Muon Capture and the Hypothesis of Partially Conserved Axial-Vector Current*

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We present the results of a new calculation of the muon-capture rates in hydrogen and He^3, with special emphasis on the estimate of the induced pseudoscalar form factor F_P. Two values of F_P based on the Nambu and the Gell-Mann–Lévy versions of the PCAC hypothesis are given, and the corresponding capture rates are compared with experimental data.

I. INTRODUCTION

THE main purpose of the study of muon capture in hydrogen and nuclei is to establish the general principles of weak interactions such as muon-electron universality, V-A interaction, and the hypotheses of conserved vector current (CVC)[1] and partially conserved axial-vector current (PCAC).[2]

The experimental study of muon capture in hydrogen has always been performed with the use of liquid hydrogen; thus the theoretical analysis is complicated by the difficulties in solving the molecular physics problems involved. Recently, a preliminary result of the experimental study of muon capture in gaseous hydrogen has been reported.[3] This eliminates the difficulties arising from the molecular physics problems.

The conventional analysis of the muon capture in nuclei has been based on the impulse approximation, and the calculated capture rates are in general rather sensitive to the details of the nuclear wave functions used. This difficulty can be avoided by the use of the elementary-particle treatment of nuclei developed previously.[4] The capture rate of the reaction $\mu^-+He^3\to H^3+\nu$ has been most carefully measured, and a more accurate experiment is currently under consideration.

For the reasons described above, the present paper re-examines and refines the previous calculations of the capture rate in hydrogen[5] and He^3 (Ref. 6) by using the latest experimental data available for the calculation of the capture rates. In particular, we calculate the value of the induced pseudoscalar form factor F_P with the use of the Gell-Mann–Lévy version[7] of PCAC. We also present an estimate of F_P based on the Nambu version[8] of PCAC. It is found that two estimates of F_P are different by 11% in the case of hydrogen and by 40% in the case of He^3. The capture rates are not very sensitive to the values of F_P but more improved data could certainly determine which of the two versions is correct.

II. CAPTURE-RATE FORMULA

The matrix elements for the muon-capture process $\mu^-+N_i\to N_f+\nu$ is given by

$$\langle\nu,N_f|H_w(0)|\mu^-,N_i\rangle=\frac{G\cos\theta_C}{\sqrt{2}}(\bar{u}_\nu\gamma_\alpha(1+\gamma_5)u_\mu)\times\langle N_f|V_\alpha^{(-)}+A_\alpha^{(-)}|N_i\rangle,\quad(1)$$

where G $(=1.02\times10^{-5}/m_p^2)$ is the weak-coupling constant, θ_C is the Cabibbo angle ($\cos\theta_C=0.98$), and $V_\alpha^{(-)}$ and $A_\alpha^{(-)}$ are the vector and axial-vector hadron weak currents, respectively. For the cases of interest, i.e., $\mu^-+p\to n+\nu$ and $\mu^-+He^3\to H^3+\nu$, the matrix elements $\langle N_f|V_\alpha^{(-)}|N_i\rangle$ and $\langle N_f|A_\alpha^{(-)}|N_i\rangle$ are given by

$$\langle N_f|V_\alpha^{(-)}|N_i\rangle=\bar{u}_f[\gamma_\alpha F_V(q^2;N_i\to N_f)-(\sigma_{\alpha\beta}q_\beta/2m_p)F_M(q^2;N_i\to N_f)]u_i,\quad(2a)$$

$$\langle N_f|A_\alpha^{(-)}|N_i\rangle=\bar{u}_f[\gamma_\alpha\gamma_5F_A(q^2;N_i\to N_f)+(i(m_i+m_f)/m_\pi^2)q_\alpha\gamma_5F_P(q^2;N_i\to N_f)]u_i,\quad(2b)$$

$$q_\alpha=(p_f-p_i)_\alpha,$$

where we have treated the nuclei as elementary particles.[4] In Eq. (2), F_V, F_M, F_A, and F_P are, respectively, vector, weak-magnetism, axial-vector, and induced pseudoscalar form factors which contain in this treatment all of the information regarding the effects of nuclear structure, and u_f and u_i are spinors describing, in the nuclear case, the motion of the nucleus as a whole. We have also assumed that the weak hadron currents are first-class currents,[9] so that the scalar and tensor form factors F_S and F_T do not appear in Eq. (2).

