Scattering of Medium-Energy Alpha Particles. II. Microscopic Analysis of Elastic Scattering

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A microscopic description of the elastic scattering of medium-energy α particles is given in terms of an optical potential calculated from the nuclear-matter distribution and an effective α-nucleon interaction. It is shown that the differential cross section is very sensitive to changes in the range parameter of the effective interaction, and that this sensitivity is reflected in the magnitude predicted for the strong-absorption radius. It is also shown that this microscopic analysis can lead to an A dependence for the strong-absorption radius of $1.5A^{1/3}$, in accordance with the general trend observed in optical-model and direct phase analyses, although the fits to the data are not yet sufficiently accurate to yield very precise results for the strong-absorption radii of individual nuclei.

1. INTRODUCTION

THE microscopic description of the inelastic scattering of nucleons by nuclei has long been recognized as a valuable tool for the investigation of nuclear structure.[1] This method has been used at energies above 100 MeV, where the impulse approximation makes plausible the use of the free two-body interaction,[2] and at medium energies, where a modified effective interaction is used.[3] The inelastic scattering of α particles at medium energies has also been described by means of a microscopic model using an effective α-nucleon interaction.[4,5] The main interest in such studies is the investigation of the overlap of the initial and final nuclear wave functions, i.e., the transition density, and of the mode of excitation,[1,6] although it is not easy to disentangle the deficiencies of the nuclear wave functions from deficiencies in the description of the interaction of the projectile with the nucleus.

Within the framework of the distorted-wave Born approximation, the microscopic description of inelastic scattering requires a knowledge of the wave functions for elastic scattering of the projectile, and this requirement is normally met by the use of the phenomenological optical potential. This procedure is formally correct and implies that there are no free parameters in the treatment of the elastic scattering. It may also be argued[6] that this procedure allows an accurate treatment of multiple scattering in the description of the elastic scattering while the error in the treatment of multiple scattering in the off-diagonal or inelastic part of the interaction may be compensated for by modification of the effective two-body interaction. However, this arbitrary adjustment of the effective interaction for inelastic scattering leads to a certain inconsistency in the treatment of elastic and inelastic scattering. It has been suggested[5] that a microscopic description of elastic scattering could help to remove the uncertainties in the parameters of the two-body interaction, and that the self-consistency criterion could be useful for the description of scattering of strongly absorbed projectiles,[6] for which ambiguities in the strengths of the optical potentials are known to exist.

The microscopic description of elastic scattering is well established in the case of electron scattering, where the formalism[7] allows a direct connection to be made between the cross section and the charge-density distribution in the ground state. The charge density may be described phenomenologically in terms of a Fermi distribution or a more elaborate parametrization, but recent work[8] has revealed the power of a fully microscopic description in which the charge distribution is constructed from nuclear wave functions through a single-particle model. A semimicroscopic formalism for the elastic scattering of nucleons from nuclei has recently been developed[9] in which the real part of the optical potential is derived from an effective two-nucleon interaction and a Fermi parametrization of the nuclear-matter distribution. The behavior of the nuclear-matter distribution required to fit nucleon scattering from ^{208}Pb is in good agreement, at least in the transition region, with that predicted from a single-particle model.[10] This new analysis of nucleon scattering[9] has indicated that nuclear-size information can be extracted from the optical potentials for nucleon scattering. A corresponding result cannot be expected for α-particle scattering owing to the ambiguities in the optical potential which cause the equivalent radius to be

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[9] G. W. Greenlees, Y. C. Tang, and G. J. Pyle, Phys. Rev. Letters **17**, 33 (1966); Phys. Rev. **171**, 1115 (1968).
[10] L. R. B. Elton, Phys. Letters **26B**, 689 (1968).

a function of the depth of the potential.[11] For α-particle scattering it is the strong-absorption radius which is the significant size parameter,[11,12] but the connection between this parameter and the nuclear-matter distribution is as yet unclear.

