potential; the other part arises from the combined effect of the potential outside the core, and the core. This second part can be obtained from the solution of a t-matrixlike equation with a well-behaved kernel, and for S waves this t-matrixlike equation becomes the equation for the t matrix of a potential which is obtained by shifting the potential outside the core into the origin. The total on-shell, S-wave scattering amplitude has been shown to be equal to the sum of the hard-core amplitude plus a phase factor times the amplitude due to the shifted potential. We note in passing that many of the results of scattering theory which depend on the potential being well behaved can be applied to the t-matrixlike Eq. (2.23); e.g., one could apply Weinberg's[13] analysis of the Born series to the iterative solution of this equation.

Starting from the separated form of the t matrix it has been possible to find a separable expansion of the t matrix. This consists of two parts: The separable expansion of the pure hard-core t matrix, and a part which arises from the separable expansion of the t-matrixlike operator $\tau(s)$ [see (3.23)]. For each of the two separable expansions we have used an expansion of the type suggested by Weinberg.[13] Of course, it is possible that there might be better separable approximations than those obtained by truncating the Weinberg series. One possibility we are studying is to approximate the pure hard-core t matrix by the hard-shell t-matrix introduced by Puff,[10] and then to expand the difference as a sum of separable terms.

In our application of the separable expansion, we have only considered the S-wave part of the t matrix. In their work with Yukawa potentials, Ball and Wong[14] have found that the effect of the higher partial waves is small. Whether this is true or not for hard-core potentials remains to be demonstrated. The most practical way of doing this is probably by using perturbation theory,[19] since adding more terms to the separable expansion would make the size of the matrix to be diagonalized unreasonably large.

In conclusion, we note that the separable expansions we have considered could be of use in many-body theory; e.g., in solving the Bethe-Goldstone[20] equation. The author is now studying this possibility.

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Theory of (a, xn) Reactions*

DANIEL SPERBER
Department of Physics and Astronomy, Rensselaer Polytechnic Institute, Troy, N. Y. 12181

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A new method for the study of (a, xn) reactions is presented ($a=n, p, \alpha, \gamma$, HI). The method is applicable to reactions in which first a compound nucleus is created and later x neutrons are evaporated. The method allows the calculation of excitation functions for the emission of a specified number of neutrons, average neutron energies, and neutron spectra. The equations governing a succession of neutron emissions are derived from first principles. The effect of γ decay is incorporated. The effect of spin on decay rates is treated rigorously. The theory is used to calculate the excitation function for the reactions $Ag^{109}(\alpha, n)In^{112m}$, $Ag^{109}(\alpha, n)In^{112g}$, and $Ag^{109}(\alpha, 2n)In^{111}$. The comparison between measured and calculated values is discussed.

I. INTRODUCTION

In the present paper reactions are considered, such that a projectile a (a may be a neutron, proton, α particle, γ ray, or heavy ion) strikes a target to create a compound nucleus. The compound nucleus decays, emitting x neutrons in succession. Attention is focused on the evaluation of the excitation function for the emission of the ith neutron, the average energy of this neutron, and the neutron spectrum. The evaluation is based on the statistical model. The effect of γ emission between successive evaporations is considered. The method is valid for the reaction in which highly excited states and states with a wide spin spectrum are reached, provided the statistical model is valid.

Jackson[1] considered reactions in which neutrons are emitted in succession. Sikkeland[2] generalized Jackson's method to incorporate angular momentum effects. Vandenbosch *et al.*,[3] addressed themselves to a similar problem. These approaches are based on a Monte Carlo calculation. Other studies of evaporations and cascades using Monte Carlo methods have been re-

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ported.[4–10] Excitation functions which were analyzed using the statistical model have been published.[11–13] The analysis usually rests on various limiting approximations. Several studies of neutron evaporation, mainly when one neutron is evaporated, can be found in the literature.[14–18] More recently, similar studies for charged particles have been undertaken.[19–22] Grover[23] suggested a method for treating neutron evaporation which considers γ-ray competition.

