action as caused by fluctuations[6] concentrated in the surface shell ($l=l_{max}$) permits one to distinguish two main parts of nuclei: (i) the *inert core*, and (ii) the *strongly interacting nuclear surface* containing nucleons with $l=l_{max}$, holes, and fluctuations. These conclusions follow from the analysis of Figs. 1–3 and Tables I and II.

4. This analysis shows that the inert-core parameters are fixed constants for different nuclei, so that those of practical importance (for the specification of the interaction properties of different nuclei) are the three nuclear surface parameters α_0, κ_L, γ. This is distinctly less than the usual number of parameters of local optical potentials.

5. More extensive analysis of these parameters for different nuclei will supply material for possible correlation between the structure of the surface layer of a nucleus and its interaction with particles, and particularly for investigation of the role of the holes and density fluctuations in such interactions.

ACKNOWLEDGMENTS

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Study of the J Dependence in the (d, He^3) Reaction*

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The J dependence of the angular distributions in the (d, He^3) reactions was experimentally observed at small angles in the $2p$ proton pickup. This J dependence is qualitatively reproduced by distorted-wave calculations which indicate that the effect arises mainly from the deuteron spin-orbit potential. The calculated angular distributions are strongly dependent on the Q value of the reactions and the incident deuteron energy. The calculations also show that the J dependence is localized at the surface of the nucleus, and is very sensitive to the volume of the real part of the deuteron optical potential. This sensitivity may be understood by considering the role of the spin-orbit term in the optical potential. The sensitivity of the calculated angular distributions to the other parameters is investigated.

I. INTRODUCTION

SINCE the first discovery[1] of the J dependence of the angular distributions in the (d, p) reaction, similar effects have been observed in many other direct reactions. The J dependence in the (d, He^3) reaction was first observed by Preedom, Newman, and Hiebert.[2] They found a difference between the angular distributions of $2p_{1/2}$ and $2p_{3/2}$ proton pickup in the region from 50° to 80° at an incident deuteron energy of 34.4 MeV. This difference was qualitatively reproduced by the ordinary distorted-wave Born-approximation (DWBA) theory, and was found to arise from the deuteron spin-orbit potential.

The J dependence of the $l=1$ transition at small angles (15°–30°) was observed in the course of our spectroscopic study of the (d, He^3) reaction on even-even Mo isotopes. This observation is important because at such small angles the direct interaction should predominate and distorted-wave calculations are expected to be reliable and rather insensitive to the choice of optical-potential parameters. In this paper the experimental evidence for such a J dependence is described. It is also shown that the J dependence is correctly given by the distorted-wave calculations. The changes in the shapes of the calculated curves in response to variations of the individual parameters are discussed in detail.

II. EXPERIMENTS AND COMPARISONS WITH THE DISTORTED-WAVE THEORY

The details of the experiments and analysis were described in a previous paper.[3] The 23-MeV deuteron beam from the Argonne cyclotron was used. The targets were isotopically enriched metallic foils, and

* Work performed under the auspices of the U.S. Atomic Energy Commission.

[1] L. L. Lee, Jr., and J. P. Schiffer, Phys. Rev. Letters **12**, 108 (1964); Phys. Rev. **136**, B405 (1964).

[2] B. M. Preedom, E. Newman, and J. C. Hiebert, Phys. Letters **22**, 657 (1966).

[3] H. Ohnuma and J. L. Yntema, Phys. Rev. **177**, 1695 (1969).

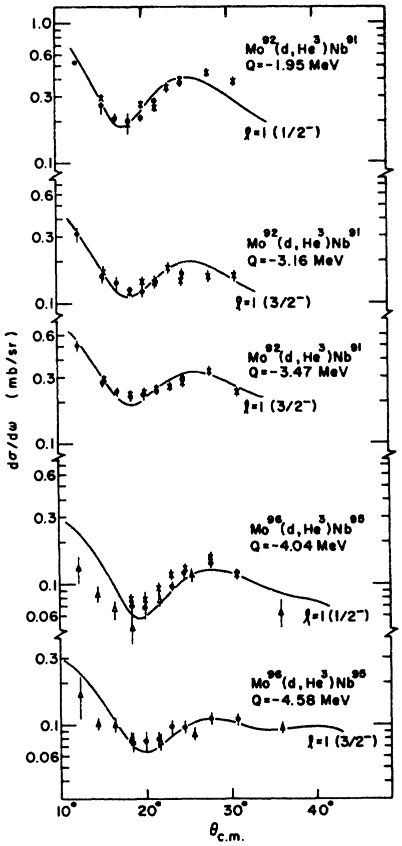

FIG. 1. Angular distributions of $l=1$ transitions in the Mo^{92}(*d*, He^3)Nb^{91} and Mo^{96}(*d*, He^3)Nb^{95} reactions. The curves are the results of distorted-wave calculations.

the detection system consisted of surface-barrier Si detectors.

