VI. DISCUSSION

The theoretical results for the capture rate given in Table I are to be compared with the recent experimental results

$$\Gamma(\mu^- + p \to n + \nu;\ \mu p \text{ singlet})]_{\text{expt.}} = 640 \pm 70\ \text{sec}^{-1}$$
[Bologna-CERN (Ref. 3)],

$$\Gamma(\mu^- + \text{He}^3 \to \text{H}^3 + \nu)]_{\text{expt.}} = 1505 \pm 46\ \text{sec}^{-1} \quad (44)$$
[Berkeley (Ref. 25)],
$$= 1465 \pm 67\ \text{sec}^{-1}$$
[Carnegie (Ref. 26)].

The over-all agreement between theory and experiment is gratifying and must be viewed as lending general support to the belief in the validity of V-A interaction, muon-electron universality, CVC, and PCAC.

The capture rate is not very sensitive to the values of F_P and the present experimental errors are still too large to eliminate one or the other version of PCAC. However, in view of the recent spectacular success of the Gell-Mann–Lévy version of PCAC in the application of current algebra to various problems of elementary-particle physics, we believe that the capture rates with $G_P^{\text{G-L}}$ and $F_A(0;\ p\to n)=1.23$, as given in the last column of Table I, are the best theoretical values at the present time.

Note added in proof. The final result of the experiment of Ref. 3 has been reported as

$$\Gamma(\mu^- + p \to n + \nu;\ \mu p \text{ singlet})]_{\text{expt}} = 651 \pm 57\ \text{sec}^{-1}$$

(Phys. Rev., to be published) which is in excellent agreement with our theoretical estimate (d) in Table I.

TABLE I. Theoretical capture rates in units of sec^{-1}. The capture rates (a), (b), (c), and (d) are, respectively, for G_P^{N} and $F_A\times(0;\ p\to n)=1.18$, $G_P^{\text{G-L}}$ and $F_A(0;\ p\to n)=1.18$, G_P^{N} and $F_A\times(0;\ p\to n)=1.23$, and $G_P^{\text{G-L}}$ and $F_A(0;\ p\to n)=1.23$.

Process	Capture rate (a)	Capture rate (b)	Capture rate (c)	Capture rate (d)
$\mu^- + p \to n + \nu$	625	613	662	654
$\mu^- + \text{He}^3 \to \text{H}^3 + \nu$	1449	1525	1449	1525

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Pole Dominance and a Low-Energy Theorem in the Radiative Decays of Charged Kaons*

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A low-energy theorem is derived for the structure-dependent axial-vector form factor in the radiative decay $K\to l+\nu+\gamma$ in the soft-kaon approximation. Corrections of the order of (m_K^2/m_V^2) $(V=\rho,\ \omega,\ \varphi)$ are obtained in the pole-dominance approximation. In each approximation, the model predictions of both (i) asymptotic $SU(3)$ and (ii) current mixing are investigated. The quantity $|\gamma_K|\equiv|a_K(0)/F(0)|$ is calculated in both approximations and in both models. It is found that the soft-kaon result is shifted upward by approximately 20%; the separation between the models in the two approximations is of the same order of magnitude.

I. INTRODUCTION

THE techniques of current algebra have recently been used[1,2] to study the radiative decays of charged pions. In particular a low-energy theorem for the structure-dependent axial-vector part of the radiative decay $\pi\to l+\nu+\gamma$ has been derived both in the soft-pion approximation[1] and in the pole-dominance approximation (PDA).[2] In this paper we consider the extension of such techniques to the analogous radiative decay $K\to l+\nu+\gamma$ where the theoretical situation is much less clear. One of the bases for our interest in such a calculation is the expectation that the PDA calculation of, say,[1] $dA/d\nu(q^2=0,\ \Delta^2=0)$, in this case, might prove substantially different from the SKA (soft-kaon approximation) result because of hardly negligible "correction terms" of the order of m_K^2/m_V^2 $(V=\rho,\ \omega,\ \varphi)$. At the same time, we are not aware of any experimental

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[1] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **19**, 859 (1967).

[2] S. G. Brown and G. B. West, Phys. Rev. **168**, 1605 (1968).

data on this decay mode as yet, although such an experiment would appear to be feasible.[3]

In Sec. II we outline the soft-kaon approach to this problem. There, two expressions for the sum rule for $a_K(\nu=0)=dA/d\nu(0,0)$ are derived, depending on whether results of asymptotic $SU(3)$ obtained by Das, Mathur, and Okubo,[4] or results of the current-mixing model of Oakes and Sakurai[5] are used.

