

<sup>1</sup>Quantization in a large sphere would be required to give the sum in Eq. (1.1) a meaning for the continuum states.

<sup>2</sup>H. F. Hameka, J. Chem. Phys. **47**, 2728 (1967); and erratum in J. Chem. Phys. **48**, 4810 (1968).

<sup>3</sup>See Eq. (2.3) of this article.

<sup>4</sup>L. Hostler, J. Math. Phys. **5**, 591 (1964).

<sup>5</sup>Herbert Buchholz, *Die Konfluente Hypergeometrische Funktion* (Springer-Verlag, Berlin, Göttingen, Heidelberg, 1953).

<sup>6</sup>Here the formula  $z\Gamma(z) = \Gamma(1+z)$  is required. See Whittaker and Watson, *A Course of Modern Analysis* (Cambridge University Press, Cambridge, England,

1927), 4th ed., p. 237.

<sup>7</sup>Ref. 6, p. 236.

<sup>8</sup>Ref. 5, Eqs. (1a) and (2), p. 208.

<sup>9</sup>Ref. 5, Eq. (7), p. 12.

<sup>10</sup>However, Hameka expresses his result in terms of the function  $f(v)$  defined in Sec. II, Sub. 3, Eq. (2.24), instead of the function  $g(v)$ . As one can show by integrating by parts, these two functions are related as follows:

$$v^{-1}(e^v - 1) + f(v) = 1 + g(v).$$

<sup>11</sup>Ref. 5, Eq. (25a), p. 22, and Eq. (24a), p. 21.

<sup>12</sup>Ref. 6, p. 240.

## Spin Orientation of Photoelectrons Ejected by Circularly Polarized Light\*

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Photoelectrons emitted by Cs vapor exposed to circularly polarized light with  $2750 \lesssim \lambda \lesssim 3150$  Å should have spin orientation  $\gtrsim 85\%$  parallel to the light's angular momentum. Measurements of this orientation along the spectrum may be represented by a parameter  $x$  defined theoretically in terms of a Hilbert transform of the spectrum of unperturbed dipole matrix elements weighted by the effect of spin-orbit interaction. The theory of this parameter is adapted from Fermi's interpretation of doublet intensity ratios and from Seaton's related interpretation of photoabsorption minima. Semiquantitative estimates are drawn from experimental data on photoabsorption.

### I. INTRODUCTION

Considerable efforts have been devoted recently to the production of spin-oriented electrons by collision with atoms and molecules,<sup>1</sup> utilizing the spin-orbit interaction, and by photoeffect on spin-oriented atoms.<sup>2</sup> The experimental study of these processes can provide information on the dynamics of electrons in atomic systems. The possibility of providing intense sources of spin-oriented electrons for high-energy accelerators has lent particular interest to these efforts.

This paper points out that, owing to a combination of known circumstances, irradiation of Cs vapor by a broad spectral band of circularly polarized light must yield photoelectrons with nearly complete spin orientation. This spectral band extends from the proximity of the photoelectric threshold (say, from  $\sim 3150$ ) to  $\sim 2750$  Å. The effect results from the influence of the (weak) spin-orbit coupling on the light absorption in the continuum adjoining the principal series, an influence that has been studied long ago by Fermi<sup>3</sup> for the

discrete spectrum and by Seaton<sup>4</sup> for the continuum. Operationally, the phenomenon considered here differs from that observed in Ref. 2 through replacement of the preorientation of target atoms by the prepolarization of the incident light beam; light polarization can be efficient in the range of quartz optics. This phenomenon should occur in all alkali vapors and perhaps in other materials; Cs merely seems to afford the most convenient opportunity for observations.

A criterion for selecting circumstances that favor the production of spin-oriented electrons has been emphasized by Kessler.<sup>5</sup> Orientation by elastic scattering occurs when the scattering amplitude approaches zero. The approach to zero magnifies the relative difference between the amplitudes for alternative mutual orientations of spin and orbit, a difference that otherwise tends to escape attention. Indeed, these two scattering amplitudes have *small values of opposite sign* over a limited range of scattering angles near their points of zero, thus enabling their interference to *suppress* one of the spin orientations in the labor-

atory frame. Similar circumstances occur in the photoabsorption by alkalis (or other atoms<sup>6</sup>) whose amplitude is known to pass through a point of zero along the energy spectrum.<sup>4,7</sup> Here again different amplitudes occur for alternative spin-orbit couplings; their interference can suppress photoemission with a particular spin orientation.

