incentive for more extensive measurements in pion-nucleon, kaon-nucleon, and nucleon-nucleon reactions, in high-energy photoproduction, as well as in a number of interesting nuclear reactions belonging to what has come to be called intermediate-energy physics.

The idea of a phase-band analysis occurred to me while attending the Conference on Pion-Nucleon Scattering at Irvine. I am grateful to the University of California at Irvine, and to Gordon Shaw in particular, for organizing this stimulating meeting. I had a number of very valuable discussions about this problem with Paul Csonka. I am also grateful to David L. Bridges for a rapid and imaginative performance of the numerical computations, and to the Statistical Laboratory and Computing Center of the University of Oregon for cooperation.

PHYSICAL REVIEW VOLUME 177, NUMBER 5 25 JANUARY 1969

Self-Consistent Multiple-Quark-Scattering Analysis and Its Application to Elastic pp Scattering*

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(Received 22 July 1968)

We discuss a viewpoint for interpreting elastic scattering of hadrons in terms of a picture in which the hadrons behave as if they are comprised effectively of A distinct subparticles Q which contribute essentially individually to multiple-internal-scattering processes within the hadrons. The formalism of this multiple-internal-scattering picture is developed and applied to the analysis of elastic pp scattering. The discontinuities in the slope of the pp differential cross section at large momentum transfers are interpreted as transitions between domains of momentum transfer that are dominated by successively higher-order multiple-scattering contributions. The structure of the pp cross section is fitted in good detail with a self-consistent analysis that circumvents the necessity for conjectures about the wave functions of internal motion of the subparticles Q within p by exploiting simpler and more direct conditions and conjectures on the effective generalized form factors and scattering amplitudes. The higher-order multiple-scattering contributions are self-consistently calculated in terms of the effective QQ scattering amplitude determined from the region of the first slope. The analysis distinguishes among subparticle models to yield an essentially exclusive fit to the experimental data with the value of the quark number of the proton $A=3$, thereby affording corroborative evidence in favor of the SU_3 quark model from a non-group-theoretical, dynamical basis. Our results are compatible with quarks of very small, or even pointlike, spatial extension as compared to the effective electromagnetic radius of the proton.

I. INTRODUCTION

ELASTIC pp (proton-proton) scattering experiments indicate that the differential cross section at sufficiently high energies tends toward a characteristic structure that is more or less energy-independent, suggestive of a semiclassical regime where diffractive effects are perhaps dominant. The structure of the differential cross section shows, in addition to a narrow forward diffraction peak, several succeeding domains at larger momentum transfers in each of which the cross section decreases at a successively slower rate.[1] The transitions between these successive domains are rather abrupt, suggestive of contributions from different orders of physical processes. All hadron elastic-scattering cross sections share these principal characteristics to some degree and our analysis should apply generally as well as to the pp case. We discuss here a viewpoint for interpreting these elastic scattering data in terms of the proton, or other hadron, behaving as if it were comprised effectively of A internal subnucleonic particles Q which contribute effectively individually to multiple-scattering processes within the proton. The successive domains of the differential cross section are to be identified as contributions of successive orders of multiple internal diffraction scattering of the Q's.

Antecedents of the Multiple-Internal-Scattering Picture

Several antecedent analyses have contributed to the suggestion of the multiple-internal-scattering picture: (i) The differential cross sections for elastic scattering of high-energy protons from light nuclei also exhibit breaks in the angular distributions that are similar to those in pp scattering, and recently these data have been successfully analyzed in terms of multiple internal nucleon-nucleon diffraction scattering within the nucleus.[2] (ii) Analysis of several hadronic total cross sec-

* Supported in part by National Science Foundation, Grant No. GP-8924. Computer facilities supported by National Science Foundation, Grant No. G-22294.
† National Science Predoctoral Fellow.
[1] A. D. Krisch, Phys. Rev. Letters **19**, 1149 (1967). Further references to pp experiments and analyses may be found from this reference.

[2] R. H. Bassel and C. Wilkin, Phys. Rev. Letters **18**, 871 (1967); W. Czyz and L. Lesniak, Phys. Letters **24B**, 227 (1967).

tions in terms of the nonrelativistic SU_3-quark model in which hadrons are viewed as prescribed bound states of SU_3 quarks (and antiquarks), each of which contributes additively certain quark-quark total cross sections, has been discussed by Franco.[3] Franco concludes that multiple-scattering contributions dominate the pn and pK^+ total cross sections. Furthermore, he suggests that triple- and higher-order multiple-scattering contributions are significant. (iii) Analyses of the high-energy elastic pp differential cross section tend to confirm the conjecture of Wu and Yang that it should be proportional, at least at low-momentum transfers, to the fourth power of the electromagnetic form factor of the proton.[4] This suggests that the effective distributions of the source densities of electromagnetic and hadronic forces within the proton are quite similar. (iv) The main characteristics of all these antecedent analyses are present in the multiple-internal-scattering picture of high-energy elastic collisions of hadrons, and this picture itself has already lead to interesting consistency relations among various hadron form factors and diffraction-scattering cross sections from which the pion form factor and $\pi\pi$ diffraction-scattering cross section have been determined.[5] The results of analysis of these consistency relations in the first-order, or single-quark-scattering approximation are in good agreement with other theoretical predictions and the present experimental evidence that the pion form factor is similar to that of the proton and that the $\pi\pi$ diffraction-scattering cross section is much like that of πp diffraction scattering.[6]

Prospects for the Multiple-Scattering Analysis

The intimacy of the multiple-internal-scattering analysis proposed here and the compound quark model is self-evident.[7] The subparticles of our analysis, which we carefully refer to for now only with the label Q, may have to be quarklike since their number A is definite, however, no specific details of the individual Q need be presupposed for now: (i) Although we shall ultimately relate to, and build upon, the SU_3-quark model, it should be possible with this analysis to discriminate among composite subparticle models in general by determining the effective number A of subparticles per hadron. (ii) It should be possible also to determine the effective differential cross sections for quasi-free QQ scattering. (iii) This analysis should furthermore yield information on the effective size and binding of the Q's, since it involves body form factors of the distributions of the Q's within the hadrons that can be compared with the electromagnetic form factors of the hadrons in order to infer information about the electromagnetic and hadronic form factors of the individual Q's themselves. In this paper we attempt to apply our analysis in all these ways to the case of pp scattering for which the most extensive and detailed information is available. In the sequel we will apply this analysis within the context of the SU_3-quark model to the analysis of, and correlations among pp, $p\bar{p}$, πp, $\pi\pi$, Kp, etc. scattering and form-factor data.

