Hall Effect of a Compensated Magnetic Metal Proportional to MB^2 in the High-Field Limit*

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(Received 10 May 1968)

The Smit asymmetric scattering on magnetic impurities is introduced into the spherical two-band model of a compensated metal. Hall resistivity ρ_H, ohmic resistivity ρ, and Hall angle ϕ_H are calculated. In the high-field limit $\omega_c\tau\gg1$, we find $\rho_H\propto MB^2/c$, $\rho\propto B^2/c$, $\tan\phi_H\propto M$, where M is the magnetization and c the impurity concentration. The more usual $\rho_H=R_0B+R_sM$ is found to hold only for low fields or uncompensated metals.

THE Hall resistivity ρ_H of a magnetic polycrystal is defined by

$$\rho_H=\tfrac{1}{2}[\rho_{yx}(B,M)-\rho_{yx}(-B,-M)];\quad \rho_{yx}=E_y/j_x. \tag{1}$$

The current density $\mathbf{j}$ is parallel to the x axis. The magnetization $\mathbf{M}$ and induction $\mathbf{B}$ are parallel to the z axis. Usually, ρ_H may be written[1]

$$\rho_H=R_0B+R_sM \qquad \text{(mks units)}. \tag{2}$$

As shown by various authors,[2] the Boltzmann equation is correct only to lowest order in the dimensionless parameter $\hbar/\tau E_f$, where τ is the electron relaxation time, and E_f the Fermi energy. For example, nonclassical second-order contributions to R_s are very important in iron at room temperature or above (small τ) and a visualization of the phenomena becomes difficult. On the other hand, the Boltzmann equation should be quite adequate in the case of very dilute alloys at low temperature; in that case, R_s is first order in $\hbar/\tau E_f$ and is caused by "asymmetric scattering" on impurities, as introduced first by Smit[3]: Because of a left-right asymmetry in differential cross section, electrons tend to pile up on one side of the sample. This simple mechanism is somewhat similar to the Mott scattering of polarized free electrons on free atoms, and is a consequence of spin-orbit interaction; usually R_s and R_0 are then predicted[2–4] to obey

$$R_s\propto c, \qquad dR_0/dc=0, \tag{3}$$

where c is the impurity concentration.

The purpose of the present work is to investigate the consequences of Smit asymmetric scattering at arbitrary values of $\omega_c\tau$, where ω_c is the cyclotron frequency. We will find $\rho_H\propto MB^2/c$ for a compensated magnetic metal in the high-field limit $\omega_c\tau\gg1$; hence Eqs. (2) and (3) fail completely in that case. We will also find unusual linear magnetoresistance terms in the low-field limit.

The simplest model of a compensated metal consists of two spherical bands, containing, respectively, n_e electrons and $n_h=n_e$ holes per unit volume. If $\mathbf{p}_e$ and $\mathbf{p}_h$ are the total momenta of the electron and hole bands, per unit volume, we have in the stationary state

$$\begin{aligned}0&=d\mathbf{p}_e/dt=e_en_e\mathbf{E}+\mathbf{j}_e\times\mathbf{B}-(m_e/e_e)[s^e]\mathbf{j}_e,\\ 0&=d\mathbf{p}_h/dt=e_hn_h\mathbf{E}+\mathbf{j}_h\times\mathbf{B}-(m_h/e_h)[s^h]\mathbf{j}_h,\\ \mathbf{j}&=\mathbf{j}_e+\mathbf{j}_h, \qquad e_e=-e_h, \qquad e_e<0.\end{aligned} \tag{4}$$

The collision terms contain the linear operators $[s^e]$ and $[s^h]$, having the dimension of a relaxation frequency. Since $\mathbf{M}\parallel z$, the off-diagonal elements $s_{xy}{}^e=-s_{yx}{}^e$ and $s_{xy}{}^h=-s_{yx}{}^h$ describe the asymmetric scattering and are proportional to M. They determine how much momentum along y is gained on the average by an electron traveling along the x axis and colliding with an impurity. Of course, the diagonal elements $s_{xx}{}^e=s_{yy}{}^e>0$ and $s_{xx}{}^h=s_{yy}{}^h>0$ are responsible for most of the ohmic resistance ρ, and exist even in a nonmagnetic metal. All elements of $[s^e]$ and $[s^h]$ are proportional to c.