As noted above, parameters describing the influence of spin-orbit coupling on the photoabsorption by alkali vapors have been identified and evaluated by Fermi<sup>3</sup> and Seaton.<sup>4</sup> Observation and measurement of the spin orientation of photoelectrons along the spectrum should provide a much more detailed determination of these parameters.

A comprehensive discussion of the relationships among different observables, presented in Sec. 3 in the current framework of atomic theory, may be warranted in advance of experimental verification.

The main treatment in this paper ignores interactions with the nuclear spin, which tend to reduce the degree of electron-spin orientation. These interactions should have a small effect in view of their weakness. Relevant data are presented in the Appendix to this paper together with an indication of procedures suited to treat the influence of multiple interactions upon the electron-spin orientation.

## II. SPIN-ORIENTATION FORMULA

To calculate the spin orientation yield we consider the matrix of the dipole operator  $x + iy = r \sin\theta \exp(i\varphi)$  which interacts with circularly polarized light. The relevant matrix elements pertain to transitions of a single electron from the  $6s$  discrete ground state of Cs to  $\epsilon p$  continuum states, with alternative values of the initial and final spin-orientation quantum numbers  $m_s$  and  $m_s'$ . The Schrödinger equation for the continuum states, including the influence of spin-orbit interaction is separable for eigenstates of the total angular momentum  $j'$ . However, degenerate eigenstates with  $j' = \frac{3}{2}$  and  $\frac{1}{2}$  exist for each energy eigenvalue  $\epsilon$ . The final states of interest to us, with specified values of  $m_s'$  can be constructed by superposition of states with the alternative values of  $j'$ . [The final-state orbital number  $m_l'$  is not of interest, since eventual integration over all directions of photoemission is implied, but its value is anyhow fixed at  $1 - (m_s' - m_s)$ .]

Spin-orbit coupling is absent in the initial  $s$  state, of course. Nor does it affect the angular integration over the  $\sin\theta \exp(i\varphi)$  factor of the dipole operator. This integration connects the initial  $s$  state with  $m_l = 0$  to a  $p$  state with  $m_l = 1$  and contributes to the complete matrix elements of a spin-independent factor  $(\frac{2}{3})^{1/2}$ . The radial integral,

$$R(\epsilon, j') = \int_0^\infty P(\epsilon p j'; r) r P(6s \frac{1}{2}; r) dr \quad (1)$$

does instead depend on  $j'$ , since spin-orbit coupling in the interior of the atom draws in the nodes of the wave function with  $j' = \frac{1}{2}$  to some extent and pushes out those for  $j' = \frac{3}{2}$ . The desired matrix elements, in a representation with spin and orbit uncoupled, result from a superposition of transition matrix elements to  $j' = \frac{3}{2}$  and  $\frac{1}{2}$  weighted by Wigner coefficients,

$$(m_l' m_s' | r \sin\theta e^{i\varphi} | 0 m_s) = \sum_{j'} (1 \frac{1}{2} m_l' m_s' | 1 \frac{1}{2} j' m') R(\epsilon, j') (1 \frac{1}{2} j' m' | 1 \frac{1}{2} 1 m_s) \sqrt{\frac{2}{3}}. \quad (2)$$