Survey of Presentation and Results

The fundamental formalism of the multiple-internal-scattering analysis and the dynamical assumptions upon which it is based are assembled and specified in Sec. II. In Sec. III is discussed the self-consistency of our calculational method whereby the necessity for conjecture about the details of the internal wave functions is circumvented through exploitation of simpler and more direct conditions and conjectures on the more directly and physically relevant phenomenological form factors and scattering amplitudes. The results of the application of our analysis to pp scattering are presented and discussed in Sec. IV and their consequences are discussed in Sec. V.

The results indicate support for the SU_3-quark model from a dynamical, but non-group-theoretical basis in that impressive agreement with the experimental data is obtained with $A=3$ for the proton while $A=2$ is absolutely excluded by the data and $A=4$, or greater, is probably excluded. The results are consistent with the picture of small, loosely bound quarks; quarks as large as half the size of the proton in effective electromagnetic radius are excluded and very small, or even pointlike quarks are compatible. The fundamental assumption of multiple-internal-diffraction scattering is supported by obtaining an estimate that the effects of intermediate excited states between internal scatterings do not seriously alter the main results. The assumption of the diffractive nature of the elementary scattering processes is, on the other hand, somewhat challenged by the fact that complete diffractive minima are not observed in the cross section and rather strong momentum-transfer-dependence of the phase of the elementary scattering amplitude is required in order to fit the data near the breaks between different multiple-scattering orders. Improvements of these and some other details of the multiple-scattering picture and broader implications and extensions of its domain of applicability are discussed in Sec. V.

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[3] V. Franco, Phys. Rev. Letters **18**, 1159 (1967).

[4] T. T. Wu and C. N. Yang, Phys. Rev. **137**, B708 (1965).

[5] E. Shrauner, Phys. Rev. Letters **20**, 1258; **20**, 1550(E) (1968).

[6] These results seem to be changed to about half this large a pion radius and about twice this broad a $\pi\pi$ diffraction peak as consequences of analyzing the consistency relations with higher-order multiple-quark-scattering effects instead of only first-order or single-quark scattering as in Ref. 5; E. Shrauner, L. Benofy, and D. W. Cho, sequel to Ref. 5 and the present work (to be published).

[7] The multiple-scattering picture within the context of the SU_3-quark model has been considered by D. R. Harrington and A. Pagnamenta, Phys. Rev. Letters **18**, 1147 (1967); E. Shrauner, Bull. Am. Phys. Soc. **13**, 49 (1968), and Ref. 5 above; E. Shrauner, L. Benofy, and D. W. Cho, Ref. 6.

II. FORMULATION OF MULTIPLE-INTERNAL-SCATTERING ANALYSIS

Assumptions

In addition to the basic assumption that large-angle hadron scattering be viewed as the cumulative effect of multiple internal scatterings of constituent Q's, we also assume that the probability distribution for each single QQ scattering is so diffractively narrow that longitudinal momentum transfers may be ignored in first approximation. We assume that we may ignore or average over spin and isospin effects as well as effects on the basic hadronic interactions due to internal motions of Q's within the initial and final states of the hadron.

Formalism

A formalism suitable for our analysis under these assumptions may be obtained as a generalization of that applied to high-energy proton-deuteron scattering by Franco and Glauber.[8] These authors have described the details of the development of the basic multiple-scattering formalism and we recapitulate here only enough of that development to establish the notation and the general picture for our generalized multiple-scattering formalism.

Basic QQ scattering at very high energy is described by the semiclassical scattering amplitude in the impact-parameter representation by

$$f(\mathbf{q})=\frac{i}{2\pi}\int e^{i\mathbf{q}\cdot\mathbf{b}}(1-e^{i\chi(\mathbf{b})})d^2b\,, \tag{1}$$

where $\mathbf{q}$ is the transverse momentum transfer, $\mathbf{b}$ is the impact parameter, and $\chi(\mathbf{b})$ is the high-energy generalization of the phase shift $2\delta_l$. The amplitude $f(\mathbf{q})$ is normalized so that the differential cross section is

$$d\sigma/dt=\pi|f(\mathbf{q})|^2,\quad t\simeq-\mathbf{q}^2. \tag{2}$$

Formula (1) shows $f(\mathbf{q})$ as the Fourier transform of the profile function, $\Gamma(\mathbf{b})$:

$$\Gamma(\mathbf{b})\equiv\frac{1}{2\pi i}\int e^{-i\mathbf{q}\cdot\mathbf{b}}f(\mathbf{q})d^2q\,. \tag{3}$$

Scattering of a single Q from a composite of A similar Q's is obtained under the assumption that the cumulative generalized phase shift and amplitude change is given by[8]

$$\begin{aligned} e^{i\chi(\mathbf{b},\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3,\cdots\mathbf{s}_A)}&\equiv1-\Gamma(\mathbf{b},\mathbf{s}_1,\mathbf{s}_2,\cdots\mathbf{s}_A)\,,\\ &\cong e^{i\chi(\mathbf{b}-\mathbf{s}_1)+i\chi(\mathbf{b}-\mathbf{s}_2)\cdots i\chi(\mathbf{b}-\mathbf{s}_A)}\cong\prod_{\alpha=1}^{A}[1-\Gamma(\mathbf{b}-\mathbf{s}_\alpha)]\,, \end{aligned} \tag{4}$$

[8] V. Franco and R. J. Glauber, Phys. Rev. **142**, 1195 (1966). This reference contains further references to Glauber's development of multiple-scattering analysis.

