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Self-Consistent Calculation of the ρ-Meson Electromagnetic Mass Splitting*

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Under the assumption that the electromagnetic (E.M.) mass splittings within the pseudoscalar and vector-meson octets self-consistently generate each other (at least in part), we compute the $\rho^{\pm}-\rho^0$ mass difference in terms of the vector-pseudoscalar-pseudoscalar (VPP) coupling strength $g^2/4\pi$ and the vector-vector-pseudoscalar (VVP) coupling strength $h^2/4\pi$. The requirement that g^2 and h^2 be in reasonable agreement with experiment strongly favors a small absolute value for $M_{\rho^{\pm}}-M_{\rho^0}$, with a negative value not ruled out.

I. INTRODUCTION

In a previous paper,[1] we showed, using a model developed previously for the spin-$\frac{1}{2}$ baryon octet,[2] that the medium strong (M.S.) mass splittings within the vector- and pseudoscalar-meson octets generate each other in a self-consistent manner. It will be the purpose of this work to use the model of Ref. 1 in order to compute the electromagnetic (E.M.) mass splittings within the above octets just as the model of Ref. 2 was used in the computation of the spin-$\frac{1}{2}$ baryon E.M. mass splittings.[3] Thus we will compute the four mass differences $M_{\rho^{\pm}}-M_{\rho^0}$, $M_{K^{*0}}-M_{K^{*\pm}}$, $\mu_{\pi^{\pm}}-\mu_{\pi^0}$, and $\mu_{K^{\pm}}-\mu_{K^0}$. Because of the presence of the elastic-form-factor (E.F.F.) contributions (e.g., one-photon exchange), the calculations will be only partially self-consistent. Before proceeding, we will comment on a few previous attempts.

The π mass splitting can be computed by itself in various ways, the most obvious of which is to compute the one-photon-exchange diagram for the pion self-energy which shifts the π^+ upward with respect to the π^0. This calculation is of course divergent and requires either a cutoff or form factor to achieve a finite result. Thus considerable interest was aroused by the work of Das *et al.*[4] in which a finite result, in good agreement with experiment, and with no cutoff parameter was arrived at through the use of the Weinberg[5] sum rules (provided the pion mass was taken to be zero).[6] The K mass difference is much more complicated, however, because the physical value is opposite in sign to that obtained from the one-photon-exchange model. In fact, the method used by Das *et al.* fails when extended to the K mass difference although this may be due to the fact that the σ term is important[7] or to a possible inadequacy in the extrapolation to zero kaon mass.

The case of the ρ meson may perhaps be the most interesting of all since there exists some evidence that $M_{\rho^0}>M_{\rho^{\pm}}$,[8] even though it is a $\Delta I=2$ mass difference and all other known $\Delta I=2$ mass differences have the charged components more massive than the neutral components. If this is confirmed, it will be in conflict with most of the previous calculations although there exists a possibility that inclusion of the $\Delta I=2$ ρ^0 mass shift in the field-algebra model[9] might account for it,[10] this being somewhat unclear because the expressions diverge.

In our model, we assume that the fraction of the E.M. mass splittings with the vector and pseudoscalar octets not caused by the E.F.F. is self-consistently generated. Thus the sign reversals of the K, K^*, and possibly the ρ will be obtained through the feedback mechanism. A calculation which exhibits some similarities to ours has been done by Sakuma.[11] This author, however, uses the one-channel approach[3] (in which each mass splitting is computed independently) and hence has to face the diffculty pointed out by Barton[12] in that the kaon mass difference develops a singularity as the strong VPP coupling constant $g^2/4\pi$ is allowed to vanish. This difficulty will not arise in our model, because we use the multichannel approach.[3]