We enter in this formula the values of the Wigner coefficients, leave out the factor  $(\frac{2}{3})^{1/2}$ , simplify the notation by defining

$$R_3 = R(\epsilon, \frac{3}{2}), \quad R_1 = R(\epsilon, \frac{1}{2}), \quad (1')$$

and rewrite the right-hand side of (2) in matrix form:

$$\begin{array}{c} \begin{array}{c} m_s' \\ \diagdown \\ m_s \end{array} \\ \begin{array}{c} \frac{1}{2} \\ -\frac{1}{2} \end{array} \end{array} \begin{array}{c} -\frac{1}{2} \\ \\ \\ \end{array} \left| \begin{array}{cc} R_3 & (\frac{2}{3})^{1/2} R_3 (\frac{1}{3})^{1/2} - (\frac{1}{3})^{1/2} R_1 (\frac{2}{3})^{1/2} \\ 0 & (\frac{1}{3})^{1/2} R_3 (\frac{1}{3})^{1/2} + (\frac{2}{3})^{1/2} R_1 (\frac{2}{3})^{1/2} \end{array} \right| = \left| \begin{array}{c} R_3 \\ 0 \end{array} \begin{array}{c} 2^{1/2} (R_3 - R_1)/3 \\ (R_3 + 2R_1)/3 \end{array} \right| \quad (3)$$

Notice how, in the absence of spin-orbit interaction,  $R_3$  and  $R_1$  would be equal and (3) would be propor-

tional to the unit matrix and would yield no spin selection. *Full spin selection*, with exclusion of  $m_s' = -\frac{1}{2}$ , results on the contrary when

$$R_3 + 2R_1 = R(\epsilon, \frac{3}{2}) + 2R(\epsilon, \frac{1}{2}) = 0 \quad (4)$$

According to (3) the degree of spin orientation is

$$P = \frac{\sum_{m_s} \left[ (m_l' \frac{1}{2} |x + iy| 0m_s)^2 - (m_l' - \frac{1}{2} |x + iy| 0m_s)^2 \right]}{\sum_{m_s} \left[ (m_l' \frac{1}{2} |x + iy| 0m_s)^2 + (m_l' - \frac{1}{2} |x + iy| 0m_s)^2 \right]}$$

$$= \frac{R_3^2 + \frac{2}{9}(R_3 - R_1)^2 - \frac{1}{9}(R_3 + 2R_1)^2}{R_3^2 + \frac{2}{9}(R_3 - R_1)^2 + \frac{1}{9}(R_3 + 2R_1)^2} = \frac{2(R_3 - R_1)(2R_3 + R_1) + (R_3 - R_1)^2}{(2R_3 + R_1)^2 + 2(R_3 - R_1)^2} = \frac{1 + 2x}{2 + x^2} \quad (5)$$

where

$$x(\epsilon) = (2R_3 + R_1)/(R_3 - R_1) = [2R(\epsilon, \frac{3}{2}) + R(\epsilon, \frac{1}{2})] / [R(\epsilon, \frac{3}{2}) - R(\epsilon, \frac{1}{2})] \quad (6)$$

may serve as the main parameter that characterizes the spin orientation.

The trend of  $P(x)$  is shown in Fig. 1 and by the following values

$$x: \quad (-x \gg 1) \quad -2 \quad -1 \quad -\frac{1}{2} \quad 0 \quad \frac{1}{2} \quad 1 \quad 2 \quad 3 \quad \gg 1$$

$$P: \quad -2/x \quad -\frac{1}{2} \quad -\frac{1}{3} \quad 0 \quad \frac{1}{2} \quad \frac{8}{9} \quad 1 \quad \frac{5}{8} \quad \frac{7}{11} \quad 2/x \quad .$$

Figure 1 also shows a tentative scale of photon wavelengths (i. e., of  $\epsilon$ ) paralleling the scale of  $x$ , estimated from data discussed in Sec. 3. The immediate purpose of experiments would be to measure  $P$  as a function of  $\epsilon$  and thereby to determine  $x(\epsilon)$  by solving (5). The significance of  $x$  will be discussed in Sec. 3.

Notice also, for purposes of analysis, that the total cross section for photoabsorption is proportional to the denominator of (5) and can be represented by

$$\sigma(\epsilon) \propto (2R_3 + R_1)^2 + 2(R_3 - R_1)^2 = (R_3 - R_1)^2(2 + x^2) \quad (7)$$

Thus the total cross section measures in effect the *sum* of two terms, while the spin orientation measures their *ratio*; analogous circumstances occur in electron scattering.

