Direct Electromagnetic Generation of Transverse Acoustic Waves in Metals

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We present a theory to describe the direct generation of transverse acoustic waves in metals by electromagnetic radiation incident on the metal surface and to describe the converse effect, both in the presence and in the absence of an external magnetic field directed normally to the metal surface. This theory can be used to predict the fraction of the incident power which goes into the generated wave. The predictions of this theory are shown to be in substantial agreement with the experimental data reported by Houck *et al.* and others on the direct generation of transverse acoustic waves by radio-frequency radiation in aluminum in the presence of a magnetic field. However, when the predictions of this theory for the direct generation of transverse acoustic waves by microwave radiation in indium are compared with the experimental data of Abeles and others, the predicted insertion loss is found to be several orders of magnitude below that reported. The theory is developed from Maxwell's equations and the equation of motion of the lattice for a free-electron, semi-infinite metal with an electromagnetic field and/or a shear acoustic wave incident on the metal surface. The only forces acting on the ions in the metal are those forces present in the bulk metal or applied externally; that is, the theory is devoid of any forces on the ions resulting from electron scattering at the metal surface. The theory can be easily understood by imagining the unfilled half-space to be occupied by another piece of the same metal and introducing a current sheet and/or a shearing force in a plane containing the origin. The equations which describe the electric field and the ionic displacement field in the metal can then be derived in the context of an infinite metal in a manner which is analogous to that frequently used in deriving the expression for the surface impedance of a metal in the extreme anomalous limit.

I. INTRODUCTION

RECENTLY, experimental evidence has been presented[1–5] for the direct excitation of transverse acoustic waves by radio-frequency and microwave electromagnetic fields in metals, both in the normal and superconducting states, and for the converse effect, the direct excitation of electromagnetic fields by transverse acoustic waves in metals. Direct excitation of transverse acoustic waves by rf radiation has been observed in the presence of a magnetic field applied perpendicularly to the metal surface in Al,[1] Nb, and Sn [2] above the helicon absorption edge and in Ag, Al, and PbTe [3] below the helicon absorption edge. Also, direct excitation of transverse acoustic waves by microwaves has been found to occur in indium films in the absence of a magnetic field.[4,5] Quinn[6] has given a brief theoretical treatment of this problem in which he finds that acoustic waves can indeed be generated directly in a normal metal by an rf field incident on the surface of the metal; he assumes that an applied magnetic field is present and directed normally to the surface of the metal, although this assumption is not necessary to the theoretical treatment of the problem.

In Sec. II A of this paper we shall develop expressions relating the electric field to the ionic displacement field in a semi-infinite normal metal where either an electromagnetic field or a transverse acoustic wave, or both, is normally incident on the surface of the metal in the absence of any externally applied magnetic fields. This development parallels that used by Reuter and Sondheimer[7] to obtain an expression for the surface impedance, except that it has been modified to include the ions by including collision drag effects and by taking the total current density to be the sum of the electronic current density and the ionic current density, rather than simply the electronic current density. The ionic current density can be related to the electric field in the metal by use of the equation of motion for the ions.[8]

Using these relations we shall then develop, in Secs. II B and C, expressions which describe the coupling between the electromagnetic field and the transverse acoustic wave in a metal for two model configurations. The first model will consist of an electromagnetic field incident normally on the surface of a semi-infinite metal; we shall obtain an expression for the rate of energy transfer to the acoustic wave. The second model will consist of a transverse acoustic wave incident normally on the metal surface; we shall obtain an expression for the rate of electromagnetic energy transfer across the boundary. In Sec. II D we shall generalize the results obtained in the prior three parts to include the presence of a magnetic field applied normally to the metal surface.

Sec. III we shall adopt the parameters and experimental configurations given in Refs. 4 and 3 and compare the predictions of the theory given in Sec. II to the experimental results. Section IV will be devoted to

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a discussion of these results and the conclusions which may be drawn from them.

II. THEORY

A. Development of Equations Coupling the Electric Field in the Metal to the Ionic Displacement Field

We shall consider a semi-infinite metal with its surface in the xy plane and the positive z axis directed toward the interior of the metal. The metal will be assumed to consist of a free-electron gas embedded in an isotropic background of positively charged ions which is able to sustain shear acoustic waves. It will further be assumed that both electromagnetic radiation and transverse acoustic waves are incident normally on the surface of the metal. The electric field $E(z)e^{i\omega t}$ in the metal will be taken to be in the x direction and the magnetic field $H(z)e^{i\omega t}$ in the y direction. Although we shall have occasion to discuss experiments which involve an external magnetic field directed normally to the surface of the metal, we choose, for reasons of simplicity of presentation, to develop the theory in the absence of an applied magnetic field and then to outline the generalization of the resultant theory to include the presence of an external magnetic field. Deleting the factor $e^{i\omega t}$ in these and all subsequent equations, Maxwell's equations take the form

$$-\frac{dH}{dz}=\frac{i\omega}{c}E+\frac{4\pi}{c}J, \qquad \frac{dE}{dz}=-\frac{i\omega}{c}H, \tag{1}$$

where $J(z)$ is the total current density. Upon eliminating $H(z)$ from these equations, one obtains

$$\frac{d^2E}{dz^2}+\frac{\omega^2}{c^2}E=\frac{4\pi i\omega}{c^2}J. \tag{2}$$

The total current density $J(z)$ will be taken to be the sum of the electronic current density $J_e(z)$ and the ionic current density $J_i(z)$. The ionic current density is given by

$$J_i(z)=n_0e\partial\xi/\partial t=n_0ei\omega\xi(z), \tag{3}$$

where $\xi(z,t)=\xi(z)\exp(i\omega t)$ is the displacement in the x direction of an ion at position z and time t. We have taken the charge on an ion to be $\bar{z}e$, where e is the absolute value of the charge on an electron, and n_0 to be the number of free electrons per unit volume. The electronic current density is shown in Appendix A to be

$$J_e(z)=2\pi e^2h^{-3}m^2\bar{v}^2\int_{-\infty}^{\infty}k\left(\frac{(z-t)}{l}\right)\times\left[E(t)-\left(\frac{i\omega m}{e\tau}\right)\xi(t)\right]dt, \tag{4}$$

where

$$k(u)=\int_0^{\pi/2}d\theta\,\sin^3\theta\,\sec\theta\,\exp[-(1+i\omega\tau)|u|\sec\theta]. \tag{5}$$