Observed $l=1$ angular distributions for Mo^{92} (*d*, He^3) Nb^{91} and Mo^{96} (*d*, He^3) Nb^{95} are shown in Fig. 1.

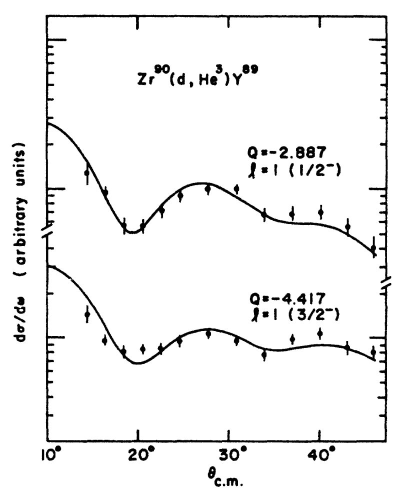

FIG. 2. Angular distributions of $l=1$ transitions in the Zr^{90}(*d*, He^3)Y^{89} reaction together with the calculated angular distributions.

TABLE I. Optical-model and bound-state parameters used in the DWBA calculations.

Parameter	He^3	*d*	Bound state
V (MeV)	173	105	a
W (MeV)	18	···	
r_0 (F)	1.14	1.06	1.2
a (F)	0.723	0.86	0.65
r' (F)	1.65	1.42	
a' (F)	0.8	0.65	
W' (MeV)	···	54	
V_{so} (MeV)	···	6	$\lambda_{so}=25$
r_{so} (F)	···	1.06	···
r_c (F)	1.4	1.3	1.4

[a] Adjusted to give the transferred proton a binding energy of −5.494 MeV.

In addition to the well-known $\frac{1}{2}^-$ first excited states, at least one state that shows an $l=1$ angular distribution was observed in each isotope. Almost all of the available $p_{1/2}$ strength is exhausted by the first excited state as discussed below, and the $l=1$ transitions to the higher states are as strong as those to the first excited state. Therefore the only reasonable explanation for the higher $l=1$ states is that $p_{3/2}$ proton pickup was observed as well as the $p_{1/2}$ proton pickup. (Transitions with $l=3$, which probably correspond to the $f_{5/2}$ neutron pickup, were also observed in all isotopes.) It is very noticeable that all these $p_{3/2}$ angular distribu-

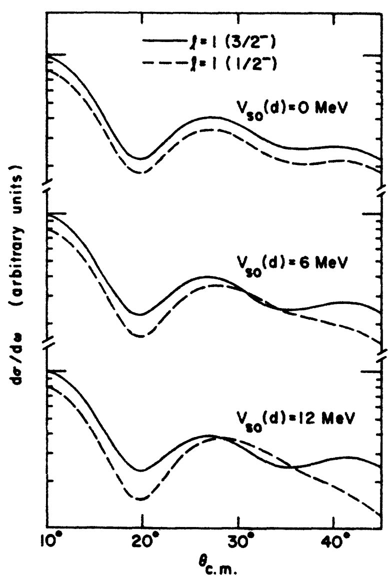

FIG. 3. Dependence of the calculated angular distributions on V_{so}. The calculations were made for 23-MeV deuterons incident on Mo^{92} for which the Q value was taken to be −3.16 MeV. The other parameters of the optical-model potential are listed in Table I.

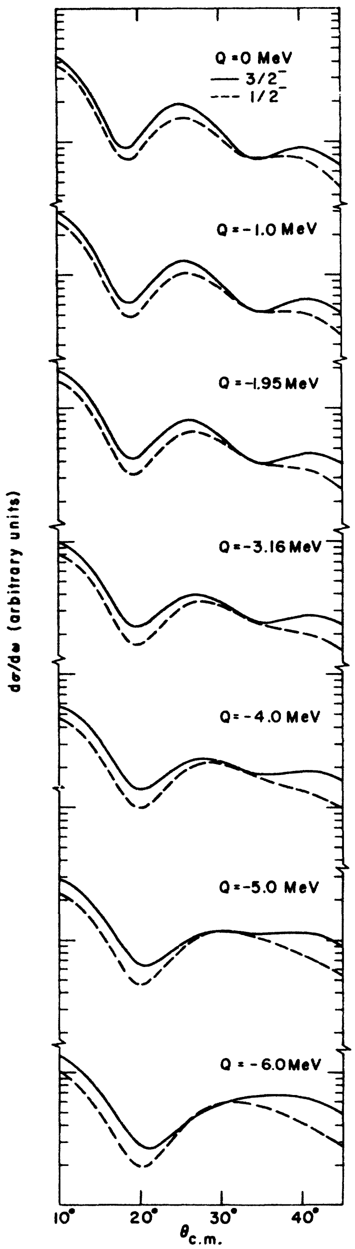

FIG. 4. Dependence of the calculated angular distributions on the Q value of the reactions. The curves were calculated for 23-MeV deuterons incident on Mo^{92}. The optical-potential parameters are listed in Table I.

tions show shallower minima than the $p_{1/2}$ angular distributions in spite of the fact that the Q values are quite different.