### III. DISCUSSION

*Ab initio* calculation of the separate radial integrals  $R_3$  and  $R_1$  and thereby of the orientation parameter  $x(\epsilon)$  is, of course, possible. However the

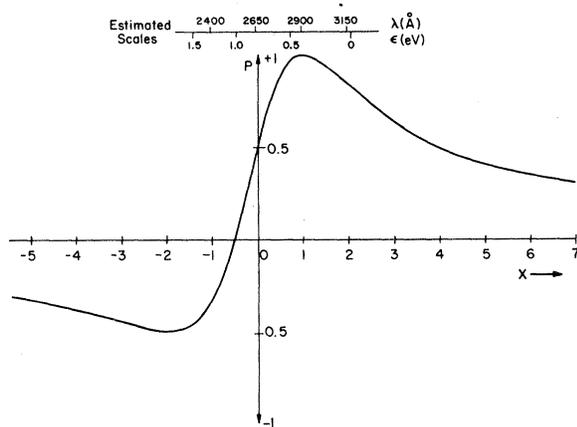


FIG. 1. The orientation function  $P(x)$ , Eq. (5); scale of  $x$  estimated in Sec. 3.

integration over  $r$  involves extensive cancellations. The attendant loss of accuracy appears more severe in view of our particular interest in the difference  $R_3 - R_1$ . Recent extensive analysis<sup>7,8</sup> shows that even the theoretical calculations of *total* photoabsorption for Cs and for related atoms in the range of interest are only in qualitative agreement with experiment.

The alternative approach of Fermi and Seaton aims directly at the evaluation of the difference  $R_3 - R_1$  in terms of the level splitting and of related effects of spin-orbit coupling. Besides removing an obvious source of inaccuracy, this approach points up the direct connection between the observable parameter  $x(\epsilon)$  and the spin-orbit interaction. It also emphasizes how  $x(\epsilon)$  depends critically not only on this interaction but also on the spectrum of photoabsorption.

Here we compare briefly the characteristics of the continuum wave functions with  $j' = \frac{3}{2}$  and  $\frac{1}{2}$  which make the integrals  $R_3$  and  $R_1$  appreciably different from one another and then adapt the Fermi-Seaton approach to the calculation of  $x(\epsilon)$ . Only a semi-quantitative estimate of  $\epsilon$  will be made.

The radial integrals  $R$  for the alkalis have small

values in atomic units, because continuum (or discrete) wave functions  $P(\epsilon p j'; r)$  have a node in the region of their maximum overlap with the ground-state wave function  $P(ns; r)$ ; extensive cancellation results in the course of integration.<sup>4</sup> The coordinate  $r_n$  of this node decreases with increasing energy  $\epsilon$ . It passes through some critical value  $r_c$  such that  $R=0$  at an energy not far in excess ( $\sim 1$  eV) of the photoelectric threshold, for all alkalis from Na to Cs. As the wave functions  $P(\epsilon p j'; r)$  oscillate with increasing  $r$ , the function with  $j' = \frac{3}{2}$  lags in phase with respect to  $j' = \frac{1}{2}$ , because the spin-orbit interaction potential which acts in the region of small  $r$  is attractive for  $j' = \frac{1}{2}$  and repulsive for  $j' = \frac{3}{2}$ . Accordingly  $r_n(\epsilon, \frac{3}{2})$  exceeds  $r_n(\epsilon, \frac{1}{2})$  at each  $\epsilon$  and will reach the critical value  $r_c$  for a higher value of  $\epsilon$ , which we call  $\epsilon_3$ , than the value  $\epsilon_1$  corresponding to  $j' = \frac{1}{2}$ . We conclude that  $R(\epsilon, \frac{3}{2})$  and  $R(\epsilon, \frac{1}{2})$  differ primarily by a shift in their energy scale of amount  $\sim \epsilon_3 - \epsilon_1 > 0$ . The phase lag of  $P(\epsilon p \frac{3}{2}; r)$  also brings about a quantum defect lag

$$\Delta\tau(\epsilon_n) = \tau(np \frac{1}{2}) - \tau(np \frac{3}{2}) \quad (8)$$

between the doublet  $p$  levels in the discrete spectrum, which amounts to 0.03 for Cs.