In these equations, m is the electronic mass, $-e$ is the electronic charge, and $\bar{v}$ is the velocity of an electron on the Fermi surface. The electronic mean free path is $l=\bar{v}\tau$, where τ is an average relaxation time for an electron on the Fermi surface. In deriving Eq. (4), as is pointed out in Appendix A, we have used the assumption that the electrons are specularly reflected at the surface of the metal. Furthermore, in order to define the electric field and the ionic displacement field on the negative z axis, we have set

$$E(-z)=E(z) \quad \text{and} \quad \xi(-z)=\xi(z). \tag{6}$$

When Eqs. (3) and (4) are substituted in Eq. (2), one finds that

$$\frac{d^2E}{dz^2}+\frac{\omega^2}{c^2}E=\frac{4\pi i\omega}{c^2}\left\{\frac{2\pi e^2m^2\bar{v}^2}{h^3}\int_{-\infty}^{\infty}k\left(\frac{z-t}{l}\right)\times\left[E(t)-\frac{i\omega m}{e\tau}\xi(t)\right]dt+n_0i\omega e\xi\right\}. \tag{7}$$

In Eq. (7) we shall make the substitutions

$$x=z/l \quad \text{and} \quad y=t/l \tag{8}$$

and delete the factor l from the dependent variables, i.e., $E(lx)=E(x)$, etc. We shall also define the parameter α according to

$$\alpha=8\omega m^2(\pi e\bar{v}/c)^2(l/h)^3=\tfrac{3}{2}(l/\delta)^2, \tag{9}$$

where $\delta=c/(2\pi\omega\sigma)^{1/2}$ is the classical skin depth. The classical conductivity $\sigma=n_0e^2\tau/m$ is related to the electron plasma frequency ω_p by the relation $\omega_p{}^2\tau=4\pi\sigma$. With these definitions and substitutions, Eq. (7) becomes

$$\frac{d^2E}{dx^2}+\frac{\omega^2l^2}{c^2}\left(E+\frac{m}{e}\omega_p{}^2\xi\right)=i\alpha\int_{-\infty}^{\infty}k(x-y)\left[E(y)-\frac{i\omega m}{e\tau}\xi(y)\right]dy. \tag{10}$$

Assuming the functions $E(x)$ and $\xi(x)$ to be continuous everywhere, to tend toward zero as $|x|$ becomes large, to be absolutely integrable over the real axis, and to be differentiable in all orders everywhere except at $x=0$, we may define the Fourier transforms

$$E(q)=\int_{-\infty}^{\infty}E(x)e^{-iqx}dx,$$

$$\xi(q)=\int_{-\infty}^{\infty}\xi(x)e^{-iqx}dx, \tag{11}$$

and

$$\kappa(q)=\int_{-\infty}^{\infty}k(x)e^{-iqx}dx.$$

In the limit $\omega\tau\ll1$ we shall make use of the limiting forms[7]

$$\lim_{q\to0}\kappa(q)=\tfrac{4}{3}(1-\tfrac{1}{5}q^2)$$

and (12)

$$\lim_{|q|\to\infty}\kappa(q)=\frac{\pi}{|q|},$$

together with the observation that $|\kappa(q)|$ decreases monotonically from $\frac{4}{3}$ to 0 as $|q|$ goes from 0 to ∞. If we define

$$\lim_{x\to0^+}\frac{dE}{dx}=\mu \quad\text{and}\quad \lim_{x\to0^+}\frac{d\xi}{dx}=\zeta, \tag{13}$$

it immediately follows that

$$\lim_{x\to0^-}\frac{dE}{dx}=-\mu \quad\text{and}\quad \lim_{x\to0^-}\frac{d\xi}{dx}=-\zeta.$$

With these definitions, the Fourier transform of Eq. (10) is

$$\begin{aligned}[-q^2+(\omega l/c)^2-i\alpha\kappa(q)]E(q)&\\ =2\mu-(\omega\omega_pl/c)^2(m/e)[1-\tfrac{3}{4}\kappa(q)]\xi(q).&\end{aligned} \tag{14}$$

The equation of motion of the ions[8] is

$$\begin{aligned}-M\omega^2\xi(z)=Ms^2(d^2\xi/dz^2)&\\ +\bar{z}eE(z)+(\bar{z}m/\tau)(\langle v\rangle-u),&\end{aligned} \tag{15}$$

where M is the ionic mass and s is the velocity of transverse sound in the metal. The average electron velocity in the x direction, $\langle v\rangle$ is related to the electronic current density by $J_e(z)=-n_0e\langle v\rangle$, and the ionic velocity in the x direction is $u=\partial\xi/\partial t=i\omega\xi$. The final term on the right side of Eq. (15) represents the collision force of the electrons on the ions, where it has been assumed that the scattering of the electrons is diffuse in the system of coordinates in which the ions are locally at rest. After making the change of variables described in Eq. (8), the Fourier transform of Eq. (15) is

$$\begin{aligned}\{q^2-(\omega l/s)^2[1-i\gamma(1-\tfrac{3}{4}\kappa(q))]\}\xi(q)&\\ =-2\zeta+(\bar{z}el^2/Ms^2)(1-\tfrac{3}{4}\kappa(q))E(q),&\end{aligned} \tag{16}$$

where we have used the relation $m\bar{v}/\hbar=(3\pi^2n_0)^{1/3}$ and we have set $\gamma=(\bar{z}m/M)(\omega\tau)^{-1}$.