In the present paper, the equations governing a succession of neutron emissions are derived from first principles, using a time-dependent approach. These equations are integrated over time to obtain time-independent equations. The use of the solutions of the time-independent equations for the calculation of observable quantities is fully justified. The decay rates for neutron and γ emission are evaluated rigorously.

The decay rate for neutron emission is based on the work of Weisskopf[24] and Ewing and Weisskopf.[25,26] The probability depends on, among others things, the level density and the phase space available for the emitted neutrons. In many of the papers previously quoted, angular momentum effects were incorporated insofar as the density of levels and the coupling of spins is concerned, but were omitted insofar as phase space is concerned. However, the effect of angular momentum on the phase space available to the emitted neutrons is considerable. Therefore, in the present paper, the effect of angular momentum on phase space is included. This inclusion of angular momentum allows the analysis of experimental results using realistic parameters.

In the present work, no parameters are adjusted. All parameters are adopted from other sources. Since there are no restrictive assumptions, all quantities are calculated as accurately as input data allows within the framework of the statistical, compound-nucleus model.

TABLE I. Experimental and theoretical cross sections for the reaction $Ag^{109}(\alpha, n)In^{112\,g}$.

α-particle energy in MeV in c.m. system	Experimental cross section in b	Theoretical cross section in b
9.7	0.0007±0.0004	0.001
11.5	0.016±0.006	0.022
13.8	0.09±0.03	0.087
15.2	0.09±0.03	0.092
17.5	0.06±0.02	0.048
19.5	0.04±0.02	0.019

In Sec. II the theory is discussed. In Sec. III the power of the method is demonstrated by a sample calculation and comparison with experiment.

II. THEORY

In this section the theory of (a, xn) reactions is discussed in detail. It is shown that one of the most important functions required for the evaluation of the observables in such reactions is a function closely related to the level-occupation function. First, an integral equation for this function is set up, and a method for solving this equation is outlined. Second, it is shown how the observable quantities are calculated. The kernel in the integral equation depends on neutron and γ decay rates. Therefore, a method for the evaluation of these rates is presented. Both decay rates depend on the nuclear density of levels, so that this subject is briefly reviewed. Finally, the cross section for the creation of the compound nucleus and the related original spin distribution which enters the present theory are discussed.

A. Occupation of Levels

It is later shown that the observable quantities depend on a function closely related to the function representing the occupation of levels, following the emission of a specified number of neutrons. Now an integral equation for this function is derived. A method for the solution of this equation is suggested.

Let $y_{i,k}(E, J, t)$ be the function representing the occupation of levels at a time t after the formation of the original compound nucleus following the emission of i neutrons and $k\gamma$ rays, irrespective of order. Here E is the energy of excitation above the ground state and J, the spin of the nucleus.

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The following equation is satisfied by the functions $y_{i,k}(E,J,t)$:

$$\frac{\partial y_{i,k}(E,J;t)}{\partial t}=(1-\delta_{i,0})\sum_{J'}\int_{E+B_i}^{E_0-\bar{B}_{i-1}} y_{i-1,k}(E',J',t)\,S^n(E',J';E,J)\,dE'-(1-\delta_{i,x})\,y_{i,k}(E,J;t)$$
$$\times\sum_{J'}\int_0^{E-B_{i+1}} S^n(E,J;E',J')\,dE'+(1-\delta_{k,0})\sum_{J'}\int_E^{E_0-\bar{B}_i} y_{i,k-1}(E',J';t)\,S^\gamma(E',J';E,J)\,dE'$$
$$-y_{i,k}(E,J;t)\sum_{J'}\int_0^E S^\gamma(E,J;E',J')\,dE'. \quad (1)$$

