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Spin Echoes in Terms of Spin Waves*

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A general theory of linearized spin echoes in an interacting Fermi liquid is developed by expressing the results of the usual type of echo experiment directly in terms of the spin-wave normal modes of the liquid as derived from the Landau theory. The possibility of obtaining further information by observing the echoes produced by a spatially nonuniform magnetization is briefly discussed.

I. DISCUSSION

A characteristic property of interactions between particles in a neutral Fermi liquid is that they lead to a whole spectrum of spin-wave–like excitations in the presence of a uniform magnetic field. [1] (In metals these also mix with cyclotron modes of the system.)

For some time spin echoes have been used as a tool to investigate spin excitations in liquid He^3 and dilute He^3-He^4 solutions.[2] Recently Leggett and Rice[3] have shown that a variation of the usual spin-echo experiment should yield further information about the Fermi liquid parameters.

The question therefore arises as to the explicit relationship between spin-echo experiments and the natural spin-wave–like modes of the Fermi liquid.

In this paper we give a general derivation of this relationship valid for long waves (low field gradient) under conditions where a linear approximation in the echo magnetization may be applied.

We also show how it can be further generalized to deal with a rather idealized echo experiment in which a nonuniform polarizing pulse ("90°" pulse) could be used to study diffusion of shorter wavelength modes of the system.

The gist of the derivation is as follows: The linearized Landau kinetic equation derived by Silin[1] for a Fermi liquid in a uniform magnetic field may be solved for long and medium long waves in terms of a set of eigenmodes with frequencies $\Omega_l(q)$ of different wave numbers q and different spherical harmonic character l measuring the phase relations between precessing spins on different parts of the Fermi surface. In the spin-echo experiment a magnetic field with a small gradient along the field direction (z axis) is applied to the liquid. The effect of this field, $H_z(z)=H_0+Gz$, is to mix up modes of different wave number q, since the Gz term may be rewritten as $iG\,\partial/\partial q_z$. The effect of the mixing is to modify the time dependence of the mode q, l, from $\exp[i\Omega_l(q)t]$ to a more complicated time dependence containing a factor $\exp[\psi(q,l,t)]$. In the long-wave limit $\psi(q,l,t)$ is given directly in terms of $d^2\Omega_l(q)/dq^2$, and in fact only depends on the $l=0$ mode in this limit. For an interacting system $d^2\Omega/dq^2$ is, in general, complex, the imaginary part corresponding to the coefficient of the usual $\nabla^2 m$ term (m the magnetization) in the diffusion equation. In an echo experiment in which there is a large transverse magnetization Leggett has recently shown[4] that there will be additional contributions to the above time dependence which are nonlinear in the transverse magnetization amplitude. The present derivation only deals with the linear case, which will be valid under conditions of small transverse magnetization produced by a small-angle ($\varphi \ll 90^\circ$) polarizing rf pulse.

When one comes to consider shorter wavelength modes, a new physical consideration enters: for $q \neq 0$ the eigenmodes of the system in the uniform field now invoke mixing of different spherical harmonic components of the magnetization. Since a polarizing pulse does not distinguish particles on different parts of the Fermi surface, this means that an initial distribution of magnetization, nonuniform in space but uniform in phase over the Fermi surface, is in fact a linear combination of modes of different l. For q not too large (compared with inverse mean free path) only the $l=0$ and $l=1$ modes need be considered. However, these precess at different rates and hence would decay at different rates in a weakly nonuniform applied field. We therefore predict that under idealized conditions a sufficiently nonuniform (measured on the scale of mean free path) initial magnetization distribution will produce echoes decaying as a linear combination of a pair of modes with different diffusion coefficients. Some estimates of this effect are given at the end of the paper. The conclusion is that it is likely to be rather difficult to observe within the present state of low-temperature technology.

II. SPIN WAVES IN A UNIFORM FIELD

In order to deal with the nonuniform case, we first need explicit solutions for the spin-wave modes of the Fermi liquid in a uniform field. These have been worked out by various people[1,5,6] (in connection with the problem in metals); we here repeat the salient features that we need for the present simpler case of the neutral Fermi liquid.