B. Direct Generation of Transverse Acoustic Waves by Electromagnetic Radiation

In this part we shall use Eqs. (14) and (16) to calculate the ratio of the power transferred to the acoustic wave to the power incident on the surface when electromagnetic radiation is normally incident on the surface of a semi-infinite metal. The Poynting theorem,

$$\partial\left(\int_V udv\right)\Big/\partial t=-\int_V\mathbf{J}\cdot\mathbf{E}dv-\int_S\mathbf{N}\cdot\mathbf{dS}, \tag{17}$$

where $u=(\mathbf{E}\cdot\mathbf{D}+\mathbf{H}\cdot\mathbf{B})/8\pi$ is the electromagnetic energy density, $\mathbf{N}=c(\mathbf{E}\times\mathbf{H})/4\pi$ is the Poynting vector, and the volume integrations are to be carried out over a volume of space V which is enclosed by the surface S, is interpreted to mean that the electromagnetic power transferred to the volume occupied by the semi-infinite metal is equal to the power transferred across the metal surface, given by the surface integral of the Poynting vector, less the volume integral of $\mathbf{J}\cdot\mathbf{E}$. This integral can be divided into two terms, i.e.,

$$\int_V\mathbf{J}\cdot\mathbf{E}dv=\int_V\mathbf{J}_e\cdot\mathbf{E}dv+\int_V\mathbf{J}_i\cdot\mathbf{E}dv, \tag{18}$$

where the first term on the right represents the Joule heating of the electrons. If we assume that no energy is lost by the transverse acoustic wave, then the second term on the right represents the power added to the transverse acoustic wave. Admitting the possibility of complex fields and currents, and noting that the fields and currents in the metal are functions only of distance along the coordinate axis normal to the metal surface, the power added to the transverse acoustic wave per unit surface area is

$$P_{11}=\tfrac{1}{8}l\int_{-\infty}^{\infty}[J_i(x)E^*(x)+J_i^*(x)E(x)]dx. \tag{19}$$

Since there are assumed to be no currents in the half-space not occupied by the metal, and since the reflectivity of a metal surface is quite close to unity, the electromagnetic power incident on the metal surface per unit area is

$$P_{10}=c|H(0)|^2/8\pi. \tag{20}$$

The ratio of the power transferred to the transverse acoustic wave to the electromagnetic power incident on the metal surface is then $\eta_1=P_{11}/P_{10}$.

If Eq. (3) is substituted into Eq. (19), we obtain

$$P_{11}=n_0i\omega el(16\pi)^{-1}\int_{-\infty}^{\infty}[\xi(q)E^*(q)-\xi^*(q)E(q)]dq, \tag{21}$$

where we have made use of the inverse Fourier transforms of Eq. (11), that is,

$$E(x)=(2\pi)^{-1}\int_{-\infty}^{\infty}E(q)e^{iqx}dq,\ \text{etc.} \tag{22}$$

Since, in the model we are presently using, the metal surface is not constrained mechanically, the stress and, hence, the strain at the surface are zero, and therefore the parameter ζ in Eq. (16) is identically zero. Also, since the existence of an ionic displacement field depends on the presence of the electric field, the significant driving term on the electric field in the metal is the gradient of the electric field at the metal surface μ; thus, we shall neglect the second term on the right of Eq. (14). We shall also neglect the term in Eq. (14) which results from the displacement current for the same reasons given in Ref. 7 for neglecting this term. Upon substituting Eqs. (14) and (16) into Eq. (21) subject to the approximations just described, we obtain

$$P_{11}=n_0 i\omega e^2 l^3|\mu|^2 I_1/(4\pi M s^2) \tag{23}$$

with

$$I_1=\int_{-\infty}^{\infty} dq\left\{\frac{(1-\frac{3}{4}\kappa(q))}{\{q^2-(\omega l/s)^2[1-i\gamma(1-\frac{3}{4}\kappa(q))]\}|q^2+i\alpha\kappa(q)|^2}-\text{c.c.}\right\}, \tag{24}$$

where c.c. means the complex conjugate of the preceding term. Finally, Maxwell's equations may be used to obtain the relation

$$c\mu=i\omega l H(0), \tag{25}$$

and Eqs. (20) and (23) may be used to obtain

$$\eta_1=(i/2\pi)(\bar{z}m/M)(\omega_p{}^2\omega^3 l^5/s^2c^3)I_1. \tag{26}$$

C. Direct Generation of Electromagnetic Radiation by Transverse Acoustic Waves

We shall again use Eqs. (14) and (16) to calculate the ratio of the electromagnetic power transferred across the metal surface to the acoustic power entering the metal when a transverse acoustic wave is normally incident on the surface of a semi-infinite metal from a semi-infinite insulator having dielectric constant equal to unity. From the Poynting theorem we immediately observe that the electromagnetic power per unit area transferred out of the metal across the metal surface is given by the Poynting vector evaluated at the surface of the metal, i.e.,

$$P_{21}=(c/16\pi)[E(x)H^*(x)+E^*(x)H(x)]|_{x=0}. \tag{27}$$

The acoustic flux of the incident transverse acoustic wave entering the metal is $\frac{1}{2}sC_{44}|\partial\xi/\partial z|^2$, where C_{44} is the shear elastic stiffness constant. This elastic stiffness constant is related to the transverse sound velocity according to $s=(C_{44}/\rho)^{1/2}$, where $\rho=n_0M/\bar{z}$ is the density of the metal. Thus, the acoustic power entering the metal per unit surface area is

$$P_{20}=Mn_0s^3|\zeta|^2/(2\bar{z}l^2), \tag{28}$$

where we have made use of Eq. (13). The ratio of the electromagnetic power leaving the metal to the acoustic power entering the metal is then $\eta_2=P_{21}/P_{20}$.

