PHYSICAL REVIEW VOLUME 177, NUMBER 2 10 JANUARY 1969

Ultrasonic Attenuation of Shear Waves in Superconducting Lead

B. C. DEATON
Applied Research Laboratory, Fort Worth Division of General Dynamics, Fort Worth, Texas 76101

(Received 25 July 1968)

Ultrasonic attenuation measurements have been made for shear waves propagating along the [001] and [110] directions in high-purity single crystals of lead at frequencies 10–50 Mc/sec and temperatures 2–15°K. It was found that the temperature-dependent dislocation attenuation is only a small fraction of the total electronic attenuation of shear waves at the transducer voltages used, and therefore reliable data on the attenuation in the superconducting state can be obtained. The rapid-fall electromagnetic attenuation just below the transition temperature was analyzed on the basis of Fossheim's model, and bulk values of the London penetration depth were obtained for the two directions of propagation. The results are in reasonable agreement with thin-film data. The residual attenuation remaining at temperatures below the rapid-fall region was analyzed on the basis of the BCS equation, using the zero-temperature energy gap $2\Delta(0)$ as an adjustable parameter. The apparent energy gap $2\Delta(0)$ is frequency-dependent for $\mathbf{q}\|[110]$ and $\boldsymbol{\varepsilon}\|[1\bar{1}0]$, varying from $4.4kT_c$ at 10 Mc/sec to $3.4kT_c$ at 50 Mc/sec. For $\mathbf{q}\|[110]$ and $\boldsymbol{\varepsilon}\|[001]$, for $\mathbf{q}\|[001]$ and $\boldsymbol{\varepsilon}\|[110]$, and for $\mathbf{q}\|[001]$ and $\boldsymbol{\varepsilon}\|[010]$, the data indicate essentially frequency-independent energy gaps of $3.1kT_c$, $3.4kT_c$, and $3.4kT_c$, respectively. In addition to the above calculations of $2\Delta(0)$ using the ordinary technique of fitting the low-temperature portion of the data with the exponential in the BCS equation, the data were also fitted over the entire temperature range with BCS curves, using $2\Delta(0)$ to adjust the fit. The resulting values of $2\Delta(0)$ were somewhat larger than those given above, with the relative values remaining about the same. The range of gap values observed is in good agreement with the recent tunneling data of Rochlin.

I. INTRODUCTION

Of the several methods for determining the anisotropy of the energy gap in superconductors, the ultrasonic attenuation technique has been one of the most useful, having been applied with reasonable success for a number of type-I superconductors.[1] In the strong-coupling materials Pb and Hg, however, the ultrasonic method has been ineffective in uniquely determining values of the energy gap because of the occurrence of anomalous effects in the attenuation of compressional waves.[2–5] The anomalous behavior in Pb consists of sound-wave-amplitude-dependent attenuation presumably caused by the motion of dislocations. In the case of Hg[3], and possibly also Pb[4], there seems to be some other as yet unexplained mechanism causing an anomalous frequency dependence of the compressional attenuation in the superconducting state.

In the present investigation, measurements have shown that by studying the attenuation of shear waves in Pb, much more reliable data on the attenuation in the superconducting state can be obtained since in this case the temperature-dependent dislocation attenuation is only a small fraction of the total electronic attenuation. The attenuation data are found to agree closely with theory in the superconducting and normal states, and for this reason the data are analyzed and interpreted in the customary manner. The temperature dependence of the shear-wave attenuation has been measured at frequencies 10–50 Mc/sec in the [001] and [110] crystal directions for shear polarizations $\boldsymbol{\varepsilon}$ along major crystallographic directions. The data are analyzed in terms of existing theories and are compared with results from other experiments.