We have attempted to develop a microscopic model for the elastic scattering of α particles from nuclei. The nuclei chosen for study are the calcium isotopes, since for these nuclei both α-particle scattering[12,13] and electron scattering[14] has been studied in detail. We have calculated an optical potential by folding an effective α-nucleon interaction into the nuclear-matter distribution obtained by fitting[14] elastic electron scattering and the neutron and proton separation energies. These optical potentials are then used to calculate the cross sections for elastic scattering, and the parameters of the α-nucleon potential are adjusted to yield a best fit to the data. The approximations made in the derivation of the optical potential (see Sec. 2) are such that the calculated potentials should more correctly be used in a coupled-channels calculation. This has also been done for elastic and inelastic scattering from ^{42}Ca and ^{50}Ti but will be reported separately.[15] In the present work we consider only the elastic scattering, and attempt to determine whether a microscopic analysis is feasible and whether it can be used to interpret the behavior of the strong-absorption radius.

2. FORMALISM

We follow the formalism and notation of Kerman, McManus, and Thaler,[2] but neglect all corrections of order $1/A$. The Hamiltonian of the system is

$$H=H_N+H_\alpha+T_\alpha+V, \qquad (1)$$

where H_N is the Hamiltonian of the nucleus, H_α is the internal Hamiltonian, T_α is the kinetic energy operator for the α particle, and

$$V=\sum_i v_i(\mathbf{r}_\alpha, \mathbf{r}_i) \qquad (2)$$

is the sum of the two-body interactions between the α particle and each target nucleon. The energy of the system is

$$E_n=K+\epsilon_n,$$

where K is the kinetic energy of the α particle and ϵ_n is the excitation energy of the nucleus. It is assumed that the internal energy of the α particle is unchanged, and this energy is therefore set equal to zero. With this notation the optical potential is given by the multiple scattering expansion

$$U_{00}=U_{00}{}^0+\sum_{n=0} U_{0n}(\alpha_n-U_{nn})^{-1}U_{n0}+\cdots, \qquad (3)$$

where the propagator is

$$(\alpha_n)^{-1}=\mathcal{A}/(E_n-H_N-H_\alpha-T_\alpha+i\epsilon) \qquad (4)$$

and $\mathcal{A}$ is the antisymmetrization operator for the nuclear states. If we neglect the off-diagonal terms in which the intermediate states correspond to excited states of the nucleus, we obtain the lowest-order approximation for the optical potential $U_{00}\approx U_{00}{}^0$, which is given by

$$U_{00}{}^0=\langle 0\,|\sum_i t_i\,|\,0\rangle, \qquad (5)$$

where t_i is the effective two-body scattering operator

$$t_i=v_i+v_i(\alpha)^{-1}t_i. \qquad (6)$$

The scattering matrix for this optical potential is

$$T^0=U_{00}{}^0+U_{00}{}^0(\alpha_0)^{-1}T^0, \qquad (7)$$

which shows that in this approximation the intermediate states in which the nucleus remains in its ground state are properly taken into account.

In a coordinate representation the scattering operator t is nonlocal and energy-dependent. It also differs from the free interaction. In view of these difficulties, we follow the standard procedure in the microscopic description of inelastic scattering and replace $\sum t_i$ by an effective interaction $V_{\rm eff}$. If we now define the nuclear-matter distribution as

$$\rho_m(\mathbf{r})=\rho_p(\mathbf{r})+\rho_n(\mathbf{r}), \qquad (8)$$

where

$$\rho_p(\mathbf{r})=\langle 0\,|\sum_{i=1}^{z}\delta(\mathbf{r}-\mathbf{r}_i)\,|\,0\rangle$$

and similarly for ρ_n, with the normalization

$$\int \rho_m(\mathbf{r})\,d\mathbf{r}=A,$$

then the optical potential is given by

$$U_{\rm opt}(\mathbf{R})\approx U_{00}{}^0(\mathbf{R})\approx\int \rho_m(\mathbf{r})\,V_{\rm eff}(\mathbf{r}-\mathbf{R})\,d\mathbf{r}, \qquad (9)$$

where $\mathbf{R}$ is the distance between the center of mass of the α particle and the center of mass of the nucleus. Thus, the finite size of the α particle is taken into account through $V_{\rm eff}$, but the structure and polarizibility of the α particle are neglected. If $V_{\rm eff}$ is real, Eq. (9) defines only the real part of the optical potential.

