Magnetic Interactions in a Warm Plasma*

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A comparison of several kinetic equations with electromagnetic interactions is made. In particular the effects of assuming the Darwin Hamiltonian are investigated, and it is concluded that the role of the magnetic forces may be of importance in the stability problem, since they seem to manifest tendencies opposite to those of the electrostatic forces in the limiting cases of long and short wavelengths. The possibility of the existence of density waves at or close to equilibrium, due to the presence of the $(v/c)^2$ interactions, is pointed out. Transport calculations will lead to the same results as those in the calculations of Trubnikov, although it may be expected that if cutoffs are eliminated as in previous work, the results will differ.

I. INTRODUCTION

Several papers[1–5] have explored recently the effects of including electromagnetic interactions for the kinetic theory of a plasma. Some authors employ the Klimontovich formalism and its generalizations,[1,2,5] while others use the resolvent method of Prigogine and his coworkers.[3,4] With the exception of the work of Hakim, these authors have started from the viewpoint that independent electromagnetic fields must necessarily be introduced. The present author has favored a particle approach that avoids the divergences inherent in the independent field method, and since the work has remained within an approximation for which the Hamiltonian and the initial value problem are well defined, any difficulties associated with the problem of radiation have been avoided.[6–8] The results obtained are described as being applicable to a plasma for which relativistic $(v/c)^2$ interactions must be examined.

Trubnikov[9] has been critical of the use of the Darwin Hamiltonian, and he has indicated that use of the Darwin Lagrangian is more appropriate. We examine previous work with the Darwin Hamiltonian, and propose that in contrast to Trubnikov's result, the transverse interaction, rather than being rapidly damped, may lead to density waves. A comparison of several other results with our results is made, and it is found that close comparison exists provided that retardation effects are ignored. This comparison generalizes the argument used earlier to show how the relativistic Landau equation reduces with the neglect of retardation in the $(v/c)^2$ approximation.

Further investigation shows that the pair distribution function exhibits oscillatory behavior. At small distances the effective interaction reduces to the Darwin result, but at larger distances the effective interaction drops off rather slowly.

Finally, note is made of similarity with the transport coefficient calculation of Trubnikov (with the neglect of corresponding terms).

II. GENERALIZATION OF LENARD-BALESCU RING APPROXIMATION

The ring approximation $(v/c)^2$ generalization of the Lenard-Balescu equation, for an arbitrary spherically symmetric distribution, was obtained earlier:[7]

$$\frac{\partial\varphi_1(\vec{P}_1)}{\partial t}=16\pi^3e^4C\int d\vec{P}_2\int d\vec{l}\,\vec{l}\cdot\frac{\partial}{\partial\vec{P}_1}\frac{\delta(\vec{l}\cdot\vec{g}_{12})}{l^4}$$
$$\times\left(\frac{1}{|\epsilon|^2}+\frac{P_{1\perp}{}^2P_{2\perp}{}^2}{2(mc)^4|\eta|^2}\right)\vec{l}\cdot\left(\frac{\partial}{\partial\vec{P}_1}-\frac{\partial}{\partial\vec{P}_2}\right)\varphi_1(\vec{P}_1)\varphi_1(\vec{P}_2)\,, \tag{2.1}$$

where
$$\epsilon\equiv1+(4\pi ie^2C/l^2)\int d\vec{P}_3\times\delta_-(\vec{l}\cdot\vec{g}_{13})\vec{l}\cdot\frac{\partial\varphi_1(\vec{P}_3)}{\partial\vec{P}_3} \tag{2.2}$$

and
$$\eta\equiv1-(4\pi ie^2C/l^2)\int d\vec{P}_3\times\delta_-(\vec{l}\cdot\vec{g}_{13})\frac{P_{3\perp}{}^2}{2(mc)^2}\vec{l}\cdot\frac{\partial\varphi_1(\vec{P}_3)}{\partial\vec{P}_3}\,. \tag{2.3}$$

The resulting kinetic equation obtained as Eq. (4.1) in Ref. 9 has the same form as that above, except it is defined in terms of velocity variables. To order $(v/c)^2$, they would be exactly the same, except that the quantity corresponding to η is given by

$$\mu=\eta+(\omega_0/lc)^2\,. \tag{2.4}$$

For both equations the ϵ are identical. The η given in (2.3) may be compared with what Trubnikov refers to in the following as the true quantity,

$$\mu_{\text{true}}=\mu-(\omega/lc)^2\,. \tag{2.5}$$

This last result and the corresponding kinetic equation in which retardation is taken into account has the same form as (2.1) and has been obtained by Silin, Klimontovich, and Shafranov.[10] The latter authors assume (all the while neglecting radiation) an approximation which says the particles move in straight-line trajectories; the possible inconsistency in treating retardation between charged particles in this way has been mentioned in an earlier paper.[8] However, we do wish to point out the error in Ref. 9 in assuming that the

Darwin approximation in some way takes into account retardation. That this is not so may be seen by observing that the approximation may be derived from expansion while assuming (by one derivation) either retarded, advanced, or symmetric interactions.