In Eq. (1), the quantities $S^n(E,J,E',J')$ and $S^\gamma(E,J,E',J')$ are the probabilities per unit time per unit energy range (decay rates) for the emission of a neutron and γ ray, respectively. The explicit forms for the above decay rates are discussed later. In Eq. (1), B_i is the binding energy of the ith neutron and $\bar{B}_i$ is the sum of the binding energies of the first i neutrons. Also, in this equation, E_0 is the excitation energy of the original compound nucleus above its ground state. The total occupation of levels after the emission of i neutrons and any number of γ rays, $y_i(E,J;t)$, is written as a sum of terms such that

$$y_i(E,J;t)=\sum_k y_{i,k}(E,J;t). \quad (2)$$

An integro-differential equation for the total occupation of levels $y_i(E,J;t)$ can be obtained from Eqs. (1) and (2),

$$\frac{\partial y_i(E,J;t)}{\partial t}=(1-\delta_{i,0})$$
$$\times\sum_{J'}\int_{E+B_i}^{E_0-\bar{B}_{i-1}} y_{i-1}(E',J';t)\,S^n(E',J';E,J)\,dE' \quad (3)$$
$$-(1-\delta_{i,x})\,y_i(E,J;t)\sum_{J'}\int_0^{E-B_{i+1}} S^n(E,J;E',J')\,dE'$$
$$+\sum_{J'}\int_E^{E_0-\bar{B}_i} y_i(E',J';t)\,S^\gamma(E',J';E,J)\,dE'$$
$$-y_i(E,J;t)\sum_{J'}\int_0^E S^\gamma(E,J;E',J')\,dE'.$$

The integrodifferential Eq. (3) for the function $y_i(E,J;t)$ is now transformed to an integral equation, for a time-independent function $Z_i(E,J)$ closely related to the function $y_i(E,J;t)$. It is shown later in this paper that the knowledge of the function $Z_i(E,J)$ is sufficient for the present purpose. For convenience, the neutron width $\Gamma_n(E,J)$, the γ width $\Gamma_\gamma(E,J)$, the total width $\Gamma_t(E,J)$, and the function $Z_i(E,J)$ are introduced below:

$$\Gamma_n(E,J)=\hbar\sum_{J'}\int_{B_i}^E S^n(E,J;E',J')\,dE', \quad (4)$$

$$\Gamma_\gamma(E,J)=\hbar\sum_{J'}\int_0^E S^\gamma(E,J;E',J')\,dE', \quad (5)$$

$$\Gamma_t(E,J)=\Gamma_n(E,J)+\Gamma_\gamma(E,J), \quad (6)$$

$$Z_i(E,J)=\int_0^\infty y_i(E,J;t)\,\Gamma_t(E,J)\,dt. \quad (7)$$

Also, the branching ratio for neutron emission $T^n(E,J;E',J')$ and γ emission $T^\gamma(E,J;E',J')$ are introduced, so that

$$S^n(E,J;E',J')=\Gamma_n(E,J)\,T^n(E,J;E',J')/\hbar, \quad (4')$$
$$S^\gamma(E,J;E',J')=\Gamma_\gamma(E,J)\,T^\gamma(E,J;E',J')/\hbar. \quad (5')$$

The integrodifferential Eq. (3) is reduced to an integral equation by integrating Eq. (3) over time from $t=0$ to $t=\infty$ and using Eqs. (4)–(7). Thus, one obtains

$$Z_i(E,J)=Z_{i,0}(E,J)+\sum_{J'}\int_E^{E_0-\bar{B}_i} Z_i(E',J')$$
$$\times\frac{\Gamma_\gamma(E',J')}{\Gamma_t(E',J')}\,T^\gamma(E',J';E,J)\,dE'. \quad (8)$$

Here

$$Z_{i,0}(E,J)=\hbar[y_i(E,J;t=0)-y_i(E,J;t=\infty)]$$
$$+(1-\delta_{i,0})\sum_{J'}\int_{E+B_i}^{E_0-\bar{B}_{i-1}} Z_{i-1}(E',J')$$
$$\times\frac{\Gamma_n(E',J')}{\Gamma_t(E',J')}\,T^n(E',J';E,J)\,dE. \quad (9)$$