To confirm our observation, the $l=1$ angular distributions from the Zr^{90} (d, He^3) Y^{89} reaction were carefully measured. It is well established[4] that the ground state of the Y^{89} is $\frac{1}{2}^-$ and the state at 1.50 MeV is $\frac{3}{2}^-$. These two states are separated from the nearby known states by at least 200 keV, and it is very unlikely that the angular distributions are smeared out by the contributions from unknown states that are not resolved in the present experiment. The observed $l=1$ angular distributions are shown in Fig. 2. There definitely is a difference in the minima around 20°.

The distorted-wave calculations were carried out with the code JULIE[5] to see whether this effect could be reproduced by the conventional theory. The optical-model parameters used are listed in Table I. The deuteron parameters are those obtained[6] from the

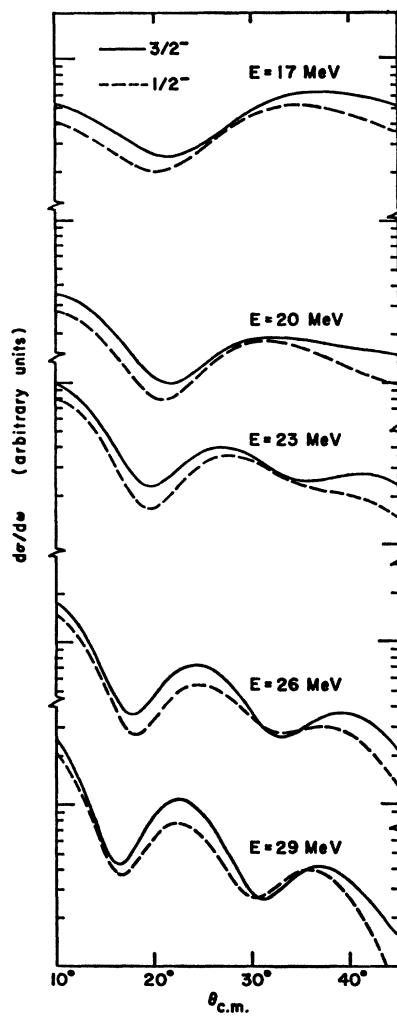

FIG. 5. Dependence of the calculated angular distributions on deuteron energy. The curves were calculated for Mo^{92} and a Q value of −3.16 MeV. The optical-potential parameters of Table I were used.

[4] S. M. Shafroth, P. N. Trehan, and D. M. Van Patter, Phys. Rev. **129**, 704 (1963); D. M. Van Patter and S. M. Shafroth, Nucl. Phys. **50**, 113 (1964); J. Alster, D. C. Shreve, and R. J. Peterson, Phys. Rev. **144**, 999 (1966); B. M. Preedom, E. Newman, and J. C. Hiebert, *ibid.* **166**, 1156 (1968).

[5] R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL-3240, 1963 (unpublished).

[6] J. L. Yntema, H. Ohnuma, and H. T. Fortune (private communication).

TABLE II. Spectroscopic factors for the $Mo^{92,96}(d, He^3)Nb^{91,95}$ reaction. The value of C^2S for each level was calculated under each of four assumptions: zero-range local (ZRL), finite-range local (FRL), zero-range nonlocal (ZRNL), and finite-range nonlocal (FRNL).

Isotope	E (MeV)	l	J^π	Spectroscopic factor C^2S ZRL	FRL	ZRNL	FRNL
Nb^{91}	0	4	$\frac{9}{2}^+$	3.4	3.1	2.7	2.3
	0.10	1	$\frac{1}{2}^-$	1.8	1.6	1.4	1.2
	1.31	1	$(\frac{3}{2}^-)$	1.5	1.4	1.0	1.0
	1.62	1	$(\frac{3}{2}^-)$	3.1	2.8	2.3	2.1
	1.85	3	$(\frac{5}{2}^-)$	6.5	5.6	4.9	3.8
Nb^{95}	0	4	$\frac{9}{2}^+$	3.8	3.5	2.9	2.6
	0.23	1	$\frac{1}{2}^-$	2.2	1.9	1.6	1.4
	0.77	1	$(\frac{3}{2}^-)$	2.5	2.1	1.8	1.6
	0.98	3	$(\frac{5}{2}^-)$	3.0	2.6	2.1	1.8

analysis of the elastic scattering of deuterons on Fe^{54} between 12 and 18 MeV. The He^3 parameters are approximately the same as those Gibson *et al.*[7] obtained by analyzing the elastic scattering of the 37.7- and 43.7-MeV He^3 particles on nuclei from Ca^{40} to Zr^{90}. The bound-state parameters are the usual ones. The solid curves in Figs. 1 and 2 show the results of the calculations.