where $\mathbf{b}$ is the impact parameter of the single Q relative to the center of mass (c.m.) of the composite A, and $\mathbf{s}_\alpha$ is the projection transverse to the collision axis of the locus vector $\mathbf{r}_\alpha$ of the αth constituent Q of A relative to the c.m. of A. Writing for now $\varphi_a(\{\mathbf{r}_\alpha\})$ and $\varphi_b(\{\mathbf{r}_\alpha\})$ for the initial (a) and final (b) effective internal wave-function amplitudes for the probability of the αth constituent Q to be at the location $\mathbf{r}_\alpha$ relative to the c.m. of A, and then after integrating and dividing out the overall c.m. factor, $(2\pi)^3\delta^3(\mathbf{P}_a-\mathbf{P}_b-\mathbf{q})$, we have

$$\begin{aligned} f_{ba}(\mathbf{q})=\frac{i}{2\pi}\int e^{i\mathbf{q}\cdot\mathbf{b}}\varphi_b^*\\ \times\left\{1-\prod_{\alpha=1}^{A}\left[1-\frac{1}{2\pi i}\int e^{-i\mathbf{q}_\alpha\cdot(\mathbf{b}-\mathbf{r}_\alpha)}f(\mathbf{q}_\alpha)d^2\mathbf{q}_\alpha\right]\right\}\\ \times\varphi_a d\tau_A d^2b\,, \end{aligned} \tag{5}$$

where

$$d\tau_A\equiv\delta^3\left(\frac{1}{A}\sum_{\alpha=1}^{A}\mathbf{r}_\alpha\right)\left(\prod_{\alpha=1}^{A}d^3\mathbf{r}_\alpha\right),$$

$\mathbf{q}_\alpha$ is the momentum transfer at the internal scattering with the αth constituent Q of A, and $\mathbf{q}_\alpha\cdot(\mathbf{b}-\mathbf{s}_\alpha)=\mathbf{q}_\alpha\cdot(\mathbf{b}-\mathbf{r}_\alpha)$ because $\mathbf{s}_\alpha$ is the transverse projection of $\mathbf{r}_\alpha$ and $\mathbf{q}_\alpha$ is also transverse. Here $\mathbf{b}$ is the impact parameter of the single Q relative to the c.m. of A. We call attention to the fact that $\mathbf{r}_\alpha$ enters only through the integral

$$\begin{aligned} \Phi_{Aba}(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3,\cdots q_A)\equiv\int\exp\Big(i\sum_{\alpha=1}^{A}\mathbf{q}_\alpha\cdot\mathbf{r}_\alpha\Big)\\ \times\varphi_b^*(\{\mathbf{r}_\alpha\})\varphi_a(\{\mathbf{r}_\alpha\})d\tau_A \end{aligned} \tag{6}$$

which is the generalized many-Q-body (transition) form factor of A.

Scattering of a composite of A Q's from a composite of B similar Q's is easily obtained because each constituent Q of B has the same amplitude $f_{ba}(\mathbf{q})$ above for scattering from the composite A. Letting primed variables refer to the internal coordinates of B and unprimed ones for A, we obtain by merely putting (5) back into itself:

$$\begin{aligned} f_{ba;b'a'}(\mathbf{q})=\frac{i}{2\pi}\int e^{i\mathbf{q}\cdot\mathbf{b}'}\varphi_{b'}'^*\\ \times\left\{1-\prod_{\beta=1}^{B}\left[1-\frac{1}{2\pi i}\int e^{-i\mathbf{q}_\beta'\cdot(\mathbf{b}'-\mathbf{r}_\beta')}f_{ba}(\mathbf{q}'_\beta)d^2q_\beta'\right]\right\}\\ \times\varphi'_{a'}d\tau_B'd^2b'\,, \end{aligned} \tag{7}$$

where $\mathbf{q}_\beta'$ is the momentum-transfer to the βth Q in B and $\mathbf{b}'$ is the impact parameter between the c.m. of B and the c.m. of A. Doing the trivial integrations over $\mathbf{b}$ and $\mathbf{b}'$ and expanding the binomial products shown in the integrands of (5) and (7), we have

$$f_{ba;b'a'}(\mathbf{q})=\int \sum_{\beta=1}^{B}\binom{B}{\beta}\left(\frac{i}{2\pi}\right)^{\beta-1}$$
$$\times\Phi_{Bb'a'}(\mathbf{q}_1',\mathbf{q}_2',\cdots\mathbf{q}_\beta',0,0,\cdots)\delta^2(\mathbf{q}-\sum_{n=1}^{\beta}\mathbf{q}_n')$$
$$\times\prod_{n=1}^{\beta}\left[d^2q_n'\sum_{\alpha=1}^{A}\binom{A}{\alpha}\left(\frac{i}{2\pi}\right)^{\alpha-1}\Phi_{Aba}(\mathbf{q}_1,\mathbf{q}_2,\cdots\mathbf{q}_\alpha,0,\cdots)\right.$$
$$\left.\times\delta^2(\mathbf{q}_n'-\sum_{m=1}^{\alpha}\mathbf{q}_m)(\prod_{m=1}^{\alpha}f(\mathbf{q}_m)d^2\mathbf{q}_m)\right].\quad(8)$$

The multiple-scattering series is obtained by recombining the product of sums in (8) into a sum of terms ordered according to the numbers of $f(\mathbf{q})$ factors,

$$f_{ba;b'a'}(\mathbf{q})\equiv\sum_{n=1}^{AB}f^{(n)}(\mathbf{q})\quad(1\le n\le AB).\qquad(9)$$

The single-scattering contribution is simply

$$f^{(1)}(\mathbf{q})=AB\Phi_{Aba}(\mathbf{q},0,\cdots0)\Phi_{Bb'a'}(\mathbf{q},0,\cdots0)f(\mathbf{q}).\quad(10)$$

The double-scattering contribution is

$$f^{(2)}(\mathbf{q})=\frac{i}{2\pi}\int d^2q_1f(\mathbf{q}_1)f(\mathbf{q}-\mathbf{q}_1)$$
$$\times\left\{B\binom{A}{2}\Phi_{Bb'a'}(\mathbf{q},0,\cdots)\Phi_{Aba}(\mathbf{q}-\mathbf{q}_1,\mathbf{q},0,\cdots)\right.$$
$$+A^2\binom{B}{2}\Phi_{Bb'a'}(\mathbf{q}-\mathbf{q}_1,\mathbf{q}_1,0,\cdots)\sum_\gamma\Phi_{Ab\gamma}(\mathbf{q}_1,0,\cdots)$$
$$\left.\times\Phi_{A\gamma a}(\mathbf{q}-\mathbf{q}_1,0,\cdots)\right\}.\quad(11)$$