Compressional wave-attenuation data in superconductors are usually analyzed using the equation derived in the Bardeen-Cooper-Schrieffer (BCS) theory,[6]

$$\alpha_s/\alpha_n = 2/(e^{\Delta(T)/kT}+1)\,, \qquad (1)$$

where α_s and α_n are the attenuation coefficients in the superconducting and normal states, respectively, $2\Delta(T)$ is the temperature-dependent energy gap which has a zero-temperature value of $3.52kT_c$, and T_c is the transition temperature. This equation was derived for an isotropic superconductor with $ql \gg 1$ but has been shown to be independent of ql,[7] where q is the magnitude of the sound-wave vector and l is the electron mean free path. Even though Eq. (1) was derived for the isotropic case, it has been successfully applied to anisotropic superconductors by treating the limiting energy gap $2\Delta(0)$ as a parameter which can vary with propagation direction.[8]

In the case of shear-wave attenuation, quite different behavior is observed. When $ql>1$, there is a very sharp drop in attenuation just below T_c followed by a more gradual decrease. The rapid-fall region is caused by the onset of the Meissner effect and the shorting out of part

---

[1] D. H. Douglass, Jr., and L. M. Falicov, in *Progress in Low Temperature Physics*, edited by C. J. Gorter (North-Holland Publishing Co., Amsterdam, 1964), Vol. IV, pp. 97–193.

[2] R. E. Love and R. W. Shaw, Rev. Mod. Phys. **136**, 260 (1964); R. E. Love, R. W. Shaw, and W. A. Fate, Phys. Rev. **138**, A1453 (1965); B. R. Tittmann and H. E. Bommel, Phys. Rev. Letters **14**, 296 (1965).

[3] R. L. Thomas, H. C. Wu, and N. Tepley, Phys. Rev. Letters **17**, 22 (1966); R. L. Thomas (private communication).

[4] B. C. Deaton, Phys. Rev. Letters **16**, 577 (1966).

[5] W. A. Fate, R. W. Shaw, and G. L. Salinger, Phys. Rev. **172**, 413 (1968).

[6] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).

[7] T. Tsuneto, Phys. Rev. **121**, 402 (1961).

[8] R. W. Morse, T. Olsen, and J. D. Gavenda, Phys. Rev. Letters **3**, 15 (1959).

of the electromagnetic inteaction,[9–11] while the residual region is found to obey the BCS relationship given by Eq. (1), and is attributable to collision-drag and shear-deformation effects.[11] Frequency and temperature measurements in the various regions of attenuation allow the determination of the superconducting energy gap, the London penetration depth,[12] and the relative amounts of shear deformation and collision-drag attenuation.[11] In principle, then, it is possible to investigate several parameters simultaneously by measuring the attenuation of shear waves in a superconductor, one of these being the energy gap. While determination of the gap anisotropy using shear data is not well established, previous data have indicated reasonable agreement with compressional wave results[9] and it is on this basis that the present data on the energy gap are presented.

II. EXPERIMENTAL

The experiments were performed using conventional single-ended pulse-echo ultrasonic techniques. The sample holder which was used is shown in cross section in Fig. 1.[13] The sample is placed in a conventional holder suspended at the end of a thin-wall stainless-steel tube, and the ultrasonic seal is made in the usual manner at room temperature with Dow Corning high-vacuum grease or Nonaq stopcock grease. This holder is then placed in a vacuum can constructed of 1-in.-diam stainless-steel tubing with the only vacuum seals consisting of O-ring seals outside the cryostat at room temperature. The temperature of the helium bath is maintained near 1°K, and ~50 mTorr of helium exchange gas, in addition to the brass fingers shown, is sufficient heat leak to keep the sample at bath temperature. The sample temperature is raised above that of the bath by a manganin-wire heater. This holder conveniently allows continuous coverage of the temperature range from 1–77°K, and offers the added feature of fitting easily in a conventional tailed Dewar between the pole faces of an electromagnet.

Sample temperatures were determined by an encapsulated Honeywell germanium resistance thermometer which was calibrated below 4.2°K using the vapor pressure of the helium bath, and above 4.2°K at the superconducting transition points of high-purity Pb and Nb single crystals. In addition the germanium thermometer had been previously calibrated using a paramagnetic salt, and the separate calibrations matched very well. The accuracy of the germanium thermometer is believed to be ±0.01°K below 4.2°K and ±0.03°K between 4.2 and 10°K. Temperature equilibrium was found to be established very quickly after a temperature change and typical temperature cycles over the interval 1–10°K required about 2 h.