II. CALCULATION OF M

As in Ref. 3, it will be convenient to utilize the U-spin formalism, thus assuming that the E.M. mass splittings are independent of the M.S. interaction. The U-spin multiplets are given by

$$\pi'=\begin{pmatrix}K^0\\ \tilde{\pi}^0\\ \bar{K}^0\end{pmatrix}, \tag{1a}$$

$$K'=\begin{pmatrix}\pi^-\\ K^-\end{pmatrix}, \tag{1b}$$

$$\eta'=(\tilde{\eta}),$$

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[1] S. L. Cohen, Phys. Rev. **174**, 1867 (1968).
[2] S. L. Cohen and C. R. Hagen, Phys. Rev. **149**, 1138 (1966).
[3] S. L. Cohen and C. R. Hagen, Phys. Rev. **157**, 1344 (1967).
[4] T. Das *et al.*, Phys. Rev. Letters **18**, 759 (1967).
[5] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967).
[6] G. C. Wick and B. Zumino, Physics Letters **25B**, 479 (1967).
[7] C. L. Cook *et al.*, Phys. Rev. Letters **20**, 295 (1968).
[8] J. Pisut and M. Roos, Nucl. Phys. **B6**, 325 (1968).
[9] B. W. Lee and H. T. Nieh, Phys. Rev. **166**, 1507 (1968).
[10] J. Schwinger, Phys. Rev. Letters **20**, 516 (1968).
[11] T. Sakuma, Progr. Theoret. Phys. (Kyoto) **38**, 1404 (1967).
[12] G. Barton and D. Dare, Phys. Rev. **150**, 1220 (1966).

with

$$\tilde{\eta}=-\tfrac{1}{2}\eta-\tfrac{1}{2}\sqrt{3}\pi^0, \tag{1d}$$

$$\tilde{\pi}^0=\tfrac{1}{2}\sqrt{3}\eta-\tfrac{1}{2}\pi^0, \tag{1e}$$

and

$$\rho=\begin{pmatrix} K^{*0} \\ \tilde{\rho}_0 \\ \bar{K}^{*0} \end{pmatrix}, \tag{2a}$$

$$K^{*\prime}=\begin{pmatrix} \rho^- \\ K^{*-} \end{pmatrix}, \tag{2b}$$

$$\omega^{8\prime}=(\tilde{\omega}^8), \tag{2c}$$

with

$$\tilde{\omega}^8=-\tfrac{1}{2}\omega^8-\tfrac{1}{2}\sqrt{3}\rho^0, \tag{2d}$$

$$\tilde{\rho}^0=\tfrac{1}{2}\sqrt{3}\omega^8-\tfrac{1}{2}\rho^0. \tag{2e}$$

The ω^8 is the octet component of the ω-φ system.[13] The mass splittings of the U-spin multiplets can be found from the U-spin formalism[3] and are given by

$$\mu_1'^2-\mu_3'^2=-\tfrac{2}{3}(2a+b), \tag{3a}$$

$$\mu_2'^2-\mu_3'^2=-\tfrac{1}{3}(a+2b), \tag{3b}$$

$$M_1'^2-M_3'^2=-\tfrac{2}{3}(2c+d), \tag{3c}$$

$$M_2'^2-M_3'^2=-\tfrac{1}{3}(c+2d), \tag{3d}$$

where μ_i' $(i=1\cdots3)$ denotes the masses of the π', K', and η' respectively, M_α' $(\alpha=1\cdots3)$ denotes the masses of the ρ', $K^{*\prime}$, and ω_8', respectively, and a, b, c, d are constants. In terms of the physically observed particles, one has

$$\mu_{K^\pm}^2-\mu_{K^0}^2=a, \tag{4a}$$

$$\mu_{\pi^0}^2-\mu_{\pi^\pm}^2=\tfrac{1}{2}b, \tag{4b}$$

$$M_{K^{*\pm}}^2-M_{K^{*0}}^2=c, \tag{4c}$$

$$M_{\rho^0}^2-M_{\rho^\pm}^2=\tfrac{1}{2}d, \tag{4d}$$

for which $a\cdots d$ may be determined.

To lowest order, we write[(1)...(3)]

$$\mu_i'^2=\bar{\mu}^2+\sum_\alpha\left[\frac{\partial\mu_i'^2}{\partial M_\alpha'^2}\right]_{M',\mu'}(M_\alpha'^2-M'^2)+\delta_{i2}E_P, \tag{5a}$$

$$M_\alpha'^2=\bar{M}^2+\sum_i\left[\frac{\partial M_\alpha'^2}{\partial\mu_i'^2}\right]_{M',\mu'}(\mu_i'^2-\mu'^2)+\delta_{\alpha2}E_V, \tag{5b}$$