Fermi's work<sup>3</sup> on the relative intensities of alkali doublet lines interpreted the experimental fact that the intensity ratio of the Cs  $7p$  doublet lines approaches 4 instead of having the usual statistical weight value of 2. This fact implies that

$$R_3 \sim \sqrt{2}R_1, \text{ i. e., } x \sim 10 \text{ at } \epsilon = \epsilon_7 = -1.2 \text{ eV.} \quad (9)$$

Seaton<sup>4</sup> interpreted the occurrence of a deep but nonzero minimum of photoabsorption in terms of Eq. (7); he pointed out that insofar as  $R_3 - R_1$  varies slowly with  $\epsilon$  the minimum of  $\sigma(\epsilon)$  nearly coincides with the zero point of  $2R_3 + R_1$ . The tentative assumption  $R_3 - R_1 \sim \text{const}$  in the proximity of  $2R_3 + R_1 = 0$  enables one to establish a scale of  $x(\epsilon)$  in this proximity by fitting (7) to experimental data. Curve 2 on p. 115 of Ref. 8 shows  $\sigma(\epsilon)$  to have its minimum at  $\epsilon \sim 0.78$  eV, corresponding to  $\lambda = 2660 \text{ \AA}$ , and to increase by  $\sim 20\%$  within 0.3 eV on either side. This datum implies  $|dx/d\epsilon| \sim 2(\text{eV})^{-1}$  and  $|dx/d\lambda| \sim 0.004 \text{ \AA}^{-1}$ ; the latter value has been used most tentatively in Fig. 1. The inaccuracy of any extrapolation of this scale is pointed up by the discrepancy between (9) and the value suggested by the scale for  $x(-1.2 \text{ eV})$ .

Fermi interpreted, in effect, the expression  $2R_3 + R_1$  as the radial matrix element obtained by replacing in Eq. (1)  $P(\epsilon p j'; r)$  with a wave function  $P_0(\epsilon p; r)$  calculated disregarding spin-orbit coupling. This amounts to setting

$$2R(\epsilon, \frac{3}{2}) + R(\epsilon, \frac{1}{2}) = 3R_0 \\ = 3 \int_0^\infty P_0(\epsilon p; r) r P(6s \frac{1}{2}; r) dr. \quad (10)$$

Fermi then utilized perturbation theory to calculate the difference

$$R_3 - R_1 = R(\epsilon, \frac{3}{2}) - R(\epsilon, \frac{1}{2}) \\ = \sum_n' R_0(n) (\epsilon - \epsilon_n)^{-1} \int_0^\infty P_0(np; r) \\ \times \frac{3}{4} \left( \frac{\hbar}{mc} \right)^2 \frac{1}{r} \frac{dV}{dr} P_0(\epsilon p; r) dr, \quad (11)$$

where the  $\sum_n'$  includes the continuum as well as the discrete spectrum with exclusion of the zero point of  $\epsilon - \epsilon_n$  and where  $-V(r)$  represents the electron's potential energy. Finally Fermi considered that  $r^{-1}dV/dr$  is appreciable only for small  $r$  where the wave function's shape depends hardly at all on the electron's energy, to an extent negligible for the present purposes. Thereby he expressed the integral in (11) in terms of the experimental doublet splitting by an argument which can be formulated as follows.

Consider wave functions  $\bar{P}_0(\epsilon p; r)$  normalized at the origin by a convention independent of  $\epsilon$ , e. g., such that

$$\bar{P}_0(\epsilon p; r) = r^{l+1} (1 + \dots). \quad (12)$$

These wave functions exist in the range of small to moderate  $r$  values for all energies  $\epsilon$ , in the continuum and discrete ranges, and are practically independent of energy variations  $\Delta\epsilon$  in the range of  $r$  where  $\epsilon + V(r) \gg \Delta\epsilon$ . With such wave functions one could calculate the matrix element

$$\bar{W}(\epsilon', \epsilon) = \frac{3}{4} \left( \frac{\hbar}{mc} \right)^2 \int_0^\infty \bar{P}_0(\epsilon' p; r) \\ \times \frac{1}{r} \frac{dV}{dr} \bar{P}_0(\epsilon p; r) dr. \quad (13)$$

Fermi's argument implies that  $\bar{W}$  is independent of  $\epsilon$  and  $\epsilon'$  over a rather wide range because  $r^{-1}dV/dr$  is significant only where  $V(r)$  is large. Accordingly we shall treat  $\bar{W}$  as a constant.