The functions $y_i(E,J;t=0)$ and $y_i(E,J;t=\infty)$ can be easily determined. At $t=0$, the only nonvanishing function $y_i(E,J;t=0)$ is the one corresponding to the level-occupation of the original compound nucleus prior to neutron and γ emission $y_c(E,J)$ such that

$$y_i(E,J;t=0)=\delta_{i,0}\,y_c(E,J). \quad (10)$$

Later the determination of $y_c(E,J)$ is discussed. On the other hand, at $t=\infty$ the only nonvanishing function $y_i(E,J;t)$ is the one corresponding to the ground state of the final nucleus which can not decay any more:

$$y_i(E,J;t=\infty)=0,\quad \text{for } E>0. \tag{11}$$

Equations (9)–(11) allow one to rewrite Eq. (8) explicitly as

$$Z_0(E,J)=\hbar y_c(E,J)+\sum_{J'}\int_E^{E_0} Z_0(E',J') \times\frac{\Gamma_\gamma(E',J')}{\Gamma_t(E',J')}T^\gamma(E',J';E,J)dE, \tag{12}$$

$$Z_i(E,J)=\sum_{J'}\int_{E+B_i}^{E_0-\bar{B}_{i-1}} Z_{i-1}(E',J') \times\frac{\Gamma_n(E',J')}{\Gamma_t(E',J')}T^n(E',J';E,J)dE' + \int_E^{E_0-\bar{B}_i} Z_i(E',J')\frac{\Gamma_\gamma(E',J')}{\Gamma_t(E',J')}T^\gamma(E',J';E,J)dE', \quad \text{for } i>0. \tag{13}$$

By inspecting Eqs. (12) and (13), it is seen that $Z_{0,0}(E,J)$ is essentially $\hbar y_c(E,J)$ and $Z_{i,0}(E,J)$ for $i>0$ can be determined from $Z_{i-1}(E,J)$, so that the equation for $Z_i(E,J)$ can be solved in succession. The equation for each of the $Z_i(E,J)$ is solved using the method of successive approximation as previously suggested[27] and applied.[28]

B. Calculation of Observable Quantities

Now it is shown how observable quantities are calculated when the functions $Z_i(E,J)$ are known. First, $\nu_i(E_0,\epsilon;t)$, the probability of emitting the ith neutron with an energy ϵ up to the time t after the beginning of neutron emission, is calculated. It is easy to see that the function $\nu_i(E_0,\epsilon;t)$ satisfies the relation

$$\frac{\partial\nu_i(E_0,\epsilon;t)}{\partial t}=\sum_{J,J'}\int_{\epsilon+B_i}^{E_0-\bar{B}_{i-1}} y_{i-1}(E',J';t) \times S^n(E',J';E'-\epsilon,J)dE'. \tag{14}$$

However, since the time of measurement is long, compared to the time of neutron and γ emission, one measures $\nu_i(E_0,\epsilon;t)$ at $t=\infty$. This can be obtained by integrating Eq. (14) over time from $t=0$, when there are no neutrons, to $t=\infty$. This integration yields

$$\nu_i(E_0,\epsilon)=\nu_i(E_0,\epsilon;t=\infty)\sum_{J,J'}\int_0^\infty\int_{\epsilon+B_i}^{E_0-\bar{B}_{i-1}} y_{i-1} \times(E',J';t)S^n(E',J';E'-\epsilon,J)dE'dt$$
$$=\hbar^{-1}\sum_{J,J'}\int_{\epsilon+B_i}^{E_0-\bar{B}_{i-1}} Z_{i-1}(E',J') \times\frac{\Gamma_n(E',J')}{\Gamma_t(E',J')}T^n(E',J';E'-\epsilon,J)dE'. \tag{15}$$

Second, the total probability of emitting the ith neutron $N_i(E_0)$ is determined by integrating $\nu_i(E_0,\epsilon)$ over energy. One obtains

$$N_i(E_0)=\int_0^{E_0-\bar{B}_i}\nu_i(E_0,\epsilon)d\epsilon. \tag{16}$$

In a similar way, the average energy for the ith neutron $\langle E_i\rangle$ can be written as

$$\langle E_i\rangle=[N_i(E_0)]^{-1}\times\int_0^{E_0-B_i}\epsilon\nu_i(E_0,\epsilon)d\epsilon. \tag{17}$$