The expressions (8) and (11) are not manifestly symmetric under interchange of target and projectile, $A\rightleftharpoons B$. However, this symmetry may be recovered simply by performing the sum over intermediate states as represented by γ in (11). This operation is made transparent by putting the generalized form factors into the explicit form (6) and using the completeness of the set of states represented by the wave-functions $\{\varphi_\gamma\}$. For example,

$$\sum_\gamma\Phi_{Ab\gamma}(\mathbf{q}_1,0,\cdots)\Phi_{A\gamma a}(\mathbf{q}-\mathbf{q}_1,0,\cdots)$$
$$=\frac{1}{A}\Phi_{Aba}(\mathbf{q},0,\cdots)+\frac{A-1}{A}\Phi_{Aba}(\mathbf{q}_1,\mathbf{q}-\mathbf{q}_1,0,\cdots).\quad(12)$$

This reduction of such products of the generalized form factors of a given hadron summed over the complete set of intermediate states into a linear sum of the generalized form factors of that hadron will be referred to later when we truncate the sums over intermediate states at the ground states in order to estimate the effects of the dynamical contributions of the excited states.

The symmetrized form of the double-scattering contribution is

$$f^{(2)}(\mathbf{q})=\frac{i}{2\pi}\int d^2q_1f(\mathbf{q}_1)f(\mathbf{q}-\mathbf{q}_1)$$
$$\times\left\{\left[\binom{B}{1}\binom{A}{2}\Phi_{Bb'a'}(\mathbf{q},0,\cdots)\Phi_{Aba}(\mathbf{q}-\mathbf{q}_1,\mathbf{q}_1,0,\cdots)\right.\right.$$
$$+\binom{B}{2}\binom{A}{2}\Phi_{Bb'a'}(\mathbf{q}-\mathbf{q}_1,\mathbf{q}_1,0,\cdots)$$
$$\left.\left.\times\Phi_{Aba}(\mathbf{q}-\mathbf{q}_1,\mathbf{q}_1,0,\cdots)\right]+[A\rightleftharpoons B]\right\}.\quad(13)$$

The triple-scattering contribution is similarly

$$f^{(3)}(\mathbf{q})=\left(\frac{i}{2\pi}\right)^2\int d^2q_1d^2q_2f(\mathbf{q}_1)f(\mathbf{q}_2)f(\mathbf{q}-\mathbf{q}_1-\mathbf{q}_2)\left\{\left[\binom{B}{1}\binom{A}{3}\Phi_{Bb'a'}(\mathbf{q},0,\cdots)\Phi_{Aba}(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}-\mathbf{q}_1-\mathbf{q}_2,0,\cdots)\right.\right.$$
$$+6\binom{B}{2}\binom{A}{3}\Phi_{Bb'a'}(\mathbf{q}_1,\mathbf{q}-\mathbf{q}_1,0,\cdots)\Phi_{Aba}(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}-\mathbf{q}_1-\mathbf{q}_2,0,\cdots)$$
$$+3\binom{B}{3}\binom{A}{3}\Phi_{Bb'a'}(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}-\mathbf{q}_1-\mathbf{q}_2,0,\cdots)\Phi_{Aba}(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}-\mathbf{q}_1-\mathbf{q}_2,0,\cdots)$$
$$\left.\left.+2\binom{B}{2}\binom{A}{2}\Phi_{Bb'a'}(\mathbf{q}_1,\mathbf{q}-\mathbf{q}_1,0,\cdots)\Phi_{Aba}(\mathbf{q}_2,\mathbf{q}-\mathbf{q}_2,0,\cdots)\right]+[A\rightleftharpoons B]\right\}\quad(14)$$

and so on for the contributions of higher-order multiple scattering.

III. SELF-CONSISTENCY

Self-Consistency of the Form Factors

It has already been noted that the formulas of our analysis require explicitly only the effective generalized form factors Φ, and not the wave functions which contain all possible physical information—considerably more detail than we need for our analysis. As is the case for many of the most directly interesting applications of composite subparticle models, such as quark models, the complete wave function is not the thing that is directly required in our formalism. This is fortunate, because otherwise exchange symmetry often forces such severe restrictions on the allowed wave functions that they yield highly structured form factors in contradistinction to experiment.[9] Therefore, in the absence of extensive other independent information, it seems preferable to attempt to conjecture about the behavior of the effective generalized form factors that are directly involved in this analysis rather than to attempt to conjecture about the more detailed wave functions which are much less directly and less completely indicated by the available phenomenological information. The generalized form factor Φ in the formula (6) is a function of $A-1$ variables and should be mathematically equivalent via Fourier transformation to the inner product of the wave functions averaged over all other degrees of freedom except the spatial variables. There seems to be no need to try to account for variables that are completely averaged out or are otherwise inconsequential in the semiclassical high-energy regime. Only the *effective* form factors of the average spatial distributions are required for our semiclassical analysis. Just as we ultimately would with the wave functions, we rely upon the self-consistency as tested by agreement with experiment to justify our conjectures about the form factors.

There is a certain amount of other independent information that can be incorporated and built upon. If (generalized) charge independence is a good symmetry of the Q's so that they are all equivalent, then the many-Q-body form factors Φ must be symmetric with respect to exchanges among the Q's comprising a given hadron. For elastic scattering, the initial and final state of the hadron are the ground state, and so $a=b$ will be dropped from Φ_{Aba}. The single-Q-body form factor of the hadron that is required for our analysis is related to its usual electromagnetic form factor $G_A{}^E$ as

$$G_A{}^E(\mathbf{q})=\Phi_A(\mathbf{q},0,\cdots)G_Q{}^E(\mathbf{q}), \tag{15}$$

where $G_Q{}^E(\mathbf{q})$ is the electromagnetic form factor of Q. Referring to the single-scattering amplitude (10), it is immediately apparent in exactly what way the multiple-subparticle-scattering picture manifests the conjecture of Wu and Yang that cross sections for the elastic scattering of hadrons should be proportional to the squares of their electromagnetic form factors.[4]

The degree of success obtained in comparisons of the Wu-Yang conjecture of the proportionality (at low momentum transfers) of the fourth power of the electromagnetic form factor of the proton with the differential cross section for elastic pp scattering indicates a strong similarity between the effective distributions within the proton of the source densities of electromagnetic and hadronic forces:

$$G_p{}^E(\mathbf{q})\cong G_p{}^H(\mathbf{q}). \tag{16}$$

This means in terms of our analysis that the electromagnetic and hadronic source densities within Q are also similar [see Eq. (15) above]:

$$G_Q{}^E(\mathbf{q})\cong G_Q{}^H(\mathbf{q}). \tag{17}$$

The form factors of Q itself do not appear explicitly in our formalism, although the $G_Q{}^H(\mathbf{q})$ are presumably incorporated within the effective QQ scattering amplitude $f(\mathbf{q})$ which we use directly. Still it will be seen that some information about the form factors of the Q's can be inferred from our analysis.