The boundary condition $j_y=0$, added to the Eqs. (1) and (4), gives

$$\tan\phi_H=E_y/E_x=-\left[u_e\left(\frac{S_{xy}{}^e-B}{S_{xx}{}^e}\right)+u_h\left(\frac{S_{xy}{}^h-B}{S_{xx}{}^h}\right)\Big/(u_e+u_h)\right], \tag{5}$$

$$\rho=E_x/j_x=[(u_e+u_h)(1+\tan^2\phi_H)]^{-1}, \tag{6}$$

$$\rho_H=E_y/j_x=\tan\phi_H/(u_e+u_h)(1+\tan^2\phi_H), \tag{7}$$

where the tensors $[S^e]=m_e[s^e]/e_e$ and $[S^h]=m_h[s^h]/e_h$ have the dimension of a magnetic induction (reciprocal of a mobility), and where the field-dependent conductivities u_e and u_h are defined by

$$u_e=\frac{e_en_e/S_{xx}{}^e}{1+[(S_{xy}{}^e-B)/S_{xx}{}^e]^2}, \tag{8}$$

$$u_h=\frac{e_hn_h/S_{xx}{}^h}{1+[(S_{xy}{}^h-B)/S_{xx}{}^h]^2}. \tag{9}$$

In the low-field limit $|\omega_e\tau_e|=|B/S_{xx}{}^e|\ll1$, $|\omega_h\tau_h|=$

* Work supported by the U.S. Army Research Office (Durham).

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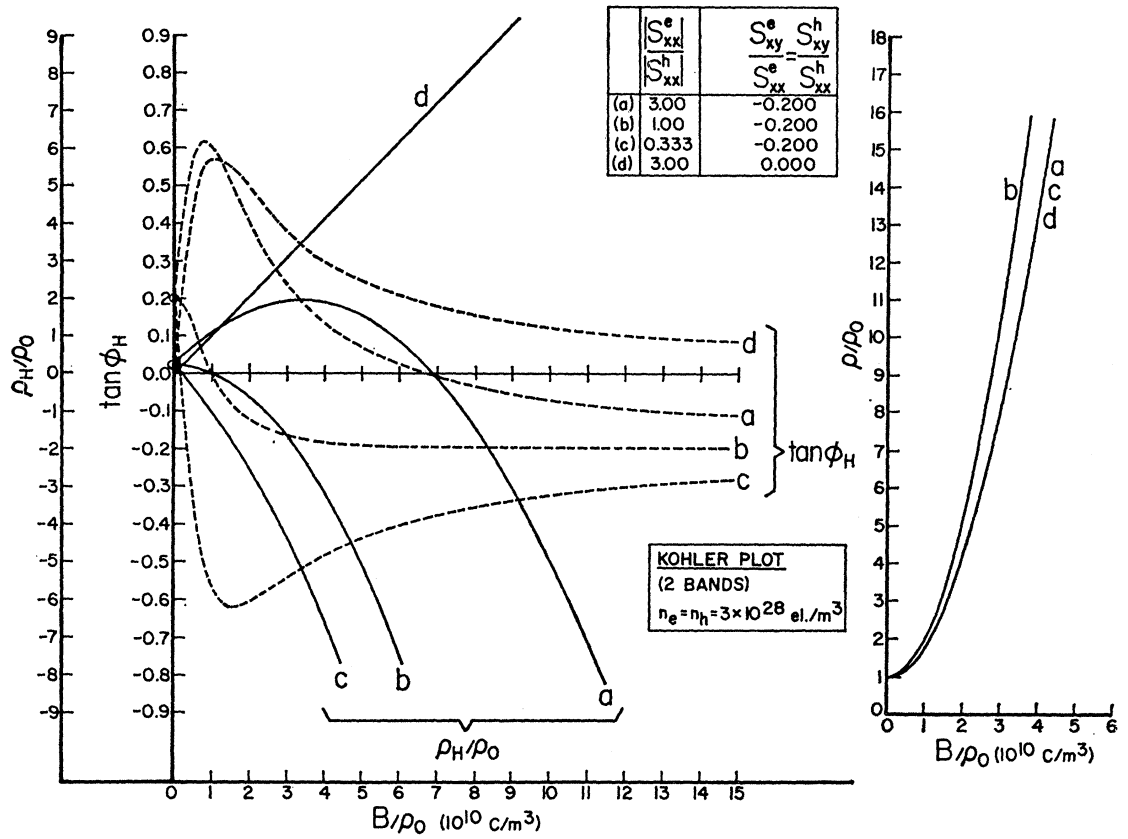