From Eq. (9) for the optical potential we obtain the usual relation between the mean-square radii of the potential, the nuclear-matter distribution, and the two-body interaction, i.e.,

$$\langle r^2\rangle_{\rm Opt}=\langle r^2\rangle_{\rm ND}+\langle r^2\rangle_{\rm TB}, \qquad (10)$$

[11] D. F. Jackson and C. G. Morgan, Phys. Rev. **175**, 1402 (1968).
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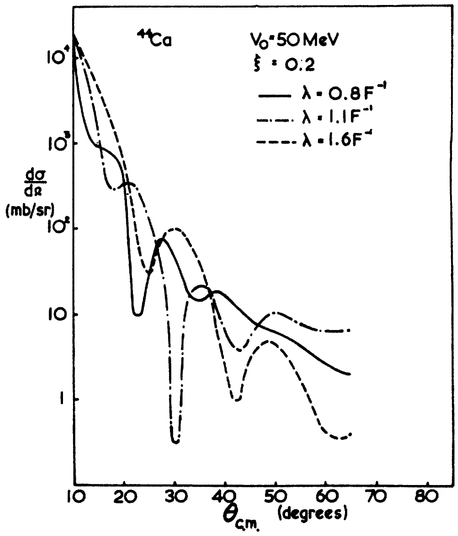

FIG. 1. The elastic cross section calculated for 42-MeV α particles on ^{44}Ca using the α-nucleon parameters shown on the figure.

or, if we introduce the equivalent radius $R_{\rm Eq}$ of the potential,

$$R_{\rm Eq}{}^2=\tfrac{5}{3}\langle r^2\rangle_{\rm ND}+\tfrac{5}{3}\langle r^2\rangle_{\rm TB}. \qquad (11)$$

For comparison we include the corresponding formulas for electron scattering. In this case, $V_{\rm eff}$ is given by the known electron-proton interaction, and the Coulomb part of the interaction can be written as[7]

$$V_{\rm eff}=-e\int[\rho_{\rm op}(\mathbf{r})/|\mathbf{r}-\mathbf{R}|]d\mathbf{r},$$

where $\rho_{\rm op}$ is the charge-density operator whose matrix element in the ground state yields the nuclear charge density $\rho_p(\mathbf{r})$. The optical potential for elastic electron

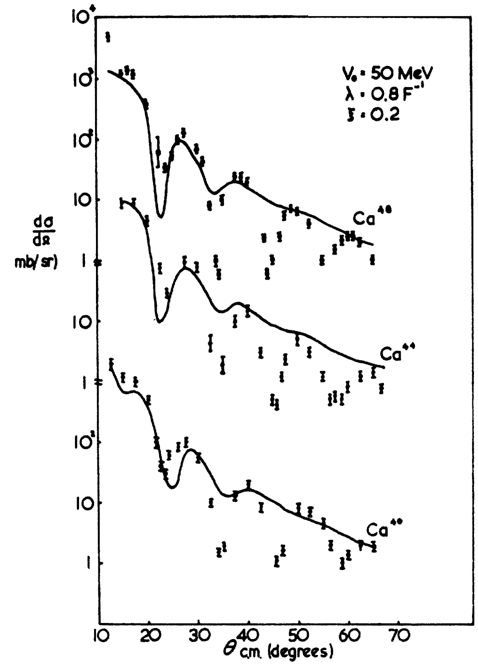

FIG. 3. The elastic cross sections calculated for 42-MeV α particles on ^{40,44,48}Ca using the same α-nucleon parameters as in Fig. 2. The data is again that of Peterson (Ref. 13).

scattering is then given by

$$U(R)=-4\pi e^2\left[(R)^{-1}\int_0^R\rho_p(r)r^2dr+\int_R^\infty\rho_p(r)r\,dr\right].$$