The agreement between the experiments and the calculations is quite good, although the experimentally observed effect seems slightly more pronounced than the calculated one. The nonlocal finite-range, nonlocal zero-range, and local finite-range calculations were also tried with the local-energy approximation.[8]

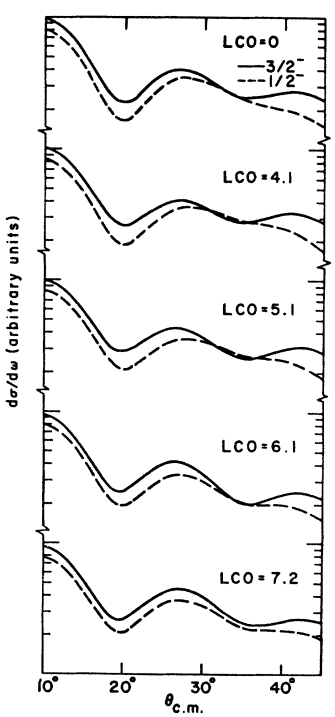

FIG. 6. Effect of using a lower cutoff on the radial integrals. The calculations were made for Mo^{92}, with a deuteron energy of 23 MeV, $Q=-3.16$ MeV, and the optical-potential parameters of Table I.

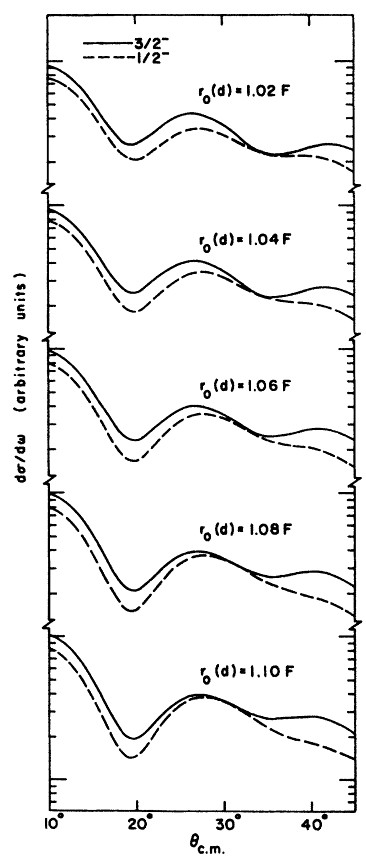

FIG. 7. Changes in the distorted-wave calculations in response to variation of $r_0(d)$.

[7] E. F. Gibson, B. W. Ridley, J. J. Kraushaar, and M. E. Rickey, Phys. Rev. **155**, 1194 (1967).

[8] F. G. Perey and B. Buck, Nucl. Phys. **32**, 353 (1962); J. C. Hiebert, E. Newman, and R. H. Bassel, Phys. Rev. **154**, 898 (1967).

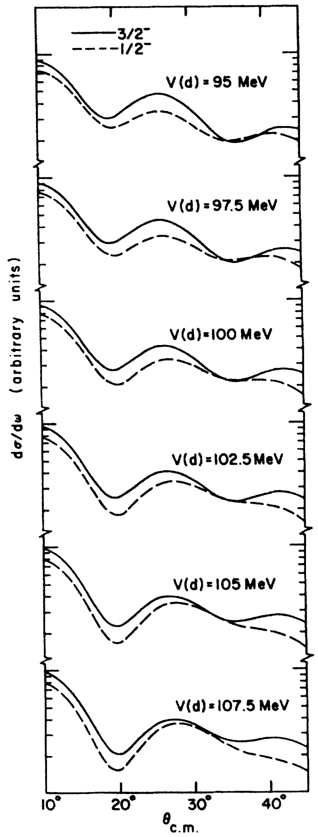

FIG. 8. Changes in the calculated angular distributions in response to variation of $V(d)$.

Nonlocalities β $(d)=0.54$ and β $(\mathrm{He}^3)=0.2$, and the range parameter 1.54 F were used in the calculations. Both the shapes of the angular distributions and the relative spectroscopic factors of the states were virtually independent of the assumptions, but the calculated cross sections varied over a range of almost 50%. Table II gives the spectroscopic factors for the low-lying states of Nb^{91} and Nb^{95}, as calculated from the Mo^{92} (d, He^3) Nb^{91} and Mo^{96} (d, He^3) Nb^{95} data on the basis of each of the four assumptions. The normalization factor[9] used here was 2.6. Here one can see that under any assumption the spectroscopic factors for the second and the third $l=1$ transitions are too large to allow a $\frac{1}{2}^-$ assignment for these levels.