The generated electromagnetic wave will be assumed to consist of a plane wave of frequency ω propagating in the dielectric medium. Therefore, since the electric and magnetic fields must be continuous across the metal-insulator boundary, we have

$$E(0)=H(0). \tag{29}$$

This result may be inserted into Eq. (27) to obtain

$$P_{21}=(c/8\pi)|E(0)|^2. \tag{30}$$

Since the existence of an electric field in the metal depends on the presence of an ionic displacement field, the significant driving term on the ionic displacement is the strain at the metal surface; thus, we shall neglect the second term on the right side of Eq. (16). The boundary condition on the electromagnetic fields given by Eq. (29) and the Maxwell relation given by Eq. (25) can be used in Eq. (14) to relate the gradient of the electric field at the metal surface to the electric field at the surface. It can then be observed that neglect of the term μ in Eq. (14) introduces an error in the electric field at the metal surface which is of order $cZ/2\pi$, where Z is the surface impedance of the metal. For a typical metal and for frequencies in the microwave range or below, this error is several orders of magnitude less than unity, and hence we shall neglect the term μ in Eq. (14). If we neglect the displacement current and substitute Eqs. (14) and (16), subject to the above approximations, into Eq. (30), we obtain

$$P_{21}=(c/8\pi)(\omega\omega_p l/c)^4(m/\pi e)^2|\zeta|^2|I_2|^2, \tag{31}$$

where

$$I_2=\int_{-\infty}^{\infty} dq \times\frac{(1-\frac{3}{4}\kappa(q))}{\{q^2-(\omega l/s)^2[1-i\gamma(1-\frac{3}{4}\kappa(q))]\}(q^2+i\alpha\kappa(q))}. \tag{32}$$

Finally, combining Eqs. (28) and (31) we find that

$$\eta_2=\pi^{-2}(\bar{z}m/M)(l^2/sc)^3\omega_p{}^2\omega^4|I_2|^2. \tag{33}$$

D. Generalization of the Theory to Include an External Magnetic Field

In this part we shall discuss the modifications of the theory that are necessary when an external magnetic field of magnitude B directed normally to the surface of the metal is added to the model discussed in Secs. II A, B, and C. Each equation in this part will be numbered in such a manner that it may be easily identified with its counterpart in the original theory. In the p resence of a magnetic field it will be convenient to describe the fields and currents in the metal by their left- and right-circularly polarized components; thus, $E_\pm(z) = E_x(z) \pm iE_y(z)$, etc., where the subscripts refer to the x and y components of the electric field vector in the metal. As before, the wave equation is

$$\frac{d^2E_\pm}{dz^2}+\frac{\omega^2}{c^2}E_\pm=\frac{4\pi i\omega}{c^2}J_\pm, \qquad (2')$$

where $J_\pm = J_{e\pm}+J_{i\pm}$. The circular components of the ionic current density are

$$J_{e\pm}=2\pi e^2h^{-3}m^2\bar{v}^2\int_{-\infty}^{\infty}k_\pm((z-t)/l)\times[E_\pm(t)-(i\omega e/m\tau)\xi_\pm(t)]dt, \qquad (4')$$

with

$$k_\pm(u)=\int_0^{\pi/2}d\theta\,\sin^3\theta\,\sec\theta\times\exp\{-[1+i(\omega\mp\omega_c)\tau]|u|\,\sec\theta\}, \qquad (5')$$

where $\omega_c=eB/mc$ is the cyclotron frequency of the electrons. The Fourier transform of Eq. (5′) is

$$\kappa_\pm(q)=\int_{-\infty}^{\infty}k_\pm(x)e^{-iqx}dx; \qquad (11')$$

in the limit $\omega\tau\ll1$ and $\omega_c\tau\gg1$ we shall make use of the limiting forms

$$\lim_{q\ll\omega_c\tau}\kappa_\pm(q)=\tfrac{4}{3}(1\mp i\omega_c\tau)^{-1}$$

and $\qquad (12')$

$$\lim_{q\gg\omega_c\tau}\kappa_\pm(q)=\frac{\pi}{|q|}.$$

If we define

$$\lim_{x\to0^+}\frac{dE_\pm}{dx}=\mu_\pm,\ \text{etc.}, \qquad (13')$$

the Fourier transform of the wave equation is

$$[-q^2+(\omega l/c)^2-i\alpha\kappa_\pm(q)]E_\pm(q)=2\mu_\pm-(\omega\omega_p l/c)^2(m/e)(1-\tfrac{3}{4}\kappa_\pm(q))\xi_\pm(q). \qquad (14')$$

The equation of motion for the ions is

$$-M\omega^2\xi_\pm=Ms^2(d^2\xi_\pm/dz^2)+\bar{z}eE_\pm\pm(\bar{z}e\omega B/c)\xi_\pm+(\bar{z}m/\tau)(\langle v\rangle_\pm-u_\pm), \qquad (15')$$

where we have included a term on the right side of Eq. (15′) to take into account the Lorentz force on the ions due to the magnetic field. The Fourier transform of Eq. (15′) is

$$\{q^2-(\omega l/s)^2[1\pm\Omega_c/\omega-i\gamma(1-\tfrac{3}{4}\kappa_\pm(q))]\}\xi_\pm(q)=-2\zeta_\pm+(\bar{z}el^2/Ms^2)(1-\tfrac{3}{4}\kappa_\pm(q))E_\pm(q), \qquad (16')$$

where $\Omega_c=\bar{z}eB/Mc$ in the cyclotron frequency of the ions. At this point we should mention that if $\zeta_\pm$ in Eq. (14′) is set equal to zero, then the solution of Eqs. (14′) and (16′) for $\xi_\pm(q)$ is identical to that obtained by Quinn,[6] and if $\mu_\pm$ in Eq. (16′) is also set equal to zero, then the solution of these equations for $\xi_\pm(q)$ is identical to that obtained by Quinn and Rodriguez.[8]

In order to derive an expression for the ratio η_1 we shall require that the electromagnetic field incident on the metal surface be plane polarized, i.e., $E_y(0)=H_x(0)=0$. The electromagnetic power incident on a unit area of the metal surface is then given by Eq. (20) and the power added to the acoustic wave per unit area of exposed surface is

$$P_{11}=\tfrac{1}{16}l\int_{-\infty}^{\infty}[J_{i+}(x)E_+^*(x)+J_{i-}(x)E_-^*(x)+\text{c.c.}]dx. \qquad (19')$$

