Vortex Fluctuations in Superconducting Thin-Film Bridges*

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The convection of the self-field vortices into the interior of a superconducting thin-film bridge is shown to generate a shot-noise voltage as well as a steady-state voltage. A basic relation is developed between the time correlation function for fluctuations in bridge voltage and the space correlation functions for the vortex number densities. Vortex motion is treated by a rudimentary plasma-dynamic model, and the results are used to derive a practical expression for the spectral intensity of the noise voltage.

I. INTRODUCTION

WHEN a thin-film superconductor enters the mixed state[1] between the pure superconducting state and the pure normal state, it is an experimental fact that a voltage drop develops in the direction of transport current flow.[2] This behavior is quite different from ohmic resistance. It is now generally attributed to the following mechanism: Magnetic flux enters the film in the form of quantized vortices of superelectrons. The circulating supercurrent of a vortex produces a magnetic field along the vortex axis. In the lowest energy state each vortex has a circulation corresponding to a flux quantum $\phi_0 = h/2e$. The vortex is believed to have a normal core on the order of the coherence length in extent, surrounded by a much larger whirl of supercurrent extending radially out a distance on the order of the penetration depth.[3]

If the vortices are pinned by lattice imperfections so they are unable to move, the transport current will encounter virtually no resistance. As the transport current exceeds a certain threshold the vortices move over the pinning sites and develop a motional electric field. The motion is subject to viscous drag and is therefore dissipative.

According to Hunt,[4] it is likely that the initial breakdown of the superconducting state with increasing current occurs as the result of this dissipative motion of the vortices. Moreover, Anderson[5] has suggested that vortex pinning is not completely rigid even for currents below the threshold. Because of thermal activation the vortices move slowly over the pinning sites at finite temperatures.

The rationale behind the present investigation of noise is this: If the weak link of a thin-film quantum interferometer is wide enough to contain a number of vortices, then its operation is intrinsically noisy; that is, the very mechanism which transports flux into and out of the interferometer loop generates voltage fluctuations. These fluctuations are in the nature of shot noise due to the quantized character of the moving vortices.

There are also voltage fluctuations from the thermal motion of the normal electrons, but at the low frequencies of present interest this noise is effectively shorted out by the inertial reactance of the superelectrons.

The following is a heuristic treatment of voltage fluctuations due to vortex motion across a thin-film bridge in the absence of an applied field. The approach is essentially that used in initial-value treatments of noise.[6] Effects having to do with the detailed structure of the vortex are not considered. The vortices are tacitly assumed to be adequately described by Boltzmann statistics, although presumably these excitations are Bosons.

In Sec. II a basic relation is developed between the time autocorrelation function for the voltage fluctuations and the vortex number densities. The notion of a vortex propagator is introduced as a mathematical convenience.

Next, consideration is given to the dynamics of viscous vortex motion. Inertial, Hall, and Magnus effects are neglected.[3] Section III is concluded with an expression for the autocorrelation function.

Section IV considers the spectral intensity of the voltage fluctuations. Finally, the relevance of the analytical results to the meager experimental evidence is discussed.

II. BASIC RELATIONS

The geometrical model of the thin-film bridge to be considered is shown in Fig. 1. A narrow bridge of length l, width w, and thickness d connects two much more extensive portions of the film. A Cartesian coordinate system with unit vectors $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ is centered in the bridge as indicated.

The bridge is supposed to carry a transport current I in the $\mathbf{e}_y$ direction. As the current is increased to a supercritical value, pairs of vortices of opposite circulation are created by the self-field at the edges of the bridge.

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[1] M. Tinkman, Phys. Rev. 129, 2413 (1963); R. D. Parks, J. M. Mochel, and L. V. Surgent, Jr., Phys. Rev. Letters 13, 3312 (1964); R. D. Parks, in *Low Temperature Physics LT9*, edited by J. G. Daunt, D. O. Edwards, F. J. Milford, and M. Yaqub (Plenum Press, Inc., New York, 1965), Part A, p. 34.