FIG. 1. Kohler functions of a compensated magnetic metal, for a given value of M, according to the spherical two-band model. Curves a, b, c are for three different values of the mobility ratio of electrons and holes, in the presence of asymmetric scattering ($S_{xy}^e/S_{xx}^e = S_{xy}^h/S_{xx}^h = -0.200$). Curve d is for the same mobility ratio as a, but without asymmetric scattering ($S_{xy}^e/S_{xx}^e = S_{xy}^h/S_{xx}^h = 0.000$). The curves d for ρ_H/ρ and for $\tan\phi_H$ are the only curves on the drawing which go through the origin.

$|B/S_{xx}^h| \ll 1$, Eqs. (5)–(7) become, if we assume $|S_{xy}^e/S_{xx}^e| \ll 1$ and $|S_{xy}^h/S_{xx}^h| \ll 1$,

$$\tan\phi_H \approx -\left(\frac{\sigma_e}{\sigma_e+\sigma_h}\right)\left(\frac{S_{xy}^e - B}{S_{xx}^e}\right) - \left(\frac{\sigma_h}{\sigma_e+\sigma_h}\right)\left(\frac{S_{xy}^h - B}{S_{xx}^h}\right), \quad (10)$$

$$\rho \approx (\sigma_e+\sigma_h)^{-1}, \qquad \sigma_e = \frac{n_e e_e}{S_{xx}^e} > 0, \qquad \sigma_h = \frac{n_h e_h}{S_{xx}^h} > 0, \quad (11)$$

$$\rho_H \approx -\left(\frac{\sigma_e}{\sigma_e+\sigma_h}\right)^2\left(\frac{S_{xy}^e - B}{n_e e_e}\right) - \left(\frac{\sigma_h}{\sigma_e+\sigma_h}\right)^2\left(\frac{S_{xy}^h - B}{n_h e_h}\right). \quad (12)$$

Equations (5)–(12) are actually valid even if $n_e \neq n_h$.

Note that Eq. (12) is compatible with Eqs. (2) and (3). A better approximation would add to the right side of Eq. (11) small (positive or negative) linear magnetoresistance terms proportional to $S_{xy}^e B$, and $S_{xy}^h B$ for transverse fields, and also the more usual positive terms in B^2.

In the high-field limit $|B/S_{xx}^e| \gg 1$, $|B/S_{xx}^h| \gg 1$, Eqs. (5)–(7) become, if the metal is compensated ($n_e = n_h$),

$$\tan\phi_H = \left(\frac{|S_{xx}^e|}{|S_{xx}^e| + |S_{xx}^h|}\right)\left(\frac{S_{xy}^e}{S_{xx}^e}\right) + \left(\frac{|S_{xx}^h|}{|S_{xx}^e| + |S_{xx}^h|}\right)\left(\frac{S_{xy}^h}{S_{xx}^h}\right), \quad (13)$$

$$\rho = \frac{B^2}{n_e|e_e|\,(|S_{xx}^e| + |S_{xx}^h|)(1+\tan^2\phi_H)}, \quad (14)$$

$$\rho_H = \frac{B^2\tan\phi_H}{n_e|e_e|\,(|S_{xx}^e| + |S_{xx}^h|)(1+\tan^2\phi_H)}. \quad (15)$$

Since all elements of $[S^e]$ and $[S^h]$ are proportional to the impurity concentration c, and since S_{xy}^e and S_{xy}^h are proportional to M, the Eqs. (13)–(15) imply for a compensated magnetic metal in the high-field limit

$$\tan\phi_H \propto M^1 B^0 c^0, \quad (16)$$

$$\rho \propto M^0 B^2/c, \quad (17)$$

$$\rho_H \propto M^1 B^2/c. \quad (18)$$

In Eq. (18), the expression MB^2 is odd with respect to simultaneous M and B reversal, as required by Eq. (1). Because of the oddness, it cannot be confused with the transverse-even voltage $\propto M^0B^2/c$, which exists in every compensated single crystal[5,6] with nonspherical bands. Because of c in the denominator, the ρ_H of Eq. (18) can be observed only in very pure materials. When $B\to\infty$ or $c\to 0$, any additional term of the form R_0B becomes negligible as compared to Eq. (18). Note that ϕ_H is independent of both field and impurity concentration, at high fields. In the special cases $S_{xx}^e=-S_{xx}^h$ or $S_{xy}^e/S_{xx}^e=S_{xy}^h/S_{xx}^h$, then the high-field [Eq. (13)] and low-field values [Eq. (10)] of ϕ_H have opposite sign and equal magnitude.