The $l=1$ angular distributions were also calculated with the He^3 optical-potential parameters obtained by Bingham and Halbert.[10] Among four sets of parameters given in Ref. 9, set C failed to reproduce the difference in the minima around 20°, but sets A, B, and D more or less show that the minimum near 20° is shallower for the $p_{3/2}$ pickup. No significant J dependence at forward angles was predicted nor experimentally observed in the (d, t) reaction on Mo isotopes at the same energy.

III. DEPENDENCE OF THE EFFECT ON PARAMETERS

Since the experimentally observed J dependence was at least qualitatively reproduced by the distorted-wave theory, the dependence of the calculated results on various input parameters was investigated.

At first, to study the origin of the J dependence, the spin-orbit terms in the bound-state potential and the deuteron optical potential were alternately switched off. It was found that the spin-orbit term in the bound-state potential does change the relative cross sections somewhat for the $2p_{1/2}$ and $2p_{3/2}$ proton pickups, but does not give a difference in the shapes of the angular distributions. In contrast, the spin-orbit term in the deuteron optical potential makes the shapes different for the $2p_{1/2}$ and $2p_{3/2}$ pickup and the effect is enhanced

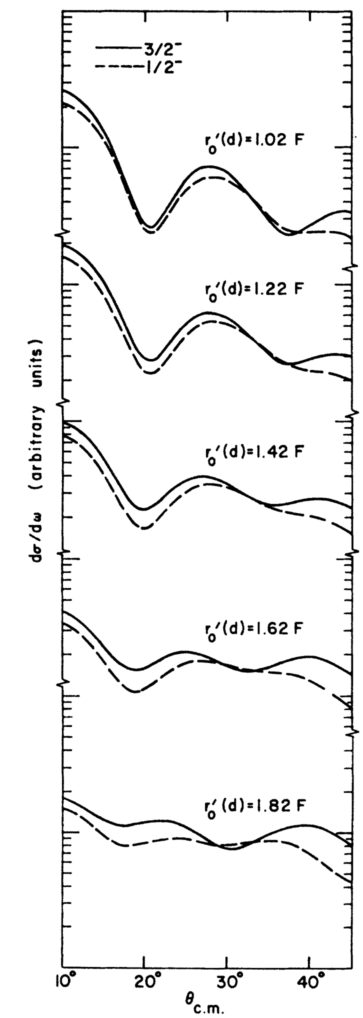

FIG. 9. Changes in the calculated angular distributions in response to variation of $r_0'(d)$.

[9] R. H. Bassel, Phys. Rev. **149**, 791 (1966).
[10] C. R. Bingham and M. L. Halbert, Phys. Rev. **158**, 1085 (1967).

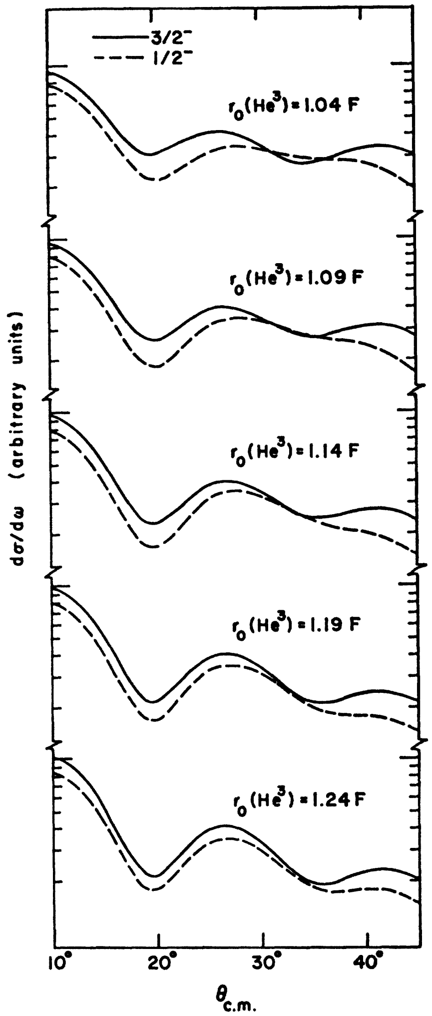

FIG. 10. Changes in the calculated angular distributions in response to variation of $r_0(He^3)$.

as the depth of the deuteron spin-orbit potential is increased, as shown in Fig. 3. There was very little change in the shapes when the real spin-orbit term of a He^3 particle was added to the optical potential of Table I. In this case the radial dependence of the He^3 spin-orbit term was taken to be the same as that of the real part of the potential, and two different depths (8 and 16 MeV) were tried. The parameters of the spin-orbit potential of a He^3 particle are not yet well established, but a depth of about 8 MeV was suggested by Hodgson.[11] These calculations lead to the conclusion that the predicted *J* dependence comes from the spin-orbit potential in the deuteron optical potential. This suggests that the effect observed here has the same origin as the *J* dependence found by Preedom *et al.*[2]

The dependence of the calculated effects on the *Q* values of the reaction and on the incident energies is

[11] P. E. Hodgson (to be published).