The cross section for emitting i neutrons, $\sigma(a,E_0;i,n)$, can be written as

$$\sigma(a,E_0;i,n)=\sigma_c(a,E_0)[N_i(E_0)/\sum_i^x N_i(E_0)]. \tag{18}$$

In Eq. (18), $\sigma_c(a,E_0)$ is the cross section for the formation of the compound nucleus with projectiles a. It will be discussed later, along with a discussion of the occupation of levels of the compound nucleus $y_c(E,J)$. To obtain the excitation function, one has to calculate $\sigma(a,E_0;i,n)$ as a function of E_0. If the beam of projectile is not monochromatic, then one has to average Eqs. (16)–(18) over the range of energy of the excitation of the compound nucleus. Finally, $\nu(E_0,\epsilon)$, the total number of neutrons with energy ϵ, describes the neutron spectrum, which can be written as

$$\nu(E_0,\epsilon)=\sum_{i=1}^x\nu_i(E_0,\epsilon;i). \tag{19}$$

Again, the arguments about averaging over the energy range of the beam apply.

C. Neutron Decay Rate

Now the emission probability for neutron per unit time per unit energy range $S^n(E,I;E,I')$ is evaluated. First, this emission probability is broken into a sum of terms, each term corresponding to the emission of a neutron with a specified orbital angular momentum l and total angular momenta

$$j,\ S^n(E,J;E',J';l,j)$$

$$S^n(E,J;E',J')=\sum_{l=|J-J'|}^{J+J'}\sum_{j=l-1/2}^{j=l+1/2} S^n(E,J;E',J';l,j). \tag{20}$$

[27] D. Sperber, Nucl. Phys. **A90**, 665 (1967).
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It is customary to derive an expression for $S^n(E,J;E',J')$ by invoking the reciprocity theorem for nuclear reactions. A rigorous treatment requires the application of the reciprocity theorem to each of the $S^n(E,J;E',J';l,j)$ individually. In other words, the theorem has to be applied to each channel with a specified l and j separately. In the standard treatment[24] one considers the probability per unit time of emitting a neutron into an energy range $d\epsilon$. On the other hand, when one considers the emission probability of a neutron with a specified orbital angular momentum l and total angular momentum j, one has to consider the probability of emitting a neutron into an energy range between E and $E-d\epsilon$ with an orbital angular momentum l and total angular momentum j, $S^n(E,J;E',J',l,j)$. Here it is assumed that for a given l the probability of j being $l+\frac{1}{2}$ or $l-\frac{1}{2}$ are the same. First, an expression for $S^n(E,J;E',J',L)$ will be obtained where L takes continuous values. In the standard treatment, $S^n(E,J;E',J')$ is proportional, among others, to the phase space available to the emitted neutrons. According to the present treatment, where each channel is treated individually, the following relations is satisfied by $S^n(E,J;E',J';L)$:

$$S^n(E,J;E',J';L)\,d\epsilon dL=\frac{\sigma(\epsilon,L;E,J;E',J')}{2\pi^2R^2\hbar^3}\times\frac{L}{[1-(L/L_0)^2]^{1/2}}\frac{\rho(E',J')}{\rho(E,J)}\,d\epsilon dL. \quad (21)$$