Wave functions with the usual normalization equal the  $\bar{P}_0$  for the relevant value of  $\epsilon$  multiplied by appropriate constants,

$$P_0(np; r) = N_n \bar{P}_0(\epsilon_n p; r), \\ P_0(\epsilon p; r) = N_\epsilon \bar{P}_0(\epsilon p; r). \quad (14)$$

We can also set

$$R_0(n) = N_n \bar{R}_0(\epsilon_n) \\ = N_n \int_0^\infty \bar{P}_0(\epsilon_n p; r) r P(6s \frac{1}{2}; r) dr, \quad (15)$$

so that (11) becomes

$$R_3 - R_1 = \sum_n' \bar{R}_0(\epsilon_n) N_n^2 (\epsilon - \epsilon_n)^{-1} \bar{W} N_\epsilon. \quad (16)$$

At this point the product  $N_n^2 \bar{W}$  is recognized as the doublet splitting which can also be expressed in terms of the quantum-defect lag (8),

$$\Delta\epsilon_n = (d\epsilon_n/dn)\Delta\tau(\epsilon_n) = N_n \bar{W} N_n. \quad (17)$$

Substitution of (17) in (16) and of (15) in (10) also causes  $N_\epsilon$  to cancel out of the final result

$$x(\epsilon) = 3\bar{R}_0(\epsilon) \times \left( \sum_n' \bar{R}_0(\epsilon_n) \Delta\tau(\epsilon_n) \frac{d\epsilon_n/dn}{\epsilon - \epsilon_n} \right)^{-1}. \quad (18)$$

The factors  $\bar{R}_0(\epsilon_n)$  and  $\Delta\tau(\epsilon_n)$  have been utilized in this expression of  $x(\epsilon)$  because they are smooth functions of  $\epsilon_n$  throughout the discrete spectrum and far into the continuum. [In the continuum  $\Delta\tau(\epsilon)$  represents the phase shift difference  $\delta(\epsilon, \frac{1}{2}) - \delta(\epsilon, \frac{3}{2})$  in units of  $\pi$ .] However  $\bar{R}_0$  and  $\Delta\tau$  may not be regarded as constant within the range of  $\epsilon_n$  that contributes effectively to the  $\sum_n'$  in (18). Indeed the main information to be derived from polarization measurements concerns the value of this sum.

The factor  $d\epsilon_n/dn$  in (18) represents the slope of a smooth curve drawn through the points of a plot of  $\epsilon_n$  versus  $n$ , where  $n$  is an integer rather than the effective quantum number  $n - \tau$ .<sup>9</sup> The expression  $\epsilon_n = -I_H/(n - \tau)^2$  leads to Seaton's formula<sup>10</sup>

$$\frac{d\epsilon_n}{dn} = \frac{2I_H}{(n - \tau)^3} \left( 1 + \frac{2I_H}{(n - \tau)^3} \frac{d\tau}{d\epsilon} \right)^{-1}. \quad (19)$$

The factor  $d\epsilon_n/dn$  is designed so that the continuum portion of  $\sum_n'$  becomes the principal part dispersion integral

$$\phi \int \bar{R}_0(\epsilon') \Delta\tau(\epsilon') d\epsilon' / (\epsilon - \epsilon'). \quad (20)$$

This integral form suggests that the expression (18) of  $x(\epsilon)$  be described as the ratio of  $3\bar{R}_0(\epsilon)$  to the Hilbert transform of the product  $\bar{R}_0(\epsilon') \Delta\tau(\epsilon')$ . As expected, smallness of the doublet splitting  $\Delta\tau$  magnifies the slope of a plot of  $x$  versus  $\epsilon$  and thus narrows the spectral band within which spin orientation occurs. On the other hand, a *rapid variation* of  $\bar{R}_0(\epsilon)$  along the spectrum is essential to make the Hilbert transform of  $\bar{R}_0 \Delta\tau$  appreciable and therefore  $x(\epsilon)$  small within that band. This interpretation of the parameter  $x(\epsilon)$  is viewed as the main conclusion of the present discussion, even though it was implicit in Fermi's perturbation formula.