Let us consider first of all the extreme longwave limit. In this limit we need only discuss the time dependence of the uniform magnetization density $\vec{m}(\vec{\Omega}, t)$ appropriate to a set of particles with velocity $\vec{v}_\Omega$ moving at an angle $\vec{\Omega}$ on the Fermi surface. The Landau kinetic equations reduce in this limit[7] to

$$\frac{d\vec{m}}{dt} - \vec{m}\times\vec{\Omega}_0 - \int \frac{d\Omega}{4\pi}\zeta(\Omega, \Omega')\vec{m}(\Omega)\times\vec{m}(\Omega') = \vec{W}, \tag{2.1}$$

where $\vec{W}$ is an appropriate collision term. The third term represents the molecular field resulting from the Landau form of the interactions between particles. We can now resolve $\vec{m}$ into circular components

$$\vec{m} = \sum_{\lambda=-1}^{+1} m_\lambda \vec{\eta}_\lambda, \tag{2.2}$$

where $\vec{\eta}_\lambda$ are circular unit vectors:

$$\vec{\eta}_{\pm 1} = (1/\sqrt{2})(\vec{\eta}_x \pm i\vec{\eta}_y), \quad \vec{\eta}_0 = \vec{\eta}_z . \tag{2.3}$$

Then for $\vec{\Omega}_0 = \beta\vec{H}_0$ along the z axis one sees from (2.1) that provided $m_\pm(\Omega)$ is not strongly dependent on $\vec{\Omega}$, $dm/dt=0$, so that we can take $m_z \equiv m_0$ as an external parameter. It is convenient to express this in terms of the equilibrium Pauli value for the noninteracting system

$$m_0^{\text{eq}} = -\tfrac{1}{2}\rho\Omega_0 \tag{2.4}$$

by setting

$$m_0 = \alpha m_0^{\text{eq}} . \tag{2.5}$$

Using

$$\zeta(\Omega, \Omega') = (1/\rho)\textstyle\sum_l Z_l P_l(\cos\theta(\vec{\Omega}, \vec{\Omega}')), \tag{2.6}$$

the equation (2.1) for $\vec{m}_\pm(\vec{\Omega}, t)$, linearized in $m_\pm$, then reads

$$dm_\pm/dt \mp i\Omega_0(1+\tfrac{1}{2}\alpha Z_0)m_\pm(\Omega) \pm\tfrac{1}{2}i\alpha\rho\Omega_0\int (dr'/4\pi)\zeta(\Omega, \Omega')m_\pm(\Omega') = W_\pm . \tag{2.7}$$

Equation (2.1) is now completely diagonalized by resolving $m_\pm(\Omega)$ into Legendre polynomials in $\theta(\Omega)$ where the polar angles θ, φ are measured with respect to the z axis

$$m_\pm(\Omega) = \sum_{l=0}^{\infty} m_\pm^{\,l} P_l(\cos\theta(\Omega)) \tag{2.8}$$

and using the spherical-harmonic addition theorem. The eigenfrequencies at $q=0$ then come out, correcting Silin's result by the factor α, to

$$\Omega_l(0) = \Omega_0[1+\tfrac{1}{2}\alpha Z_0 - \alpha Z_l/2(2l+1)] + i/\tau_l , \tag{2.9}$$

where we have assumed that the harmonic components of the collision integral may be written in terms of an appropriate spin-flip collision rate

$$(2l+1)\int W_\pm(\Omega)P_l(\Omega)d\Omega/4\pi = \pm(i/\tau_l)m_\pm^{\,l} . \tag{2.10}$$

What we need for the discussion of echoes are the solutions for $q\neq 0$. Writing

$$m_\pm^{\,l}(\vec{r}) = \textstyle\sum_{\vec{q}} e^{i\vec{q}\cdot\vec{r}} m_\pm^{\,l}(\vec{q}) , \tag{2.11}$$

the effect of $q\neq 0$ is to add a term

$$i\vec{v}_\Omega\cdot\vec{q}[\vec{m}(\Omega) + \frac{\rho}{4}\int\frac{d\Omega'}{4\pi}\zeta(\Omega, \Omega')\vec{m}(\Omega')] \tag{2.12}$$

to Eq. (2.1). The first term in (2.12) results from the straightforward motion of the particles at the Fermi surface, and the second term is a sort of backflow resulting from the interaction. Because of the angular dependence (we assume $\vec{q}$ is along the z axis as this is the only case we need for discussion of echoes)

$$\vec{v}_\Omega \cdot \vec{q} = v_F\, q P_1(\cos\theta(\Omega)). \qquad (2.13)$$