with E_P denoting the E.F.F. for pseudoscalar mesons and E_V denoting the E.F.F. for vector mesons, E_P and E_V affecting only charged mesons. Using the Lehmann spectral mass sum rules [Eqs. (4a) and (4b) of Ref. 1], and performing the indicated differentiations of Eqs. (5), one obtains

$$\left[\frac{\partial\mu_i'^2}{\partial M_\gamma'^2}\right]_{M',\mu'}=-K_1\sum_j a_{ij\alpha}g_{ij\alpha}^2-2K_1'\sum_\gamma a_{i\gamma\alpha}h_{i\gamma\alpha}^2 -K_2\sum_{j,\gamma}a_{ij\gamma}g_{ij\gamma}^2\left[\frac{\partial\mu_j'^2}{\partial M_\alpha'^2}\right]_{M',\mu'}, \tag{6a}$$

$$\left[\frac{\partial M_\alpha'^2}{\partial\mu_j'^2}\right]_{M',\mu'}=-L_1\sum_\beta a_{\alpha\beta i}h_{\alpha\beta i}^2-2L_1'\sum_j a_{\alpha ji}g_{\alpha ji}^2 -L_2\sum_{j,\beta}a_{\alpha\beta j}h_{\alpha\beta j}^2\left[\frac{\partial M_\beta'^2}{\partial\mu_i'^2}\right]_{M',\mu'}. \tag{6b}$$

Here, g_{xyz} and h_{xyz} denote the PPV and PVV coupling constants, respectively, for a particle x dissociating virtually into particles y and z, a_{xyz} is the appropriate isospin factor, and $K_1\cdots L_2$ are the integrals resulting from the differentiations of the spectral functions and are given in Ref. 1. Assuming the coupling constants to be given by SU_3 in terms of the invariant couplings g and h, making the useful substitutions

$$\gamma=g^2K_2,\quad \eta=h^2L_2,\quad \alpha=g^2K_1-\tfrac{2}{3}h^2K_1',$$
$$\beta=h^2L_1-6g^2L_1',\quad \sigma=(M_1'^2-M_3'^2)/(M_2'^2-M_3'^2),$$

and inserting Eqs. (6) into Eqs. (5), one arrives at

$$\mu_1'^2-\mu_3'^2=\frac{M_2'^2-M_3'^2}{D}[8(\sigma-1)+24(3\sigma-4)\gamma-288\sigma\gamma^2]\alpha, \tag{7a}$$

$$\mu_2'^2-\mu_3'^2-E_P=\frac{M_2'^2-M_3'^2}{D}[3(\sigma-2)+12(3\sigma-8)\gamma-288\gamma^2]\alpha, \tag{7b}$$

$$M_1'^2-M_3'^2=-\frac{\mu_2'^2-\mu_3'^2}{\tilde{D}}(3.544+29.85\eta+41.42\eta^2)\beta, \tag{7c}$$

$$M_2'^2-M_3'^2-E_V=-\frac{\mu_2'^2-\mu_3'^2}{\tilde{D}}(0.330+4.86\eta+17.78\eta^2)\beta. \tag{7d}$$

The functions D and $\tilde{D}$ are given by

$$D=1+14\gamma-288\gamma^3, \tag{8a}$$

$$\tilde{D}=1+6\eta-7.111\eta^2-17.77\eta^3. \tag{8b}$$

[13] S. L. Glashow, Phys. Rev. 111, 48 (1963).

Use of the latest results for the pseudoscalar masses[14] gives $\mu_1'^2-\mu_3'^2=0.00703$ BeV^2 and $\mu_2'^2-\mu_3'^2=0.00302$ BeV^2 and hence $(\mu_1'^2-\mu_2'^2)/(\mu_2'^2-\mu_3'^2)=2.32$, which result was used in deriving Eqs. (7c) and (7d).