TABLE III. Sensitivity of the calculated *J* dependence to each of the parameters. Classification as "sensitive" (S) means that the *J* dependence can be eliminated by changing the parameter $\lesssim 5\%$ from its value in Table I, "medium" (M) means that a change of $\sim 10\%$ is required, and "insensitive" (X) means that the *J* dependence persists even for a change $>20\%$. The parameter marked *O* is the main origin of the *J* dependence.

Parameter	He^3	d	Bound state
V	X	S	
W	X		
r_0	M	S	X
a	M	X	X
r'	M	M	
a'	X	X	
W'	X	X	
V_{so}	X	*O*	X
r_{so}	X	X	X
r_c	X	X	X

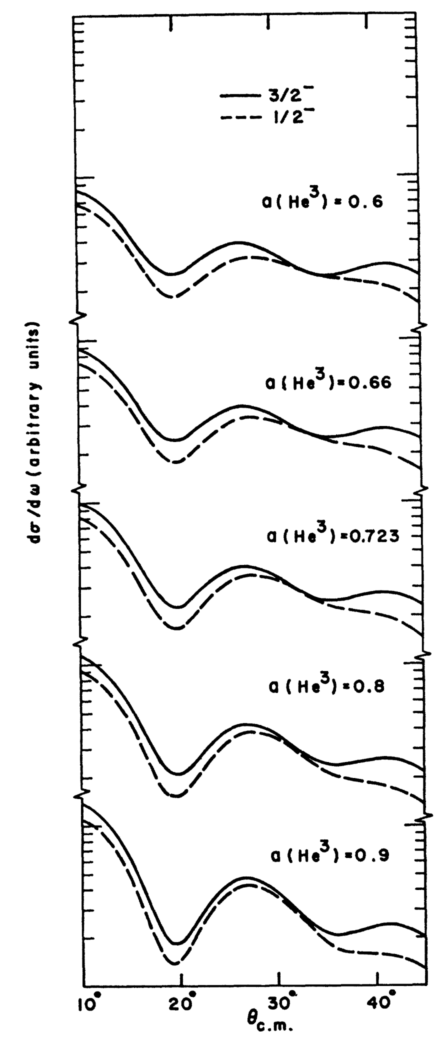

FIG. 11. Changes in the calculated angular distributions in response to variation of $a(He^3)$.

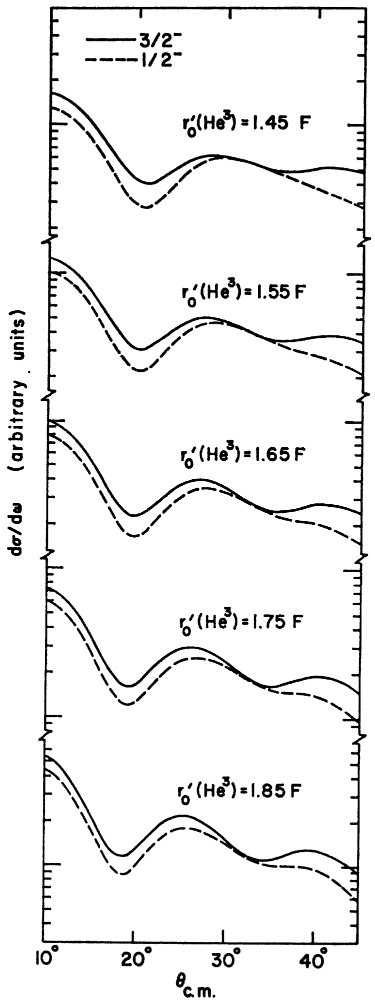

FIG. 12. Changes in the calculated angular distributions in response to variation of $r_0'(\mathrm{He}^3)$.

shown in Figs. 4 and 5, respectively. In Fig. 4 the incident deuteron energy was fixed at 23 MeV, and in Fig. 5 the Q value was fixed at -3.16 MeV. The results show that the shapes of the $l=1$ angular distributions are strongly dependent on both the Q value and the incident energy. When either of these was increased, the difference between the $p_{1/2}$ and $p_{3/2}$ first decreases, then disappears, and finally increases in the reversed sense; i.e., the $p_{3/2}$ angular distribution is deeper at the first minimum than the $p_{1/2}$ angular distribution. When the incident energy is varied, both incident and outgoing waves are changed. When the Q value is varied, the bound-state wave function and the outgoing wave are affected. This point will be discussed later. The figures show that the deuteron beam energy from the Argonne cyclotron and the Q values of the Mo (d, He^3) reactions happened to be very suitable for experimental observation of the J dependence. In the following calculations the Q value and the incident energy are fixed at -3.16 and 23 MeV, respectively, unless otherwise stated.