Equation (2.12) leads to a mixing between solutions $m_\pm{}^l(q=0)$ with different l, which is proportional to q. For longish waves this mixing can be very easily calculated by using nondegenerate perturbation theory.[8,9]

We write Eq. (2.12) as $K\vec{m}$ where K is an operator in Ω space whose matrix elements between normalized Lengendre polynomials

$$(2l+1)^{1/2} P_l(\cos\theta(\Omega))$$

are

$$\langle l|K|l'\rangle = iv_F q\langle l|P_1|l'\rangle\,[1+Z_{l'}/4(2l'+1)]. \qquad (2.14)$$

Then to lowest order in q the eigenfrequencies and eigenfunctions of the liquid are given by[10]

$$\Omega_l(\vec{q}) = \Omega_l(0) + \sum_{l'=l+1} \frac{\langle l|K|l'\rangle\langle l'|K|l\rangle}{\Omega_l(0)-\Omega_{l'}(0)} \qquad (2.15)$$

and

$$\mu_+{}^l(\vec{q},\Omega) = (2l+1)^{\frac{1}{2}} P_l(\Omega) + \sum_{l'=l\pm 1} \frac{\langle l'|K|l\rangle\,(2l'+1)^{1/2} P_{l'}(\Omega)}{\Omega_l(0)-\Omega_{l'}(0)}. \qquad (2.16)$$

For $l=0$ this gives to order q

$$\Omega_0(q) = \Omega_0 + \frac{\frac{1}{3}v_F{}^2 q^2(1+Z_0/4)(1+Z_1/12)}{\alpha\Omega_0(Z_0/2-Z_1/6)+i/\tau_1}, \qquad (2.17)$$

$$\mu_0(q) = \left(1 - \frac{qv_F(1+Z_0/4)}{\Omega_0(0)-\Omega_1(0)}\,P_1(\Omega)\right), \qquad (2.18)$$

and for $l=1$

$$\Omega_1(9) = \Omega_0(1+\alpha Z_0/2-\alpha Z_1/6) - \frac{\frac{1}{3}v_F{}^2 q^2(1+Z_0/4)(1+Z_1/12)}{\alpha\Omega_0(Z_0/2-Z_1/6)+i/\tau_1} + \frac{\frac{1}{15}v_F{}^2 q^2(1+Z_1/12)(1+Z_2/20)}{\alpha\Omega_0(Z_1/6-Z_2/10)+i(1/\tau_2-1/\tau_1)}, \qquad (2.19)$$

$$\mu_1(q) = \sqrt{3}P_1(\Omega) - \frac{qv_F}{\sqrt{3}}\,\frac{(1+Z_0/4)}{\Omega_1(0)-\Omega_0(0)} - \frac{qv_F}{\sqrt{15}}\,\frac{(1+Z_1/12)\sqrt{5}P_2(\Omega)}{\Omega_1(0)-\Omega_2(0)}. \qquad (2.20)$$

The important thing to notice is that for $q\neq 0$, even the $l=0$ mode mixes in some $\cos\theta(\Omega)$ dependence, so that if an $\vec{\Omega}$-independent magnetization is set up by an external field, this automatically has to be a mixture of $l=0$ and higher modes. For small q, mainly $l=1$ will be mixed in.