The matrix elements (1), (2a), and (2b) yield the well-known expression for the muon-capture

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[2] Y. Nambu, Phys. Rev. Letters **4**, 380 (1960); J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo Cimento **17**, 757 (1960); M. Gell-Mann and M. Lévy, *ibid.* **16**, 705 (1960).
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[4] C. W. Kim and H. Primakoff, Phys. Rev. **139**, B1447 (1965); **140**, B566 (1965).
[5] H. Primakoff, Rev. Mod. Phys. **31**, 802 (1959); P. K. Kabir, Z. Physik **191**, 447 (1966).
[6] C. W. Kim and H. Primakoff, Phys. Rev. **140**, B566 (1965).
[7] See the last paper in Ref. 2.
[8] See the first paper in Ref. 2.
[9] S. Weinberg, Phys. Rev. **112**, 1375 (1958).

rate[10]

$$\Gamma(\mu^- + N_i \to N_f + \nu) = \frac{G^2 \cos^2\theta_C \alpha^3 m_\mu^5 [Z(N_i)]^3}{2\pi^2} C(N_i)\eta_{if}^2 \times \{[G_V(q^2; N_i \to N_f)]^2 + 3[G_A(q^2; N_i \to N_f)]^2 - 2G_A(q^2; N_i \to N_f)G_P(q^2; N_i \to N_f) + [G_P(q^2; N_i \to N_f)]^2\};$$

$$\eta_{if}^2 = \left(\frac{E_\nu}{m_\mu}\right)^2 \left(1 - \frac{E_\nu}{m_\mu + m_f}\right)\left(\frac{m_i}{m_i + m_\mu}\right)^3,$$

$$E_\nu = m_\mu \left[1 + \frac{m_i^2 - m_f^2 + m_\mu^2}{2m_\mu m_i}\right]\left(1 + \frac{m_\mu}{m_i}\right)^{-1},$$

$$G_V(q^2; N_i \to N_f) = F_V(q^2; N_i \to N_f)\left(1 + \frac{E_\nu}{2m_f}\right),$$

$$G_A(q^2; N_i \to N_f) = -F_A(q^2; N_i \to N_f) - \frac{E_\nu}{2m_f}\left[F_V(q^2; N_i \to N_f) + \frac{m_f}{m_p} F_M(q^2; N_i \to N_f)\right], \tag{3}$$

$$G_P(q^2; N_i \to N_f) = \frac{E_\nu}{2m_f}\left[\frac{(m_i + m_f)}{m_\pi^2} m_\mu F_P(q^2; N_i \to N_f) - \frac{m_f}{m_p} F_M(q^2; N_i \to N_f) + F_A(q^2; N_i \to N_f) - F_V(q^2; N_i \to N_f)\right],$$

$$q^2 = m_\mu^2 + 2m_\mu\left(m_i - m_f - \frac{m_\mu^2}{2m_f}\right),$$

where $Z(N_i)$ is the charge of the nucleus N_i, α is the fine-structure constant, and $C(N_i)$ is a correction factor for the effect of the nonpoint character of the charge distribution of N_i which takes the values $C(p) = 1$ and $C(\mathrm{He}^3) = 0.965$.[6] Thus, the capture rate is completely determined by the form factors F_V, F_M, F_A, and F_P.

III. FORM FACTORS—THEORETICAL

We now proceed to calculate the values of the form factors at the appropriate momentum transfer q^2, i.e., $q^2 = 0.88 m_\mu^2$ for $\mu^- + p \to n + \nu$ and $q^2 = 0.96 m_\mu^2$ for $\mu^- + \mathrm{He}^3 \to \mathrm{H}^3 + \nu$.

The form factors associated with the vector current are obtained from the CVC hypothesis which relates F_V and F_M to the Dirac and Pauli electromagnetic form factors $F_{\mathrm{Dirac}}(q^2; N)$ and $F_{\mathrm{Pauli}}(q^2; N)$ by

$$F_V(q^2; N_i \to N_f) = F_{\mathrm{Dirac}}(q^2; N_i) - F_{\mathrm{Dirac}}(q^2; N_f), \tag{4}$$

$$F_M(q^2; N_i \to N_f) = F_{\mathrm{Pauli}}(q^2; N_i) - F_{\mathrm{Pauli}}(q^2; N_f). \tag{5}$$