In Sec. III we derive an expression for $A(q^2,\Delta^2)$ in PDA, after calculating the various form factors by the dispersion-theoretic technique of Das, Mathur, and Okubo.[6] The SKA results for $dA/d\nu(0,0)$ are then merely the limiting values $(m_K{}^2\rightarrow 0)$ of the corresponding PDA expressions. We find that although individually large, the aggregate effect of the correction terms on $|\gamma_K|\equiv|a_K(0)/F(0)|$ [$F(\nu)$ is the structure-dependent vector form factor] is just a shift to larger values of the order of 20% of the soft-kaon result in both models.[4,5] Moreover, the differences in the model[4,5] predictions are of the same order of magnitude. The remaining source of uncertainty in the numerical estimates of $|\gamma_K|$ in the two models[4,5] resides, of course, in the calculation of $|F(0)|$; there we have chosen to relate the VVP couplings by nonet symmetry.[7]

II. LOW-ENERGY THEOREM IN THE DECAY $K^+\rightarrow l^++\nu+\gamma$: SOFT-KAON APPROXIMATION (SKA)

Following Ref. 1 we write

$$M_{\mu\nu}=(2k_0)^{1/2}\int d^4x\, e^{-iq\cdot x}\times\langle 0|T\{V_\mu{}^{(\mathrm{em})}(x),[V_{3\nu}{}^1(0)+A_{3\nu}{}^1(0)]\}|K^+(k)\rangle, \quad (1)$$

with the axial vector part of $M_{\mu\nu}$ (for arbitrary q^2) being given by

$$M_{\mu\nu}{}^A(\nu,q^2)=A(\nu,q^2)\delta_{\mu\nu}+B(\nu,q^2)q_\nu k_\mu+C(\nu,q^2)q_\mu q_\nu+D(\nu,q^2)q_\mu k_\nu+E(\nu,q^2)k_\mu k_\nu. \quad (2)$$

Using current algebra and taking the soft-kaon limit $k\rightarrow 0$, one finds

$$M_{\mu\nu}{}^A(\nu=0,q^2)=-[\Delta_{\mu\nu}{}^{As}(q)-\Delta_{\mu\nu}{}^{V(3)}(q)-\Delta_{\mu\nu}{}^{V(8)}(q)]/F_K, \quad (3)$$

where the decay constant F_K is defined by

$$\langle 0|A_{3\nu}{}^1(0)|K^+(k)\rangle=-ik_\nu F_K/(2k_0)^{1/2}, \quad (4)$$

and

$$\Delta_{\mu\nu}{}^{As}(q)=i\int d^4x\, e^{-iq\cdot x}\times\langle 0|T\{A_\mu{}^{(4+i5)}(x),A_\nu{}^{(4-i5)}(0)\}|0\rangle, \quad (5)$$

$$\Delta_{\mu\nu}{}^{V(\alpha)}(q)=i\int d^4x\, e^{-iq\cdot x}\times\langle 0|T\{V_\mu{}^{(\alpha)}(x),V_\nu{}^{(\alpha)}(0)\}|0\rangle. \quad (6)$$

Analogous to the result obtained in Ref. 1, one finds

$$A(\nu=0,q^2)=-\frac{1}{F_K}\int\frac{\rho_{As}(m^2)-\rho_{V(3)}(m^2)-\rho_{V(8)}(m^2)}{q^2+m^2}dm^2, \quad (7)$$

$$C(\nu=0,q^2)=-\left[\frac{1}{F_K}\int\frac{\rho_{As}(m^2)-\rho_{V(3)}(m^2)-\rho_{V(8)}(m^2)}{m^2(q^2+m^2)}dm^2+\frac{F_K}{q^2+m_K{}^2}\right]. \quad (8)$$

Vector-meson dominance implies that

$$A(0,0)=\left[\int dm^2\frac{\rho_{V(3)}(m^2)+\rho_{V(8)}(m^2)-\rho_{As}(m^2)}{m^2}\right]\Big/F_K=\left(\frac{G_\rho{}^2}{m_\rho{}^2}+\frac{G_\varphi{}^2}{m_\varphi{}^2}+\frac{G_\omega{}^2}{m_\omega{}^2}-\frac{G_{K_A}{}^2}{m_{K_A}{}^2}\right)\Big/F_K, \quad (9)$$

with[4]

$$\frac{G_\rho{}^2}{m_\rho{}^2}=\frac{G_\varphi{}^2}{m_\varphi{}^2}+\frac{G_\omega{}^2}{m_\omega{}^2}, \quad (10)$$

and Weinberg's first spectral function sum rule,[8]

$$2\frac{G_\rho{}^2}{m_\rho{}^2}-\frac{G_{K_A}{}^2}{m_{K_A}{}^2}=\frac{G_{K^*}{}^2}{m_{K^*}{}^2}-\frac{G_{K_A}{}^2}{m_{K_A}{}^2}=F_K{}^2, \quad (11)$$

implies that

$$A(0,0)=F_K. \quad (12)$$

The sum rule which then emerges from the straightforward manipulation of the preceding material (as in

[3] We wish to thank Professor S. L. Meyer for some discussions on this point.

[4] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **18**, 761 (1967); **19**, 470 (1967).

[5] R. J. Oakes and J. J. Sakurai, Phys. Rev. Letters **19**, 1266 (1967).

[6] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **19**, 1067 (1967).

[7] S. Okubo, Phys. Letters **5**, 165 (1963).

[8] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967). Note that in this paper G_ρ is defined (as in Ref. 4) by

$$G_\rho\epsilon_\nu{}^{(\rho)}(q)/(2q_0)^{1/2}=\langle 0|V_\nu{}^{(3)}(0)|\rho^0(q)\rangle=(1/\sqrt{2})\langle 0|V_{2\nu}^1(0)|\rho^+(q)\rangle,$$

with $(G_\rho{}^2/m_\rho{}^2)=F_\pi{}^2$; also $\int dm^2\rho_4(m^2)=\frac{1}{2}G_{K^*}{}^2$, as in the first, but *not* the second, of Ref. 4.