For a constant value of S_{xy}^e/S_{xx}^e and S_{xy}^h/S_{xx}^h (and, therefore, a constant value of M), the general equations (5)–(7) imply the following "Kohler's rules"[7]:

$$\tan\phi_H=f(B/\rho_0), \tag{19}$$

$$\rho/\rho_0=g(B/\rho_0), \tag{20}$$

$$\rho_H/\rho_0=h(B/\rho_0), \tag{21}$$

where ρ_0 is the resistivity at $B=0$, given by Eq. (11).

Examples of Kohler functions of a compensated magnetic metal are shown on Fig. 1 for three different values of the ratio of mobilities of electrons and holes. Note the finite asymptotic value -0.2 of $\tan\phi_H$ and the parabolic behavior of ρ_H and of ρ at high fields, in agreement with Eqs. (16)–(18). This contrasts with the vanishing $\tan\phi_H$ value and linear ρ_H obtained in the absence of asymmetric scattering (curve d).

If the metal is not compensated ($n_e\neq n_h$), it can be shown from Eq. (7) that Eq. (18) is replaced in the high-field limit by an equation similar to Eq. (2):

$$\rho_H=R_0B+r\,\mathrm{sign}(B)+R_sM, \tag{22}$$

where the constant r may exist even for a nonmagnetic metal.

In the case of a saturated ferromagnet, the last two terms would be difficult to differentiate experimentally, especially since Kohler's rule implies that both constants r and R_s are proportional to c. This difficulty is present in the interpretation of existing ρ_H measurements in dilute nickel[4] alloys. Note also that the high-field and low-field values of R_0 and R_s are usually different.

The present theory may be generalized to a model with an arbitrary number of spherical Fermi surfaces of holes and of electrons, labelled by the index i. Then Eq. (5) becomes, for example,

$$\tan\phi_H=-\frac{\sum_i u_i[(S_{xy}^i-B)/S_{xx}^i]}{\sum_i u_i}, \tag{23}$$

and Eqs. (16)–(18) still hold at high fields if the metal is compensated ($\sum_i e_in_i=0$). The behavior in low fields and at $\omega_c\tau\approx 1$ is rather complicated.

Since the present theory assumes spherical Fermi surfaces, it can be applied to real metals only for **B** directions without open orbits (or to polycrystals, where open orbits usually exist in only a few grains).

Among the ferromagnetic transition metals, only iron is compensated.[8] The Kohler function for the Hall resistivity of iron at 4.2°K has been investigated by Berger and Majumdar,[9] who reported considerable curvature at $\omega_c\tau\approx 1$. However, our most recent measurements up to 70 kG indicate that the curvature decreases considerably at higher $\omega_c\tau$ values, and give an upper limit of 0.03 for the asymptotic value of $|\tan\phi_H|$. The spin-orbit parameter of ferrous metals is rather small, and Smit[3] estimates theoretically that $|\tan\phi_H|\approx 10^{-2}$ only, for nickel alloys in the low-field limit. This can be taken as a rough prediction of S_{xy}^e/S_{xx}^e in similar materials [see Eq. (10)]. More suitable materials with higher spin-orbit parameter are probably the ferromagnetic rare-earth metals, or very dilute rare-earth impurities (with local moment) dissolved in a compensated nonmagnetic metal and magnetized by an external field.

Asymmetric scattering involves in a selective manner scattering states located at the Fermi level and, therefore, constitutes a tool to study the perturbation of these states by spin-orbit interaction. A similar tool is provided by measurements of the ferromagnetic anisotropy of resistivity.[10]

We would like to thank R. Huguenin, A. Ehrlich, and S. Soffer for discussions on these topics.

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