[2] I. Giaever, Phys. Rev. Letters 16, 50 (1966); P. R. Solomon, *ibid.* 16, 59 (1956).

[3] P. Nozières and W. F. Vinen, Phil. Mag. 14, 667 (1966); J. Bardeen and M. J. Stephen, Phys. Rev. 140, A1197 (1965).

[4] T. K. Hunt, Phys. Rev. 151, 325 (1966).

[5] P. W. Anderson, Phys. Rev. Letters 9, 309 (1962).

[6] A. van der Ziel, *Fluctuation Phenomena in Semiconductors* (Academic Press Inc., New York, 1959), p. 31.

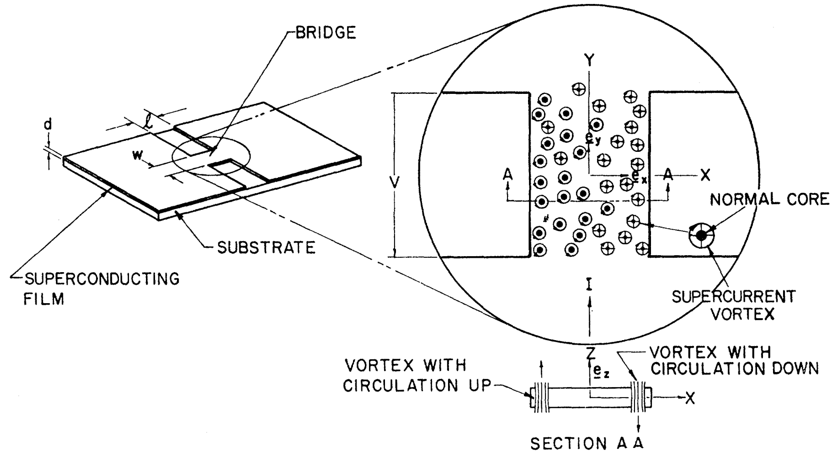

FIG. 1. Thin-film bridge.

The appearance of the vortices tends to produce a more uniform current distribution and to lower the free energy of the bridge. Under the influence of Lorentz forces[3] the vortices are driven inward. Somewhere in the interior these counter-circulating vortices eventually annihilate one another.

For an observer in the frame of the film lattice, the motion of the magnetic field associated with the unpinned vortices gives rise to a motional electric field given by

$$\mathbf{E}\approx -\mathbf{V}^{+}\times\mathbf{B}^{+}-\mathbf{V}^{-}\times\mathbf{B}^{-}. \qquad (1)$$

The drift velocities are $\mathbf{v}^{+}$ for the vortices with circulation up ($+\mathbf{e}_z$ direction) and $\mathbf{v}^{-}$ for those with circulation down ($-\mathbf{e}_z$), as indicated in Fig. 1. The corresponding magnetic flux densities, the averages of the microscopic fields, are $\mathbf{B}^{+}=n^{+}\phi_0\mathbf{e}_z$ and $\mathbf{B}^{-}=-n^{-}\phi_0\mathbf{e}_z$, where n^{+} and n^{-} are the areal number densities of vortices of each circulation.

At the low frequencies of present interest the total electric field virtually vanishes everywhere in the film except within the penetration depth and within the vortices. That is, as a result of the motional electric field a charge distribution appears, the irrotational electric field of which tends to cancel the motional field given by expression (1).