Ref. 1) is

$$a_K(\nu=0)\equiv\frac{dA}{d\nu}(0,0)$$
$$=-\left[\frac{1}{F_K}\int\frac{\rho_{A_s}(m^2)-\rho_{V(3)}(m^2)-\rho_{V(8)}(m^2)}{m^4}dm^2+\tfrac{1}{3}F_K\langle r^2\rangle_K\right],\quad(13)$$

where the derivative of the kaon electromagnetic form factor $f_K(q^2)$ is given by

$$f_K'(0)=-\tfrac{1}{6}\langle r^2\rangle_K$$
$$=-\left(\frac{G_\rho G_{\rho KK}}{m_\rho^4}+\frac{G_\varphi G_{\varphi KK}}{\sqrt{3}m_\varphi^4}+\frac{G_\omega G_{\omega KK}}{\sqrt{3}m_\omega^4}\right),\quad(14)$$

and may be further reduced to[4]

$$f_K'(0)=-\frac{1}{2}\left[\frac{1}{m_\rho^2}+\frac{1}{m_\omega^2}+\left(\frac{m_\rho^2G_\varphi^2}{m_\varphi^2G_\rho^2}\right)\frac{(m_\omega^2-m_\varphi^2)}{m_\varphi^2m_\omega^2}\right].\quad(15)$$

Thus,

$$a_K(\nu=0)=-F_K\left\{\frac{1}{F_K^2}\left(\frac{G_{K_A}^2}{m_{K_A}^4}-\frac{G_\rho^2}{m_\rho^4}-\frac{G_\varphi^2}{m_\varphi^4}-\frac{G_\omega^2}{m_\omega^4}\right)+\left[\frac{1}{m_\rho^2}+\frac{1}{m_\omega^2}+\left(\frac{m_\rho^2G_\varphi^2}{m_\varphi^2G_\rho^2}\right)\frac{(m_\omega^2-m_\varphi^2)}{m_\varphi^2m_\omega^2}\right]\right\}.\quad(16)$$

Expression (16) may be simplified somewhat if we accept (i) the proposal of Das, Mathur, and Okubo[4] that $\int dm^2\rho_{\alpha\beta}(m^2)$ satisfy the Gell-Mann–Okubo formula, or (ii) the proposal of Oakes and Sakurai[5] (OS) that $\int dm^2m^{-4}\rho_{\alpha\beta}(m^2)$ satisfy the Gell-Mann–Okubo formula. In case (i) we find[9]

$$[a_K(\nu=0)]_{\rm DMO}$$
$$=-F_K\left\{\left[\frac{(m_{K^*}^2/m_{K_A}^2)}{(m_{K_A}^2-m_{K^*}^2)}-\frac{(F_\pi/F_K)^2}{m_\rho^2}-\frac{(F_\pi/F_K)^2}{3m_\varphi^2m_\omega^2}[3(m_\varphi^2+m_\omega^2)+m_\rho^2-4m_{K^*}^2]\right]+\left[\frac{1}{m_\rho^2}+\frac{1}{m_\omega^2}+\frac{1}{m_\varphi^2}-\frac{1}{3}\frac{(4m_{K^*}^2-m_\rho^2)}{m_\varphi^2m_\omega^2}\right]\right\},\quad(17)$$

[9] For calculational purposes we do not assume the existence of the κ meson in this paper. [However, see K. Kang, Phys. Rev. Letters 21, 857 (1968), for arguments on the other side.] Thus from Ref. 4 we have, for example,

$$G_{K^*}^2=G_{K_A}^2,$$

and

$$\frac{G_{K_A}^2}{m_{K^*}^2}\left(1-\frac{m_{K^*}^2}{m_{K_A}^2}\right)=F_K^2,$$

where it is assumed that $m_{K_A}=1320$ MeV.

while in case (ii), where[5]

$$\frac{G_\omega^2}{m_\omega^4}+\frac{G_\varphi^2}{m_\varphi^4}=\frac{1}{3}\left[4(\tfrac{1}{2}G_{K^*}^2)\frac{1}{m_{K^*}^4}-\frac{G_\rho^2}{m_\rho^4}\right],\quad(18)$$

we find[10]

$$[a_K(0)]_{\rm OS}=-F_K\left[\frac{(m_{K^*}^2/m_{K_A}^2)}{(m_{K_A}^2-m_{K^*}^2)}-\frac{(F_\pi/F_K)^2}{m_\rho^2}-\frac{(F_\pi/F_K)^2}{3m_{K^*}^2m_\rho^2}(4m_\rho^2-m_{K^*}^2)+\frac{2}{3}\left(\frac{1}{m_\rho^2}+\frac{2}{m_{K^*}^2}\right)\right].\quad(19)$$