III. SPIN ECHOES: LONG-WAVE LIMIT

What happens in the usual spin-echo experiment is that an initial spatially uniform, polarizing pulse (which may be 90° or some other angle) rotates the equilibrium polarization m_z to produce a component $m^+(\vec{r})$ which is independent of Fermi-surface angle. This is allowed to precess in a very slightly nonuniform field $H_z(r) = H_0 + Gz$. The effect of spin diffusion is then to lead to an irreversible decay of m^+ which is separated from the reversible dephasing produced by $H_z(r)$ by use of a later 180° pulse to produce an echo. The way we study the effect of spin diffusion here is to resolve $m^+(r,t)$ into its Fourier components

$$m^+(\vec{r},t) = \sum e^{i\vec{q}\cdot\vec{r}} m^+(q,t). \qquad (3.1)$$

In the absence of the small nonuniformity, the resulting time dependence in the linear regime would be totally determined by resolving the initial $m^+(q,0)$ into its eigencomponents, or

$$m^+(q,0) = \sum_l m^+(q,l)\mu_l(q,\Omega), \qquad (3.2)$$

where the $\mu_l(q,\Omega)$ are the eigenvectors as given in (2.18) and (2.20), leading to a resulting time dependence

$$m^+(\vec{r},t) = \sum e^{i\vec{q}\cdot\vec{r}} \sum_l m^+(q,l) \times \mu_l(\vec{q},\Omega) e^{i\Omega_l(q)t}. \qquad (3.3)$$

The effect of the nonuniform field is to add a term

$$\pm\beta G(\partial m^\pm(q,t)/\partial q_z) \qquad (3.4)$$

to the equations of motion (2.7) and (2.12).

In the $q\to 0$ limit the admixture of higher modes to the $l=0$ mode tends to zero, so we only need consider the $l=0$ mode eigenfrequency:

$$m^+(\vec{r},t) = \sum_q e^{i\vec{q}\cdot\vec{r}} m^+(\vec{q}, l=0) e^{i\Omega_0(\vec{q})t}. \qquad (3.5)$$

The effect of (3.4) on (3.5) is to introduce an additional t dependence into the equation of motion since $\partial\Omega_0(q)/\partial q_z \neq 0$ which results from the mixing of modes q and $q+\delta q$.

We now show that for small field gradients G, this additional time dependence is taken care of by replacing $\exp[i\Omega_0(q)t]$ by a factor $\exp[i\psi_0(q,t)]$, so that Eq. (3.5) becomes

$$m^+(\vec{r},t) = \sum_q e^{i\vec{q}\cdot\vec{r}} \tilde{m}^+(\vec{q}, l=0, t) e^{i\psi_0(\vec{q},t)}, \qquad (3.6)$$

where

$$\psi_0(\vec{q},t) = \Omega_0(\vec{q})t - \tfrac{1}{2}\Omega_0{}'(\vec{q})g\Omega_0 t^2 + \tfrac{1}{6}\Omega_0{}''(\vec{q})g^2\Omega_0{}^2 t^3. \qquad (3.7)$$

with $g = \beta G/\Omega_0$ and $\Omega_0^{(n)}(q) = (\partial^n/\partial q_z^n)\Omega_0(q)$. Substituting (3.7) into the equation of motion (2.7) with (2.12) and (3.4) added in, and using the fact that $\Omega_0(q)$ is the eigenfrequency for (2.7) and (2.12) without (3.4), one then finds up to second order in the gradient of the applied field

$$\left(\frac{\partial}{\partial t} \pm g\Omega_0 \frac{\partial}{\partial q_z}\right) \tilde{m}(\mathrm{q}, t) = 0, \qquad (3.8)$$

so that $\tilde{m}_{\pm}(q, t)$ is completely determined in terms of the initial form $\tilde{m}(q) = \tilde{m}_{\pm}(q, t=0)$ by

$$\tilde{m}_{\pm}(\vec{\mathrm{q}}, t) = \tilde{m}(\vec{\mathrm{q}}, (q_z \pm g\Omega_0 t)). \qquad (3.9)$$

The important limit which has been taken in (3.7) is to expand $\Omega(q)$ about the particular q of interest in a Taylor series in q. In the long-wave limit $\Omega'(q) \propto q_z$, so that this term in the exponent does not contribute in the limit of a uniform magnetization for which $\tilde{m}(q) \approx \delta(q)$. In this limit the next effective term in the expansion beyond Ω'' is smaller by the ratio

$$g^2 \Omega^{(iv)}(q)(\Omega_0 t)^2/\Omega''(q), \qquad (3.10)$$

where t is the time of observation. Now $\Omega^{(iv)}/\Omega''$ is a microscopic quantity of order of magnitude p_F^2 times a dimensionless interaction strength. Hence for small field gradients (typically 1 G/cm) $(g/p_F)^2$ is of order 10^{-18}, and hence even if t is 10^6 Zeeman periods (Ω_0^{-1}) this correction will be infinitesimally small, so that the last term in (3.7) completely determines the decay of the magnetization.