The over-all consequence of vortex flow is then to establish a potential drop along the bridge in the direction of the current. Averaged over the time τ_0 it takes a vortex to drift across the bridge, the potential difference is

$$V=-\frac{1}{\tau_0}\int_{-w/2}^{+w/2}\frac{dx}{|V_x|}\int_{-l/2}^{+l/2}E_y dy. \qquad (2)$$

Recognizing that the magnitude of the drift velocity is independent of the direction of vortex circulation, one finds upon substitution of (1) into expression (2)

$$V=\frac{\phi_0}{\tau_0}\int_{-w/2}^{+w/2}\int_{-l/2}^{+l/2}(n^{+}+n^{-})dxdy. \qquad (3)$$

Result (3) is, then, the basic relation between the bridge voltage drop and the number densities of the vortices. It will be used to obtain the autocorrelation function[7] for fluctuations in the voltage drop along the bridge.

Consider now fluctuations $n_1^{+}(\mathbf{r},t)$ and $n_1^{-}(\mathbf{r},t)$ in the number densities of the unpinned vortices; that is, let $n^{+}(\mathbf{r},t)=n_0^{+}(r)+n_1^{+}(\mathbf{r},t)$ and $n^{-}(\mathbf{r},t)=n_0^{-}(\mathbf{r})+n_1^{-}(\mathbf{r},t)$, where $n_0^{+}(\mathbf{r})$ and $n_0^{-}(\mathbf{r})$ are the steady-state values and $\mathbf{r}\equiv(x,y)$.

Similar resolution of the voltage drop into a steady-state value V_0 and fluctuation $V_1(t)$ leads, according to expression (3), to the autocorrelation function

$$\langle V_1(t)V_1(0)\rangle=\frac{\phi_0^2}{\tau_0^2}\int\int d\mathbf{r}d\mathbf{r}'\langle[n_1^{+}(\mathbf{r},t)+n_1^{-}(\mathbf{r},t)]\times[n_1^{+}(\mathbf{r}',0)+n_1^{-}(\mathbf{r}',0)]\rangle. \qquad (4)$$

The integrals are over the bridge area.

The fluctuations in the vortex number densities are assumed to be describable by linearized phenomenological transport equations of the form

$$L^{+}\left(\frac{\partial}{\partial\mathbf{r}},\frac{\partial}{\partial t}\right)n_1^{+}(\mathbf{r},t)=0$$

for the vortices with circulation up and

$$L^{-}\left(\frac{\partial}{\partial\mathbf{r}},\frac{\partial}{\partial t}\right)n_1^{-}(\mathbf{r},t)=0 \qquad (5)$$

for those of opposite circulation. The linear differential operators L^{+} and L^{-} are discussed later.

Equations (5) are not, of course, the most general linear forms one might assume. Coupling terms could be added as well as a pair of equations for the pinned vortices, to mention only a few possible refinements.

As they stand, Eqs. (5) can be implicitly solved for the number densities by Laplace transformation with re-

[7] A. van der Ziel, *Noise* (Prentice-Hall, Inc., Englewood Cliffs, N. J., 1954), p. 311.

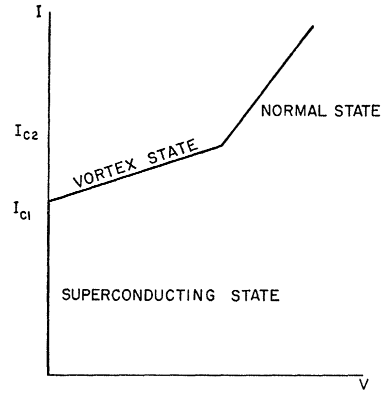

FIG. 2. Schematic I-V characteristic of thin-film bridge.

spect to time and Fourier transformation with respect to space coordinates. The results are

$$n_1^{\pm}(\mathbf{r},t)=\int_{-\infty}^{+\infty} G^{\pm}(\mathbf{r}-\mathbf{r}',t)n_1^{\pm}(\mathbf{r}',0)d\mathbf{r}', \tag{6}$$

in which the functions

$$G^{\pm}(\mathbf{r}-\mathbf{r}',t)\equiv\frac{1}{(2\pi)^2}\mathfrak{L}^{-1}\int_{-\infty}^{+\infty}\frac{e^{-i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}d\mathbf{k}}{L^{\pm}(\mathbf{k},s)} \tag{7}$$

are introduced for mathematical convenience. For brevity, the notation $n^{\pm}$ is used to indicate that the expressions apply to both vortex species.