In case (ii) we need the results,[11]

$$G_\varphi^2=\frac{m_\varphi^4m_\omega^2}{m_\rho^2(m_\omega^2-m_\varphi^2)}\times\left(\frac{4}{3m_{K^*}^2}-\frac{1}{3m_\rho^2}-\frac{1}{m_\omega^2}\right)G_\rho^2\simeq1.43G_\rho^2,$$
$$G_\omega^2=\frac{m_\omega^4m_\varphi^2}{m_\rho^2(m_\varphi^2-m_\omega^2)}\times\left(\frac{4}{3m_{K^*}^2}-\frac{1}{3m_\rho^2}-\frac{1}{m_\varphi^2}\right)G_\rho^2\simeq0.24G_\rho^2.\quad(20)$$

We shall present numerical calculations of the ratio[12] $|\gamma_K|\equiv|a_K(0)/F(0)|$ in the next section, where the "corrections" of the order of (m_K^2/m_V^2) $(V=\rho,\omega,\varphi)$ in PDA are taken up.

III. POLE-DOMINANCE APPROACH TO THE DECAY $K^+\to l^++\nu+\gamma$

In the pole-dominance approach to the decay $K^+\to l^++\nu+\gamma$ we are concerned with the retarded amplitude

$$M_{\mu\nu}{}^A=\int d^4x\,e^{-iq\cdot x}\times\langle0|\theta(x_0)[V_\mu{}^{(\rm em)}(x),A_{3\nu}{}^1(0)]|K^+(k)\rangle(2k_0)^{1/2},\quad(21)$$

[10] As in case (i) we assume the validity of the second spectral sum rule in the subgroup $SU(2)\times SU(2)$.

[11] Our notation follows that of the second of Ref. 4.

[12] In view of the experimental situation we confine ourselves here solely to numerical estimates of this quantity.

with absorptive part,

$$\begin{aligned}\mathrm{Abs}M_{\mu\nu}{}^{A}=&-i\pi\sum_{\lambda}\langle 0|V_\mu{}^{(\mathrm{em})}(0)|\rho^0(q);\lambda\rangle 2(m_\rho{}^2+\mathbf{q}^2)^{1/2}\langle\rho^0(q);\lambda|A_{3\nu}{}^1(0)|K^+(k)\rangle(2k_0)^{1/2}\delta(q^2+m_\rho{}^2)\\&-i\pi\sum_{\lambda}\langle 0|V_\mu{}^{(\mathrm{em})}(0)|\varphi(q);\lambda\rangle 2(m_\varphi{}^2+\mathbf{q}^2)^{1/2}\langle\varphi(q);\lambda|A_{3\nu}{}^1(0)|K^+(k)\rangle(2k_0)^{1/2}\delta(q^2+m_\varphi{}^2)\\&-i\pi\sum_{\lambda}\langle 0|V_\mu{}^{(\mathrm{em})}(0)|\omega(q);\lambda\rangle 2(m_\omega{}^2+\mathbf{q}^2)^{1/2}\langle\omega(q);\lambda|A_{3\nu}{}^1(0)|K^+(k)\rangle(2k_0)^{1/2}\delta(q^2+m_\omega{}^2)\\&+i\pi\langle 0|A_{3\nu}{}^1(0)|K^+(\Delta)\rangle 2(m_K{}^2+\mathbf{\Delta}^2)^{1/2}\langle K^+(\Delta)|V_\mu{}^{(\mathrm{em})}(0)|K^+(k)\rangle(2k_0)^{1/2}\delta(\Delta^2+m_K{}^2)\\&+i\pi\sum_{\lambda}\langle 0|A_{3\nu}{}^1(0)|K_A{}^+(\Delta);\lambda\rangle 2(m_{K_A}{}^2+\mathbf{\Delta}^2)^{1/2}\langle K_A{}^+(\Delta);\lambda|V_\mu{}^{(\mathrm{em})}(0)|K^+(k)\rangle(2k_0)^{1/2}\delta(\Delta^2+m_{K_A}{}^2),\quad(22)\end{aligned}$$

where $\Delta_\mu=(k-q)_\mu$. On substituting the definitions[13]

$$\begin{aligned}\langle 0|V_\mu{}^{(\mathrm{em})}(0)|\rho^0(q)\rangle&=G_\rho\epsilon_\mu{}^{(\rho)}(q),\\\langle 0|V_\mu{}^{(\mathrm{em})}(0)|\varphi,\omega(q)\rangle&=(1/\sqrt{3})G_{\varphi,\omega}\epsilon_\mu{}^{(\varphi,\omega)}(q),\quad(23)\\\langle 0|A_{3\nu}{}^1(0)|K_A{}^+(\Delta)\rangle&=-G_{K_A}\epsilon_\nu{}^{(K_A)}(\Delta),\end{aligned}$$

and the relevant form factors tabulated in the Appendix in Eq. (22), one finds

$$\begin{aligned}\mathrm{Abs}A(q^2,\Delta^2)&=\pi G_\rho P_1{}^{(\rho)}(\Delta^2)\delta(q^2+m_\rho{}^2)+\pi\frac{G_\varphi}{\sqrt{3}}P_1{}^{(\varphi)}(\Delta^2)\delta(q^2+m_\varphi{}^2)\\&\quad+\pi\frac{G_\omega}{\sqrt{3}}P_1{}^{(\omega)}(\Delta^2)\delta(q^2+m_\omega{}^2)\\&\quad+\pi G_{K_A}Q_1(q^2)\delta(\Delta^2+m_{K_A}{}^2).\quad(24)\end{aligned}$$