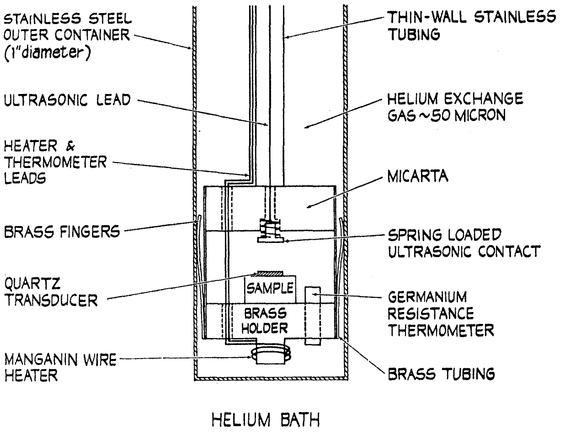

FIG. 1. Cross section of the low-temperature sample holder used to vary the temperature from 1 to 77°K.

An example of a typical recorder tracing of relative attenuation as a function of sample temperature (i.e., the output from the germanium thermometer) is shown in Fig. 2. In this particular case, the sound propagation vector **q** was along [001] and shear polarization **ε** was along [110] at a frequency of 30 Mc/sec. The attenuation in the normal state was measured while a transverse magnetic field of 800 G was applied. Problems encountered in obtaining the zero-field normal-state attenuation in compressional wave Pb studies[14] were not present in the shear measurements and the curve shown for α_n very closely approximates the data corrected to zero magnetic field. It was observed that at frequencies of 30 Mc/sec and above, the normal-state attenuation

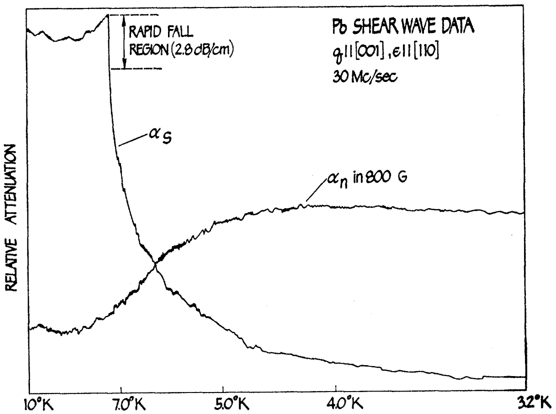

FIG. 2. Recorder tracing of the relative attenuation as a function of sample temperature for **q** along [001] and **ε** along [110] at 30 Mc/sec. The normal-state curve has been displaced vertically for clarity of presentation.

---

[9] A. R. Mackintosh, in *Proceedings of the Seventh International Conference on Low-Temperature Physics, 1960*, edited by G. M. Graham and A. C. H. Hallet (The University of Toronto Press, Toronto, 1961).

[10] L. T. Claiborne and R. W. Morse, Phys. Rev. **136**, A893 (1964).

[11] J. R. Leibowitz, Phys. Rev. **136**, A22 (1964).

[12] K. Fossheim, Phys. Rev. Letters **19**, 81 (1967).

[13] The sample holder is similar to a design suggested by L. T. Claiborne.

[14] W. A. Fate and R. W. Shaw, Phys. Rev. Letters **19**, 230 (1967).

decreased with temperature at the lower temperatures, an anomalous phenomenon observed previously in In by Sinclair.[15]

The Pb single crystal used in the measurements was grown from 99.9999% pure starting material by James F. Kirn of the Virginia Institute for Scientific Research. Various pieces of the original sample were cut and planed with an electric-discharge machine to lengths suitable for shear-wave measurements (0.2–0.5 cm). The crystals were cut so that sound could be propagated along the [001] and [110] crystal directions and were oriented by back-reflection x-ray techniques. AC-cut quartz transducers were used for generating shear waves.