Employing expression (6), one can rewrite the autocorrelation function (4) as

$$\langle V_1(t)V_1(0)\rangle =\frac{\phi_0^2}{\tau_0^2}\iint_{\text{bridge}} d\mathbf{r}d\mathbf{r}'\int_{-\infty}^{+\infty} d\mathbf{r}''\langle[G^+(\mathbf{r}-\mathbf{r}'',t)n_1^+(\mathbf{r}'',0) +G^-(\mathbf{r}-\mathbf{r}'',t)n_1^-(\mathbf{r}'',0)][n_1^+(\mathbf{r}',0)+n_1^-(\mathbf{r}',0)]\rangle. \tag{8}$$

In considering the correlations for the vortex number densities one should take account of the long-range interactions of the vortices.[8] For sufficiently dilute vortex systems the Debye-Hückel theory of Coulomb correlations may be employed to estimate vortex pair correlations.[9] The results are not very different from the δ-function autocorrelation that applies in the absence of interactions. Moreover, in the self-field case of interest here, where vortices of both circulation appear, the autocorrelations and crosscorrelations due to pair interactions tend to cancel. As a first approximation, then, the vortex number-density correlations will be assumed to be adequately given by

$$\langle n_1^{\pm}(\mathbf{r}'',0)n_1^{\pm}(\mathbf{r}',0)\rangle = n_0^{\pm}(\mathbf{r}'')\delta(\mathbf{r}''-\mathbf{r}')$$
and
$$\langle n_1^{+}(\mathbf{r}'',0)n_1^{-}(\mathbf{r}',0)\rangle = 0. \tag{9}$$

Insertion of correlations (9) into expression (8) for the voltage autocorrelation yields

$$\langle V_1(t)V_1(0)\rangle=\frac{\phi_0^2}{\tau_0^2}\iint_{\text{bridge}} d\mathbf{r}d\mathbf{r}'[G^+(\mathbf{r}-\mathbf{r}',t)n_0^+(\mathbf{r}') +G^-(\mathbf{r}-\mathbf{r}',t)n_0^-(\mathbf{r}')]. \tag{10}$$

This section is concluded by noting that the functions $G^{\pm}(\mathbf{r}-\mathbf{r}',t)$ are very similar to the so-called wave-function propagator sometimes found convenient in quantum-mechanical discussions. Here the functions $G^{\pm}(\mathbf{r}-\mathbf{r}',t)$ characterize the dynamical behavior of the vortex system.

III. VORTEX PROPAGATORS AND AUTOCORRELATION

To determine the average number densities $n_0^{\pm}(\mathbf{r})$ and the transform operators $L^{\pm}(\mathbf{k},s)$ the dynamics of vortex motion must be considered. The simplest model which accounts for the gross features of vortex dynamics in the self-field case is one having only two components, that is, unpinned up-circulation vortices and unpinned down-circulation vortices. The frequencies of practical interest are sufficiently low that inertial effects are negligible, so the motion is assumed to be dominated by the viscosity.[10] Vortex pinning and annihilation are considered to be adequately described by two parameters: a vortex lifetime and a vortex pinning fraction.