Here, R is the radius of the nucleus and $\sigma(\epsilon,L;E,J;E',J')$ is the inverse cross section for exciting a nucleus at an energy E' and spin J' to a nucleus with an energy E and spin J by absorbing a neutron with orbital angular momentum L. The quantity L_0 in Eq. (21) is

$$L_0=R(2m\epsilon)^{1/2}. \quad (22)$$

It is now shown that if the spin dependence of the inverse cross section and the density of levels are neglected and the summation over L in Eq. (20) is replaced by integration over L from $L=0$ to $L=L_0$, Eq. (21) reduces to the standard expression. The arguments which do not enter the calculation in this approximation are suppressed. Let $S^n(E,E')$ be the decay rate for emitting a neutron from a state with energy E to a state with an energy E', $\sigma(\epsilon;E,E')$ be the cross section for the inverse process, and $\rho(E)$ be the energy-dependent density of levels; then

$$S^n(E,E')\,d\epsilon=\frac{\sigma(\epsilon;E;E')}{2\pi^2R^2\hbar^3}\frac{\rho(E')}{\rho(E)}\,d\epsilon\int_0^{L_0}\frac{LdL}{[1-(L/L_0)^2]^{1/2}}$$
$$=\frac{\sigma(\epsilon;E;E')}{\pi\hbar^3}\frac{\rho(E')m\epsilon}{\rho(E)}\,d\epsilon. \quad (23)$$

This indeed is the standard expression.[24]

In the usual treatment, one considers both spin states of the neutron. In the present treatment the case for $j=l+\frac{1}{2}$ and the case for $j=l-\frac{1}{2}$ are treated separately, since the cross section for the inverse process is j-dependent.

The function $S^n(E,J,E,J';L,j)$ is evaluated for a continuous variable L. However, the quantization of angular momentum requires the knowledge of an equivalent form of the function for integer values of l. This is achieved by the following definition:

$$S^n(\epsilon,J;E',J';l,j)\,d\epsilon=\int_{(l-1/2)\hbar}^{(l+1/2)\hbar}S^n(E,J;E',J';L,j)\,dL$$
$$=\frac{\sigma(E,l,j;E,J;E',J')l_0^2}{8\hbar R^2\pi^4}\times\left\{\left[1-\left(\frac{l-\frac{1}{2}}{l_0}\right)^2\right]^{1/2}-\left[1-\left(\frac{l+\frac{1}{2}}{l_0}\right)^2\right]^{1/2}\right\}\times\frac{\rho(E',J')}{\rho(E,J)}\,d\epsilon,\quad\text{for}\quad l\leq l_0 \quad (24a)$$

and

$$S^n(E,J;E',J';l,j)=0,\quad\text{for}\quad l>l_0. \quad (24b)$$

In Eq. (24)

$$l_0=L_0/\hbar. \quad (25)$$

The cross section for the inverse reaction is written as[29]

$$\sigma(\epsilon,l,j;E',J';E,J)=\pi(2j+1)T_{l,j}(\epsilon)\lambda^2. \quad (26)$$

D. γ Decay Rate

The emission probability per unit time per unit energy range $S^\gamma(E,J;E',J')$ is now evaluated. First, the emission term is broken into two terms: The first term is due to electric multipole transitions $S_E{}^\gamma(E,J;E',J')$ and the second is due to magnetic transitions, so that

$$S^\gamma(E,J;E',J')=S_E{}^\gamma(E,J;E',J')+S_M{}^\gamma(E,J;E',J'). \quad (27)$$

Each of the two terms in Eq. (27) is written as a sum of terms corresponding to a radiation of a specified multipolarity. [This is the equivalent of Eq. (20) for neutrons.] For example, $S_E{}^\gamma(E,J;E',J')$ is written as[29]

$$S_E{}^\gamma(E,J;E',J')=\frac{8\pi^2e^2}{\hbar c}\sum_{l=|J-J'|}^{J+J}S_E(E,J;E',J',l). \quad (28)$$

Here,

$$S_E{}^\gamma(E,J;E',J';l)=\sum_{M,M'}\frac{(l+1)}{l[(2+1)!!]^2}\left(\frac{E-E'}{\hbar c}\right)^{2l+1}\times|\langle JM|Q_{M-M'}{}^l|J'M'\rangle|^2\rho(E',J'). \quad (29)$$

[29] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons Inc., New York, 1952).