It is informative to compare the work of several other authors. Hogan and Lewis[2] and Dupree[1] use the Klimontovich[10] approach and introduce fields as independent quantities; the occurrence of infinite self-energies is unavoidable therefore, and one must drop these infinite terms. However, the kinetic equation obtained by Hogan and Lewis is essentially that obtained in work on the relativistic Landau equation.[8] The results are identical provided one effectively neglects the $(\vec{l}\cdot\vec{v}/c)$ term in the denominator of the transverse interaction term,[11] we get

$$\frac{\partial\varphi_1(\vec{P}_1)}{\partial t}=16\pi^3e^4C\int d\vec{P}_2\int d\vec{l}\,\vec{l}\cdot\frac{\partial}{\partial\vec{P}_1}\times\delta(\vec{l}\cdot\vec{g}_{12})\left(1+\frac{P_{1\perp}{}^2P_{2\perp}{}^2}{2(mc)^2}\right)\times\vec{l}\cdot\left(\frac{\partial}{\partial\vec{P}_1}-\frac{\partial}{\partial\vec{P}_2}\right)\varphi_1(\vec{P}_1)\varphi_1(\vec{P}_2)\;. \tag{2.6}$$

Had the appropriate retardation factor for straight line trajectories been included in the scalar potential as well, the result of the previous authors[2] would have been the same as the two-cycle equation of Mangeney.[3] However, it is argued in Ref. 8 that the inclusion of retardation without a corresponding examination of the radiation problem may not be valid.

The kinetic equation of Dupree[1] also has the same form as (2.1).[12] If for the arbitrary spherically symmetric case one neglects the corresponding $(\vec{l}\cdot\vec{v}/c)$ term in the denominator of the transverse interaction term [this corresponds also to neglecting the second term on the right-hand side of (2.5)] the result is not quite the same (the longitudinal term in both cases reproduces the Lenard-Balescu result). Although (2.3) differs somewhat from Dupree's term, we note that if one takes

$$m^{-1}(\vec{l}\cdot\vec{P}_1)\equiv\omega\sim 0$$

in (2.3) and in the corresponding term of Dupree's result, it leads to the equation (when $\eta\sim 0$)

$$l^2c^2=\omega_0{}^2\;. \tag{2.7}$$

Trubnikov argues that in the $\omega\sim 0$ limit, one must have

$$l^2c^2=-\,\omega_0{}^2\;, \tag{2.8}$$

and he is led to conclude that transverse plasma waves of low frequency are damped in space. Here, then, is an important difference in that (2.7) would be consistent with the idea of transverse plasma-density waves at low frequencies, while (2.8) precludes their existence.

We would like to further comment on the relation

$$(lc/\omega)^2=1-(\omega_0/\omega)^2 \tag{2.9}$$

which is used by Trubnikov to justify (2.8). It would appear that one can not use this high-frequency relation to justify a result for which $\omega\sim 0$. According to Landau,[13] this should be used only if $\omega\gg\omega_0$. However, even if it may be used at low frequencies, it is questionable whether it is valid when transverse magnetic effects are taken into account.

In further comparison, we would like to indicate that one may get Trubnikov's results in a straightforward but somewhat *ad hoc* fashion by modifying the momentum-dependent interaction with a screening factor with range proportional to $r_R\equiv c/\omega_0$, and obtain results in ring approximation. This was one of the procedures in Ref. 6, but there the range was taken as the Debye radius.