III. RESULTS

A. Amplitude-Dependent Effects

In order to ascertain what effect amplitude-dependent attenuation might have on the present data, measurements of the total attenuation of compressional and shear waves were made at several frequencies and peak-to-peak pulse amplitudes, using a Sperry exponential attenuation comparator. The results of such measurements are shown in Fig. 3 at ~2.2°K for sound along [110] and polarization along [001]. Amplitude effects of up to ~4 dB/cm are observed at the highest pulse amplitudes at 50 Mc/sec. On the other hand, the total electronic attenuation of shear waves at these frequencies (~50 dB) is large enough that the amplitude-dependent attenuation represents a small fraction (~10%) of the total attenuation even at maximum pulse amplitude. Since the data were taken for transducer voltages below those voltages indicated by arrows in Fig. 3, it is believed that temperature-dependent amplitude effects are too small to appreciably affect the data. These measurements do not, however, preclude the presence of amplitude-independent, temperature-dependent effects which could be caused by the presence of dislocations. The possibility of this mechanism was ruled out by Fate[16] in his measurements on the basis of the good agreement of the normal-state data with the free-electron model.

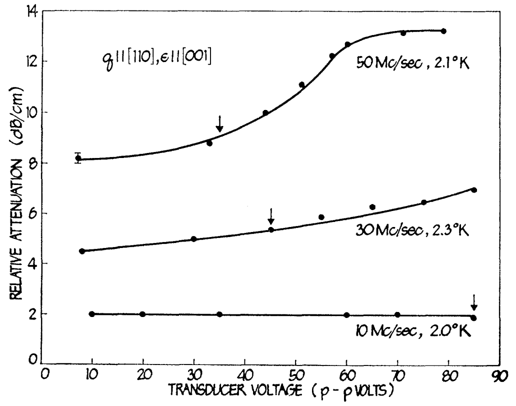

FIG. 3. Relative attenuation as a function of peak-to-peak transducer voltage for **q** along [110] and **ε** along [001] for 10, 30, and 50 Mc/sec. The arrows indicate the maximum voltages used in the energy-gap measurements.

The presence of some amplitude-dependent attenuation in these crystals is found to explain part of the anomalous frequency behavior observed in a previous investigation of compressional-wave attenuation in Pb.[4] In that case the total electronic attenuation was small enough that the amplitude effects were a considerable fraction of the total attenuation. It has not been possible to reach low enough amplitude levels to ascertain if the amplitude effects can account for all of the anomalous behavior; on the basis of measurements in Hg,[3] some other mechanism seems to be acting which causes an anomalously large drop in relative attenuation just below T_c at high frequencies.

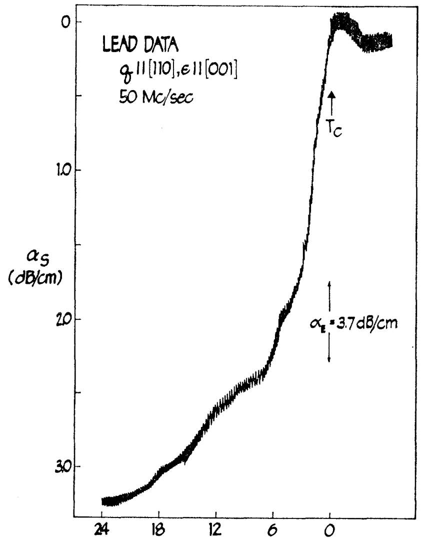

FIG. 4. Recorder tracing of the attenuation in the temperature range just below the transition temperature for **q**∥[110] and **ε**∥[001].

B. Shear-Wave Data

Data were taken for sound propagation along two of the three principal symmetry axes. For sound propagation vector **q**∥[001], measurements were made for shear direction **ε** parallel to [110] and [010]. For sound along [110], measurements were made with **ε** along the [001] and [1$\bar{1}$0] directions. As mentioned earlier, the shear-wave data in the superconducting state consist of two distinct regions, a rapid fall in α_s just below the transition temperature, followed by a more gradual falloff. The drop just below T_c is not discontinuous, but occurs

---

[15] A. C. E. Sinclair, Proc. Phys. Soc. (London) **92**, 962 (1967).