The description appropriate to such a model is essentially a two-dimensional plasma-dynamic approximation. In this approximation the spread in velocities due to thermal motion is largely ignored. The vortex "plasma" is described by the usual Eulerian variables: number densities $n^+(\mathbf{r},t)$, $n^-(\mathbf{r},t)$ and mass average velocities $\mathbf{v}^+(\mathbf{r},t)$, $\mathbf{v}^-(\mathbf{r},t)$, all assumed to be single-valued functions of space, $\mathbf{r}\equiv(x,y)$, and time. For the unpinned vortices the equations of continuity are then

$$\frac{\partial}{\partial t}n^{\pm}+\frac{\partial}{\partial \mathbf{r}}\cdot n\mathbf{v}^{\pm}+\frac{1}{\tau}n^{\pm}=0, \tag{11}$$

where τ is an effective lifetime for moving vortices and is determined by pinning and pair annihilation.[11]

[8] J. Pearl, in *Low Temperature Physics LT9*, edited by J. D. Daunt, D. O. Edwards, F. J. Milford, and M. Yaqub (Plenum Press, Inc., New York, 1965), Part A, p. 566.

[9] G. L. McCone and P. G. Thiene, Office of Naval Research Report No. ASTIA 816213, 1967 (unpublished). Copies available from Department of Defense Communications.

[10] M. J. Stephen and J. Bardeen, Phys. Rev. Letters **14**, 112 (1965); M. Tinkman, *ibid.* **13**, 804 (1964); R. Deltour, M. Tinkham, Phys. Letters **23**, 183 (1966); Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. **139**, A1163 (1965); J. Bardeen, Phys. Rev. Letters **13**, 747 (1964).

[11] J. R. Clem, Phys. Letters **22**, 125 (1966).

Since the vortex flow is considered to be viscosity-limited, the equations of motion are[3]

$$\nu_c n^+ M_0 \mathbf{v}^+ - (\mathbf{j}-\mathbf{j}_{c1})\times \mathbf{e}_z n^+ \phi_0 d = 0$$
and
$$\nu_c n^- M_0 \mathbf{v}^- + (\mathbf{j}-\mathbf{j}_{c1})\times \mathbf{e}_z n^- \phi_0 d = 0. \qquad (12)$$

Here, ν_c is an effective rate for relaxation of vortex motion against the film lattice. The effective mass of a vortex is M_0. The first terms in each equation are the viscous drag force per unit area. The second terms are simply the Lorentz force per unit area due to the transport current of density $\mathbf{j}$. The critical current $\mathbf{j}_{c1}$ marks the onset of the vortex state, as indicated schematically in Fig. 2 by I_{c1}, the corresponding total current through the bridge. The current I_{c2} is the beginning of the normal state.

If it is assumed that the steady-state current density is spatially constant, Eqs. (12) give

$$\mathbf{v}_0^+ = -\mathbf{v}_0^- \equiv \mathbf{i}v_0 = \mathbf{i}(\phi_0/M_0\nu_c w)(I-I_{c1}). \qquad (13)$$

The magnetic flux density at the edge of the thin-film bridge is readily estimated to be

$$B_z = \pm(\mu_0 I/2\pi w)\ln(w/2d), \quad x=\mp\tfrac{1}{2}w. \qquad (14)$$

The corresponding average values of the number densities of the free vortices are

$$n_0^+ = n_0^- = f(\mu_0 I/2\pi\phi_0 w)\ln(w/2d), \quad |x| = \tfrac{1}{2}w, \qquad (15)$$

where f is the fraction of vortices which are unpinned. Equations (11) are easily integrated to obtain the spatial distribution of the average number densities. The results are

$$n_0^+(\mathbf{r}) = \frac{f\mu_0 I \ln(w/2d)}{2\pi\phi_0 w} e^{-(x+w/2)/w}$$
and
$$n_0^-(\mathbf{r}) = \frac{f\mu_0 I \ln(w/2d)}{2\pi\phi_0 w} e^{(x-w/2)/w}, \qquad (16)$$

when account is taken of boundary values (15).