Proceeding as before, we find that

$$\eta_1=(i/4\pi)(\bar{z}m/M)(\omega_p^2\omega^3l^5/s^2c^3)(I_{1+}+I_{1-}), \qquad (26')$$

where

$$I_{1\pm}=\int_{-\infty}^{\infty}dq\left\{\frac{(1-\frac{3}{4}\kappa_\pm(q))}{\{q^2-(\omega l/s)^2[1\pm\Omega_c/\omega-i\gamma(1-\frac{3}{4}\kappa_\pm(q))]\}\,|q^2+i\alpha\kappa_\pm(q)|^2}-\text{c.c.}\right\}. \qquad (24')$$

Similarly, in order to define an expression for the ratio η_2 we shall require that the transverse acoustic wave incident on the metal surface be plane polarized, i.e., $\zeta_y=0$. The acoustic flux entering the metal is then given by Eq. (28) and the electromagnetic power transferred across a unit area of the metal surface is

$$P_{21}=(ic/32\pi)(E_+(x)H_+^*(x)-E_-(x)H_-^*(x)-\text{c.c.})|_{x=0}. \qquad (27')$$

Proceeding as before, we find that

$$\eta_2=(2\pi^2)^{-1}(\bar{z}m/M)(l^2/sc)^3\omega_p^2\omega^4[|I_{2x}|^2+|I_{2-}|^2], \qquad (33')$$

where

$$I_{2\pm}=\int_{-\infty}^{\infty}dq\frac{(1-\frac{3}{4}\kappa_\pm(q))}{\{q^2-(\omega l/s)^2[1\pm\Omega_c/\omega-i\gamma(1-\frac{3}{4}\kappa_\pm(q))]\}(q^2+i\alpha\kappa_\pm(q))}. \qquad (32')$$

III. COMPARISON WITH EXPERIMENT

A. No Magnetic Field

In this section we shall specialize the theory developed in Sec. II A, B, and C to the experimental conditions described by Abeles.[4] In this experiment microwaves in a resonant rectangular cavity were incident upon an indium film several thousand angstroms in thickness which composed a portion of one of the walls of the cavity. The indium film had been evaporated on a germanium rod which extended behind the cavity. Thus, microwaves incident on the indium film generated transverse acoustic waves which traveled down the Ge rod, were reflected, and traveled back to the In film. These acoustic waves then generated microwave radiation which was detected in the cavity as an echo occurring at a time after the initial pulse corresponding to the transit time of a transverse sound wave up and down the Ge rod. For the 9.3-GHz microwaves used in this experiment, the insertion loss, defined as the ratio of the amplitude of the echo to the amplitude of the initial pulse, was observed to be 10^{-13}. The parameters appropriate to the In film[4] are $s=1.28\times10^5$ cm/sec, $\bar{z}m/M=0.48\times10^{-5}$, $\omega_p{}^2=1.2\times10^{32}$ sec^{-2}, $\bar{v}=1.21\times10^8$ cm/sec, and $\tau=2\times10^{-3}$ sec. Assigning this value to the relaxation time τ leads to a value of the parameter α which is of order unity; however, the poles of the integrals I_1 and I_2 are not easily obtained when α is of order unity. Hence, we shall examine the regions $\alpha\ll1$ and $\alpha\gg1$ in the expectation, since α is proportional to the cube of the relaxation time, that one of these cases will correctly describe the experimental data.

We shall consider this experiment in two parts. In the first part we discuss the generation of transverse acoustic waves by the microwave radiation incident on the indium film. As a model for this portion of the experiment we shall extend the thickness of the In film to infinity and ignore all the energy-loss mechanisms available to the acoustic wave. In the context of this model the ratio of the acoustic power generated in the In film to the microwave power incident on the film is η_1 given by Eq. (26). The details of the evaluation of the integral I_1 are outlined in note 1 of Appendix B; for the parameters just given, we have $\omega l/s\gg1$, $\gamma\ll1$, and $\beta\gg1$, where

$$\beta=(\omega l/s)(\alpha\pi)^{-1/2}. \tag{34}$$

We find that

$$I_1=-2\pi i(s/\omega l)^5 \tag{35}$$

for both the regions $\alpha\ll1$ and $\alpha\gg1$, and hence, for both these regions we have

$$\eta_1=(\bar{z}m/M)(\omega_p/\omega)^2(s/c)^3. \tag{36}$$

We now consider the second part of this experiment, that is, the generation of microwave radiation by the transverse acoustic waves which return to the In film upon reflection from the far end of the Ge rod. As a model for this portion of the experiment we shall take the In-Ge interface as the boundary surface, i.e., the $z=0$ plane, and extend the thickness of the In film to infinity. In making this approximation we assume that the microwave power emitted at the In-Ge interface is the same as the power emitted by the film into the cavity. In the context of this model, the ratio of the microwave power entering the cavity to the acoustic power entering the metal is η_2 given by Eq. (33). The details of the evaluation of the integral I_2 are outlined in note 2 of Appendix B; we find that

$$I_2=-\pi(s/\omega l)^3\times i,\quad \alpha\ll1$$
$$\times\tfrac{2}{3}\beta(1-i/\sqrt{3}),\quad \alpha\gg1 \tag{37}$$

and hence

$$\eta_2=\left(\frac{\bar{z}m}{M}\right)\left(\frac{\omega_p}{\omega}\right)^2\left(\frac{s}{c}\right)^3\times1,\quad \alpha\ll1$$
$$\times16\beta^2/27,\quad \alpha\gg1. \tag{38}$$

If we assume that the attenuation of the transverse acoustic wave in traveling up and down the Ge rod is negligible compared to the power reductions involved in transferring energy between the microwave field and the acoustic wave, then the ratio of the electromagnetic power leaving the film to the electromagnetic power incident on the film is $\eta_1\eta_2$. Thus, from Eqs. (36) and (38) we obtain, for the parameters given above,

$$\eta_1\eta_2=1.8\times10^{-22}\times1,\quad \alpha\ll1$$
$$\times43,\quad \alpha\gg1.$$

Even when allowance is made for the enhancement factor[4] of the cavity, this numerical result is still several orders of magnitude smaller than the insertion loss of 10^{-13} reported by Abeles.