From Eq. (24) we first determine $A(q^2,\Delta^2)$ up to "unknown" pole terms in q^2:

$$\begin{aligned}A(q^2,\Delta^2)&=-\frac{G_{K_A}Q_1(q^2)}{m_{K_A}{}^2+\Delta^2}+\sum_{V=\rho,\varphi,\omega}\frac{C_V}{m_V{}^2+q^2}\\&=-\frac{(G_{K_A}{}^2/F_K)}{m_{K_A}{}^2+\Delta^2}-\frac{G_{K_A}}{m_{K_A}{}^2+\Delta^2}\sum_V\left(\frac{q^2+m_{K_A}{}^2}{q^2+m_V{}^2}\right)\\&\quad\times\frac{g_VG_s{}^{K,V}}{(m_V{}^2-m_{K_A}{}^2)}+\sum_V\frac{C_V}{m_V{}^2+q^2},\quad(25)\end{aligned}$$

where

$$g_V=G_\rho,\quad(1/\sqrt{3})G_{\varphi,\omega},\quad(26)$$

and then fix the pole residues C_V ($V=\rho,\varphi,\omega$) by reference to the absorptive part of A as given by (24). Thus,

$$\begin{aligned}C_\rho&=\frac{G_\rho{}^2}{F_K}-\frac{G_\rho G_{K_A}G_s{}^{K,\rho}}{m_{K_A}{}^2-m_\rho{}^2},\\C_{\varphi,\omega}&=\frac{G_{\varphi,\omega}{}^2}{F_K}-\frac{G_{\varphi,\omega}G_{K_A}G_s{}^{K,\varphi,\omega}}{m_{K_A}{}^2-m_{\varphi,\omega}{}^2},\end{aligned}\quad(27)$$

[13] We now dispense with kinematical factors [e.g., $2(m_\rho{}^2+\mathbf{\Delta}^2)^{1/2}$] and polarization sums, for simplicity.

and

$$\begin{aligned}A(q^2,\Delta^2)=&-\frac{(G_{K_A}{}^2/F_K)}{m_{K_A}{}^2+\Delta^2}-\sum_{V=\rho^0,\varphi,\omega}\left(\frac{q^2+m_{K_A}{}^2}{q^2+m_V{}^2}\right)\frac{1}{m_{K_A}{}^2+\Delta^2}\\&\times\frac{G_{K_A}g_VG_s{}^{K,V}}{(m_V{}^2-m_{K_A}{}^2)}+\sum_{V=\rho^0,\varphi,\omega}\frac{(G_V{}^2/F_K)}{m_V{}^2+q^2}\\&+\sum_{V=\rho^0,\varphi,\omega}\frac{G_{K_A}g_VG_s{}^{K,V}}{m_V{}^2+q^2}\frac{1}{(m_V{}^2-m_{K_A}{}^2)},\quad(28)\end{aligned}$$

with

$$\frac{dA(0,0)}{d\nu}=-\frac{2G_{K_A}{}^2}{m_{K_A}{}^4F_K}-2\sum_{V=\rho^0,\varphi,\omega}\frac{G_{K_A}g_VG_s{}^{K,V}}{(m_V{}^2m_{K_A}{}^2)(m_V{}^2-m_{K_A}{}^2)}.\quad(29)$$

On making use of the Eqs. (A29–A31) derived in the Appendix, one finds

$$\begin{aligned}a_K(0)=-F_K\Bigg\{&\frac{1}{F_K{}^2}\Bigg[\frac{G_{K_A}{}^2}{m_{K_A}{}^4}\left(2-\frac{\eta_\rho}{\eta_\rho+\eta_{K_A}}-\frac{1}{2}\right)\\&-\frac{2\eta_{K_A}}{\eta_\rho+\eta_{K_A}}\frac{G_\rho{}^2}{m_\rho{}^4}-\frac{G_\varphi{}^2}{m_\varphi{}^4}-\frac{G_\omega{}^2}{m_\omega{}^4}\Bigg]\\&+4\left[\frac{\eta_{K_A}}{\eta_\rho+\eta_{K_A}}\frac{G_\rho G_{\rho KK}}{m_\rho{}^4}+\frac{1}{2}\frac{G_\varphi G_{\varphi KK}}{\sqrt{3}m_\varphi{}^4}+\frac{1}{2}\frac{G_\omega G_{\omega KK}}{\sqrt{3}m_\omega{}^4}\right]\Bigg\}\\&+\mathrm{terms}\{O(\eta_{K_A}-\eta_V),O(\eta_\omega-\eta_\varphi)\},\quad(30)\end{aligned}$$

where

$$\eta_\alpha=1-\frac{m_K{}^2}{m_\alpha{}^2},\quad(\alpha=\rho,\varphi,\omega,K_A);\quad(31)$$

of course, in the limit $m_K{}^2\to 0$, we recover the soft-kaon limit, Eq. (16).