In this case also, the multiple-scattering terms involving intermediate excited states of nucleus are neglected, but recent attempts to estimate the corrections have indicated that they are small.[16]

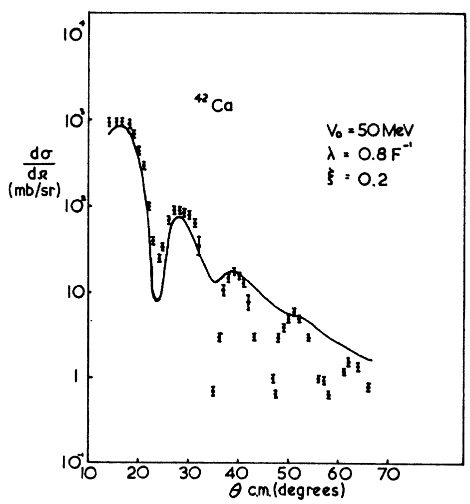

FIG. 2. The elastic cross section calculated for 42-MeV α particles on ^{42}Ca using the α-nucleon parameters shown on the figure. The data is that of Peterson (Ref. 13).

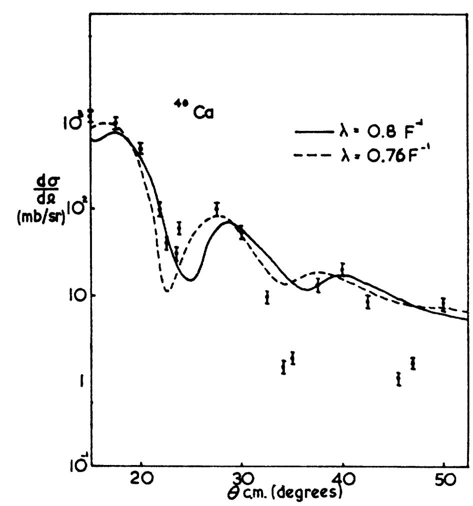

FIG. 4. The elastic cross section calculated for 42-MeV α particles on ^{40}Ca using different values of the α-nucleon range parameter λ.

[16] G. Rawitscher, Phys. Rev. **151**, 846 (1966).

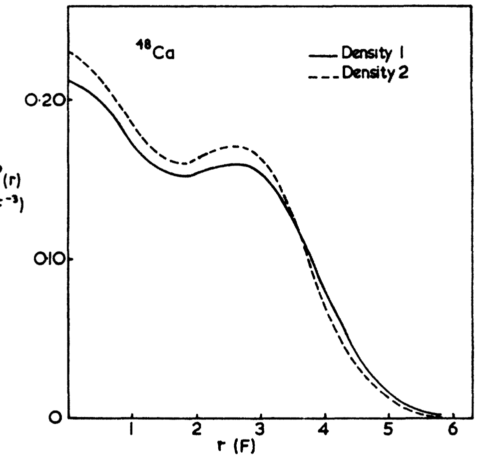

FIG. 5. The nuclear-matter distributions used for calculations on ^{48}Ca. The difference between the two distributions is explained in the text.

3. DETAILS OF CALCULATION

The optical potential defined by Eq. (9) has been evaluated numerically in a subroutine of a standard optical-model code. The nuclear-matter distributions for the calcium isotopes ^{40,44,48}Ca are also calculated numerically from the single-particle potentials given by Elton and Swift.[14] Since for each isotope these potentials yield a charge density in agreement with the electron scattering data, and single-particle energies for both protons and neutrons in agreement with the known separation energies, we take the view that these potentials yield the best available result for the nuclear-matter distribution. A similar set of single-particle potentials is not available for ^{42}Ca, so that for this