III. POSSIBILITY OF TRANSVERSE DENSITY WAVES

The results of an earlier paper[6] on the equilibrium statistical mechanics of relativistic interaction corrections showed that with a screened interaction one could expect negligible contribution to order $(v/c)^2$ for all but the highest densities and temperatures. When the ring summation for the exact (no screening) Darwin interaction was performed, the result led to an integral whose principal value vanished. In light of that result and subsequent work on the nonequilibrium problem, we have come to the conclusion that the presence of real axis singularities, while possibly indicative of insufficiency of the ring approximation, may point to some interesting physical consequences due to relativistic interaction corrections. One interpretation of the vanishing result is that to order $(v/c)^2$ there is no contribution in the equilibrium situation.

Next, we see what follows from the nonequilibrium situation. It is simple to show[14] that (2.1) satisfies the H theorem:

$$\frac{\partial s}{\partial t}=\text{const}\times\int d\vec{P}_1\int d\vec{P}_2\int d\vec{l}\,[\delta(\vec{l}\cdot\vec{g}_{12})/l^4][1/|\epsilon|^2+P_{1\perp}{}^2P_{2\perp}{}^2/2(mc)^4|\eta|^2]\times[\vec{l}\cdot\vec{\partial}_{12}\varphi_1(\vec{P}_1)\varphi_1(\vec{P}_2)]^2/\varphi_1(\vec{P}_1)\varphi_1(\vec{P}_2)\geqslant 0\;, \tag{3.1}$$

where $s\equiv\int d\vec{P}_1 C\varphi_1(\vec{P}_1)\ln[C\varphi_1(\vec{P}_1)]$.

Thus in the stable condition, the plasma will come to equilibrium, and the Maxwell-Boltzmann or the Jüttner distribution will satisfy (3.1).

Examining further the situation close to equilibrium, we have from (2.2) and (2.3)

$$\epsilon = 1 + \frac{\omega_0^2 \beta m}{l^2}\left[1 - \frac{\omega(2\beta m)^{1/2}}{l}\psi\left(\frac{\omega(2\beta m)^{1/2}}{l}\right)\right] - \frac{i\omega}{l}\left(\frac{\omega_0}{l}\right)^2 (\beta m)^{\frac{3}{2}}\left(\frac{\pi}{2}\right)^{1/2} e^{-\frac{1}{2}\beta m(\omega/l)^2}, \tag{3.2}$$

and

$$\eta = 1 - \left(\frac{\omega_0}{lc}\right)^2\left[1 - \frac{\omega(2\beta m)^{1/2}}{l}\psi\left(\frac{\omega(2\beta m)^{1/2}}{l}\right)\right] + \frac{i\omega}{l}\left(\frac{\omega_0}{lc}\right)^2\left(\frac{\beta m}{2\pi}\right)^{1/2} e^{-\frac{1}{2}\beta m(\omega/l)^2}, \tag{3.3}$$

where the Maxwell-Boltzmann distribution has been used; ψ is the well-known error function. It is apparent from (3.3) that if one takes $\omega \sim 0$, then setting $\eta = 0$ leads to (2.7). The zeros that occur here are the same singularities encountered in the equilibrium problem.[6] One can also examine the $l \to 0, \infty$ limits. In the long-wavelength limit, we get

$$\eta = 1 + \left(\frac{\omega_0}{\omega}\right)^2 \frac{1}{\zeta} + i\frac{\omega_0^2 \omega}{(lc)^3}\left(\frac{\zeta}{2\pi}\right)^{1/2} e^{-\frac{1}{2}\beta m(\omega/l)^2}, \tag{3.4}$$

where $\zeta \equiv mc^2/kT$. This result would have followed in the $l \to 0$ limit for any spherical symmetric distribution.[15] Now one expects that the roots of (3.4) have imaginary parts, and thus it is possible for the exponential to dominate. The only real ω consistent with $\eta = 0$ in this limit is $\omega = 0$, when either of the corresponding limits $c \to \infty$ or $T \to 0$ are taken. Note also that if only $T \to 0$ is taken, then (3.4) can not vanish and $\eta \to 1$. For $c \to \infty$, there is no transverse contribution whatever.

In the short-wavelength limit $l \to \infty$ (3.3) becomes

$$\eta = 1 - (\omega_0/lc)^2 . \tag{3.5}$$

One then retrieves (2.7) as a zero of η.

The above limiting cases show that the transverse behavior is the opposite of what follows from the longitudinal relation given by (3.2).