Finally, on applying a uniform 180° pulse at time $t = t_{180}$, m^+ is converted to m^- so that the magnetization "winds back" following the equation of motion for m^-, and at the echo time we have

$$m(\vec{\mathrm{r}}, 2t_{180}) = \sum_{\vec{\mathrm{q}}} e^{i\vec{\mathrm{q}}\cdot\vec{\mathrm{r}}}\, m(q) \times \exp\{i[\psi(q, t_{180}) - \psi^*(q, t_{180})]\} \qquad (3.11)$$

so that in the long-wave $(q \to 0)$ limit, the echo is attenuated by the factor

$$\exp[-i(\Omega_0'' - \Omega_0''^*) g^2 \Omega_0^2 t_{180}^3]. \qquad (3.12)$$

Substituting from (2.17) we therefore find

$$\ln|m(2t_{180})/m(0)| = \tfrac{2}{3} D_{\mathrm{eff}}\, g^2 \Omega_0^2 t_{180}^3, \qquad (3.13)$$

where

$$D_{\mathrm{eff}} = \mathrm{Im}\left(\frac{\frac{1}{3} v_F^2 (1 + Z_0/4)(1 + Z_1/12)}{\alpha\Omega_0(Z_0/2 - Z_1/6) + i/\tau_1}\right). \qquad (3.14)$$

For an initial 90° pulse, all the magnetization is rotated out of the z axis so that $\alpha = 0$ and the diffusion coefficient D_{eff} is given by the usual formula

$$D_{\mathrm{eff}} = \tfrac{1}{3} v_F^2 \tau_D (1 + Z_0/4), \qquad (3.15)$$

where $\tau_D = (1 + Z_1/12)\tau_1$. (3.16)

For a pulse at angle ϕ, as pointed out by Leggett and Rice, $\alpha = \cos\theta/(1 + Z_0/4)$ and Eq. (3.14) may be rewritten as

$$D_{\mathrm{eff}} = \mathrm{Im}\left\{\frac{1}{3} v_F^2 \left(1 + \frac{Z_0}{4}\right)\left[2\Omega_0 \cos\phi \times \left(\frac{1}{1 + Z_1/12} - \frac{1}{1 + Z_0/4}\right) + \frac{i}{\tau_D}\right]^{-1}\right\}. \qquad (3.17)$$

This is in agreement with the formula for the linearized effective diffusion coefficient derived by Leggett and Rice[11] which may now be seen to be none other than the imaginary part of the coefficient of q^2 in the low-q expansion of the $l = 0$ eigenfrequency for the Fermi liquid.

IV. ECHOES PRODUCED BY A NONUNIFORM SPIN DISTRIBUTION

The theory of spin echo used in Sec. III also allows us to discuss the time development of a nonuniform magnetization. Again we limit discussion to the linearized approximation. We discuss here a highly idealized case in which the initial polarizing 90° rf has a phase in the x-y plane which is strongly spatially nonuniform along the z axis; then the resulting initial magnetization $m^+(\vec{\mathrm{r}}, t=0)$ will contain non-negligible Fourier components for $q \neq 0$. We now consider the situation where this strongly nonuniform initial magnetization is allowed to precess in the very weakly nonuniform applied field (3.4). This means we are restricted to the case of an initial 90° pulse, since any other polarizing angle would lead to a residual nonuniform molecular field which would tend to lead to rapid q mixing and decay of the echoes. In the idealized case that G is kept small, however, the time dependence of m^+ will still be of the t^3 type found in (3.7). The presence of $q \neq 0$ will now introduce the possibility of observing the coupling to the higher $l \neq 0$ spin-wave modes given in (3.2). For q not too large we consider just $l = 0$ and $l = 1$ modes. Then the ratio of $l = 1$ to $l = 0$ initial amplitudes is determined by the conditions that the initial distribution is not Ω dependent. Using Eqs. (2.18) and (2.20) this gives

$$m_1^+(\vec{\mathrm{q}}, t=0) = A_1(\vec{\mathrm{q}}) m_0^+(\vec{\mathrm{q}}, t=0), \qquad (4.1)$$

where to order q

$$A_1(\vec{\mathrm{q}}) = \frac{q v_F}{\sqrt{3}} \frac{1 + Z_0/4}{\Omega_0(0) - \Omega_1(0)}. \qquad (4.2)$$