In Eq. (29), $\langle JM | Q_{M-M'}{}^{l} | J'M' \rangle$ are the model-dependent nuclear matrix elements for the components for the multipole tensor of order l. Expressions similar to (28) and (29) hold for magnetic multipole transitions. In the present discussion, only contributions from dipole and quadrupole radiations are considered. The nuclear matrix element for electric dipole radiation is calculated using the single-particle model.[30–32] The amount of quadrupole admixture is discussed later.

E. Density of Levels

The density of levels $\rho(E,J)$ appearing in all the expressions for decay rates is written as a product of a spin-dependent and an energy-dependent terms,[33–36]

$$\rho(E,J)=(2J+1)\rho(E)\exp[-(J+\tfrac{1}{2})^2/2\sigma^2]. \quad (30)$$

In Eq. (30), σ^2 is the spin cut-off parameter.[35] The spin cut-off parameter is related to the nuclear moment of inertia $\mathcal{I}$, and the nuclear temperature T by[36]

$$\sigma^2=\mathcal{I}T/\hbar^2. \quad (31)$$

Alternatively,[33] σ^2 can be expressed as

$$\sigma^2=gT\langle m^2\rangle, \quad (32)$$

where g is the number of proton and neutron single-particle levels per MeV, and $\langle m^2\rangle$, the mean square of the magnetic quantum numbers of the excited particles. It has been suggested[37–40] that for every specified energy there is a corresponding spin J_M such that at this energy there are no states with spin values higher than J_M, and the density of levels vanishes for states with spin higher than J_M. This property of the density of levels has been included in this study. The maximum spin is related to the energy in the following way:

$$J_M=(2E\mathcal{I})^{1/2}/\hbar. \quad (33)$$

Nuclear densities of levels have been discussed extensively by many authors.[33–36,41–46] Shell effects and pairing have been considered. In the present paper, the form suggested by Lang and LeCouteur[41] has been used. Following Lang and LeCouteur, the energy-dependent term in the density of levels $\rho(E')$ is

$$\rho(E')=C(E'+T)^{-5/4}\exp(2aE')^{1/2}. \quad (34)$$

Here, the temperature T and the excitation E' are related by

$$E'=aT^2-T, \quad (35)$$

TABLE II. Experimental and theoretical cross sections for the reaction $Ag^{109}(\alpha, n)In^{112\,m}$.

α-particle energy in MeV in c.m. system	Experimental cross section in b	Theoretical cross section in b
9.7	0.0005±0.0002	0.0000
11.5	0.0140±0.004	0.0300
13.8	0.18±0.06	0.1700
15.2	0.26±0.06	0.2500
17.5	0.30±0.07	0.2900
19.5	0.21±0.05	0.1400

TABLE III. Experimental and theoretical cross section for the reaction $Ag^{109}(\alpha, 2n)In^{111}$.

α-particle energy in MeV in c.m. system	Experimental cross section in b	Theoretical cross section in b
13.6	0.006±0.005	0.00
15.4	0.10	
17.1	0.30	0.31
18.6	0.52	
20.0	0.69±0.07	0.64
21.2	0.81	
22.5	0.88	0.88
24.0	1.03±0.09	1.02
25.3	1.10	
25.5	0.97	
26.6	0.95	
28.1	0.90±0.08	0.88
29.1	0.78	
30.2	0.61	
31.1	0.53	0.49
31.9	0.48	0.35
32.9	0.36	
33.6	0.32	
34.4	0.27±0.04	0.15
35.2	0.24	
36.1	0.19	0.06
37.0	0.15±0.02	0.03
38.1	0.12	0.00

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where

$$E'=E, \quad \text{for odd-odd nuclei} \tag{36a}$$

$$E'=E-\Delta, \quad \text{for odd-}A\text{ nuclei} \tag{36b}$$

$$E'=E-2\Delta, \quad \text{for even-even nuclei.} \tag{36c}$$

F. Compound-Nucleus Properties

The results of the present theory depend on the cross section for the formation of the compound nucleus, as is shown in Eq. (18). Also, the knowledge of the level-occupation function of the compound $y_c(E,J)$ is required, as seen from Eq. (10). The cross section for the formation of the compound nucleus $\sigma(\epsilon,l,j;E,J;E',J')$ is written as[29]

$$\sigma(\epsilon,l,j;E,J;E',J')=\pi\lambda\!\!\!^{-2}(2j+1)T_{l,j}(\epsilon). \tag{37}$$