Linearized about the steady-state values n_0^+, n_0^- and $\mathbf{v}_0^+ = -\mathbf{v}_0^- \equiv \mathbf{i}v_0$, Laplace-transformed in time and Fourier-transformed in space, the equations of continuity (11) appear as

$$(ik_x v_0 + s + 1/\tau)n_1^+(\mathbf{k},s) = n_1^+(\mathbf{k},0)$$
and
$$(-ik_x v_0 + s + 1/\tau)n_1^-(\mathbf{k},s) = n_1^-(\mathbf{k},0). \qquad (17)$$

Comparing Eqs. (17) with Eqs. (5), it is apparent that for the simplified vortex model assumed here, the transform operators are

$$L^+(\mathbf{k},s) = ik_x v_0 + s + 1/\tau$$
and
$$L^-(\mathbf{k},s) = -ik_x v_0 + s + 1/\tau. \qquad (18)$$

The vortex propagators defined by expressions (7) are now readily evaluated. The results are

$$G^+(\mathbf{r}-\mathbf{r}', t) = e^{-t/\tau}U(t/\tau_0)\delta(x-x'+v_0\tau)\delta(y-y')$$
and
$$G^-(\mathbf{r}-\mathbf{r}', t) = e^{-t/\tau}U(t/\tau_0)\delta(x-x'-v_0\tau)\delta(y-y'), \qquad (19)$$

where $U(t/\tau_0)$ is a unitary function defined as

$$U(t/\tau_0) = 1, \quad 0 < t/\tau_0 < 1$$
$$= 0, \quad t/\tau_0 < 0, \quad t/\tau_0 > 1.$$

The bridge transit time τ_0 is determined from expression (13) to be

$$\tau_0 = w/v_0 = M_0 \nu_c w^2/\phi_0(I-I_{c1}). \qquad (20)$$

Finally, the desired autocorrelation function for the voltage fluctuations is obtained by inserting the average number densities (16) and the vortex propagators (19) into expression (10). The result of the integration over the bridge area is

$$\langle V_1(t)V_1(0)\rangle = \frac{f\mu_0 I l \phi_0 \tau \ln(w/2d)}{\pi\tau_0^3}[e^{-t/\tau} - e^{-\tau_0/\tau}] \times e^{-t/\tau}U\left(\frac{t}{\tau_0}\right). \qquad (21)$$

Result (21) may be rewritten in terms of the average voltage drop along the bridge. From expression (3) the average voltage corresponding to the number density distributions given by (16) is

$$V_0 = \frac{f\mu_0 I l \tau \ln(w/2d)}{\pi\tau_0^2}[1 - e^{-\tau_0/\tau}]. \qquad (22)$$

Eliminating the bridge current from expression (21) by means of (22), one finds

$$\langle V_1(t)V_1(0)\rangle = \frac{V_0\phi_0}{\tau_0}\frac{e^{-t/\tau} - e^{-\tau_0/\tau}}{1 - e^{-\tau_0/\tau}} U\left(\frac{t}{\tau_0}\right). \qquad (23)$$

In Sec. IV the spectral intensity corresponding to the voltage autocorrelation function will be considered.

IV. DISCUSSION

The spectral intensity of the voltage fluctuations follows readily from the autocorrelation function (23). The spectral intensity $W(\nu)$ is defined such that $W(\nu)d\nu$ is the mean-square voltage fluctuation in the frequency interval $d\nu$ at the frequency ν.

According to the Wiener-Khintchine theorem[12] the spectral intensity is the Fourier cosine transform of 4

[12] A. van der Ziel, *Noise* (Prentice-Hall, Inc., Englewood Cliffs, N. J., 1954), p. 316.

times the autocorrelation function; that is,

$$W(\nu)=4\int_0^\infty \langle V_1(t)V_1(0)\rangle \cos 2\pi\nu t dt. \quad (24)$$