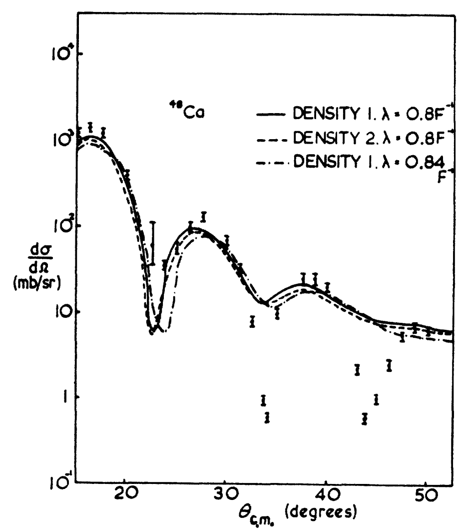

FIG. 6. The elastic cross section calculated for 42-MeV α particles on ^{48}Ca using the two distributions shown in Fig. 5 and various values of the α-nucleon length parameter λ.

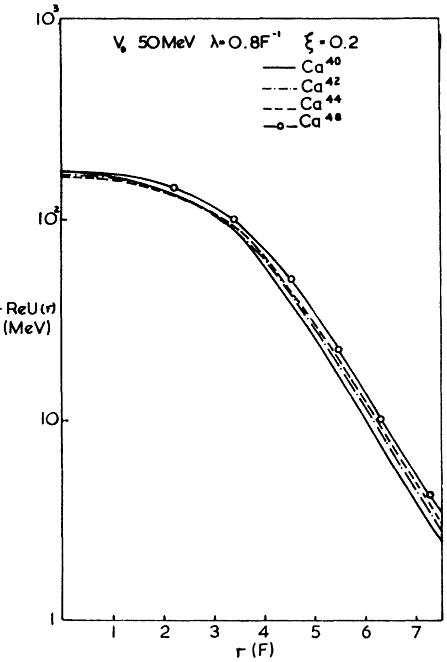

FIG. 7. The optical potentials for elastic α-particle scattering from the calcium isotopes calculated with the fixed set of α-nucleon parameters.

isotope we have used the potentials for ^{44}Ca but have adjusted the depth of the potential for the $f_{7/2}$ neutrons to give the correct separation energy. The proton and neutron distributions so obtained are very similar in ^{40}Ca, but the addition of neutrons in the $f_{7/2}$ shell causes the neutron distribution to extend beyond the proton distribution in the surface region of the heavier isotopes.

In the microscopic description of inelastic α-particle scattering, a variety of forms have been tried for the α-nucleon interaction.[4] In those cases where the Gaussian potential, which fits the low-energy two-body scattering

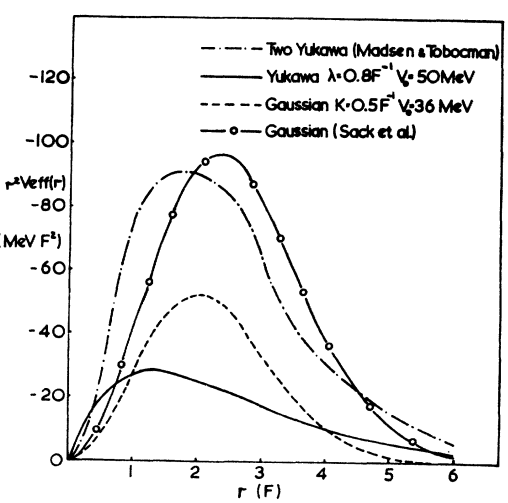

FIG. 8. Comparison of the effective α-nucleon interactions used in this and other calculations.

TABLE I. Results obtained for strong-absorption radii.