Furthermore, F_{Dirac} and F_{Pauli} are given in terms of the charge and magnetic form factors F_{Ch} and F_{Mag} as

$$F_{\mathrm{Dirac}}(q^2; N) = \left[Z(N)F_{\mathrm{Ch}}(q^2; N) + \frac{q^2}{4m_p^2}\mu(N)F_{\mathrm{Mag}}(q^2; N)\right] \times \left(1 + \frac{q^2}{4m_p^2}\right)^{-1},$$

$$F_{\mathrm{Pauli}}(q^2; N) = \left[\mu(N)F_{\mathrm{Mag}}(q^2; N) - \frac{Z(N)}{A(N)}F_{\mathrm{Ch}}(q^2; N)\right] \times \left(1 + \frac{q^2}{4m_p^2}\right)^{-1}, \tag{6}$$

where $\mu(N)$ and $A(N)$ are the magnetic moment and mass number of the nucleus N. The form factors $F_{\mathrm{Ch}} \times (q^2; N)$ and $F_{\mathrm{Mag}}(q^2; N)$ are, then, obtained from appropriate electron-N scattering experiments, so that F_V and F_M can be explicitly determined.

At the present there exists no theory to calculate unambiguously the form factor $F_A(q^2; N_i \to N_f)$ without a recourse to the impulse approximation. On the basis of a comparison of the expression for $\langle N_f | A_\alpha^{(-)} | N_i \rangle$ given in Eq. (2b) and the impulse-approximation expression of $\langle N_f | A_\alpha^{(-)} | N_i \rangle$ with meson-exchange corrections, it has been shown[6,11] that

$$\frac{F_A(q^2; N_i \to N_f)}{F_A(0; N_i \to N_f)} = \left[\frac{F_A(q^2; p \to n)}{F_A(0; p \to n)} \bigg/ \frac{F_V(q^2; p \to n)}{F_V(0; p \to n)}\right] \frac{F_M(q^2; N_i \to N_f)}{F_M(0; N_i \to N_f)} = \left(\frac{1 + q^2/M_V^2}{1 + q^2/M_A^2}\right)^2 \times \frac{F_M(q^2; N_i \to N_f)}{F_M(0; N_i \to N_f)}, \tag{7}$$

where we have used the experimental dipole-fit form factors

$$[F_{V,A}(q^2; p \to n)/F_{V,A}(0; p \to n)] = (1 + q^2/M_{V,A}^2)^{-2}$$

[10] Equation (3) is correct within neglect of terms $\approx E_\nu^2/4m_p^2$, $E_\nu^2/4m_i^2$, $\cdots$.

[11] In Ref. 6, it was assumed that

$$\left[\frac{F_A(q^2; p \to n)}{F_A(0; p \to n)} \bigg/ \frac{F_V(q^2; p \to n)}{F_V(0; p \to n)}\right] \cong 1.$$

with $M_V^2=0.71$ BeV^2 (Ref. 12) and $M_A^2=0.99$ BeV^2.[13] The value of $F_A(0; N_i\rightarrow N_f)$ can be obtained from the ft value of $N_f\rightarrow N_i+e^-+\bar{\nu}$.

It now remains to specify the form factor

$$F_P(q^2; N_i\rightarrow N_f).$$

The estimate of F_P is based on the PCAC hypothesis. Various hypotheses would predict different values of F_P, the Nambu version[8] and the Gell-Mann–Lévy version[7] being the well-known examples. First, we discuss the formula [Eq. (11) below] which has been used in the previous works.[6] In a way similar to that in Eq. (7), it can be shown[6] that

$$\frac{F_P(q^2; N_i\rightarrow N_f)}{F_P(0; N_i\rightarrow N_f)}=\left[\frac{F_P(q^2; p\rightarrow n)}{F_P(0; p\rightarrow n)}\Big/\frac{F_A(q^2; p\rightarrow n)}{F_A(0; p\rightarrow n)}\right]\frac{F_A(q^2; N_i\rightarrow N_f)}{F_A(0; N_i\rightarrow N_f)}. \quad (8)$$

With the use of the approximate relations

$$F_P(q^2; p\rightarrow n)\cong\frac{1}{1+q^2/m_\pi^2}F_P(0; p\rightarrow n)$$
$$F_A(q^2; p\rightarrow n)\cong F_A(0; p\rightarrow n) \quad \text{for} \quad |q^2|\lesssim m_\pi^2 \quad (9)$$

and the Goldberger-Treiman relation in the form[6,14]

$$F_P(0; N_i\rightarrow N_f)\cong -F_A(0; N_i\rightarrow N_f), \quad (10)$$

Eq. (8) becomes

$$F_P(q^2; N_i\rightarrow N_f)=-\frac{1}{1+q^2/m_\pi^2}F_A(q^2; N_i\rightarrow N_f). \quad (11)$$