Due to the fact that the ρ mass splitting is not well known and the K^* mass difference value given in Ref. 14 has a sizable error ($\sim$65%), our approach will differ from the procedure of Refs. 1–3. For a fixed K^* mass difference, we will allow $M_{\rho^\pm}-M_{\rho^0}$ to vary, computing values for $g^2/4\pi$ and $h^2/4\pi$ for each value of the ρ mass difference. The object will be to find the range of values $M_{\rho^\pm}-M_{\rho^0}$ which gives $g^2/4\pi$ and $h^2/4\pi$ in reasonable agreement with experiment. Unfortunately, the parameters E_P and E_V are arbitrary so they will also be varied to see their effect on $g^2/4\pi$ and $h^2/4\pi$. For $M_{K^{*0}}-M_{K^{*\pm}}$, in addition to the stated result of 6.3±4.1 MeV we shall take the values 4.0 MeV and 8.0 MeV to see the effect on the results. Setting $r=(\mu_1'^2-\mu_3'^2)/(\mu_2'^2-\mu_3'^2-E_P)$, $t=(M_1'^2-M_3'^2)/(M_2'^2-M_3'^2-E_V)$, $H=M_2'^2-M_3'^2$, and forming the quotients Eq. (7a)/Eq. (7b) and Eq. (7c)/Eq. (7d) gives expressions for γ and η:

$$\gamma=\frac{24(3\sigma-4)-12(3\sigma-8)\pm\{[24(3\sigma-4)-12(3\sigma-8)]^2+1152(\sigma-r)[8(\sigma-1)-3(\sigma-2)r]\}^{1/2}}{576(\sigma-r)}, \tag{9a}$$

$$\eta=\frac{4.86t-29.85\pm\{[4.86t-29.85]^2-4(3.544-0.330t)(41.42-17.78t)\}^{1/2}}{2(41.42-17.78t)}. \tag{9b}$$

The definitions of α and β together with the expressions $\gamma=g^2K_2$ and $\eta=h^2L_2$ give the following consistency condition on the integrals which will determine the cutoff parameter Λ^2:

$$\beta\gamma K_1K_1'L_2+\tfrac{3}{2}\alpha\gamma K_1L_1L_2+\tfrac{3}{2}\alpha^2K_2L_1L_2 +\alpha\beta K_2K_1'L_2-\alpha\eta K_2K_1'L_1-\tfrac{2}{3}\beta K_2K_1'^2=0. \tag{10}$$

A second consistency condition involving L_1', is of lesser interest because L_1' is logarithmically divergent while the other five are quadratically divergent and hence one would not expect L_1' to have the same cutoff as the others.

III. RESULTS AND DISCUSSION

Before carrying out any calculations, it is well to see if there are any limitations on the values of M_ρ, E_ρ, and E_V (other than E_P, $E_V>0$), and to review the experimental situation for $g^2/4\pi$ and $h^2/4\pi$. One sees from Eq. (9b) that in order to have $\eta\geqslant 0$ (which is necessary if one is to have $h^2/4\pi\geqslant 0$) t must lie in the range $2.33\leqslant t\leqslant 10.7$. Taking $M_\rho=765$ MeV(Refs. 8, 15) we find that the restriction on t requires $M_{\rho^\pm}\geqslant 763.8$ MeV for $M_{K^{*0}}-M_{K^{*\pm}}=6.3$ MeV, $M_{\rho^\pm}\geqslant 764.2$ MeV for $M_{K^{*0}}-M_{K^{*\pm}}=4$ MeV and $M_{\rho^\pm}\geqslant 763.4$ MeV for $M_{K^{*0}}-M_{K^{*\pm}}=8$ MeV. Thus we see immediately that the large negative value $M_{\rho^\pm}-M_{\rho^0}=-2.4\pm2.1$ MeV found by Pisut[8] can certainly not be accommodated in our model. Further, a few calculations will show that certain ranges of r are *excluded* for certain intervals of σ. In particular, we cannot have $0\leqslant r\leqslant\infty$ for $\sigma<2.0$, $0\leqslant r\leqslant 8(\sigma-1)/3(\sigma-2)$ for $2.0\leqslant\sigma<2.33$, and $0\leqslant r\leqslant 2.33$, $\sigma\leqslant r\leqslant 8(\sigma-1)/3(\sigma-2)$ for $\sigma\geqslant 2.33$.