In order to make some estimate of, say, $|\gamma_K|$, it is necessary to derive an expression for $F(\nu)$, the structure-dependent vector form factor, which is given by

$$\begin{aligned}M_{\mu\nu}{}^{V}(q^2=0,\Delta^2)&=\int d^4x\,e^{-iq\cdot x}\langle 0|T\{V_\mu{}^{(\mathrm{em})}(x),V_{3\nu}{}^1(0)\}|K^+(k)\rangle\Big|_{(q^2=0)}\\&=F(\nu)\epsilon_{\mu\nu\lambda\sigma}q_\lambda k_\sigma.\quad(32)\end{aligned}$$

TABLE I. Values of the parameter $|\gamma_K|$ in the DMO and OS models for soft and hard kaons.

	$\|\gamma_K\|$ (SKA)	$\|\gamma_K\|$ (PDA)
DMO model	0.38	0.48
OS model	0.44	0.58

In the pole approximation one finds

$$|F(\nu)|=\left|\sum_{V=\rho^0,\varphi,\omega}\frac{g_V G_{K^*}G_{K^*KV}}{m_V^2(m_{K^*}^2-m_K^2-2\nu)}\right|. \quad (33)$$

We evaluate the unknown couplings G_{K^*KV} in (33) by assuming nonet symmetry,[7,14] so that $G_{K^*K\rho}=(1/\sqrt{2})G_{K^*K\varphi}=G_{K^*K\omega}=(\frac{1}{2})G_{\rho\pi\omega}$, with[15] $m_\pi^2 G_{\rho\pi\omega}^2/(4\pi)=0.40$. Note that because of the conventions of Ref. 14, where

$$\varphi=-\cos\theta\,\omega_8+\sin\theta\,\omega_1,\qquad \omega=\sin\theta\,\omega_8+\cos\theta\,\omega_1, \quad (34)$$

one has, rather,[16]

$$|F(0)|=G_{\rho\pi\omega}\frac{G_{K^*}}{2(m_{K^*}^2-m_K^2)}\times\left[\frac{G_\rho}{m_\rho^2}-(\tfrac{2}{3})^{1/2}\frac{G_\varphi}{m_\varphi^2}-(\tfrac{1}{3})^{1/2}\frac{G_\omega}{m_\omega^2}\right]. \quad (35)$$

The SKA and PDA predictions for $|\gamma_K|$ are compared in Table I for both models of interest.[17]

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APPENDIX

We give here the pole-dominant solutions for the matrix elements occurring in Abs $M_{\mu\nu}{}^A$ [Eq. (22)] after the method of Ref. 6.[18] Thus,

$$\langle\rho^0(q)|A_{3\nu}{}^1(0)|K^+(k)\rangle=i\epsilon_\mu{}^{(\rho)}(q)[P_1{}^{(\rho)}(\Delta^2)\delta_{\mu\nu}+P_2{}^{(\rho)}(\Delta^2)k_\mu(k+q)_\nu+P_3{}^{(\rho)}(\Delta^2)k_\mu(q-k)_\nu], \quad (A1)$$

$$P_1{}^{(\rho)}(\Delta^2)=\frac{G_\rho}{F_K}\left(\frac{\Delta^2+m_\rho^2}{\Delta^2+m_{K_A}^2}\right)\frac{G_{K_A}G_s{}^{K,\rho}}{(m_{K_A}^2-m_\rho^2)}, \quad (A2)$$

$$P_2{}^{(\rho)}(\Delta^2)=-\frac{G_{K_A}G_d{}^{K,\rho}}{2(\Delta^2+m_{K_A}^2)}, \quad (A3)$$

$$P_3{}^{(\rho)}(\Delta^2)=\frac{2G_{\rho KK}F_K}{\Delta^2+m_K^2}-\frac{G_{K_A}}{m_{K_A}^2(\Delta^2+m_{K_A}^2)}\times[G_s{}^{K,\rho}-\tfrac{1}{2}(m_\rho^2-m_K^2)G_d{}^{K,\rho}], \quad (A4)$$

and, for $V=\varphi,\omega$,

$$\langle V(q)|A_{3\nu}{}^1(0)|K^+(k)\rangle=i\epsilon_\mu{}^{(V)}(q)[P_1{}^{(V)}(\Delta^2)\delta_{\mu\nu}+P_2{}^{(V)}(\Delta^2)k_\mu(k+q)_\nu+P_3{}^{(V)}(\Delta^2)k_\mu(q-k)_\nu], \quad (A5)$$

$$P_1{}^{(\varphi)}(\Delta^2)=\frac{\sqrt{3}G_\varphi}{F_K}\left(\frac{\Delta^2+m_\varphi^2}{\Delta^2+m_{K_A}^2}\right)\frac{G_{K_A}G_s{}^{K,\varphi}}{(m_{K_A}^2-m_\varphi^2)}, \quad (A6)$$

$$P_2{}^{(\varphi)}(\Delta^2)=-\frac{G_{K_A}G_d{}^{K,\varphi}}{2(\Delta^2+m_{K_A}^2)}, \quad (A7)$$