It is interesting to note that Eq. (11) immediately follows if one uses the Nambu version[8] of PCAC. This is based on the assumption

$$\lim_{q^2\rightarrow\infty}\langle N_f|\partial_\alpha A_\alpha^{(-)}|N_i\rangle=0, \quad (12)$$

which, Nambu argues, can be satisfied by the condition

$$F_A(q^2; N_i\rightarrow N_f)+\frac{m_\pi^2+q^2}{m_\pi^2}F_P(q^2; N_i\rightarrow N_f)=0. \quad (13)$$

This immediately leads to Eq. (11). For this reason we refer to Eq. (11) as the Nambu formula, in spite of the fact that Nambu's argument may not necessarily be valid for the nuclear case.[15]

The Gell-Mann–Lévy version of PCAC[7] is based on the relation

$$\partial_\alpha A_\alpha^{(\pm)}(x)=a_\pi m_\pi^3\varphi_\pi^{(\pm)}(x), \quad (14)$$

where a_π $(=0.95)$ is the pion-decay coupling constant and $\varphi_\pi^{(\pm)}(x)$ is the pion field. Taking the matrix element of Eq. (14) between $\langle N_f|$and$|N_i\rangle$, we obtain

$$F_A(q^2; N_i\rightarrow N_f)+(q^2/m_\pi^2)F_P(q^2; N_i\rightarrow N_f)=a_\pi m_\pi^2 f_{\pi N_iN_f}(q^2)/(m_\pi^2+q^2), \quad (15)$$

where $f_{\pi N_iN_f}(q^2)$ is the πN_iN_f vertex function evaluated at q^2, so that, rewriting Eq. (15), we have

$$F_P(q^2; N_i\rightarrow N_f)=-\frac{m_\pi^2}{m_\pi^2+q^2}F_A(q^2; N_i\rightarrow N_f)\times\left\{1+\frac{m_\pi^2}{q^2}\left[1-\frac{a_\pi f_{\pi N_iN_f}(q^2)}{F_A(q^2; N_i\rightarrow N_f)}\right]\right\}. \quad (16)$$

Equation (16) shows that in the Gell-Mann–Lévy version, a knowledge of $f_{\pi N_iN_f}(q^2)$ as well as of $F_A(q^2;N_i\rightarrow N_f)$ is necessary to calculate $F_P(q^2;N_i\rightarrow N_f)$. In view of the absence of such a knowledge of $f_{\pi N_iN_f}(q^2)$, we proceed to estimate $f_{\pi N_iN_f}(q^2)$ in the following way. The value of $f_{\pi N_iN_f}(0)$ can be calculated from the experimental value of $F_A(0; N_i\rightarrow N_f)$ with the use of the Goldberger-Treiman relation[4,14]

$$F_A(0; N_i\rightarrow N_f)=a_\pi f_{\pi N_iN_f}(0), \quad (17)$$

which is obtained from Eq. (15) in the limit $q^2=0$. The value of $f_{\pi N_iN_f}(-m_\pi^2)$, i.e., the physical πN_iN_f strong coupling constant, may be obtained from an analysis of appropriate experimental data, e.g., $f_{\pi pp}(-m_\pi^2)$ from a dispersion-theoretic analysis of $\pi^\pm+p\rightarrow\pi^\pm+p$ elastic scattering experiments. Once the values of $f_{\pi N_iN_f}(0)$ and $f_{\pi N_iN_f}(-m_\pi^2)$ are known, $f_{\pi N_iN_f}(q^2)$ for $|q^2|\lesssim m_\pi^2$ may be calculated by means of a linear extrapolation of the values at the two points.

The justification of the linear extrapolation in q^2 for $|q^2|\lesssim m_\pi^2$ is based upon a dispersion-theoretic calculation of the form factor in $\langle N_f|\partial_\alpha A_\alpha^{(-)}|N_i\rangle$. Under the assumption of the unsubtracted dispersion relation, we have[16]

$$\langle N_f|\partial_\alpha A_\alpha^{(-)}|N_i\rangle=(m_i+m_f)(\bar{u}_f\gamma_5u_i)\Phi(q^2)$$

$$\Phi(q^2)=F_A(q^2; N_i\rightarrow N_f)+\frac{q^2}{m_\pi^2}F_P(q^2; N_i\rightarrow N_f)$$
$$=\frac{a_\pi m_\pi^2 f_{\pi N_iN_f}(-m_\pi^2)}{q^2+m_\pi^2}+\frac{a_\beta m_\beta^2 f_{\beta N_iN_f}(-m_\beta^2)}{q^2+m_\beta^2}, \quad (18)$$

[12] See, for example, G. Weber in *Proceedings of the 1967 International Symposium on Electron and Photon Interactions at High Energy, Stanford, 1967* (Stanford Linear Accelerator Center, Stanford, Calif., 1967).