[16] W. A. Fate, Phys. Rev. **172**, 402 (1968). The author would like to thank Dr. Fate for communicating his results prior to publication.

in a temperature interval of the order[12] of 20 mdeg and can be related to the screening of the induced fields by the superconducting electrons. The temperature dependence of this attenuation caused by the electromagnetic interaction can be used to measure the London penetration depth $\lambda_L(0)$ at a single wave vector. The ultrasonic attenuation remaining after the sharp drop is expected to obey the BCS theory and thus provide a measure of the energy gap in the superconducting state. Each of these two regions of attenuation will be discussed individually in the following sections.

1. *Rapid-Fall Region*

Attenuation data in the region just below T_c were taken for all the configurations studied and were analyzed according to the theoretical model developed by Fossheim.[12] These measurements required the continuous recording of the attenuation falloff in the first 50 mdeg below T_c, so that the temperature dependence and the total amount of electromagnetic attenuation could be determined. Typical results are shown in Fig. 4 for $\mathbf{q}\|[110]$ and polarization along [001]. These data are somewhat qualitative because of the difficulty of controlling the temperature just below T_c. The rate of change of temperature was regulated by varying the current in the heater and was of the order of 2 mdeg/min.

If the results in Fig. 4 are analyzed in terms of Fossheim's model, the experiment is found to determine the London penetration depth at absolute zero, $\lambda_L(0)$, in terms of an integral over the effective zone on the Fermi surface. The present data were analyzed on the basis of Fermi momenta determined from magnetoacoustic studies of Pb as reported by Rayne.[17] It was implicitly assumed that the parts of the Fermi surface which contribute to the magnetoacoustic oscillations also are the effective zones in superconducting attenuation. From Rayne's results, Fermi momenta of 0.83×10^8 cm^{-1} and 0.52×10^8 cm^{-1} were used for $\mathbf{q}\|[110]$, while 1.03×10^8 cm^{-1} and 0.40×10^8 cm^{-1} were used for $\mathbf{q}\|[001]$. These values correspond to orbits on the third-zone electron surface in Pb, and are the two distinct momenta observed in the above crystal directions with magnetic field aligned in configurations which approximate the present zero-field data. The theoretical fitting parameter from Fossheim's model is σ_{2s}/σ_{1n}, where σ_{2s} is the imaginary part of the conductivity in the superconducting state and σ_{1n} is the real part of the conductivity in the normal state. This parameter is related to the electromagnetic part of the electronic attenuation in the superconducting state α_E, and the reiterative fitting procedure also establishes the zero level of α_E. The data in each crystal direction for all frequencies studied could be fit to $\sigma_{2s}/\sigma_{1n}=170\Delta T$, where ΔT is defined as T_c-T. The results are given in Table I, where a value of $\lambda_L(0)$ is calculated for each of the Fermi momenta given above. The bulk values of $\lambda_L(0)$ are higher than the free-electron value of 1.46×10^{-6} cm but agree reasonably with the data of Lock,[18] who found $\lambda_L(0)=3.9\times 10^{-6}$ cm from thin-film measurements.

TABLE I. Values of the London penetration depth obtained from the rapid-fall electromagnetic attenuation data.

Configuration	Assumed Fermi momentum p_c (10^8 cm^{-1})	$\lambda_L(0)$ (10^{-6} cm)
$\mathbf{q}\|[110]$, $\boldsymbol{\varepsilon}\|[001]$	0.52	4.38
	0.83	2.75
$\mathbf{q}\|[001]$, $\boldsymbol{\varepsilon}\|[110]$	0.40	5.71
	1.03	2.22

2. *Residual Region*

The residual attenuation α_R of shear acoustic waves can be separated into two distinct contributions using the frequency dependence of the attenuation in the superconducting state.[11] For $ql \gg 1$, the residual region is caused by a collision-drag term α_C, and a contribution α_D caused by the deformation of the Fermi surface by the passage of the shear wave. In the limit as ql becomes very large, α_C/α_n decreases as the reciprocal of the frequency while α_D/α_n becomes a constant independent of ql. The two components can thus be separated by frequency measurements of α_R/α_n.

Previous investigations have indicated that the ratio α_D/α_n is approximately 0.10 for Al[11] and near 0.35 in the case of In.[19] In the present investigation of Pb over the rather narrow frequency range 10–70 Mc/sec, much larger deformation contributions than those seen in Al and In are suggested. Whether or not the ratio α_D/α_n is actually anomalously large in Pb can be established only through further studies at higher frequencies.