Choice of the generalized form factor Φ is made in such a way as to enable adjustment for the size of the Q continuously to the limit of point Q's by the variation of a single parameter. The single-Q form factor becomes equal to the observed electromagnetic form factor in the limit of point Q's for which $G_Q{}^E(\mathbf{q})=G_Q{}^H(\mathbf{q})=1$. Since the observed electromagnetic form factor of the proton is well fitted by the Wilson-Hofstadter dipole function,[10] we choose the parameterization

$$\Phi_p(\mathbf{q},0,\cdots)=(1+\mathbf{q}^2/\mu^2)^{-2}, \tag{18}$$

for which the point-Q limit corresponds to the value of the parameter $\mu^2=0.71$ (BeV/c)^2 and larger values of μ^2 correspond to larger, nonpoint Q's. The generalized many-Q form factor, as given by the formula (6), satisfies translation invariance in $\mathbf{q}$ space, which means that the most general effective many-Q form factor is a function of $A-1$ independent $\mathbf{q}$ variables symmetric under exchange of any pair of Q labels, and it is equal to the single-Q form factor (18) for the case where only one $\mathbf{q}_n$ is nonzero. We choose the simplest extension of the single-Q case (18) for the effective generalized many-Q form factors

$$\Phi_p(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3,\cdots\mathbf{q}_A)=\left[1+\frac{1}{A-1}\sum_{n>m}^{A}(\mathbf{q}_n-\mathbf{q}_m)^2/\mu^2\right]^{-2}. \tag{19}$$

Checks of the conjectured form factors may be attempted in several ways. Most importantly, of course, they should lead to reasonable fits to the pp scattering data. The parameter μ^2 in the form factor (19) may be varied from the value $\mu^2=0.71$ (BeV/c)^2 corresponding to effectively pointlike Q's to larger values corresponding to larger nonpointlike Q's in order to attempt to infer the effective size of the Q's through relations such as (15). If the sums over intermediate states in such formulas as, for example, (11) and (12) were truncated by omitting all states in the sums except the ground state, the results can be compared to the results with the complete summation in order to estimate the importance of the dynamical contributions of the omitted

[9] Difficulties in reconciling tractable quark wave functions with observed form factors have been discussed by R. H. Dalitz, in *Proceedings of The Thirteenth International Conference On High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, Calif., 1967); and R. E. Kreps and J. J. de Swart, Phys. Rev. **162**, 1729 (1967).

[10] W. Albrecht *et al.*, Phys. Rev. Letters **17**, 1192 (1966).

intermediate excited states and thereby check the basic dynamical assumption of multiple-internal-diffraction scattering.

B. Self-Consistency of QQ Scattering Amplitude f

Our formalism involves the as yet unknown effective QQ scattering amplitude f. Our self-consistent program is essentially to analyze the first slope of the pp data according to the leading single-scattering contribution to the scattering amplitude $f^{(1)}$ [see Eq. (10)], and then with the effective QQ amplitude determined from the first slope, calculate the higher-order multiple-scattering contributions self-consistently in terms of this f. The QQ scattering amplitude f determined this way is a functional, not only of the phenomenological pp scattering in the region of the first slope, but of the conjectured form factors Φ and, actually, of the double- and triple-scattering contributions in the region of the first slope. The effect that results from this more completely consistent determination of f is a compensation that is roughly equivalent to about a 5% decrease of the observed effective first slope, $(d/dt)[\ln(d\sigma/dt)]$ at $t=0$, and a 5% increase of the observed value of the differential cross section in the forward direction (or, of the total cross section) that would be used in the corresponding naive fitting to just the single-scattering contribution. As mentioned before, the f so determined represents the full, effective QQ scattering amplitude and there is no need to consider separately the hadronic form factor of Q because its effects are already included in f.

We parametrize the effective scattering amplitudes in the region of the first slope as

$$f_{pp}^{(1)}(\mathbf{q})\cong(i-\alpha_{pp})\frac{\sigma_{pp}}{4\pi}e^{-\frac{1}{2}\xi\mathbf{q}^2}, \tag{20}$$

with $\mathrm{Re}\xi\cong 9-10$ $(\mathrm{BeV}/c)^{-2}$, $\alpha_{pp}\cong 0.2$–0.3 $\sigma_{pp}\cong 35$–40 mb for pp scattering. The QQ amplitude is obtained by inverting Eq. (10) to obtain

$$f(\mathbf{q})\cong A^{-2}\Phi_A{}^{-2}(\mathbf{q},0,\cdots 0)\frac{\sigma}{4\pi}(i-\alpha)e^{-\frac{1}{2}\xi\mathbf{q}^2}, \tag{21}$$

where the parameters $\mathrm{Re}\xi$, α, and σ are near the values for f_{pp} in (20), except for the small multiple-scattering corrections near the forward angle mentioned above, and ξ must have a rather large imaginary part in order to obtain agreement with the absence of observed interference minima in the cross section.