[13] The value $M_A=(0.99)^{1/2}$ BeV is an average of two measurements, $M_A=1.1\pm0.3$ BeV [T. B. Novey, Invited talk at Washington, D. C. meeting of the American Physical Society, 1968 (unpublished)] and $M_A=0.81_{-0.20}^{+0.13}$ BeV [E. C. M. Young, CERN Report No. CERN 67-12, 1967 (unpublished)].

[14] H. Primakoff, in *High Energy Physics and Nuclear Structure*, edited by G. Alexander (North-Holland Publishing Co., Amsterdam, The Netherlands, 1967).

[15] The usual pion-pole dominance is, in general, not a good assumption for nuclear processes. Thus the Nambu argument may be valid only for the value of $q^2\cong0$ for nuclear processes.

[16] See, for example, C. W. Kim and M. Ram, Phys. Rev. **162**, 1584 (1967).

where β represents the effective 3π, 5π, $\cdots$, $J^P=0^-$ contribution which we approximate by a pole of mass m_β and is *not* necessarily related to a real particle or resonance. Comparison of Eq. (18) with Eq. (15) gives

$$f_{\pi N_i N_f}(q^2)=f_{\pi N_i N_f}(-m_\pi^2)\times\left[1+\frac{a_\beta f_{\beta N_i N_f}(-m_\beta^2)}{a_\pi f_{\pi N_i N_f}(-m_\pi^2)}\left(\frac{1+q^2/m_\pi^2}{1+q^2/m_\beta^2}\right)\right]. \quad (19)$$

Since we expect $m_\beta^2 \gtrsim (3m_\pi)^2$ for nucleon and He^3-H^3 systems[17] and $|q^2|\lesssim m_\pi^2$, the quantity $(1+q^2/m_\beta^2)^{-1}$ in Eq. (19) can be expanded to give

$$f_{\pi N_i N_f}(q^2)=f_{\pi N_i N_f}(-m_\pi^2)(1+\xi)\times\left\{1+\frac{\xi}{1+\xi}\left(\frac{m_\beta^2-m_\pi^2}{m_\beta^2 m_\pi^2}\right)q^2\left(1-\frac{q^2}{m_\beta^2}+\cdots\right)\right\}, \quad (20)$$

where

$$\xi=\frac{a_\beta f_{\beta N_i N_f}(-m_\beta^2)}{a_\pi f_{\pi N_i N_f}(-m_\pi^2)}\ll 1.$$

This clearly indicates that the linear approximation in q^2 for $|q^2|\lesssim m_\pi^2$ is valid; in fact, for $q^2=m_\mu^2$, the correction is less than 1%. Eliminating the unknown parameters ξ and m_β in Eq. (20) with the use of the values $f_{\pi N_i N_f}(0)=a_\pi^{-1}F_A(0;\,N_i\to N_f)$ and $f_{\pi N_i N_f}(-m_\pi^2)$, and keeping only the term linear in q^2, we obtain

$$f_{\pi N_i N_f}(q^2)=\frac{F_A(0;\,N_i\to N_f)}{a_\pi}\times\left\{1+\left[1-\frac{a_\pi f_{\pi N_i N_f}(-m_\pi^2)}{F_A(0;\,N_i\to N_f)}\right]\frac{q^2}{m_\pi^2}\right\}\quad\text{for}\quad |q^2|\lesssim m_\pi^2, \quad (21)$$

which, of course, reduces to the Goldberger-Treiman relation of Eq. (17) in the limit $q^2=0$. Substituting Eq. (21) into Eq. (16), we finally obtain

$$F_P(q^2;\,N_i\to N_f)=-\frac{1}{1+q^2/m_\pi^2}F_A(q^2;\,N_i\to N_f)\times\left\{1+\frac{m_\pi^2}{q^2}\left(1-\frac{F_A(0;\,N_i\to N_f)}{F_A(q^2;\,N_i\to N_f)}\right)+\frac{a_\pi f_{\pi N_i N_f}(-m_\pi^2)-F_A(0;\,N_i\to N_f)}{F_A(q^2;\,N_i\to N_f)}\right\}. \quad (22)$$