3. *Energy-Gap Data*

The part of the attenuation remaining after the electromagnetic attenuation is cut off was analyzed on the basis of the BCS theory as expressed in Eq. (1). Very few efforts have been made to measure energy-gap anisotropy with shear waves,[9] but because of the previous difficulties encountered with compressional waves in Pb, any experimental results on energy-gap anisotropy in Pb are quite valuable.[20]

Analysis on the basis of Eq. (1) requires the selection of a zero of electronic attenuation which is customarily deduced from an extrapolation of α_s to absolute zero. This extrapolation was carried out using an IBM 360 computer program which determined the zero giving the best fit to the BCS equation at temperatures $T/T_c < 0.65$. This least-squares fitting procedure allowed the selection of the zero of electronic attenuation and also calculation of the zero-temperature energy gap $2\Delta(0)$

---

[17] J. A. Rayne, Phys. Rev. **129**, 652 (1963).

[18] J. M. Lock, Proc. Roy. Soc. (London) **A208**, 391 (1951).

[19] J. R. Leibowitz and K. Fossheim, Phys. Rev. Letters **17**, 636 (1966).

[20] G. I. Rochlin, Phys. Rev. **153**, 513 (1967).

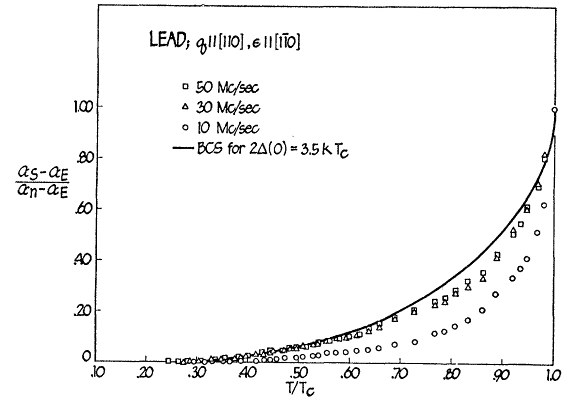

FIG. 5. Normalized residual shear-wave attenuation in Pb for **q**||[110] and **ε**||[1$\bar{1}$0] as a function of frequency.

for the particular experiment being considered. The complications of analysis introduced by the presence of the electromagnetic attenuation α_E were taken into account in the following manner. The magnitude of α_E was assumed to have a constant value (independent of temperature) for a particular frequency, this value being determined from the analysis of Sec. III B 1; this constant value was then subtracted from α_s and α_n and the analysis was carried out using $(\alpha_s-\alpha_E)/(\alpha_n-\alpha_E)$ for the attenuation ratio in the BCS equation. It was further assumed that T_c coincided with the temperature at the end of the electromagnetic falloff. Since α_E is quite small for Pb, the effect of subtracting α_E from α_s and α_n is not appreciable, and the final reduced curves depend only slightly on this correction. In fact, $2\Delta(0)$ data obtained by ignoring α_E entirely and plotting α_s/α_n directly varied only ~4% from the corrected data.

Energy-gap data were taken for the four independent sound and polarization directions mentioned earlier. The normalized data for sound along [110] and shear along [1$\bar{1}$0] as a function of ultrasonic frequency are shown in Fig. 5. The reduced data for this configuration are found to be frequency-dependent, with the largest deviation from the weak-coupling BCS value of $2\Delta(0) = 3.5kT_c$ occurring at the lowest frequency, in agreement with recent compressional wave data on pure deformed Pb published by Fate and Shaw.[14] In Fig. 6 data for two crystal directions are shown at a frequency of 50 Mc/sec. The anisotropy between propagation along [110] and [001] is readily apparent. The other data were similar to those shown in Figs. 5 and 6, and only the energy-gap results will be presented.