Next it may be seen that the general solution of the Landau equations in the presence of the weakly nonuniform applied field is given by

$$m^{\pm}(\vec{\mathrm{r}}, t) = \sum_{\vec{\mathrm{q}}} e^{i\vec{\mathrm{q}}\cdot\vec{\mathrm{r}}} \sum_{l=0,1} \tilde{m}_l^+(q_z \pm g\Omega_0 t) \times e^{i\psi_l(\vec{\mathrm{q}}, t)}\, \mu_l(q_z \pm g\Omega_0 t, \vec{\Omega}), \qquad (4.3)$$

where $\psi_l(q,t)$ is as given in (3.7) with $\Omega_0(q)$ replaced by $\Omega_l(\vec{q})$.

Finally after a uniform 180° pulse at t_{180} (presumably applied by a different coil from the nonuniform polarizing coil), an echo will be produced whose magnitude does not depend on $\vec{\Omega}$, so the average over $\vec{\Omega}$ of the value is predicted by Eq. (4.3), denoted by $\langle m^+(r, 2t_{180})\rangle$. This is given by

$$\langle m^+(\vec{r}, 2t_{180})\rangle = \sum_{\vec{q}} e^{i\vec{q}\cdot\vec{r}}\,\tilde{m}_0{}^+(\vec{q})$$

$$\times\{e^{i(\psi_0-\psi_0{}^*)} + |A_{\vec{q}}|^2 e^{i(\psi_1-\psi_1{}^*)}\}, \quad (4.4)$$

where A_q is given in (4.2). Thus the admixture of time dependence due to the $l=1$ mode is proportional to $(ql)^2$ where $l\approx v_F\tau_D$ is the mean free path for spin-flip scattering. So for this effect to be observed, the phase nonuniformity of the polarizing pulse needs to vary appreciably over a length of order l. Under these conditions the time dependence due to the $\Omega' t^2$ term in ψ given in Eq. (3.7) will start to become appreciable if $(q/g)\Omega_0 t_{180}$ is of order unity, i.e., it will depend on the effective field uniformity achieved and the ratio of echo time t_{180} to rf period $\Omega_0{}^{-1}$.

Unfortunately, l is pretty small even in the millidegree range. The most favorable case appears to be a 5% solution of He^3 in He^4 for which l is of order 0.5 mm at 1 mdeg. Thus polarizing fields with a phase nonuniformity of about this order (i.e., a large phase change in a mm or so) would be required to see the $l=1$ mode mixing effect. For pure He^3 the mean free path is about 100 times shorter at this temperature.

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where ρ is the density of states at the Fermi surface and $\vec{\sigma}(p,r,t)$ is the spin density for quasiparticles of momentum p, defined by Silin.

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$$\begin{pmatrix} \omega-\Omega_0 & \frac{9v_F}{\sqrt{3}}\left(1+\frac{Z_1}{12}\right) \\ \frac{9v_F}{\sqrt{3}}\left(+\frac{Z_0}{4}\right) & \omega-\Omega_1 \end{pmatrix}\begin{pmatrix}\mu_0\\ \mu_1\end{pmatrix} = 0$$

with solutions

$$\omega = \tfrac{1}{2}(\Omega_0+\Omega_1) + [\tfrac{1}{4}(\Omega_0-\Omega_1)^2 + X]^{1/2},$$

where

$$X = \tfrac{1}{3}9^2 v_F{}^2(1+Z_0/4)(1+Z_1/12).$$

[11]Using (2.4), (2.5), and the formula for α, Leggett and Rice's Ω_p reduces to $2\Omega_0\cos\phi$.