We note that in the approximations, $F_A(q^2;\,N_i\to N_f)\cong F_A(0;\,N_i\to N_f)$ and $f_{\pi N_i N_f}(-m_\pi^2)\cong f_{\pi N_i N_f}(0)$, Eq. (22) reduces, with the use of Eq. (17), to the Nambu formula, Eq. (11). However, these approximations are, in general, not valid for the nuclear case and we expect the quantity in the bracket in Eq. (22) to be different from unity, i.e., the nonequivalence of the two versions of PCAC. If we set this quantity equal to unity, we would then obtain the following q^2 dependence of $F_A(q^2;\,N_i\to N_f)$;

$$F_A(q^2;\,N_i\to N_f)=F_A(0;\,N_i\to N_f)\times\left\{1-\left[\frac{f_{\pi N_i N_f}(-m_\pi^2)}{f_{\pi N_i N_f}(0)}-1\right]\frac{q^2}{m_\pi^2}\right\}\ \text{for}\ |q^2|\lesssim m_\pi^2. \quad (23)$$

Comparison of Eq. (23) with the dipole-fit form

$$F_A(q^2;\,N_i\to N_f)=F_A(0;\,N_i\to N_f)[1+(q^2/M_A^2)]^{-2}\cong F_A(0;\,N_i\to N_f)[1-(2q^2/M_A^2)]\quad\text{for}\ |q^2|\lesssim m_\pi^2 \quad (24)$$

yields

$$M_A=\sqrt{2}m_\pi\left(\frac{f_{\pi N_i N_f}(-m_\pi^2)}{f_{\pi N_i N_f}(0)}-1\right)^{-1/2}. \quad (25)$$

For the case of the nucleon, where $f_{\pi np}(-m_\pi^2)=1.43$ (Ref. 18) and $f_{\pi np}(0)=F_A(0;\,p\to n)/a_\pi=1.24$, we obtain, from Eq. (25),

$$M_A\cong 0.5\ \text{BeV}, \quad (26)$$

which is in disagreement[19] with the experimental results[13] obtained from neutrino-nucleon elastic scattering.

IV. FORM FACTORS—NUMERICAL

In this section the numerical values of the form factors are presented for the processes (A.) $\mu^-+p\to n+\nu$ and (B.) $\mu^-+He^3\to H^3+\nu$.

A. $\mu^-+p\to n+\nu$ ($q^2=0.88m_\mu^2$)

From the electron-nucleon[12] and neutrino-nucleon[13] scattering data, we have

$$F_V(q^2=0.88m_\mu^2;\,p\to n)=0.973, \quad (27)$$

$$F_M(q^2=0.88m_\mu^2;\,p\to n)=3.60, \quad (28)$$

$$F_A(q^2=0.88m_\mu^2;\,p\to n)=1.156, \quad (29)$$

where we have used $F_A(0;\,p\to n)=1.18$.

Let us denote the Nambu and Gell-Mann–Lévy formulas of F_P by F_P^{N} and $F_P^{\text{G-L}}$, respectively. We then have, from Eqs. (11) and (29),

$$F_P^{\text{N}}(q^2=0.88m_\mu^2;\,p\to n)=-0.769[-6.63F_A(0;\,p\to n)]. \quad (30)$$

[17] This can be seen from the fact that in the dispersion-theoretic treatment of the Goldberger-Treiman relation for the He^3-H^3 system, the contribution of states other than the dominant single pion pole is about 15% as in the case of the n-p system (see Ref. 23 below).

[18] J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. **35**, 737 (1963).

[19] For the case of the He^3-H^3 system, the agreement becomes even worse, because Eq. (25) and the values, $f_{\pi He^3H^3}(0)=1.27$ and $f_{\pi He^3H^3}(-m_\pi^2)=-1.13$ (see Ref. 23) would predict an imaginary value of M_A.