Nucleus	$(5/3)^{1/2}\langle r^2\rangle_{\text{ND}}^{1/2}$ (F)	Range parameter λ (F^{-1})	Potential equivalent radius R_{Eq}(F)	$R_{1/2}$ (F) This work	$1.52A^{1/3}+2.24$ (F)	$R_{1/2}$ (F) Ref. 12
^{40}Ca	$1.24A^{1/3}$	0.8	5.80	7.43	7.44	6.36
		0.76	5.94	7.66		
^{42}Ca	$1.26A^{1/3}$	0.8	5.89	7.55	7.52	7.39
^{44}Ca	$1.26A^{1/3}$	0.8	5.95	7.59	7.61	7.43
^{48}Ca	$1.24A^{1/3}$	0.8	5.99	7.77	7.77	7.50
		0.84	5.87	7.43		
	$1.20A^{1/3}$ (density 2)	0.8	5.89	7.62		

data, has been used, it has proved necessary to adjust the magnitudes of the parameters away from their values in the free interaction. In this work we have chosen a simple Yukawa interaction of the form

$$V_{\text{eff}}=-V_0e^{-\lambda s}/\lambda s, \qquad \mathbf{s}=\mathbf{r}-\mathbf{R} \tag{12}$$

to give the real part of the optical potential through Eq. (9), and have taken the imaginary part of the optical potential to have the same radial behavior as the real part. The strength of the imaginary part is modified by a factor ξ, and the parameters V_0, ξ, and λ are then varied to obtain a fit to the data on the elastic scattering of 42-MeV α particles from the calcium isotopes.

In a preliminary investigation[17] we attempted to determine the range parameter λ using Eq. (10). We plotted the mean-square radii of phenomenological optical potentials in the literature as a function of $A^{2/3}$ and took the intercept at $A=0$ to be the mean-square radius of the two-body interaction. This gave a value for λ of 0.96 F^{-1}. This procedure was unsatisfactory, however, because of the ambiguities in the optical potential for α particles which cause the mean-square radius of the potential to be a function of the depth, and this same difficulty prevents determination of V_0 and ξ from comparison of volume integrals. Further, the gradient of the curve was not consistent with that predicted from Eq. (10) from the variation of $\langle r^2\rangle_{\text{ND}}$ with $A^{2/3}$.

The spin-orbit term in the α-nucleon interaction averages to zero for spin-zero nuclei. The Coulomb part of the optical potential is taken to have the usual form due to a uniformly charged sphere.

4. RESULTS AND DISCUSSION

We first attempted to find one set of values for V_0, ξ, and λ which would yield reasonable agreement with the elastic α-particle scattering from the four calcium isotopes. It was found that the variation in V_0 and ξ essentially changed only the magnitude, but that the shape of the cross section is very sensitive to the range parameter λ. The effect of variations in λ is shown in Fig. 1, from which it can be seen that a sufficient change in λ can cause a maximum in the cross section to change into a minimum. It was also found that this model is not able to produce a deep diffraction pattern at large angles, but, contrary to many of the results obtained for inelastic scattering,[4] it is possible to reproduce the spacing of the maxima. The best fit to the data with a fixed set of parameters is given by $V_0=50$ MeV, $\xi=0.2$, and $\lambda=0.08$ F^{-1}. With these parameters the best fit is obtained for ^{42}Ca and is shown in Fig. 2. The fits to the data on the other isotopes are shown in Fig. 3 and are clearly far from satisfactory by the usual standards of optical-model analyses, but in view of the simplicity of the model and the very small number of parameters

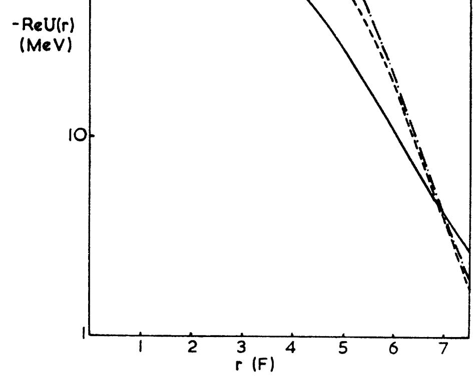

FIG. 9. Comparison of the real part of the folded potential for ^{42}Ca with some phenomenological potentials of similar depth.

[17] D. F. Jackson and V. K. Kembhavi, Rutherford High Energy Laboratory PLA Report, 1967, p. 153 (unpublished).

it contains, these results indicate that a microscopic description of the elastic scattering of α particles is indeed feasible.