The spectral intensity corresponding to autocorrelation (23) is then

$$W(\nu)=\frac{4V_0\phi_0}{(\tau_0/\tau)(1-e^{\tau_0/\tau})}\left\{2\pi\nu\tau\left[\frac{1}{4[1+(\pi\nu\tau)^2]}-\frac{1}{1+(2\pi\nu\tau)^2}\right]e^{-2\tau_0/\tau}\sin 2\pi\nu\tau_0-\left[\frac{1}{2[1+(\pi\nu\tau)^2]}-\frac{1}{1+(2\pi\nu\tau)^2}\right]e^{-2\tau_0/\tau}\cos 2\pi\nu\tau_0+\frac{1}{2[1+(\pi\nu\tau)^2]}-\frac{e^{-\tau_0/\tau}}{1+(2\pi\nu\tau)^2}\right\}. \quad (25)$$

In the low-frequency limit $\nu\to 0$ and with no annihilation or pinning ($\tau\to\infty$), (25) reduces to $W(\nu)=2V_0\varphi_0$, which is formally the same as the expression for current fluctuations in an emission-limited vacuum tube. However, the limit $\tau\to\infty$ leads to $\mathbf{B}\to 0$ and hence no transport current, which is inconsistent with the assumed model. Not only must τ be finite, but in fact τ must be of the order of or less than τ_0. Owing to the finite value of τ, $W(\nu)$ is less than $2V_0\varphi_0$ at all frequencies, provided of course that flux flow takes place in units of magnitude φ_0 rather than in bundles.

There is very little experimental data with which to compare the theoretical model. Van Gurp has made extensive studies on flux flow in thin metal foils under the influence of applied fields, so vortices of only one species were present.[13] The closest approach to self-field induced flux flow was the earlier experiment of Van Ooijen on a long thin-film cylinder of tin under the influence of a steadily increasing axial field.[14] Measurements of the spectral intensity at about 4 kHz gave the result $2V_0\varphi_0$, which is the low-frequency limiting value of expression (25) for vortices of a single species (or for both species with no annihilation). Although the experimental result has an appealing simplicity, the implication of no annihilation is somewhat surprising; and, furthermore, a remarkably short transit time over the length of the pickup coil is indicated. Measurements of the whole frequency spectrum would be of interest.

The above experiment demonstrated the incoherent flow of flux into a metal thin-film loop. The opposite extreme of highly coherent flux flow is represented by the Josephson linewidth measurements of Silver, Zimmerman, and Kamper,[15] in which the fluctuation spectrum shows no shot-noise contribution at all. The question arises as to why the spectrum is all shot noise in the one case and devoid of shot noise in the other, particularly since the energy $\frac{1}{2}\varphi_0^2/L$ of one flux unit φ_0 in the loop of inductance L is rather large compared to thermal energy in either case. The increased shot-noise contribution in the case of multiple vortex flow can be made plausible by considering a simple example. Suppose a thin-film bridge is wide enough to contain approximately two vortices at a time, which move across the bridge under the influence of appropriate boundary conditions of current and voltage. We will not concern ourselves with the exact nature of the boundary conditions, but will suppose that each vortex has a transit time τ_0, and that the voltage pulse accompanying the transit of each vortex is rectangular (with area φ_0). The postulate that the bridge contains approximately two vortices at a time means that the voltage pulses of successive vortices overlap by about half their length. As a result of this overlap, the amplitude of the voltage at the Josephson frequency f_j is greatly reduced, even in the absence of fluctuations, below the value $2\varphi_0 f_j$ appropriate to the case of vanishingly small transit time. Also, the variation of the magnetic contribution to the free energy, as a function of vortex position, is likewise reduced, so that thermal fluctuations are more effective in destroying coherence. Thus we would not expect to observe interference effects in thin-film quantum interferometers unless the bridge width is comparable to or smaller than the size of a vortex. Qualitatively, this expectation seems to be borne out by experiment, but no quantitative study has been made.

[13] G. J. Van Gurp, Phys. Rev. **166**, 436 (1968).
[14] D. J. Van Ooijen, Phys. Letters **14**, 95 (1965).
[15] A. H. Silver, J. E. Zimmerman, and R. A. Kamper, Appl Phys. Letters **11**, 209 (1967).