The value quoted in the bracket represents the corresponding value in the usual normalization of F_P as given in

$$\langle N_f|A_\alpha^{(-)}|N_i\rangle=\bar{u}_f\left[\gamma_\alpha\gamma_5F_A(q^2;N_i\to N_f)+\frac{iq_\alpha\gamma_5}{m_\mu}F_P(q^2;N_i\to N_f)\right]u_i. \quad (31)$$

Substituting the values of $f_{\pi np}(-m_\pi^2)=1.43$ (Ref. 18) and Eq. (29) into Eq. (22), we have

$$F_P^{\mathrm{G-L}}(q^2=0.88m_\mu^2;p\to n)=-0.853\,[-7.37F_A(0;p\to n)]. \quad (32)$$

We note that $F_P^{\mathrm{G-L}}(q^2=0.88m_\mu^2;p\to n)$ is larger (in magnitude) than $F_P^{\mathrm{N}}(q^2=0.88m_\mu^2;p\to n)$ by about 11% and also that the value of F_P on the basis of a dispersion-theoretic argument[20] is known to be roughly $-0.83\,[\sim-7F_A(0;p\to n)]$.

B. $\mu^-+\mathrm{He}^3\to\mathrm{H}^3+\nu$ ($q^2=0.96m_\mu^2$)

From the elastic electron-He^3 and electron-H^3 scattering data[21] and Eqs. (4) and (5), we have

$$F_V(q^2=0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)=0.811, \quad (33)$$

$$F_M(q^2=0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)=-4.69, \quad (34)$$

where we have used the experimental values of the magnetic moment, $\mu(\mathrm{He}^3)=-2.13$ and $\mu(\mathrm{H}^3)=2.98$. The measured rate of $\mathrm{H}^3\to\mathrm{He}^3+e^-+\bar{\nu}$ gives[22]

$$F_A(0;\mathrm{He}^3\to\mathrm{H}^3)=-1.207, \quad (35)$$

so that from Eq. (7)

$$F_A(q^2=0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)=-1.046. \quad (36)$$

The Nambu formula of Eq. (11) yields

$$F_P^{\mathrm{N}}(q^2=0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)=0.675. \quad (37)$$

Substitution of Eqs. (35) and (36), and the value $f_{\pi\mathrm{He}^3\mathrm{H}^3}(-m_\pi^2)=-1.13$ which has been calculated[23] from the observed Panofsky ratio

$$\Gamma(\pi^-+\mathrm{He}^3\to\mathrm{H}^3+\pi^0)/\Gamma(\pi^-+\mathrm{He}^3\to\mathrm{H}^3+\gamma),$$

into Eq. (22) yields

$$F_P^{\mathrm{G-L}}(q^2=0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)=0.400, \quad (38)$$

which is considerably smaller than the value of

$$F_P^{\mathrm{N}}(q^2=0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)$$

of Eq. (37), in fact smaller by about 40%. This is due to a rather sensitive q^2 dependence of $F_A(q^2;\mathrm{He}^3\to\mathrm{H}^3)$ which results from the nuclear structure effects. With this property of $F_P(q^2;\mathrm{He}^3\to\mathrm{H}^3)$ one can compare, in nuclear muon capture, the two versions of PCAC and determine which of the two versions is correct.

V. RESULTS

A. $\mu^-+p\to n+\nu$

The rate of the process $\mu^-+p\to n+\nu$ from μp singlet atomic system is given by[5]

$$\Gamma(\mu^-+p\to n+\nu;\mu p\text{ singlet})=\frac{G^2\cos^2\theta_C\alpha^3m_\mu^5}{2\pi^2}\eta_{pn}^2[G_V-3G_A+G_P]^2. \quad (39)$$

We use

$$\eta_{pn}^2=0.576,\quad E_\nu=99.2\text{ MeV}. \quad (40)$$

The numerical values of the form factors are given by

$$\begin{aligned}G_V(q^2=0.88m_\mu^2;p\to n)&=1.024\\ G_A(q^2=0.88m_\mu^2;p\to n)&=-1.397\\ G_P^{\mathrm{N}}(q^2=0.88m_\mu^2;p\to n)&=-0.594\\ G_P^{\mathrm{G-L}}(q^2=0.88m_\mu^2;p\to n)&=-0.639.\end{aligned} \quad (41)$$

From Eqs. (39)–(41) we obtain the capture rates as listed in Table I. We have also listed in Table I the capture rates calculated from the value, $F_A(0;p\to n)=1.23$, which is based on the recently measured value of the neutron half-life.[24]

B. $\mu^-+\mathrm{He}^3\to\mathrm{H}^3+\nu$

From the numerical values

$$\eta_{if}^2=0.824,\quad E_\nu=103.2\text{ MeV},\quad C(\mathrm{He}^3)=0.965 \quad (42)$$

and the form factors

$$\begin{aligned}G_V(q^2=0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)&=0.826\\ G_A(q^2=0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)&=1.290\\ G_P^{\mathrm{N}}(q^2=0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)&=0.602\\ G_P^{\mathrm{G-L}}(q^2=0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)&=0.448,\end{aligned} \quad (43)$$

we obtain the capture rates as listed in Table I. In this case the capture rates do not depend directly on the value of $F_A(0;p\to n)$.