As mentioned before, the shapes of the angular distributions were changed very little by introducing nonlocality and the finite-range interaction. Perey[12] pointed out that the nonlocal calculation suppresses the contribution from the nuclear interior. In order to see more clearly the contributions from different parts of the nucleus, the lower radial cutoff was introduced into the local zero-range calculation. The results are shown in Fig. 6. The difference between the shapes of the $p_{1/2}$ and $p_{3/2}$ angular distributions suddenly disappears at a lower cutoff radius of about 5 F. Therefore the effect appears to be localized at the surface of the nucleus. This may explain why the

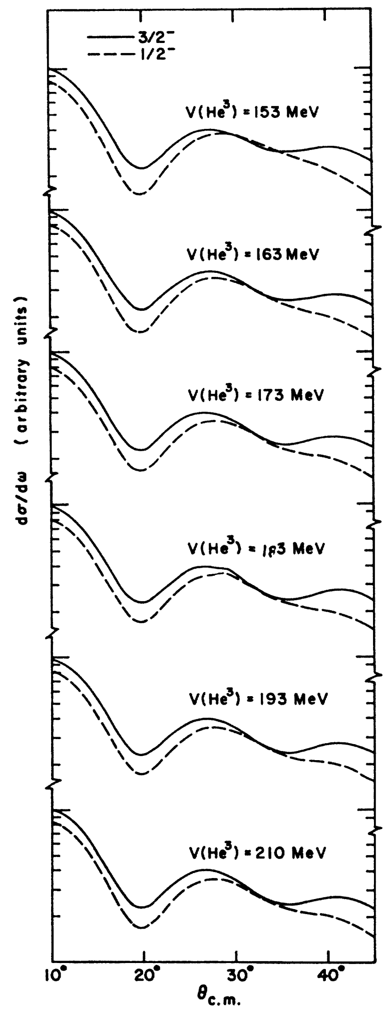

FIG. 13. Changes in the calculated angular distributions in response to variation of $V(\mathrm{He}^3)$.

[12] F. G. Perey, in *Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, 1962*, edited by E. Clementel and C. Villi (Gordon and Breach, Science Publishers, Inc., New York, 1963), p. 125.

nonlocality only slightly affects the calculated *J* dependence.

The dependence on the optical-model parameters was studied by varying each parameter around the value listed in Table I. Only one parameter was changed at a time. The radius of the real potential and of the spin-orbit potential were varied independently of each other. The results are summarized in Table III. In this table "sensitive" means that the *J* dependence can be eliminated by changing the parameter by less than 5% from its value in Table I, "medium" means that about 10% change is required to eliminate it, and "insensitive" means that the *J* dependence persists even when the parameter is changed by more than 20%. This criterion may be unfair in some cases because different parameters have different weights in the calculations. For example,

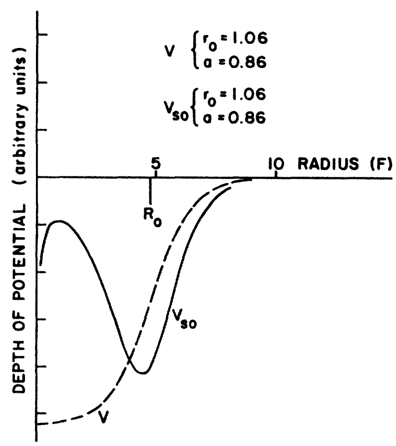

FIG. 14. Shapes of the real potential (dashed curve) and the spin-orbit potential (solid) of the deuteron optical-model potential.

a change of 10% in r_0 may have the same effect as a change of about 20% in V_0. However, the *J* dependence seems more sensitive to r_0 (He^3) than is the volume effect expected from the sensitivity of the *J* dependence to $V(He^3)$. Therefore the former parameter was classified as one to which the *J* dependence has "medium" sensitivity and the latter as one to which it is "insensitive." This will be considered again below.

Figures 7–13 show the dependence of the angular distributions on those parameters to which the *J* dependence is classified as "sensitive" and "medium," as well as on $V(He^3)$ to which it was classified as "insensitive." One interesting feature is that the *J* dependence is very sensitive to the volume of the real potential of the deuteron. The curves for the different radii with $V(d) = 105$ MeV are shown in Fig. 7, and those for the different depths with $r_0(d) = 1.06$ F in Fig. 8. As one can see in Fig. 8, the calculated *J* dependence disappears for $V \approx 100$ MeV, and for smaller values of V the effect is reversed. This turning

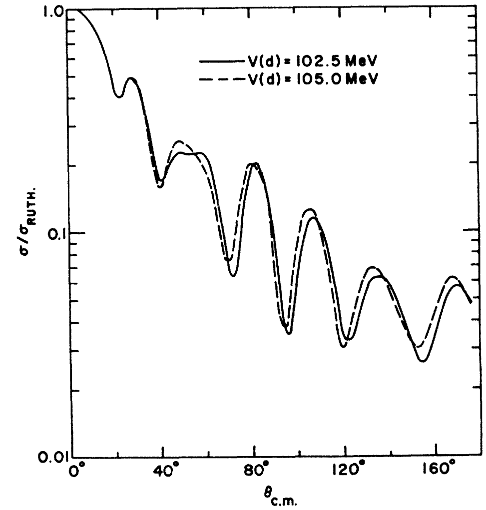

FIG. 15. Change in the calculated angular distributions of the elastic deuteron scattering in response to changing the real deuteron well depth from 102.5 to 105 MeV.

point goes down to about 98 MeV for $r_0(d) = 1.08$ F, to about 95 MeV for $r_0(d) = 1.10$ F, and so on.