The VPP coupling constant $g^2/4\pi$ may be obtained from the ρ width, $\Gamma(\rho\to 2\pi)$, using the

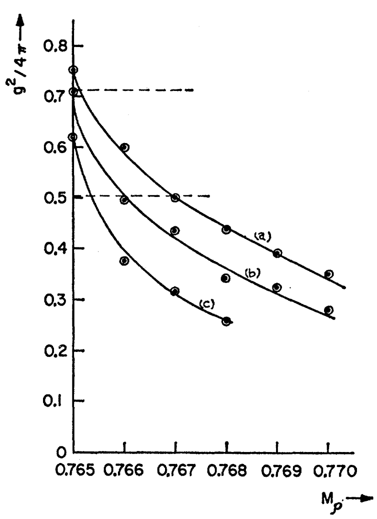

FIG. 1. Graph of $g^2/4\pi$ against $M_{\rho^\pm}$ for $M_\rho\geqslant 765$ MeV. The horizontal dashed lines encompass the present experimental uncertainty in the value of $g^2/4\pi$. The three curves correspond to the following values of $M_{K^{*0}}-M_{K^{*\pm}}$: (a) 8.0 MeV; (b) 6.3 MeV; (c) 4.0 MeV. We use $M_\rho=765$ MeV.

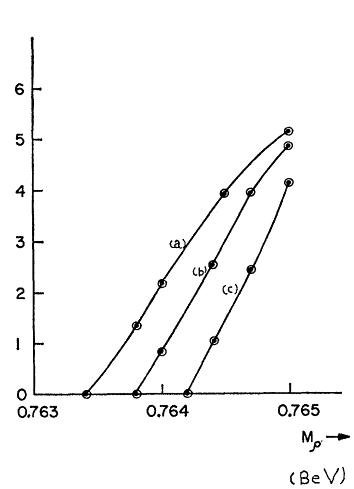

FIG. 2. Graph of $h^2/4\pi$ against $M_{\rho^\pm}$ for $M_\rho\leqslant 765$ MeV. For details see the caption of Fig. 1.

[14] A. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 81 (1968).

[15] This choice is unimportant since only the mass difference $M_{\rho^\pm}-M_{\rho^0}$ enters our calculation.

TABLE I. Computed values of $g^2/4\pi$ and $h^2/4\pi$ as a function of the ρ mass M_ρ, and the elastic form factors E_P and E_V for $M_{K^{*0}}-M_{K^{*\pm}}=6.3$ MeV.

M_ρ (MeV)	σ	E_P (BeV²)	r	γ	E_V (BeV²)	t	η	$g^2/4\pi$	$h^2/4\pi$	Λ^2 (BeV²)
770	1.79	0.010	−1.01	0.0845	0.0077	4	0.5	0.280	5.72	3.90
769	1.93	0.007	−1.77	0.0795	0.0060	3.87	0.553	0.302	7.26	3.73
					0.0062	4.00	0.5	0.294	6.46	3.76
					0.0067	4.35	0.392	0.285	4.87	3.80
		0.010	−1.01	0.101	0.0060	3.87	0.553	0.335	6.31	3.90
					0.0062	4.00	0.5	0.326	5.55	3.94
					0.0067	4.35	0.392	0.291	3.90	4.10
768	2.13	0.007	−1.77	0.103	0.0044	3.87	0.553	0.325	6.00	3.97
					0.0046	4.00	0.5	0.311	5.20	4.03
					0.0051	4.35	0.392	0.278	3.64	4.20
		0.010	−1.01	0.125	0.0044	3.87	0.553	0.356	5.41	4.12
					0.0046	4.00	0.5	0.338	4.64	4.20
					0.0051	4.35	0.392	0.297	3.18	4.42
767	2.42	0.007	−1.77	0.135	0.0030	3.87	0.553	0.435	6.14	3.94
					0.0031	4.00	0.5	0.417	5.32	4.00
					0.0035	4.35	0.392	0.395	3.93	4.08
		0.010	−1.01	0.153	0.0030	3.87	0.553	0.456	5.67	4.05
					0.0031	4.00	0.5	0.436	4.89	4.12
					0.0035	4.35	0.392	0.411	3.61	4.21
766	2.88	0.007	−1.77	0.176	0.0015	3.87	0.553	0.553	5.75	4.03
					0.0016	4.00	0.5	0.501	4.89	4.12
					0.0020	4.35	0.392	0.448	3.42	4.30
		0.010	−1.01	0.190	0.0015	3.87	0.553	0.517	5.16	4.19
					0.0016	4.00	0.5	0.495	4.47	4.26
					0.0020	4.35	0.392	0.416	3.09	4.57
765	4	any	any	0.25	0	4.00	0.500	0.712	4.89	4.13
					0.0002	4.29	0.410	0.637	3.57	4.30
764.7	4.61	0	2.33	0.194	0	4.61	0.333	0.665	3.93	3.86
		0.0002	2.50	0.183				0.626	3.93	3.86
		0.0004	2.70	0.168				0.488	3.56	4.09
764.4	5.48	0	2.33	0.152	0	5.48	0.207	0.537	2.52	3.82
764.0	7.58	0	2.33	0.0833	0	7.58	0.0749	0.345	0.855	3.63