Thus, if the longitudinal interaction tends to stabilize the plasma at long wavelengths, the magnetic interactions will not show the same tendency. Conversely, at short wavelengths, the opposite would appear to be true. If one maintains the relation (2.9) for high frequencies in the short-wavelength limit, we see that

$$\omega^2 = l^2c^2 + \omega_0^2 \sim 2\omega_0^2 . \tag{3.6}$$

This would indicate that a "resonance" may occur in this region. Recently Tidman and Dupree[16] have obtained such a resonance (to explain the observed spectrum in solar flares) from consideration of electron-electron quadrupole bremsstrahlung, although the frequency occurs closer to the second harmonic of the plasma frequency.

IV. PAIR CORRELATION FUNCTION

To further explore the equilibrium properties, and to point up the contrast with the result of Ref. 9, we further write the pair correlation function in the chain approximation (here the endpoints are fixed, rather than closed to form a ring):

$$\varphi_2 = \varphi_1(\vec{P}_1)\varphi_1(\vec{P}_2)(1 + f_{12}) , \tag{4.1}$$

$$f_{12} = -(e^2/\theta r)e^{-r/r_D} + [e^2/(mc)^2\theta r]\{\vec{P}_1 \cdot \vec{P}_2[\cos\rho - (\sin\rho)/\rho + (1-\cos\rho)/\rho^2]$$
$$+ [(\vec{P}_1\cdot\vec{r})(\vec{P}_2\cdot\vec{r})/r^2][\cos\rho - 3(\sin\rho)/\rho - 3(1-\cos\rho)/\rho^2]\} , \tag{4.2}$$

above $\rho \equiv r/r_R$, where $r_R \equiv (c/\omega_0)^{1/2}$, $\theta \equiv (kT)^{-1}$ and r_D is the Debye length. The calculation to obtain this result may be obtained by either performing the equilibrium summation of diagrams, or by solving the corresponding nonequilibrium case in the equilibrium limit.[17] The equilibrium calculation is somewhat lengthy, but not qualitatively different from earlier calculations[6], and it is not included here. Here also the nonrelativistic kinetic energy is used; if the relativistic form is used, r_R is somewhat smaller than $(c/\omega_0)^{1/2}$, and is temperature dependent.

In the $(v/c)^2$ approximation, one can write an effective interaction energy[9]

$$\epsilon_{12} = P_1^2/2m + P_2^2/2m - \theta f_{12} , \tag{4.3}$$

where f_{12} is given by (4.2).

In the limit r_R, $r_D \gg r$, the interaction term in (4.3) reduces to the Darwin interaction for two particles. As indicated earlier, the result of Trubnikov here gives exponential screening in the transverse interaction as well as in the longitudinal one.

In the limit $r_D \ll r_R \ll r$, the effective interaction energy becomes

$$-[e^2/(mc)^2 r][\vec{P}_1 \cdot \vec{P}_2 + (\vec{P}_1 \cdot \vec{r})(\vec{P}_2 \cdot \vec{r})/r^2] \cos(r/r_R) . \tag{4.4}$$

Thus the correlations are still long-ranged at distances beyond which the screened electrostatic interaction is negligible.

The conclusion with respect to the equilibrium calculation of thermodynamic quantities must be that, because of the tensor symmetry of the interaction, there is negligible contribution to order $(v/c)^2$. This is clear if one uses f_{12}, as in (4.2) (to determine the average energy for example). This is the case (as it must be) if one uses the ring summation or the pair correlation function; it is also true for the pair correlation function given by Trubnikov, although his function is qualitatively different from ours.

The wavelength associated with the density of variation for hot electron plasmas is $O(c/\omega_0)$ which is $O(10^{-2})$ cm for $\omega_0 = 10^{12}$ sec^{-1}, and $O(1)$ cm for $\omega_0 = 10^{10}$ sec^{-1}. A shell structure with maximum variation at 5 cm for a hot electron plasma with $\omega_0 \sim 10^{10}$ sec^{-1} has been reported recently.[18] The plasma in this experiment is in a region where $(v/c)^2$ corrections should be examined, although one must certainly be cautious in interpreting results where external fields also play an important role. For hot, rarified plasmas, the predicted wavelength can be many orders of magnitude larger.

The periodicity in (4.2) can lead to quite complicated structure depending on whether ρ is an even or odd, integer or half-integer, multiple of π. Such density wave structure has been recently proposed by Lin[19] in explaining galaxy structure in the very similar problem of a homogeneous self-gravitating stellar system. Wu[20] has obtained the pair correlation function which has "modulation factors" such as occur in (4.2), and indeed his gravitational (nonrelativistic) result could also be obtained by formally summing over chain diagrams [as is done to obtain (4.2)].