No evaluation of (18) is available at this time. Fermi calculated, in effect, the value of  $\sum_n'$  for  $\epsilon = \epsilon_7$  by disregarding all of its terms but the dominant one  $n = 6$ . For  $\epsilon$  in the continuum, this term will be less important. Notice finally that the key approximation of the present treatment lies in the assumption that  $\bar{W}(\epsilon', \epsilon)$  is effectively a constant. The perturbation approach is well founded since

$\Delta\tau \sim 0.03 \ll 1$  but actually unnecessary. If the spin-orbit interaction operator,  $(\hbar/mc)^2 r^{-1} dV/dr$ , were not treated as a small quantity it should be replaced in the calculations of  $R_3 - R_1$  and of  $\Delta\epsilon_n$  by a reaction operator  $K$  which represents the interaction effect to all orders. The dependence of  $x(\epsilon)$  on  $\Delta\tau(\epsilon')$  would remain unchanged because all normalization factors would still cancel out.<sup>11</sup>

#### APPENDIX. TREATMENT OF MULTIPLE INTERACTIONS<sup>12</sup>

The interaction between the optical electron of Cs and the nuclear-spin momentum  $\vec{I}$  causes the radial wave functions of angular-momentum eigenstates in (1) to depend not only on the electron quantum number  $j'$  but also on the combined quantum numbers  $F$  and  $F'$  ( $\vec{F} = \vec{j} + \vec{I}$ ) pertaining to the  $6s$  and  $\epsilon p$  states. The procedure utilized in Eqs. (2)–(5) for averaging over alternative values of  $j'$ , with weights determined by vector-coupling coefficients, becomes involved when additional momenta have to be coupled.

An alternative and speedier procedure, which relates the desired spin orientation  $P$  more directly to the fine and hyperfine splits observed in spectra, is suggested by the treatment of the radial integral  $R(\epsilon, j)$  in Sec. III, Eqs. (10) and (11). This integral can be expressed as

$$R(\epsilon, j) = R_0(\epsilon) + 2\vec{I} \cdot \vec{s} \Delta(\epsilon)/3, \quad (21)$$

where  $R_0(\epsilon)$  is defined by (10),  $\Delta = R_3 - R_1$  is represented by (11) or by (15) or by the expression in the brackets of (18) and where  $\vec{I}$  and  $\vec{s}$  are the orbital and spin-angular-momentum operators of the electron in units of  $\hbar$ . The features of (21) relevant to us are: (a) The coefficient  $\Delta(\epsilon)$  of  $\vec{I} \cdot \vec{s}$  is proportional to  $\Delta\tau$ , the observed fine-structure split of the  $\epsilon p$  quantum defect. (b) The factor  $\vec{I} \cdot \vec{s}$  need not be expressed explicitly in terms of the alternative values of  $j'$ , as required by the averaging procedure of Sec. II; an equivalent averaging of  $\vec{I} \cdot \vec{s}$  is performed automatically in a calculation of  $P$  by the method of operator traces.<sup>13</sup>

In the presence of nuclear-spin interactions, (21) should be replaced by a longer sum of terms, proportional to various scalar products of  $\vec{I}$ ,  $\vec{s}_z$ , and  $\vec{I}$ . The coefficients of terms with a factor  $\vec{I}$  will contain hyperfine-structure factors  $(\Delta\tau)_{\text{hfs}}$  analogous to the  $\Delta\tau = 0.03$  in (8) and (18) but smaller than it in the ratio of the hyperfine and fine structures observed in the spectra. Existing data<sup>14</sup> yield  $(\Delta\tau)_{\text{hfs}} = 3 \times 10^{-5}$  for the  $6s$  and  $(\Delta\tau)_{\text{hfs}} = 2 \times 10^{-6}$  for the  $p$  states of Cs. Accordingly, the terms to be added to (21) – and hence to the orientation  $P$  in (5) – owing to nuclear-spin interaction include coefficients at least three orders of magnitude smaller than  $\Delta$ .