expression

$$\Gamma(\rho \to 2\pi) = \frac{2}{3}\frac{g^2}{4\pi}\frac{(\frac{1}{4}M_\rho^2-\mu_\pi^2)^{3/2}}{M_\rho^2}. \tag{11}$$

The most probable value for the ρ width at the present time is $\Gamma_\rho \sim 130$ MeV (Refs. 16, 17) which gives $g^2/4\pi = 0.622$. On the other hand, values as large as $\Gamma_\rho = 150$ MeV,[8] giving $g^2/4\pi = 0.718$, and as small as $\Gamma_\rho = 105$ MeV, (Ref. 18) giving $g^2/4\pi = 0.503$, have been mentioned. In connection with the last value, it should be

TABLE II. Computed values for $M_{K^{*0}}-M_{K^{*\pm}}=4.0$ MeV. For details see caption of Table I.

M_ρ (MeV)	σ	E_P (BeV²)	r	γ	E_V (BeV²)	t	η	$g^2/4\pi$	$h^2/4\pi$	Λ^2 (BeV²)
768.0	1.83	0.010	−1.01	0.0897	0.0046	4	0.5	0.257	4.92	4.11
767.0	2.09	0.007	−1.77	0.0992	0.0030	3.87	0.553	0.304	5.84	4.01
					0.0031	4.00	0.5	0.292	5.28	4.07
					0.0034	4.35	0.392	0.258	3.50	4.26
		0.010	−1.01	0.121	0.0030	3.87	0.553	0.333	5.59	4.17
					0.0031	4.00	0.5	0.315	4.47	4.26
					0.0034	4.35	0.392	0.273	3.02	4.51
766.0	2.58	0.007	−1.77	0.151	0.0015	3.87	0.553	0.396	4.97	4.25
					0.0016	4.00	0.5	0.384	4.36	4.30
					0.0018	4.29	0.410	0.330	2.46	4.58
					0.0015	3.87	0.553	0.402	4.54	4.40
					0.0016	4.00	0.5	0.375	3.84	4.52
					0.0018	4.29	0.410	0.314	2.51	4.88
765.0	4	any	any	0.25	0	4.00	0.5	0.618	4.19	4.35
					0.0002	4.29	0.410	0.550	3.08	4.56
764.7	4.94	0	2.33	0.175	0	4.94	0.275	0.451	2.42	4.28
764.4	6.91	0	2.33	0.118	0	6.91	0.104	0.340	1.03	4.10

[16] M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961).
[17] S. C. C. Ting, DESY Report No. DESY-F31-681, Interner Bericht (unpublished). I would like to thank Tom Ferbel for bringing this report to my attention.
[18] V. L. Auslander *et al.*, Phys. Letters **25B**, 433 (1967).

TABLE III. Computed values for $M_{K^{*0}}-M_{K^{*\pm}}=8.0$ MeV. For details see caption of Table I.