Finally, we remark that the errors in the theoretical capture rates in Table I, which arise mainly from the uncertainties in the experimental values of the form factors and the ft values, are less than 3%.

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VI. DISCUSSION

The theoretical results for the capture rate given in Table I are to be compared with the recent experimental results

$$\Gamma(\mu^- + p \to n + \nu;\ \mu p \text{ singlet})]_{\text{expt.}} = 640 \pm 70 \text{ sec}^{-1} \quad [\text{Bologna-CERN (Ref. 3)}],$$
$$\Gamma(\mu^- + \text{He}^3 \to \text{H}^3 + \nu)]_{\text{expt.}} = 1505 \pm 46 \text{ sec}^{-1} \quad [\text{Berkeley (Ref. 25)}], \quad = 1465 \pm 67 \text{ sec}^{-1} \quad [\text{Carnegie (Ref. 26)}]. \tag{44}$$

The over-all agreement between theory and experiment is gratifying and must be viewed as lending general support to the belief in the validity of V-A interaction, muon-electron universality, CVC, and PCAC.

The capture rate is not very sensitive to the values of F_P and the present experimental errors are still too large to eliminate one or the other version of PCAC. However, in view of the recent spectacular success of the Gell-Mann–Lévy version of PCAC in the application of current algebra to various problems of elementary-particle physics, we believe that the capture rates with $G_P^{\text{G-L}}$ and $F_A(0; p \to n) = 1.23$, as given in the last column of Table I, are the best theoretical values at the present time.

Note added in proof. The final result of the experiment of Ref. 3 has been reported as

$$\Gamma(\mu^- + p \to n + \nu;\ \mu p \text{ singlet})]_{\text{expt}} = 651 \pm 57 \text{ sec}^{-1}$$

(Phys. Rev., to be published) which is in excellent agreement with our theoretical estimate (d) in Table I.

TABLE I. Theoretical capture rates in units of sec^{-1}. The capture rates (a), (b), (c), and (d) are, respectively, for G_P^{N} and $F_A \times (0; p \to n) = 1.18$, $G_P^{\text{G-L}}$ and $F_A(0; p \to n) = 1.18$, G_P^{N} and $F_A \times (0; p \to n) = 1.23$, and $G_P^{\text{G-L}}$ and $F_A(0; p \to n) = 1.23$.

Process	Capture rate (a)	Capture rate (b)	Capture rate (c)	Capture rate (d)
$\mu^- + p \to n + \nu$	625	613	662	654
$\mu^- + \text{He}^3 \to \text{H}^3 + \nu$	1449	1525	1449	1525

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Pole Dominance and a Low-Energy Theorem in the Radiative Decays of Charged Kaons*

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A low-energy theorem is derived for the structure-dependent axial-vector form factor in the radiative decay $K \to l + \nu + \gamma$ in the soft-kaon approximation. Corrections of the order of (m_K^2/m_V^2) $(V = \rho, \omega, \varphi)$ are obtained in the pole-dominance approximation. In each approximation, the model predictions of both (i) asymptotic $SU(3)$ and (ii) current mixing are investigated. The quantity $|\gamma_K| \equiv |a_K(0)/F(0)|$ is calculated in both approximations and in both models. It is found that the soft-kaon result is shifted upward by approximately 20%; the separation between the models in the two approximations is of the same order of magnitude.

I. INTRODUCTION

THE techniques of current algebra have recently been used[1,2] to study the radiative decays of charged pions. In particular a low-energy theorem for the structure-dependent axial-vector part of the radiative decay $\pi \to l + \nu + \gamma$ has been derived both in the soft-pion approximation[1] and in the pole-dominance approximation (PDA).[2] In this paper we consider the extension of such techniques to the analogous radiative decay $K \to l + \nu + \gamma$ where the theoretical situation is much less clear. One of the bases for our interest in such a calculation is the expectation that the PDA calculation of, say,[1] $dA/d\nu(q^2 = 0, \Delta^2 = 0)$, in this case, might prove substantially different from the SKA (soft-kaon approximation) result because of hardly negligible "correction terms" of the order of m_K^2/m_V^2 $(V = \rho, \omega, \varphi)$. At the same time, we are not aware of any experimental

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