B. Magnetic Field

In this section we shall specialize the extension of the theory to include an external magnetic field which was outlined in the Sec. II D, to the experimental conditions described by Houck, Bohm, Maxfield, and Wilkins.[3] In this experiment a radio-frequency field generated in a coil is independent upon an aluminum film several millimeters in thickness which is coupled acoustically to a quartz delay rod. The transverse acoustic waves generated in the metal film were detected by a piezoelectric transducer attached to the opposite end of the quartz delay rod after a time corresponding to the transit time of a shear acoustic wave down the delay rod. An external magnetic field was applied normally to the metal surface; the magnitude of this field was such that damping of the helicon by the mechanism of Doppler-shifted cyclotron resonance absorption takes place while damping of the acoustic wave by this mechanism does not occur. It is then observed that as the magnitude of the magnetic field is varied within the limitations just mentioned the power transmitted in the acoustic wave is proportional to the square of the magnetic field strength. It is also pointed out that the converse effect was observed, i.e., transverse acoustic waves generated by the transducer induced a voltage in the coil. The parameters appropriate to the Al sample[3] are $s=3.4\times10^5$ cm/sec, $\bar{z}m/M=6\times10^{-5}$, $\omega_p^2=5.6\times10^{32}$ sec^{-2}, $\omega=2\pi\times10^7$ sec^{-1}, $\omega_c(10\text{ kG})=1.8\times10^{11}$ sec^{-1}, $\bar{v}=2\times10^8$ cm/sec, and $\omega_c\tau\gg1$. These parameters imply that $\alpha\gg1$ and $\gamma\ll1$.

We shall first consider the generation of transverse acoustic waves by an rf field in the presence of an external magnetic field. If we imagine the thickness of the Al film to be extended to infinity and ignore all the energy-loss mechanisms available to the acoustic wave, then the ratio of the acoustic power generated in the Al sample to the rf power incident on the sample surface is η_1 given by Eq. (26′). The details of the evaluation of the integrals $I_{1\pm}$ are outlined in note 3 of Appendix B; for the parameters just given, we have $\Omega_c\ll\omega$, $\omega l/s\ll(\alpha\pi)^{1/3}$ and $(\alpha/\omega_c\tau)^{1/2}$, $\omega l/s<\omega_c\tau$, as is required for damping of the transverse sound wave via the mechanism of Doppler-shifted cyclotron resonance absorption to be absent, and $(\alpha\pi)^{1/3}>\omega_c\tau$, as is required for damping of the electromagnetic mode by this mechanism to be present. We find that

$$I_{1\pm}=-2\pi i(s/\omega l)(3\omega_c\tau/4\alpha)^2\,, \tag{39}$$

and hence,

$$\eta_1=(\bar{z}m/M)(c/s)(\omega_c/\omega_p)^2. \tag{40}$$

At this point it should be pointed out that if the magnetic strength is increased to well above the helicon absorption edge so that $(\alpha\pi)^{1/3}\ll\omega_c\tau$, then, as is discussed in note 3 of Appendix B, the integral $I_{1\pm}$ is unchanged from the result given in Eq. (39). Thus, the theory predicts that acoustic power generated by rf power incident on a metal surface in the presence of a magnetic field will be proportional to the square of the magnetic field strength in the region well above the helicon absorption edge as well as in the region below the absorption edge; it has been reported[2] that the acoustic power generated in Nb and Sn crystals was proportional to the square of the magnetic field intensity in magnetic fields up to 110 kG. Equation (40) does not contain the relaxation time τ and hence we conclude that the generated acoustic signal intensity should not be a function of temperature; it is reported[2] that the signal amplitude was not strongly temperature dependent.

Houck *et al.*[3] failed to observe an ultrasonic signal for magnetic fields of less than 5 kG. This field is near the acoustic absorption edge and hence one would expect the acoustic wave to be damped by the mechanism of Doppler-shifted cyclotron resonance absorption since the sample thickness is much larger than the acoustic wavelength. Although damping of the acoustic wave by this mechanism will certainly occur for the single-crystal Al samples described in Ref. 3 when the magnetic field is reduced below the acoustic absorption edge, damping by this mechanism will not be significant for the polycrystalline aluminum-foil sample which was also found to generate transverse acoustic waves. With reference to this latter sample we may let the magnetic field strength go to zero and calculate the ratio η_1 by use of Eq. (26); for this situation, noting that $\omega l/s\gg1$ and $\beta\ll1$, the calculation of the integral I_1 proceeds in a straightforward manner, and we find that

$$I_1=-2\pi i(\omega l/s)(\alpha\pi)^{-2} \tag{41}$$

and

$$\eta_1=(1/\pi^2)(\bar{z}m/M)(\omega/\omega_p)^2(\bar{v}/s)^2(c/s)\,. \tag{42}$$

Thus, the predicted ratio of the acoustic power generation for the aluminum-foil sample in the presence of a magnetic field the magnitude of which lies somewhat below the helicon absorption edge to the power generation in the absence of the magnetic field, that is, Eq. (40) divided by Eq. (42), is $(\pi\omega_c s/\omega\bar{v})^2$, and this quantity takes the value 200 for a magnetic field of 10 kG.