$$P_3{}^{(\varphi)}(\Delta^2)=\frac{2G_{\varphi KK}F_K}{\Delta^2+m_K^2}-\frac{G_{K_A}}{m_{K_A}^2(\Delta^2+m_{K_A}^2)}\times[G_s{}^{K,\varphi}-\tfrac{1}{2}(m_\varphi^2-m_K^2)G_d{}^{K,\varphi}], \quad (A8)$$

with the $P_i{}^{(\omega)}$ being given by the simple replacement $\varphi\rightarrow\omega$ in the $P_i{}^{(\varphi)}$ [Eqs. (A6)–(A8)]; also,

$$\langle K_A{}^+(q)|V_\mu{}^{(\mathrm{em})}(0)|K^+(k)\rangle=i\epsilon_\nu{}^{(K_A)}(q)[Q_1(\Delta^2)\delta_{\mu\nu}+Q_2(\Delta^2)k_\nu(k+q)_\mu+Q_3(\Delta^2)k_\nu(q-k)_\mu], \quad (A9)$$

$$Q_1(\Delta^2)=\frac{G_{K_A}}{F_K}+\sum_{V=\rho^0,\varphi,\omega}\left(\frac{\Delta^2+m_{K_A}^2}{\Delta^2+m_V^2}\right)\frac{g_V G_s{}^{K,V}}{m_V^2-m_{K_A}^2}, \quad (A10)$$

$$Q_2(\Delta^2)=\sum_{V=\rho^0,\varphi,\omega}\frac{g_V G_d{}^{K,V}}{2(\Delta^2+m_V^2)}, \quad (A11)$$

$$Q_3(\Delta^2)=\sum_{V=\rho^0,\varphi,\omega}\frac{g_V}{m_V^2(\Delta^2+m_V^2)}\times[G_s{}^{K,V}-\tfrac{1}{2}(m_{K_A}^2-m_K^2)G_d{}^{K,V}]. \quad (A12)$$

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[14] We use the conventions of Yellin [J. Yellin, Phys. Rev. 147, 1080 (1966)].

[15] G. W. Barry and J. J. Sakurai, Stanford Linear Accelerator Center SLAC Report No. 382, 1968 (unpublished).

[16] Note that in Ref. 4, $G_\varphi=G_8\cos\theta+G_1\sin\theta$ and $G_\omega=-G_8\sin\theta+G_1\cos\theta$. We also use $(G_\varphi/G_\rho)=\cos\theta$, and $(G_\omega/G_\rho)=-\sin\theta$.

[17] We have taken $(F_K/F_\pi)=1.1$ in our computations.

[18] We shall consider the application of some of these relations in a forthcoming work on the K^+-K^0 mass difference done in collaboration with K. Kang.

To the matrix elements of (A1), (A5), and (A9), we add those of the octet vector current $\langle K_A^+(q)| \times V_\mu^{(8)}(0)|K^+(k)\rangle$, and of the unitary-singlet vector current $\langle K_A^+(q)|V_\mu^{(0)}(0)|K^+(k)\rangle$,

$$\langle K_A^+(q)|V_\mu^{(8)}(0)|K^+(k)\rangle = i\epsilon_\nu^{(K_A)}(q)[R_1(\Delta^2)\delta_{\nu\mu}+\cdots], \quad (A13)$$

$$R_1(\Delta^2)=\tfrac{1}{2}\sqrt{3}\frac{G_{K_A}}{F_K} - \sum_{V=\varphi,\omega}\left(\frac{\Delta^2+m_{K_A}^2}{\Delta^2+m_V^2}\right)\frac{G_V G_s^{K,V}}{(m_{K_A}^2-m_V^2)}, \quad (A14)$$

$$R_2(\Delta^2)=\sum_{V=\varphi,\omega}\frac{G_V G_d^{K,V}}{2(\Delta^2+m_V^2)}, \quad (A15)$$

$$R_3(\Delta^2)=\sum_{V=\varphi,\omega}\frac{G_V}{m_V^2(\Delta^2+m_V^2)} \times[G_s^{K,V}-\tfrac{1}{2}(m_{K_A}^2-m_V^2)G_d^{K,V}], \quad (A16)$$

and[11]

$$\langle K_A^+(q)|V_\mu^{(0)}(0)|K^+(k)\rangle = i\epsilon_\nu^{(K_A)}(q)[S_1(\Delta^2)\delta_{\nu\mu}+\cdots], \quad (A17)$$

$$S_1(\Delta^2)=-\sum_{V=\varphi,\omega}\frac{\sigma_V G_s^{K,V}}{\Delta^2+m_V^2}, \quad (A18)$$

$$S_2(\Delta^2)=\sum_{V=\varphi,\omega}\frac{\sigma_V G_d^{K,V}}{2(\Delta^2+m_V^2)}, \quad (A19)$$

$$S_3(\Delta^2)=\sum_{V=\varphi,\omega}\frac{\sigma_V}{m_V^2(\Delta^2+m_V^2)} \times[G_s^{K,V}-\tfrac{1}{2}(m_{K_A}^2-m_K^2)G_d^{K,V}]. \quad (A20)$$