IV. RESULTS FOR pp SCATTERING AND COMPARISON WITH DATA

The self-consistent multiple-diffraction-scattering analysis described above has been applied to elastic pp scattering at high energies. Our semiclassical formalism applies to an energy-independent regime at high energies. Krisch[1] has plotted all the pp elastic cross-section data as a function of $\beta^2\mathbf{q}_\perp{}^2$, the square of the collision velocity times the transverse momentum transfer in the c.m. frame, and shown that $d\sigma/dt$ as a function of $\beta^2\mathbf{q}_\perp{}^2$ is nearly independent of energy. The energy dependence is not completely eliminated in the plot of $d\sigma/dt$ *versus* $\beta^2\mathbf{q}_\perp{}^2$, but the approximate energy independence is notable.[11] Plotting $d\sigma/dt$ *versus* $\beta^2\mathbf{q}_\perp{}^2$ is a convenient way of removing the energy-dependent shrinkage of the width of the diffraction peak toward the lower end of this energy regime. The variable $\beta^2\mathbf{q}_\perp{}^2$ becomes equal to $\mathbf{q}_\perp{}^2$ at high energies and $\mathbf{q}_\perp{}^2$ becomes equal to $|t|$, the square of the 4-momentum transfer, for finite momentum transfers at high energies. The results of our analysis can be compared with the experimental plot of $d\sigma^\dagger/dt$ *versus* $\beta^2\mathbf{q}_\perp{}^2$ by understanding the single variable $\mathbf{q}^2$ $(=\mathbf{q}_\perp{}^2)$ where it appears in both sides of the equations

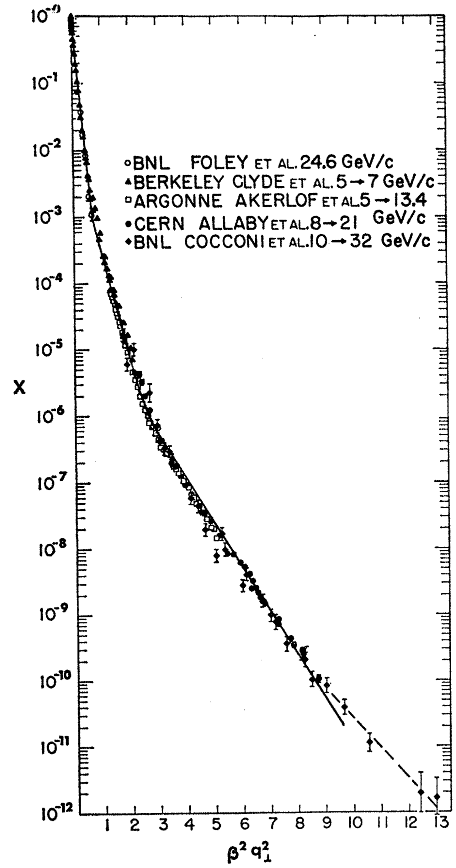

FIG. 1. Comparison with experiment of the calculated cross section; $X\equiv(d\sigma^\dagger/dt)/(d\sigma^\dagger/dt)_{t=0}$ *versus* $\beta^2\mathbf{q}_\perp{}^2$. The parameters of the calculated curve here are $\xi=(9.8-i13.0)$ $(\mathrm{BeV}/c)^{-2}$, $\mu^2=0.71$ $(\mathrm{BeV}/c)^2$, and $A=3$. The characteristic structure of the cross section is so well fitted with the third-order calculation shown here that we have taken the liberty of drawing in also the dashed extension of the curve to suggest the way in which fourth-order multiple-scattering contributions may dominate in a domain of momentum transfers greater than $\beta^2\mathbf{q}_\perp{}^2\cong 9$ $(\mathrm{BeV}/c)^2$.

[11] Krisch, Ref. 1, plots cross-section data that have been corrected for identical-particle effects, thereby obtaining somewhat better energy independence of data near 90 deg. Using his notation, we denote the corrected cross section by $d\sigma^\dagger$.

for our multiple-scattering amplitudes to be $\beta^2 \mathbf{q}_\perp^2$. This allows us to compare our results with data that includes some that perhaps belong to energies below the proper energy-regime for our analysis.

The calculations were all performed by numerical integration on an electronic computer.

Figure 1 shows the results of our analysis compared with the experimental data as used by Krisch.[11] Clearly, the main characteristics of the data are well fitted by the calculated curves. Particularly interesting is the fact that for almost all values of the parameters used for our analysis the breaks, as well as the slopes, of the cross section occur very near to the experimental ones in the plot of $d\sigma^\dagger/dt$ *versus* $\beta^2 \mathbf{q}_\perp^2$. This tends to corroborate the fundamental basis of our analysis and also the pertinence of the transverse momentum-transfer variable $\mathbf{q}_\perp^2$. Only the contributions of single, double, and triple scattering were included in our calculations presented here. The leveling trend at the highest momentum transfers may suggest significant contributions from fourth-order multiple internal scattering. The third-order calculation has saturated our present calculational capacities, and so the fourth-order effects will have to be approximated and reported later. It will be interesting to see if a definite fourth-slope region can be found, both theoretically and experimentally. It should be mentioned here that multiple-scattering characteristics do not necessarily imply a finite number of discrete subparticles; similar effects have recently been discussed within the contexts of continuous distributions of scattering matter.[12]

Figure 2 shows the A dependence that is characteristic of our analysis. As a determination of A our analy-

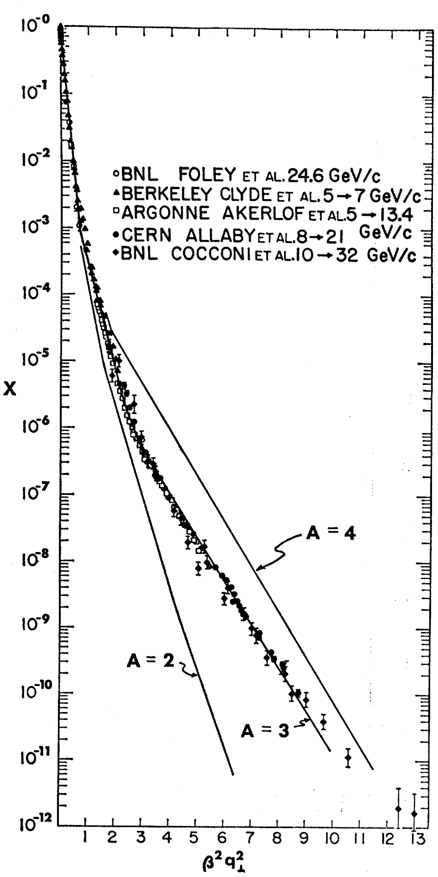

FIG. 2. A dependence in the self-consistent multiple-scattering analysis is shown in this plot of $X \equiv (d\sigma^\dagger/dt)/(d\sigma^\dagger/dt)_{t=0}$ versus $\beta^2 \mathbf{q}_\perp^2$ as a function of the parameter A holding the other parameters fixed at $\xi = (9.8 - i13.0)$ (BeV/c)$^{-2}$ and $\mu^2 = 0.71$ (BeV/c)^2. The possibility that $A = 2$ is definitely excluded. However, the curve for $A = 4$ might conceivably be lowered to where it were compatible with the data by means of employing some other ansatz for the effective form factors that were parameterized to correspond to some rather large spatial extension of the subparticles Q. (See Fig. 3 for an example of how this lowering effect behaves.) The essentially exclusive fit with the value $A = 3$ may be interpreted as affording corroborative evidence in favor of the SU_3-quark model from a nongroup-theoretical dynamical basis.