This sensitivity to the volume may not be too surprising if one considers the role of the spin-orbit term in the optical potential. The shapes of the real potential V and the (real) spin-orbit potential V used here are shown in Fig. 14. The magnitude and the sign of the spin-orbit potential are determined by the contributions from different $\boldsymbol{\sigma}\cdot\mathbf{l}$ terms in all partial waves. The (real) spin-orbit potential is then added or subtracted from the real potential, and effectively modifies the surface region of the real potential. Therefore the role of the (real) spin-orbit term may be simulated by using real potentials with slightly different radii for different *J* values. From these

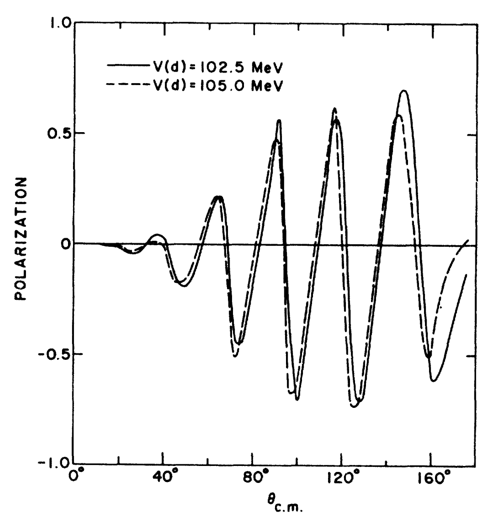

FIG. 16. Vector polarization predicted by the deuteron optical-potential parameters with $V = 105$ MeV and $V = 102.5$ MeV.

considerations one may conclude that if the J dependence is reproduced by ordinary distorted-wave calculations, and if it arises from the spin-orbit term of one particle, then the calculated results should be sensitive to the radius (i.e., to the volume) of the optical potential of that particle.

The angular distributions for elastically scattered deuterons are plotted in Fig. 15 for $V(d)=105$ MeV and 102.5 MeV. This change in $V(d)$ shifts the angular distributions by about 2° in the region from 40° to 180°, and alters the shape of the peak around 50°. The angular distribution of the vector polarizations of the elastically scattered deuterons (Fig. 16) is shifted by 2° to 6° in the same region. Corresponding changes in the real part, imaginary part, and the absolute value of the partial-wave amplitudes of the incident deuteron also occur.

The insensitivity of the J dependence to the bound-state parameters suggests that its Q-value dependence comes mainly from the dependence on the outgoing wave. This may appear to be in contradiction to the conclusion that the main origin of the J dependence is the deuteron spin-orbit term. However, some evidence for interference between the incident and outgoing channels is seen in Table III and in Figs. 10–12. These show that the geometrical parameters for the He^3 particle do have some effect on the calculated angular distributions. The dependence of the shape of the angular distributions on the geometrical parameters seems stronger than the dependence on $V(He^3)$ shown in Fig. 13. When $r_0(He^3)$ is increased from 1.14 to 1.24 F, the J dependence disappears, but it still remains when $V(He^3)$ is changed from 173 to 210 MeV. A careful analysis of the outgoing waves may clarify the extent to which such interference affects the J dependence.

IV. SUMMARY AND CONCLUSION

A J dependence was experimentally observed at small angles (~20°) in the angular distributions corresponding to $l=1$ proton pickups by the (d, He^3) reaction. This J dependence was qualitatively reproduced by distorted-wave calculations. These calculations indicate that the J dependence arises mainly from the deuteron spin-orbit potential. The calculated results are strongly dependent on the Q value of the reaction and on the incident deuteron energy. The calculations also show that the effect is localized at the surface of the nucleus. Introduction of nonlocality and the finite-range interaction does not appreciably affect the results.

The sensitivity of the calculated effect to the model parameters was investigated. It was shown that the effect is sensitive to the volume of the real part of the deuteron optical potential. This sensitivity may be understood if one remembers that the spin-orbit term effectively modifies the optical potential without spin-orbit term; and therefore the effect of the spin-orbit potential may be simulated (without a spin-orbit term) by using a different radius (volume) for different J values. There seems to be some interference between the incident and outgoing waves. However, further investigation is required to clarify the physical meaning of the observed phenomena and the calculated results.

ACKNOWLEDGMENTS

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