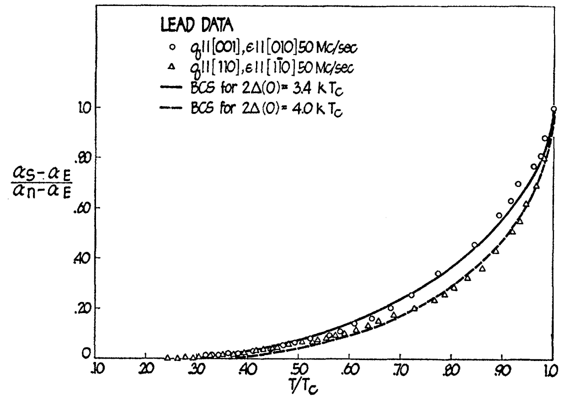

FIG. 6. Normalized residual shear-wave attenuation in Pb for the configurations **q**||[110], **ε**||[1$\bar{1}$0] and **q**||[001], **ε**||[010] at a frequency of 50 Mc/sec.

In addition to the calculations of $2\Delta(0)$ using the ordinary technique of fitting the low-temperature portion of the data to the exponential in the BCS equation, another technique was utilized. Since the above method is quite sensitive to small errors in background attenuation, and Pb has a tendency to show temperature-dependent dislocation background attenuation, the data were also fitted over the entire temperature range to BCS curves using $2\Delta(0)$ as an adjustable parameter. Results of these fits are also indicated on the reduced data of Fig. 6. The results of these analyses are shown in Table II for the sound configurations investigated. The apparent energy gap varies with ultrasonic frequency, crystal direction, polarization in the case of [110] propagation, and the method of analysis. Within the limits of error the data for **q**||[001] are frequency-independent, and since both shear polarizations are elastically equivalent for this propagation direction, these data allow a check of the internal consistency of the results. A fairly large anisotropy exists between the two different polarization directions studied for **q** along [110]. The sound velocities vary by about a factor of 2 for these cases, so that this anisotropy is obviously related to the phonon spectrum.

IV. CONCLUSIONS

A. Compressional-Wave Results

The present measurements have indicated that a small component of temperature-dependent amplitude-dependent attenuation exists in the Pb crystals employed in these studies. Although the amount of this amplitude effect is not large enough to appreciably affect the results on shear waves, it is pertinent to our previously published compressional-wave data.[4] The amplitude-dependent attenuation represents a considerable fraction of the total electronic attenuation of compressional waves. It is found that this amplitude-dependent attenuation is able to account for part of the anomalous drop in α_s/α_n at high frequencies observed earlier, but it has not been possible to reach low enough amplitudes to ascertain if the amplitude effects can account for all of the anomalous behavior. There still remains an anomalously small attenuation in the superconducting state for high frequencies in the data on Hg taken by Thomas[3] and more recently by Newcomb and Shaw.[21] The physical mechanism causing this anomaly is therefore not understood in the light of the present theories.

---

[21] C. P. Newcomb and R. W. Shaw, Phys. Rev. **173**, 509 (1968).

TABLE II. Summary of energy-gap results (in units of kT_c) from shear-wave attenuation measurements in single-crystal lead.

Sound and polarization directions	Frequency (Mc/sec)	Sound velocity (10^5 cm/sec)	$2\Delta(0)$ Extrapolation technique	$2\Delta(0)$ Curve fitting to full curve	$2\Delta(0)$ Tunneling (Rochlin)
$\mathbf{q}\parallel[110]$, $\boldsymbol{\varepsilon}\parallel[1\bar{1}0]$	10	0.66	4.4 ± 0.1	5.0 ± 0.2	3.4–4.8
	30		3.5 ± 0.1	4.0 ± 0.1	
	50		3.4 ± 0.1	4.0 ± 0.1	
$\mathbf{q}\parallel[110]$, $\boldsymbol{\varepsilon}\parallel[001]$	10	1.29	3.0 ± 0.2	3.5 ± 0.2	
	30		3.3 ± 0.1	3.5 ± 0.1	
	50		3.1 ± 0.1	3.5 ± 0.1	
$\mathbf{q}\parallel[001]$, $\boldsymbol{\varepsilon}\parallel[010]$	10	1.29	3.4 ± 0.2	3.8 ± 0.2	
	30		3.4 ± 0.1	3.6 ± 0.1	
	50		3.3 ± 0.1	3.5 ± 0.1	
$\mathbf{q}\parallel[001]$, $\boldsymbol{\varepsilon}\parallel[110]$	10	1.29	3.5 ± 0.2	3.9 ± 0.1	
	30		3.4 ± 0.1	3.6 ± 0.1	

B. Shear-Wave Results

The following conclusions can be reached on the basis of the measurement of the temperature, frequency, and crystal-direction dependence of the attenuation of shear waves propagating in superconducting Pb.