V. TRANSPORT COEFFICIENTS

As in the paper by Trubnikov, one may calculate the frictional and diffusion coefficients for a Maxwell-Boltzmann distribution. Any difference in the expressions will come from the transverse for form alone [Eq. (5.2) below]:

$$\epsilon = 1 + (1 - \omega I)/l^2 r_D^2 , \tag{5.1}$$

$$\eta = 1 - (1 - \omega I)/l^2 r_R^2 , \tag{5.2}$$

where $I \equiv I_1 + iI_2$ and I_1, I_2 are given as

$$I_1 \equiv \frac{P}{2(mc)^2} \int d\vec{P} \frac{1}{\omega - (\vec{l} \cdot \vec{P}/m)} \vec{P}_\perp \cdot \frac{\partial \varphi_1(\vec{P})}{\partial \vec{P}} , \tag{5.3}$$

and

$$I_2 \equiv \frac{\pi}{2(mc)^2} \int d\vec{P} \delta\left(\omega - \frac{\vec{l} \cdot \vec{P}}{m}\right) \vec{P}_\perp \cdot \frac{\partial \varphi_1(\vec{P})}{\partial \vec{P}} . \tag{5.4}$$

For the Maxwell-Boltzmann distributions, these have the explicit forms given in (3.2) and (3.3). If one again neglects the "nondominant" terms, the same expressions follow as in Trubnikov's paper. Again the transverse friction coefficient will not show diminishing behavior with increasing $\vec{v}$, unlike the longitudinal contribution.

Calculations as above are carried out by inserting a cutoff in a divergent integral. In a previous paper,[7] we have shown that the $(v/c)^2$ contributions may be calculated without cutoffs.[21] It is possible that results such as the transport calculations will be modified when the explicit appearance of cutoffs is avoided. A recent nonrelativistic calculation of this type has been made by Gould and DeWitt.[22]

VI. CONCLUSIONS

We have compared the results of several authors on the kinetics of a plasma with electromagnetic interactions with the results obtained using the Darwin Hamiltonian. The analysis shows close correspondence with the work of Dupree and Hogan and Lewis, but some qualitative differences arise with the work of Trubnikov. The suggestion is made that a plasma near or at equilibrium has density waves; the main characteristic of these, apart from their periodic structure, is that they lead to long-ranged effective interactions [in fact, longer in range than the screened Coulomb interaction – see Eq. (4.4)]. Finally, in defense of the use of a Hamiltonian (although it is not clear why *classical* results should differ if one uses the Darwin Lagrangian instead), we would like to comment that the route to a quantum treatment is through the Hamiltonian.[23]

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Number Density and Phase Diagram of Dilute He^3-He^4 Mixtures at Low Temperatures*

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The molar volume at the saturated vapor pressure has been determined by a dielectric method for solutions containing up to 15 atomic percent of He^3 in He^4 between 0.025 and 1.2°K. In the single-phase region there is a pronounced thermal expansion from which it is deduced that the derivative of the He^3 effective mass with respect to pressure is for low concentrations $[(0.0151 \pm 0.0006)m^*]\ \text{atm}^{-1}$. The single-phase measurements also show that at 0°K and for low concentrations, He^3 atoms occupy (1.284 ± 0.005) times the volume occupied by He^4. In the two-phase region, the concentration of the lower phase at 0°K is found to be (6.40 ± 0.07)% He^3. The He^3 chemical potential along the solubility curve has been obtained and compared with the predictions of Bardeen, Baym, and Pines (BBP). Assuming the effective interaction originally proposed by BBP, the binding energy at 0°K of a single He^3 atom in He^4 is found to be $L_3{}^0 + R(0.284 \pm 0.010)$°K, where $L_3{}^0$ is the latent heat of pure He^3 at 0°K.

1. INTRODUCTION

The experiment described here uses measurements of the capacitance of a parallel-plate capacitor immersed in liquid He^3-He^4 solutions to determine their atomic or molar number density under the saturated vapor pressure. The principal results of the experiment are (a) The determination of the limiting solubility of He^3 in He^4 at 0°K, (b) The determination of the volume occupied by a He^3 atom in liquid He^4 at finite temperatures and at 0°K, (c) The determination of the derivative of the He^3 effective mass in solution with respect to pressure. An earlier and less accurate form of the experiment was reported some time ago[1] while a brief account of the present measurements has been published in a Letter.[2]

Most of the properties of dilute solutions can be