Finally, we consider the converse effect for magnetic fields in the region somewhat below and well above the helicon absorption edge. We shall assume a model for this discussion which is identical to that used in the discussion of the direct generation of microwaves by transverse acoustic waves given in Sec. III A. The ratio of the rf power radiated by the sample to the acoustic power entering the sample is η_2 given by Eq. (33′). The integral $I_{2\pm}$ can be shown to be

$$I_{2\pm}=\pm i\pi(s/\omega l)(3\omega_c\tau/4\alpha)\,; \tag{43}$$

thus,

$$\eta_2=(\bar{z}m/M)(c/s)(\omega_c/\omega_p)^2. \tag{44}$$

IV. DISCUSSION

If we assume, as we have done, that the electrons are reflected specularly at the surface of the metal, then we can obtain Eqs. (14) and (16) in a manner which is somewhat simpler than that which we have used. When, in a semi-infinite metal with its surface in the $z=0$ plane, an electron is scattered specularly from this surface, it follows a trajectory which is the mirror image of the trajectory it would have followed if it had been allowed to pass through the metal surface into the other semi-infinite space.[9] Hence, we can extend the model we have presently adopted, consisting of a semi-infinite metal, to one consisting of an infinite metal by considering the other semi-infinite space to be filled with another piece of the same metal and defining the fields in this half-space in the manner given by Eq. (6). However, in order to treat the problem in this context one must introduce a current-density sheet in the $z=0$ plane in order to produce the correct boundary conditions on the electric field in the metal, i.e., a term of the form $2(dE(z)/dz)|_{z=0}\delta(z)$ must be added to the right side of Eq. (2). Similarly, in order to produce the correct boundary conditions on the ionic displacement field in the

[9] D. C. Mattis and G. Dresselhaus, Phys. Rev. 111, 403 (1958)

metal, one must introduce a shearing stress must be introduced in the $z=0$ plane; this is accomplished by adding a term of the form $-2(d\xi(z)/dz)|_{z=0}\delta(z)$ to the right side of Eq. (15). By taking the Fourier transforms of these modified forms of Eqs. (2) and (15), which are assumed to be valid for an infinite metal, one obtains Eqs. (14) and (16) directly.

In the Sec. III we found that the predictions of the theory developed in this paper are in substantial agreement with the experimental observations of Houck *et al.*[3] on aluminum using rf fields and an applied magnetic field. The predicted insertion loss for microwaves in the absence of a magnetic field was found, however, to be several orders of magnitude less than that observed by Abeles[4] in indium. In the theory which we have presented to describe the direct generation of transverse acoustic waves by electromagnetic radiation in metals and the converse effect, as is made clear in the preceding paragraph, the only forces acting on the ions are those which act on the ions in the bulk metal or are applied externally. Therefore, while the predictions of the theory presented here are compatible with the experimental data which have recently been reported, it seems apparent that other forces on the ions in the metal must be present which dominate those present in the bulk metal in the microwave range. In this connection we should point out that there exist other theories[4,10,11] in which a force on the ions resulting from diffuse electron scattering at the surface of the metal as introduced. Some of these authors' present calculations which indicate that for microwave frequencies the power transferred to the transverse acoustic wave from the incident electromagnetic wave due to electron scattering at the surface of the metal may be equal to or considerably larger than the power transferred to the acoustic wave due to the forces acting on the ions in the bulk metal. Finally, it should be mentioned that the theory given in this paper ignores any dependence which the amplitude of the transverse acoustic waves generated directly in a metal by electromagnetic radiation might have on the sample thickness. Although this assumption is supported by the observations reported in Ref. 3, insertion losses which decrease from 10^{-13} to 10^{-15} as the sample thickness is increased from 2000 to 10 000 Å were obtained in several recent measurements by Weisbarth[12] on In films using an experimental configuration similar to that used by Abeles.

The magnitude of the coupling between the electric field and the ionic displacement field in a metal is largely determined by the factor $1-\frac{3}{4}\kappa(q)$ which appears in Eqs. (14) and (16) and which can be interpreted as a measure of the nonlocal nature of the qth Fourier component of the electronic current-field relation in a metal. That is, in order that the generated field not be screened out by the electrons, it is necessary that a nonlocal electronic current-field relation exist for the modes at which coupling between the fields takes place. Since coupling may occur via the acoustic mode where $q=\omega l/s$ or via the electromagnetic mode where $q=l/\delta$, we must have either $l\gg\lambda_s/2\pi$, where $\lambda_s=2\pi s/\omega$ is the wavelength of the sound wave, or $l\gg\delta$, or both, for effective coupling between the fields to occur. The former inequality is seen to hold for both the experimental configurations which we have discussed; the latter inequality is simply the definition of the anomalous limit, i.e., $\alpha\gg1$.

If one assumes that the dominant poles of the integrals I_1 and I_2 correspond to the acoustic mode with $l\gg\lambda_s/2\pi$, that is, that all coupling between the ions and the electric field occurs at wavelengths corresponding to the acoustic wave as is the case for the experimental configuration of Houck *et al.*[3] and for that of Abeles[4] in the classical limit, then one finds from Eqs. (26) and (33) that η_1 equals η_2. Thus, if we define the efficiency of power generation in a metal to be the ratio of the power generated to the power incident on the metal, we are led to conclude that, under the assumptions just mentioned, the efficiency of generation of acoustic power from electromagnetic radiation is identical to the efficiency of generation of electromagnetic power from transverse acoustic waves.

In the absence of a magnetic field the ratio of the acoustic power generated in a metal to the electromagnetic power incident on the metal surface is given by Eq. (36) in the microwave region and by Eq. (24) in the rf region. Although these results were obtained for different metals, the only significant difference in the parameters used to derive these two equations is that β, given by Eq. (34) is much greater than unity in the former case and much less than unity in the latter. In terms of the ineffectiveness concept introduced by Pippard[13,14] in connection with the anomalous skin effect in metals one may define, in the extreme anomalous skin effect in metals one may define, in the extreme anomalous region, i.e., when $\alpha\gg1$, an effective skin depth $\delta_{\text{eff}}=(\delta^2 l\Lambda^{-1})^{1/3}$, where Λ is a constant of order unity; thus, the parameter β is found to be proportional to $2\pi\delta_{\text{eff}}/\lambda_s$, the constant of proportionality being of order unity. We observe that in the microwave region, where $\beta\gg1$, we have η_1 proportional to ω^{-2}; in the rf region, where $\beta\ll1$, we have η_1 proportional to ω^2. In the region where β is of order unity, we find η_1 to be of the order $(m/M)(\bar{v}/c)$.

