any angle A peak in this region of excitation is seen in both the $C^{12}(\rho, p') C^{12}$ and the B¹¹(ρ , n) C¹¹ reactions, but not in other reactions. The α decay of the 19.7-MeV state to Be⁸ (2.9) indicates that it has a $T=0$ component; however, sufficient information is not available to specify any further spectroscopic information.

2. 20.3-MeV State. The 20.3-MeV state is the only state seen in the $C^{12}(p, 2p)B^{11}$ (g.s.) reaction. It is excited relatively strongly in this reaction compared to the general continuum in the same region of the spectrum. This state if weakly excited, would be very difficult to see in the data for the C¹²(p , $p\alpha$) Be⁸ reaction due to the strong excitation of nearby levels and in some cases due to the lower-energy thresholds. No strong peak is seen in this region of the spectrum in

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- ³³ G. D. Symons and P. B. Treacy, Nucl. Phys. 46, 93 (1963).
¹⁴ L. Marquez, J. O. Laugier, R. Ballini, C. Lemeille, N. Saunier,
and J. Rey, Nucl. Phys. **A97**, 321 (1967).
¹⁶ Y. M. Shin and W. E. Stephens, Phys. Rev.
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Fro. 7. Projections of the C¹²(p , 2 p) B¹¹(g.s.) reaction onto the proton energy axis. The data for three pairs of angles has been averaged to improve the statistics. The arrow indicates excitation energy in C^{12} .

either the C¹²(α , α') C¹² or C¹²(β , β') C¹² reaction. Several other reactions do, however, exhibit structure corresponding to such a state. A $T=1$ assignment is consistent with the experimental observations but on assignment of J^{π} can be made on the basis of the existing data.

3. 21.1-MeV State. The 21.1-MeV state is seen very strongly in the $C^{12}(p, p\alpha)$ Be⁸ (2.9) reaction, but not in the C¹²(p , $p\alpha$) Be⁸ (g.s.) or C¹²(α , α') C¹² reactions. A weakly excited 21.1 MeV in either of these last two reactions would be difficult to observe; however, an assignment of an unnatural parity with $J \geq 1$ and with a large $T=0$ component could explain the above observations.

The 21.1-MeV state is also not seen in the $B^{11}(p, \gamma_0)C^{12}$ reaction, but is seen in the B¹¹(p , γ ₁)C¹² reaction. If we assume that the 21.1-MeV state has unnatural parity with $J \ge 1$ and $T=0$, then the observation of the $B^{11}(p, \gamma_1)$ C¹² reaction makes the assignment of values $\mathcal{D}(\gamma, \gamma_1)$ exaction makes the assignment of values
of J greater than 3 improbable. A 2⁻ state of C¹² could decay to the first excited 2^+ state of C^{12} by a E1 transition; but as $\Delta T=0$, E1 transitions in self-conjugate nuclei are strongly inhibited, the assignment of 2 seems improbable. A 1^+ state must decay to the 0^+ ground state by a $M1$ transition which Morpurgo's selection predicts will be strongly inhibited, but could

¹⁰ F. Bertrand (private communication).

[&]quot; F. Bertrand (private communication).
¹¹ R. G. Allas, S. S. Hanna, Luise Meyer-Schutzmeister, and R. E. Segel, Nucl. Phys. 58, 122, (1964).

¹² J. C. Overly and R. R. Borchers, Nucl. Phys. 65, 156 (1965).

decay to the 2^+ excited state by either $M1$ or $E2$ transition. A 3^+ state requires a $M3$ transition to ground state and a $M1$ or $E2$ transition to the excited state. Hence, all of the above observations are consistent with either a 1^+ or 3^+ assignment.

The 21.1-MeV state is seen only weakly in the $C^{12}(p, p')C^{12*}$ reaction. An assignment of 1⁺, $T=0$ is consistent with this observation since according to calculations by Banerjee¹⁸ those 1^+ , $T=0$ states which have strong transitions from the ground state should lie below the lowest single-particle emission threshold (about 16 MeV); thus any $T=0$, 1⁺ states seen at an excitation of 21 MeV would be expected to be weakly excited by inelastic proton scattering. Although the $C^{12}(\rho, \rho')C^{12}$ reaction may be inhibited for a $T=0, 1^+$ state, such a state might still be seen relatively strongly in the C¹²(ϕ , $\phi \alpha$)Be⁸ (2.9) reaction since the strong $\Delta T=0$ selection rule that governs the decay of this reaction should select the $T=0$ states from the general background of other states excited in the C¹²(ϕ , ϕ)['])C¹² reaction. The excitation of a 3^+ state in C^{12} by inelastic proton scattering would also be expected to be strongly inhibited, hence, similar arguments also apply in this case.

4. 22.2-MeV State. The 22.2-MeV state is seen in the $C^{12}(p, p\alpha)$ Be⁸ (g.s.) reaction; thus, it must be a natural parity state with a substantial $T=0$ component. This state is seen relatively strongly in both the $C^{12}(p, p')C^{12}$ and the $C^{12}(\alpha, \alpha')C^{12}$ reactions which is consistent with the hypothesis of a natural parity, $T=0$ state. Furthermore, a weakly excited state near 22.2 is also seen in the B¹¹(p , γ_0)C¹² reaction. The angular distribution of the γ rays shows predominantly an A_2 component, indicating that $J=1$. In addition, this state is seen in the $B^{11}(p, \gamma_1)C^{12}$ and the $C^{12}(\gamma, n)C^{11}$ reaction. The magnitude of the cross sections in both of the latter reactions indicates that these are E1 transitions giving further justification to the assignment of $J=1$. Therefore, if it is the same state which is actually seen in all of the above reactions the assignment of J^{π} = 1⁻ is highly probable. Furthermore, because of the $T=1$ selection rule on E1 transitions in self-conjugate nuclei this state probably has both $T=1$ and $T=0$ components, On the other hand, it is also possible that there are two states at 22 MeV, one with $T=0$ and the other with $T=1$. If two such states exist the J^* assignment of the $T=0$ state observed in the C¹²(p , $p\alpha$)Be⁸ reaction can only be restricted to natural parity

5. Z6-MeV State. Since the 26-MeV state is seen in the C¹²(ϕ , $\phi \alpha$) Be⁸ reaction leading to both ground state and first excited state, it must be a natural parity state with a $T=0$ component. This state is not seen in the $C^{12}(\alpha, \alpha')$ C¹² reaction. As most of the data presently available on other reactions do not extend to this excitation energy, it is not possible to specify the parameters of this state further.

SUMMARY

At bombarding energies in the region of 60 MeV the $C^{12}(\rho, \rho \alpha)$ Be⁸ reaction to the ground state and first excited state of Be' appears to proceed predominantly by a sequential mechanism. The large contribution of the quasi-elastic mechanism observed in the $(p, p\alpha)$ reactions on Li', Li', and Be' at similar energies and the small contributions of this process to the C¹²(p , $p\alpha$) Be⁸ reaction at 60 MeV is probably a result of the larger binding energy of the α particle in C¹² than in Li⁶, Li⁷, or Be⁹. The effect of the larger binding energy reduces the long-range tail of the wave function which may be the most important factor in the difference between the reaction on C^{12} and the lighter elements.

The spectroscopic information obtained indicates that the spectrum of states in the giant dipole region of C^{12} is relatively complex. Configurations other than the simple particle-hole configuration are important. Many of the states in this region also have a large $T=0$ component.

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¹⁸ M. Banerjee (private communication).