The divergence conditions, $V=\rho^0,\ \varphi,\ \omega$,

$$\langle V(q)|\partial_\nu A_{3\nu}^1(0)|K^+(k)\rangle = -\frac{2F_K m_K^2}{\Delta^2+m_K^2}G_{VKK}\epsilon^{(V)}\cdot k, \quad (A21)$$

$$\langle K_A^+(q)|\partial_\mu V_\mu^{(\mathrm{em},8,0)}(0)|K^+(k)\rangle=0, \quad (A22)$$

with

$$\sigma_\varphi=-\sigma_\omega\left(\frac{G_\omega}{G_\varphi}\right)\left(\frac{m_\varphi^2}{m_\omega^2}\right), \quad (A23)$$

are then sufficient to determine the three s-wave and three d-wave couplings, $G_s^{K,V}$, $G_d^{K,V}$ $(V=\rho^0,\ \varphi,\ \omega)$. These conditions are

$$-\frac{G_\rho}{F_K}+\frac{G_{K_A}G_s^{K,\rho}}{m_{K_A}-m_\rho^2}+2G_{\rho KK}F_K-\frac{G_{K_A}}{m_{K_A}^2} \times[G_s^{K,\rho}-\tfrac{1}{2}(m_\rho^2-m_K^2)G_d^{K,\rho}]=0, \quad (A24)$$

$$\frac{G_{K_A}}{F_K}+\sum_{V=\rho^0,\varphi,\omega}\frac{m_{K_A}^2 g_V G_s^{K,V}}{m_V^2(m_V^2-m_{K_A}^2)}+\tfrac{1}{2}(m_{K_A}^2-m_K^2) \times\sum_{V=\rho^0,\varphi,\omega}\frac{g_V G_d^{K,V}}{m_V^2}=0, \quad (A25)$$

$$-\sqrt{3}\frac{G_V}{F_K}+\frac{G_{K_A}G_s^{K,V}}{(m_{K_A}^2-m_V^2)}+2G_{VKK}F_K-\frac{G_{K_A}}{m_{K_A}^2} \times[G_s^{K,V}-\tfrac{1}{2}(m_V^2-m_K^2)G_d^{K,V}]=0, \quad (V=\varphi,\omega) \quad (A26)$$

$$-\frac{\sqrt{3}G_{K_A}}{2F_K}+\sum_{V=\varphi,\omega}\frac{G_V G_s^{K,V}}{(m_{K_A}^2-m_V^2)}+\sum_{V=\varphi,\omega}\frac{G_V}{m_V^2} \times[G_s^{K,V}-\tfrac{1}{2}(m_{K_A}^2-m_K^2)G_d^{K,V}]=0, \quad (A27)$$

$$G_\omega G_s^{K,\varphi}-G_\varphi G_s^{K,\omega} -\tfrac{1}{2}(m_{K_A}^2-m_K^2)(G_\omega G_d^{K,\varphi}-G_\varphi G_d^{K,\omega})=0, \quad (A28)$$

with

$$G_s^{K,\rho}=\left(2-\frac{m_K^2}{m_\rho^2}-\frac{m_K^2}{m_{K_A}^2}\right)^{-1}(m_\rho^2-m_{K_A}^2) \times\left[\left(1-\frac{m_K^2}{m_{K_A}^2}\right)\frac{m_{K_A}^2}{m_\rho^2 G_{K_A}}\left(-\frac{G_\rho}{F_K}+2G_{\rho KK}F_K\right) +\left(1-\frac{m_K^2}{m_\rho^2}\right)\frac{m_\rho^2}{m_{K_A}^2 G_\rho}\left(-\frac{G_{K_A}}{2F_K}\right)\right], \quad (A29)$$

and

$$\left(1-\frac{m_K^2}{m_{K_A}^2}\right)\sum_{V=\varphi,\omega}\frac{G_{K_A}G_V G_d^{K,V}}{\sqrt{3}m_V^2 m_{K_A}^2} =-\frac{G_{K_A}^2}{m_{K_A}^4 F_K}-\frac{2}{\sqrt{3}}\sum_{V=\varphi,\omega}\frac{G_{K_A}G_V G_s^{K,V}}{m_{K_A}^2 m_V^2(m_V^2-m_{K_A}^2)}; \quad (A30)$$

note that

$$\left(1-\frac{m_K^2}{m_{K_A}^2}\right)\sum_{V=\varphi,\omega}\frac{G_{K_A}G_V G_d^{K,V}}{\sqrt{3}m_V^2 m_{K_A}^2} =-\frac{G_{K_A}^2}{2m_{K_A}^4 F_K}+\sum_{V=\varphi,\omega}\left(\frac{G_V^2}{m_V^4 F_K}-\frac{2G_V G_{VKK}F_K}{\sqrt{3}m_V^4}\right) +\text{terms}\left\{O\left(\frac{m_K^2}{m_\varphi^2}-\frac{m_K^2}{m_\omega^2}\right), O\left(\frac{m_K^2}{m_{K_A}^2}-\frac{m_K^2}{m_V^2}\right)\right\}. \quad (A31)$$

The perturbations $O(m_K^2/m_\alpha^2-m_K^2/m_\beta^2)$ may be straightforwardly obtained by suitably manipulating Eqs. (A26)–(A28).