Here,

$$\epsilon=E-E'-B. \tag{38}$$

Equation (37) contains the transmission coefficients, which are calculated using an optical potential, and B, which is the binding energy. Their particular choice is discussed later. In Eq. (37), $\lambda\!\!\!^{-}$ is the wave number of incoming particles. From Eq. (37), one can obtain the level occupation of the compound nucleus. The occupation function is proportional to the cross section $\sigma(\epsilon,l,j;E,J;E',J')$. The expressions in Eq. (37) become simpler if the original nucleus is in its ground state, and one uses a monochromatic beam of energy ϵ. In this case,

$$\sigma(\epsilon,l,j;E,J;0',J')=(2j+1)\pi\lambda\!\!\!^{-2}T_{lj}(\epsilon)\delta(\epsilon+B-E). \tag{39}$$

III. NUMERICAL RESULTS AND DISCUSSION

The power of the method described in this paper is demonstrated by performing a sample calculation and comparing the results with the experiment. The reactions chosen for this comparison are $Ag^{109}(\alpha,n)In^{112B}$, $Ag^{109}(\alpha,n)In^{112g}$, and $Ag^{109}(\alpha,2n)In^{111}$. These reactions are chosen because they have been measured extensively.[47–52] The predictions of the present theory are compared with the experiment in Tables I–III.

For the reaction $Ag^{109}(\alpha,2n)In^{111}$, there is a large number of measured values of the cross section. The cross section is calculated for a smaller number of energy values. For the values of energy for which the cross section is not calculated there is a blank in the last column of Table III. In this preliminary calculation only that of excitation functions for a limited number of reactions is performed. Experimentally, the cross section for the emission of up to five neutrons is known. Excitation functions for the emission of more than two neutrons will be published later, since this is only a sample calculation. The theoretical values are compared with the experimental results of Fukushima *et al.*[52]

The parameter a appearing in the form for the density of levels is taken as $\frac{1}{10}A$ MeV^{-1} (here A is the number of nucleons) or 11 MeV^{-1}. A rigid-body moment of inertia is assumed. The gap parameter Δ is taken from the work of Nemirovsky and Adamchuk.[53]

The transmission coefficients for low-energy neutrons are taken from the work of Auerbach and Perey.[54] Auerbach and Perey calculated transmission coefficients using an optical potential, with the inclusion of spin-orbit interactions. However, their transmission coefficients are limited to neutron energies up to 5 MeV. For higher neutron energies, in this preliminary numerical calculation, values for the transmission coefficients based on the sharp cutoff approximation are used, namely,

$$T_{l,j}(\epsilon)=1, \quad \text{for } l\leq l_0 \tag{40a}$$

$$T_{l,j}(\epsilon)=0, \quad \text{for } l\geq l_0. \tag{40b}$$

The transmission coefficients for α particles are taken from the work of Huizenga and Igo.[55,56] The work of Grover[23] and Sperber[28] suggest about 20% of quadrupole admixture.

The comparison between the calculated and measured results (Tables I–III) shows satisfactory agreement for this sample calculation. This agreement which is obtained without fitting parameters, but rather by using reasonable parameters based on other work gives one confidence in the power of the present method.

In particular, it has to be remembered that Fukishima in his analysis had to use very small unrealistic values for the parameter a to obtain agreement between theory and experiment. It is believed that the worse agreement between the present theory and experiment on the high-energy neutron tail, is among others, due to the use of approximate values of the transmission coefficients as suggested by Eq. (40). However, the underestimate for cross section for the high-energy neutrons may be also due to precompound reactions. Such precompound processes have been suggested by Griffin.[57] Finally, the high-energy end of the spectrum corresponds to transitions to low-lying states for which the statistical model may not be applicable.

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