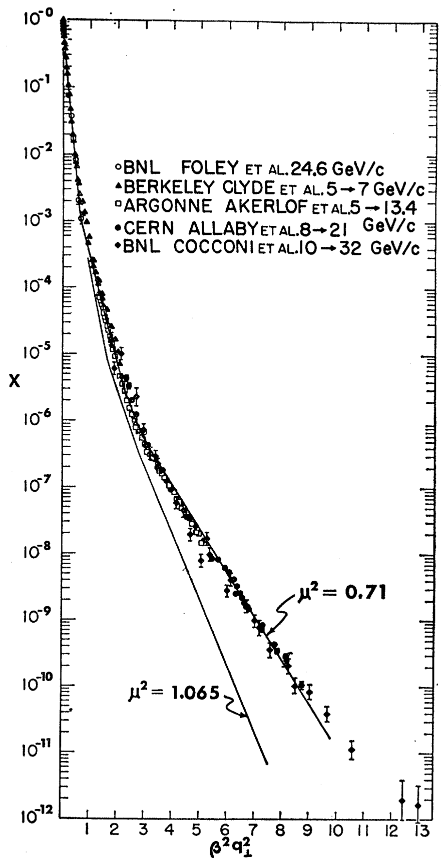

FIG. 3. The μ^2 dependence of the self-consistent multiple-scattering analysis of the cross section $X \equiv (d\sigma^\dagger/dt)/(d\sigma^\dagger/dt)_{t=0}$ versus $\beta^2 \mathbf{q}_\perp^2$. The value $\mu^2 = 0.71$ (BeV/c)^2 corresponds to effectively pointlike quarks and $\mu^2 = 1.065$ (BeV/c)^2 corresponds to quarks of larger effective spatial distribution with effective rms electromagnetic radius about 0.56 times that of the proton. Our results are compatible with quarks that have very small or even pointlike effective spatial extension. The values of the other parameters here are $\xi = (9.8 - i13.0)$ (BeV/c)$^{-2}$ and $A = 3$.

[12] T. T. Chou and C. N. Yang, Phys. Rev. Letters **20**, 1213 (1968); Phys. Rev. **170**, 1591 (1968), and (to be published); L. Durand, III, and R. Lipes, Phys. Rev. Letters **20**, 637 (1968).

sis has succeeded in excluding the possibility of $A=2$. The curve of the results with $A=2$ lies considerably below the data for the case of point Q's. Choosing form factors that correspond to Q's of larger finite extension has the effect of lowering even more the cross section in the regions of double- and higher-order multiple-scattering contributions. (See Fig. 3 below.) The results with $A=3$ and small or pointlike Q's fit the data rather well. The A dependence in the results of our analysis of pp scattering appears to support the SU_3-quark model by selecting the value $A=3$. The case with $A=4$ and point Q's seems to be excluded. However, results with A greater than 3 might be made compatible with the data if the calculated curves were lowered through the use of form factors corresponding to Q's of some rather large extension.

These types of effects are illustrated in Fig. 3 with the results for fixed $A=3$ and form factors that correspond to point Q's [$\mu^2=0.71$ (BeV/c)^2 and $G_Q{}^E(q)\cong 1$] compared to the case for Q's of effective rms radii of about half the electromagnetic radius of the proton [$\mu^2\cong 1.065$ (BeV/c)^2]. Spatial extensions of the Q's that are not small correspond in general to results in which the cross sections fall off too rapidly at large momentum transfers. Our results here are consistent with small Q's, or quarks, as are also the consistency relations among various elastic cross sections and form factors of hadrons based upon the multiple-scattering picture and the SU_3-quark model.[5]

Implicit in Eq. (10) for the leading single-scattering contribution there is the possibility that the explanation of the puzzle of the dipole behavior of the proton electromagnetic form factor is simply that the single-Q-body form factor of the proton $\Phi_p(\mathbf{q},0,\cdots)$ and the electromagnetic form factor of Q are nearly equal:

$$\Phi_p(\mathbf{q},0,\cdots)\cong G_Q{}^E(\mathbf{q})\cong(1+\mathbf{q}^2/\mu^2)^{-1} \quad [\mu^2\cong 0.71\ (\mathrm{BeV}/c)^2]. \tag{22}$$

The results of our analysis under this assumption are shown in Fig. 4. We conclude that this simple explanation is not the answer.

Phase variation of the assumed QQ scattering amplitude f has to be rather strong if pronounced minima are to be avoided at the transitions between regions of different multiple-scattering orders. Figure 5 shows the

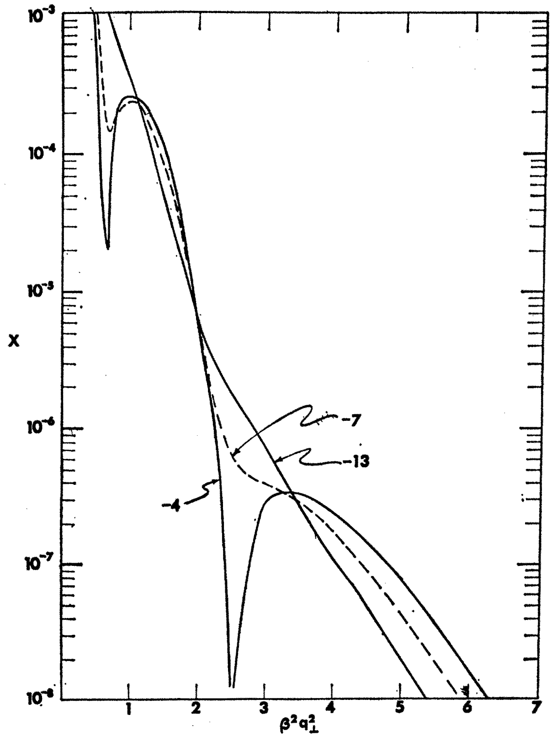

FIG. 5. Phase-variation dependence of the fundamental quark-quark scattering amplitude f must be rather large to avoid interference minima at the breaks between slopes of $X\equiv(d\sigma^\dagger/dt)/(d\sigma^\dagger/dt)_{t=0}$ versus $\beta^2\mathbf{q}_\perp{}^2$ as shown here. The curves labeled -4, -7, and -13 represent the results with the fundamental quark-quark amplitude f, as shown in Eq. (21) of the text, given with Im$\xi=-4.0$, -7.0, and -13.0 (BeV/c)$^{-2}$, respectively, and Re$\xi=9.8$ (BeV/c)$^{-2}$, $\mu^2=0.71$ (BeV/c)^2, and $A=3$.