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<sup>1</sup>See, e.g., H. Deichsel, H. Reichert, and H. Steidl, *Z. Physik* **189**, 212 (1966); K. Jost and J. Kessler, *ibid.* **195**, 1 (1966); J. Mehr, *ibid.* **198**, 345 (1967); W. Hilgner and J. Kessler, *Phys. Rev. Letters* **18**, 983 (1967); D. W. Walker, *ibid.* **20**, 827 (1968); A. C. Yates, *ibid.* **20**, 829 (1968).

<sup>2</sup>R. L. Long, Jr., W. Raith, and V. W. Hughes, *Phys. Rev. Letters* **15**, 1 (1965); V. W. Hughes, M. S. Lubell, M. Posner, and W. Raith, in *Proceedings of the Sixth International Conference on High Energy Accelerators*, Cambridge, Massachusetts, September, 1967, Cambridge Electron Accelerator (1967), p. A144.

<sup>3</sup>E. Fermi, *Z. Physik* **59**, 680 (1930), [also in *Collected Papers of Enrico Fermi* (University of Chicago Press, Chicago, Illinois 1962), p. 322].

<sup>4</sup>M. J. Seaton, *Proc. Roy. Soc. (London)* **A208**, 408 (1951); Sec. 5.

<sup>5</sup>J. Kessler, *Bull. Am. Phys. Soc.* **13**, 98 (1968).

<sup>6</sup>It suffices that a zero occurs in the amplitude of the transition that contributes most of the photoabsorption.

<sup>7</sup>J. W. Cooper, *Phys. Rev.* **128**, 681 (1962); U. Fano and J. W. Cooper, *Rev. Mod. Phys.* **40**, 441 (1968); Sec. 4.5.

<sup>8</sup>G. V. Marr, *Photo-ionization Processes in Gases* (Academic Press, Inc., New York, 1967).

<sup>9</sup>See in particular Sec. 2.4 of Fano and Cooper,

Ref. 7.

<sup>10</sup>M. J. Seaton, *Monthly Notices Roy. Astron. Soc.* **118**, 504 (1958); the denominator of (19) coincides with Eq. (86) of this reference.

<sup>11</sup>In fact, the theory could also be extended to take into account a dependence of  $\bar{W}$  (or of the more accurate reaction matrix) on  $\epsilon$  and  $\epsilon'$ , by inserting in (18) a corrective factor which multiplies  $\bar{R}_0(\epsilon')\Delta\tau(\epsilon')$  and represents the ratio  $\bar{W}(\epsilon, \epsilon')/\bar{W}(\epsilon', \epsilon')$ .

<sup>12</sup>I thank Dr. W. Raith for suggesting that the hyperfine structure effects be discussed.

<sup>13</sup>This method derives (5) from the following considerations: (i) The transition operator for photon absorption is proportional to (21), i.e., to  $x+2\vec{l}\cdot\vec{s}$ . (ii) Absorption of circularly polarized light yields an initial orientation of  $\vec{l}$  (prior to spin-orbit interaction) represented by the density operator  $\frac{1}{2}I_z(l_z+1)$ . (iii) The initial orientation of  $\vec{s}$  is random. (iv) The degree of spin orientation is represented by the final-state average  $\langle 2s_z \rangle$ . Thus one finds directly

$$P = \frac{\text{Tr} [2s_z (x+2\vec{l}\cdot\vec{s}) I_z (l_z+1) (x+2\vec{l}\cdot\vec{s})]}{\text{Tr} [(x+2\vec{l}\cdot\vec{s}) I_z (l_z+1) (x+2\vec{l}\cdot\vec{s})]} = \frac{(2x+1)}{(x^2+2)}$$

<sup>14</sup>P. Brix and H. Kopfermann, *Landolt-Börnstein Zahlenwerte und Funktionen*, edited by K. H. Hellwege (Springer-Verlag, Berlin, 1952), Vol. I, Pt. 5, pp. 36, 37.