(a) The data in Fig. 3 indicate that the amplitude-dependent attenuation is relatively small compared to the total electronic attenuation of shear waves in Pb for the transducer voltages used in the experiments. It thus appears that the present results on Pb are indicative of high-purity, strain-free behavior of the attenuation of shear waves in superconducting Pb.

(b) It has been found that careful observations of the attenuation of shear waves in Pb just below T_c lead to a determination of the London penetration depth $\lambda_L(0)$. This measurement of $\lambda_L(0)$ is a bulk value based partially on the free-electron model and partly on Fermi-surface data on Pb. The results are indicated in Table I. These values are higher than the free-electron value of 1.46×10^{-6} cm but fall reasonably close to the thin-film experimental value of 3.9×10^{-6} cm. It should be noted that the ultrasonic experiment determines a bulk penetration depth for a specific wave vector $\mathbf{q}$ for an assumed orbit on the Fermi surface, while the thin-film value probably represents a realistic average of the penetration depth over the entire Fermi surface.

(c) The relative amounts of attenuation caused by electromagnetic, collision drag, and shear-deformation mechanisms in Pb seem to be unlike those found in other materials. Using the theoretical model of Leibowitz, at frequencies such that $ql\gg1$, the amount of shear-deformation contribution to the residual attenuation remaining after the rapid fall appears to be anomalously large. This problem is currently being investigated further using higher frequencies to determine whether or not the deformation contribution is actually much larger in Pb than in In and Al.

(d) The temperature dependence of the residual attenuation in the superconducting state has been analyzed in terms of the BCS model to obtain values of the zero-temperature energy gap. The energy gap is found to vary somewhat with frequency, crystal direction polarization direction, and type of analysis as indicated in Table II. The over-all *range* of gap values obtained is in quite good agreement with the tunneling results of Rochlin[20] but is in considerable disagreement with the theoretical range of gap values calculated by Bennett,[22] on the assumption that all the anisotropy in Pb is due to the anisotropy of the phonon spectrum.

Comparison of the gap values obtained in Table II with compressional-wave values is not a straightforward procedure and the results should be taken only as indicative of the energy gap anisotropy which can exist in Pb. The values of $2\Delta(0)$ are generally somewhat smaller than the accepted strong-coupling value for Pb, $2\Delta(0)=4.3kT_c$, but the ultrasonic technique is known to measure the minimum value of $2\Delta(0)$ on the effective zone of the Fermi surface.[23]

The frequency dependence of the reduced attenuation curves indicates that the lower frequencies disagree most with BCS behavior. This is in agreement with the results of Fate *et al.*[5] on lightly cold-worked Pb and dilute alloys of Pb, but in disagreement with our earlier work on pure Pb crystals[4] and other results on Hg.[3,21] This former frequency dependence has been explained on the basis of a decrease in the electron mean free path in the superconducting state in the phonon-limited scattering region. Our data also tend to support this conclusion when analyzed in the manner used by Fate *et al.*[5] As mentioned earlier, however, a full explanation of the frequency dependence of compressional waves in Pb and Hg does not seem possible at present.

ACKNOWLEDGMENTS

The author gratefully expresses his appreciation to D. E. Gordon, R. E. Beissner, B. G. W. Yee, and C. P. Newcomb for their contributions to the various phases of this research.

---

[22] A. J. Bennett, Phys. Rev. 140, A1902 (1965).

[23] V. L. Pokrovskii, Zh. Eksperim. i Teor. Fiz. 40, 898 (1961) [English transl.: Soviet Phys.—JETP 13, 628 (1961)].