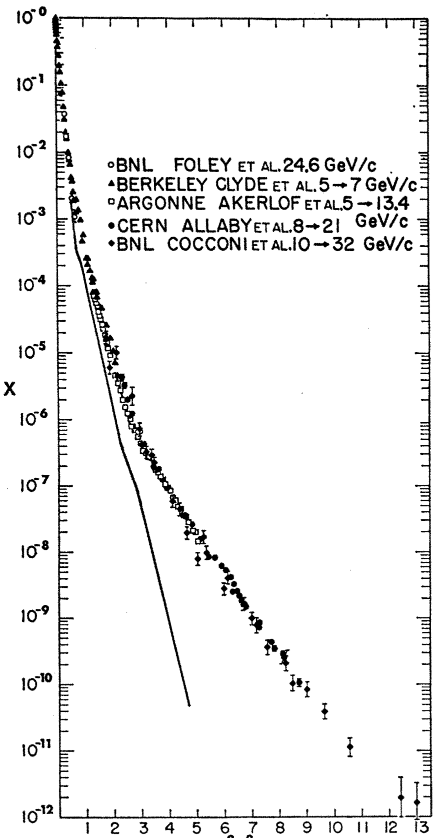

FIG. 4. The assumption that the dipole form factor of the proton is due to the approximate equality of the effective electromagnetic form factor of the quark with the effective single-quark body form factor of the proton, as represented in Eqs. (16) and (22) of the text, incorporated into the calculation of $X\equiv(d\sigma^\dagger/dt)/(d\sigma^\dagger/dt)_{t=0}$ versus $\beta^2\mathbf{q}_\perp{}^2$, is shown to be incompatible with the self-consistent multiple-quark-scattering analysis of the pp cross section. The values of the other parameters here are $\xi=(9.8-i13.0)$ (BeV/c)$^{-2}$ and $A=3$.

effects of various assumptions of linear phase variations obtained by adding an imaginary part to the parameter ξ shown in formula (21) for f. It may be too strict to try to fit all the interferences among single-, double-, and triple-scattering amplitudes with only linear dependence of the phase upon momentum transfer (squared) as we have, particularly if fourth-order scattering were also considered. The strong phase variation that is required to fit the data may indicate an inconsistency in the assumption that the internal multiple-scattering processes are essentially completely diffractive. The effect of the contributions of inelastically excited intermediate states of the proton between internal scatterings can be estimated by comparing results with and without the truncation of sums over dynamical intermediate states at the ground state. The differences which result from tests of this kind are small, suggesting that the contributions of inelastically excited intermediate states do not overwhelm our basic dynamical assumption of multiple-internal-elastic scattering, whether the internal scatterings are completely diffractive or not.

V. CONCLUSION

The self-consistent multiple-quark-scattering analysis accounts for the main characteristics of the pp elastic-scattering data. Although the analysis, as applied above, included only as high as triple-scattering contributions the results, as well as the data, strongly suggest that both theoretical and experimental investigations should be extended in an attempt to establish the existence of a domain at higher momentum-transfers [$\beta^2\mathbf{q}_\perp^2 \gtrsim 9$ $(\text{BeV}/c)^2$] in which fourth-order multiple-quark-scattering contributions dominate. The essentially exclusive fit to the experimental data with the value of the quark number of the proton, $A=3$, may be interpreted as affording corroborative evidence in favor of the SU_3-quark model from a non-group-theoretical, dynamical basis. Our results are compatible with quarks that have very small, or even pointlike effective spatial extension as compared to the effective electromagnetic distribution of the proton. The large phase-variation dependence of the fundamental quark-quark scattering amplitude that is required in order to avoid interference minima suggests that the operating assumption that this amplitude is essentially completely diffractive is inconsistent and that perhaps potential effects may need to be taken into account.[13]

The multiple-internal quark-scattering picture gives such encouraging results with only the simplest calculations for the analysis of pp elastic scattering that it merits further consideration. The interpretation of high-energy collisions of hadrons in terms of this multiple-internal quark-scattering picture extends to momentum-transfer-dependent quantities, i.e., differential cross sections and form factors, the same systematic analysis for which previously the quark model has been so surprisingly successful mainly for momentum-transfer-independent quantities, i.e., total cross sections and branching ratios. Consistency relations that correlate differential cross sections for elastic pp, $p\bar{p}$, πp, and $\pi\pi$ scattering with the form factors of the nucleon and pion have been derived on the basis of the multiple-internal quark-scattering picture.[5] These correlations have been used in lowest order to obtain the pion form factor and the $\pi\pi$ diffraction-scattering cross section. (See Ref. 5.) Further analysis is being carried to higher-order and broader application, and will be reported separately.[14]

ACKNOWLEDGMENTS

We acknowledge the contributions of the Washington University Computer Facilities and the support of our computer calculations through their National Science Foundation Grant. One of us (E.S.) wishes to acknowledge the hospitality of the Theoretical Physics Division of the Stanford Linear Accelerator Center where the early stages of this work were done, and also to acknowledge useful comments on this work by Professors Kerson Huang, Marc Ross, and P. G. O. Freund, and Dr. John Pumplin. We are grateful to Professor Alan Krisch for permission to reproduce in part the plots of the data from Ref. 1.

[13] R. C. Arnold [Argonne report, 1968 (unpublished)] gives an example of a possible way in which diffractive and potential effects might be joined in a high-energy phenomenology.

[14] E. Shrauner, L. Benofy, and D. W. Cho, Ref. 6.