An attempt to improve the fit to the data on ^{40}Ca by adjusting λ is shown in Fig. 4, and this again shows how sensitive the cross sections are to this parameter. In order to test the sensitivity of the calculated cross section to reasonable changes in the nuclear-matter distribution, we have compared the cross section for ^{48}Ca using the two distributions shown in Fig. 5. Density 1 is the density distribution obtained by the procedure described in Sec. 3 and used to calculate the cross section shown in Fig. 3. Density 2 is obtained by generating the neutron single-particle wave functions in the same potentials as the proton, but, of course, without the Coulomb potential. This yields a substantial change in the neutron distribution and hence, in the matter distribution; if a comparable change had been made in the proton distribution, the effect would have been easily detected in the analysis of elastic electron scattering or muonic x rays. However, it can be seen from Fig. 6 that the change produced in the cross section for elastic α-particle scattering is small, and unless the theoretical basis of the microscopic model is made more sophisticated so that the two-body interaction is determined more precisely, the uncertainty in the parameters of this interaction will prevent the use of this approach for the investigation of the nuclear-matter distribution. The same conclusion has already been reached[18] from an examination of the calculated potentials.

The optical potentials for the calcium isotopes calculated from the fixed set of two-body parameters are shown in Fig. 7, and the effective two-body interaction given by those parameters is compared in Fig. 8 with the Sack, Biedenharn, and Breit potential[19] and other choices for the effective interaction.[4,15] It can be seen that the double Yukawa potential of Madsen and Tobocman[4] and the Gaussian potential of Morgan[15] peak at about 2 F, and are rather similar in shape but not in magnitude. Both these potentials differ substantially from the Gaussian potential of Sack, Biedenharn, and Breit,[19] which fits the free α-nucleon scattering at low energies. The single Yukawa potential used in the present work has a long tail and when folded into the nuclear-matter distribution yields optical potentials with rather large diffuseness, as can be seen from Figs. 7 and 9, in which our folded potential for ^{42}Ca is compared with some phenomenological potentials of comparable depth.[11] For a larger value of λ, less diffuse potentials are obtained and, consequently, deeper diffraction minima appear in the calculated cross section, as can be seen from Fig. 1, but these potentials do not yield the correct spacing of the diffraction pattern. It is evident that in order to obtain a good fit to the elastic scattering data and, in particular, to obtain minima of the correct depth and spacing, the form of V_{eff} must be chosen with some care.[20] This is not surprising since V_{eff} is an approximation to the two-body t matrix and there is no reason to expect it to resemble closely the free α-nucleon potential. What is significant is the conclusion that we are able to parametrize a complicated many-body operator with a very simple potential and obtain a reasonable fit to the data. The same conclusion has been reached in the microscopic analysis of elastic nucleon scattering,[9] although in that work the imaginary part of the optical potential was treated phenomenologically and many more parameters were available for the fitting procedure.

The results obtained for the strong-absorption radii are given in Table I. In this table, the first line for each isotope gives the results obtained using the Elton-Swift matter distribution and the fixed set of parameters for the α-nucleon interaction, and examination of the corresponding strong-absorption radii shows the remarkable result that although the nuclear-matter distributions increase as $1.25A^{1/3}$, the strong-absorption radii increase as $1.52A^{1/3}$ in accordance with the general trend obtained from direct phase analyses.[21] That this result should arise from our microscopic calculation indicates that it is properly described by conventional scattering theory, and no special mechanism need be invoked to explain the behavior of the strong-absorption radii. These results for the strong-absorption radii differ by about 2–3% from the result obtained from a fit to the same data for ^{42}Ca using phenomenological potentials,[11] and from the results obtained from a similar phenomenological fit to much more accurate data on the calcium isotopes.[12] In addition, the changes we have made in the nuclear-matter distribution or in the range parameter of the α-nucleon interaction also cause changes in the strong-absorption radii of the same order of magnitude, and this provides yet more evidence that in α-particle scattering the strong-absorption radius is a sensitive size parameter whose magnitude can in principle be determined by a microscopic calculation.

ACKNOWLEDGMENTS

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