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APPENDIX A

Using the physical model described in Sec. II A we shall develop here the relationship between the x component of the electronic current-density vector $J_e(z)$ and the x components of the electric field $E(z)$ and the ionic displacement $\xi(z)$. The time dependence of these quantities, as well as all other quantities, is given by $e^{i\omega t}$. This relationship will be established by use of the electron distribution

$$f(\mathbf{v},z)=f_0(\mathbf{v})+f_1(\mathbf{v},z), \tag{A1}$$

where z is the z component of the electron position vector and $\mathbf{v}$ is the electron velocity vector. The distribution $\bar{f}_0$ is not the Fermi equilibrium distribution f_0 but rather a local equilibrium distribution.[15] It differs from f_0 in that the electrons scatter into an equilibrium distribution centered about the scattering center, i.e., the distribution $\bar{f}_0$ is centered in a coordinate system moving with the velocity $u=i\omega\xi(z)$ of the lattice. Since the ionic velocity is much smaller than the electronic velocity, we may expand $\bar{f}_0(\mathbf{v})$ about the Fermi distribution to obtain

$$\bar{f}_0(\mathbf{v})=f_0(\mathbf{v})-u(\partial f_0/\partial v_x). \tag{A2}$$

The electron distribution obeys the Boltzmann equation

$$\frac{\partial f}{\partial t}-\frac{e}{m}E(z)\frac{\partial f_0}{\partial v_x}+v_z\frac{\partial f}{\partial z}=-\frac{(f-\bar{f}_0)}{\tau}, \tag{A3}$$

where $-e$ is the electronic charge and τ is an average relaxation time for all the electrons on the Fermi surface. Neglecting terms quadratic in f_1 and $E(z)$, Eq. (A3) becomes

$$\frac{\partial f_1}{\partial z}+\frac{(1+i\omega\tau)}{v_z\tau}f_1=\frac{e}{mv_z}\frac{\partial f_0}{\partial v_x}\left[E(z)-\left(\frac{im\omega}{e\tau}\right)\xi(z)\right]. \tag{A4}$$

This equation is seen to be identical with Eq. (5) of Ref. 7, if one defines the bracketed quantity in Eq. (A4) to be $\mathcal{E}(z)$. If one proceeds in a manner analogous to that used in Ref. 7, assumes that the electrons are reflected specularly at the metal surface, and extends the electric field and ionic displacement field along the negative z axis according to the relations $E(-z)=E(z)$ and $\xi(-z)=\xi(z)$, one obtains Eqs. (4) and (5) of the text.

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APPENDIX B

Note 1. When the two terms of the integral I_1 are combined, one obtains a single integral with poles at $q=\pm(\omega l/s)(1-i\gamma/2)$ and $q=\pm(\omega l/s)(1+i\gamma/2)$, where $\kappa(q)=\pi/|q|$. If $\alpha\ll1$, there will be additional poles at $q=\pm(l/\delta)(1+i)$, where $\kappa(q)=\frac{4}{3}(1-\frac{1}{5}q^2)$; if $\alpha\gg1$, there will be additional poles at $q=(\alpha\pi)^{1/3}e^{in\pi/6}$ with $n=1$, 3, 5, 7, 9, and 11, where $\kappa(q)=\pi/|q|$. One then obtains the result $I_1=-2\pi i(s/\omega l)^5$ when the contour integration is carried out for the region $\alpha\ll1$ and one neglects $(l\gamma/50\delta)\times(\omega l/s)$ with respect to unity; an identical result is obtained when the contour integration is carried out for the region $\alpha\gg1$ and one neglects $\frac{1}{3}\beta^3\gamma$ with respect to unity.

Note 2. (a) $\alpha\ll1$. The integral I_2 will have poles at $q=\pm(\omega l/s)(1-i\gamma/2)$ with $\kappa(q)=\pi/|q|$ and at $q=\pm(l/\delta)(1-i)$ with $\kappa(q)=\frac{4}{3}(1-\frac{1}{5}q^2)$. One obtains the result $I_2=-i(s/\omega l)^3$ when the contour integration is carried out and $\omega l^2/5s\delta$ is neglected with respect to unity. (b) $\alpha\gg1$. Since $|q|\gg1$ for all the poles of I_2 in this regime, we can set $\kappa(q)=\pi/|q|$. One finds that

$$I_2=-2\beta\left(\frac{s}{\omega l}\right)^3\left[\int_0^\infty(y^2+i)^{-1}ydy+O(\beta^{-1})\right],$$

where $O(\beta^{-1})$ means terms of order β^{-1}. Hence, we find $I_2=-(\frac{2}{3}\pi\beta)(s/\omega l)^3(1-i/\sqrt{3})$ in the limit $\beta\gg1$.

Note 3. When the two terms of $I_{1\pm}$ are combined, one obtains a single integral with poles at $q=\pm(\omega l/s)\times(1-i\gamma/2)$ and $q=\pm(\omega l/s)(1+i\gamma/2)$ with $\kappa_\pm(q)=\frac{4}{3}(1\mp i\omega_c\tau)^{-1}$ and at $q=(\alpha\pi)^{1/3}e^{in\pi/6}$ with $n=1$, 3, 5, 7, 9, and 11 where $\kappa(q)=\pi/|q|$. One then obtains $I_{1\pm}=-2\pi i(s/\omega l)(3\omega_c\tau/4\alpha)^2$ when the contour integration is carried out and one neglects $(\gamma\omega l/s)(\beta/\omega_c\tau)^2$ with respect to unity. If the magnitude of the magnetic field is increased so that $(\alpha\pi)^{1/3}\ll\omega_c\tau$, the integral I_{1+} will have poles at $q=\pm(4\alpha/3\omega_c\tau)^{1/2}[1+i(2\omega_c\tau)^{-1}]$ and at q^* while the integral I_{1-} will have poles at $q=\pm(4\alpha/3\omega_c\tau)^{1/2}[(2\omega_c\tau)^{-1}+i]$ and at q^*. Both integrals will have poles at $q=\pm(\omega l/s)(1-i\gamma/2)$ and at $q=\pm(\omega l/s)(1+i\gamma/2)$ and we have $\kappa_\pm(q)=\frac{4}{3}(1\mp i\omega_c\tau)^{-1}$ for all these poles. When the contour integration is performed, the result is identical to that given above.