M_ρ (MeV)	σ	E_P (BeV^2)	r	γ	E_V (BeV^2)	t	η	$g^2/4\pi$	$h^2/4\pi$	Λ^2 (BeV^2)
770.0	1.95	0.010	-1.01	0.104	0.0077	4.00	0.500	0.351	5.81	3.88
769.0	2.10	0.007	-1.77	0.100	0.0059	3.87	0.553	0.356	6.79	3.81
					0.0061	4.00	0.5	0.350	6.04	3.83
					0.0067	4.35	0.392	0.337	4.55	3.88
		0.010	-1.01	0.118	0.0059	3.87	0.553	0.401	6.47	3.87
					0.0061	4.00	0.5	0.389	5.68	3.91
					0.0067	4.35	0.392	0.380	4.35	3.94
768.0	2.30	0.007	-1.77	0.122	0.0044	3.87	0.553	0.421	6.57	3.85
					0.0046	4.00	0.5	0.408	5.76	3.89
					0.0052	4.35	0.392	0.390	4.32	3.95
		0.010	-1.01	0.140	0.0044	3.87	0.553	0.455	6.19	3.93
					0.0046	4.00	0.5	0.439	5.39	3.98
					0.0052	4.35	0.392	0.429	4.14	4.01
767.0	2.59	0.007	-1.77	0.151	0.0029	3.87	0.553	0.494	6.23	3.92
					0.0031	4.00	0.5	0.480	5.47	3.96
					0.0035	4.29	0.410	0.436	4.07	4.10
		0.010	-1.01	0.168	0.0029	3.87	0.553	0.515	5.84	4.01
					0.0031	4.00	0.5	0.498	5.09	4.06
					0.0035	4.29	0.410	0.454	3.80	4.20
766.0	3.27	0.007	-1.77	0.205	0.0014	3.87	0.553	0.634	5.88	4.00
					0.0016	4.00	0.5	0.608	5.09	4.06
					0.0020	4.29	0.410	0.554	3.80	4.20
		0.010	-1.01	0.214	0.0014	3.87	0.553	0.622	5.52	4.09
					0.0016	4.00	0.5	0.594	4.76	4.16
					0.0020	4.29	0.410	0.523	2.90	4.37
765.0	4	any	any	0.25	0	4	0.5	0.751	5.16	4.04
					0.0003	4.29	0.410	0.677	3.80	4.20
764.5	4.83	0	2.33	0.181	0	4.83	0.293	0.700	3.91	3.71
		0.0005	2.80	0.140	0	4.83	0.293	0.546	3.94	3.70
764.0	6.25	0	2.33	0.131	0	6.25	0.150	0.548	2.17	3.62
763.8	7.17	0	2.33	0.114	0	7.17	0.0914	0.486	1.34	3.60

noted that Pisut and Roos[8] have argued that the rather low result for Γ_ρ as compared with their calculation is due to the difference in the theoretical expression used for the pionic form factor and would be $\sim$140 MeV if their form were to be used.

Determination of $h^2/4\pi$ is somewhat more complicated. An expression using the decay width for $\omega\to 3\pi$ and $\varphi\to 3\pi$ can be obtained,[11,15] giving

$$\frac{h^2}{4\pi}=\frac{113.1}{g^2/4\pi}[2.310\Gamma^{1/2}(\omega\to 3\pi)\sin\theta \pm\Gamma^{1/2}(\varphi\to 3\pi)\cos\theta]^2, \quad (12)$$

where it has been assumed that the decays take place through a ρ-π intermediate state, that $M_\rho=765$ MeV, and where θ is the ω-φ mixing angle. From the experimental values for the widths (and branching ratios)[14] we arrive at $h^2/4\pi=(5.4\pm1.3, 3.5\pm1.0)$ for $g^2/4\pi=0.622$, $h^2/4\pi=(6.7\pm1.7, 4.4\pm1.3)$ for $g^2/4\pi=0.503$, and $h^2/4\pi=(4.69\pm1.2, 3.1\pm0.9)$ for $g^2/4\pi=0.718$. The two values given in parenthesis depend on the relative sign of $h_{\varphi\rho\pi}$ and $h_{\omega\rho\pi}$ [cf. Eq. (12)].

The results of our computations are shown in Table I for $M_{K^{*0}}-M_{K^{*\pm}}=6.3$ MeV, Table II for $M_{K^{*0}}-M_{K^{*\pm}}=4.0$ MeV, and Table III for $M_{K^{*0}}-M_{K^{*\pm}}=8.0$ MeV. It is immediately evident that the cutoff parameter Λ^2 is consistently less than 5 BeV^2, the two-baryon threshold. Thus, as in Ref. 1, the baryon-antibaryon intermediate state does not contribute anything. It will be convenient to discuss the cases $M_{\rho^\pm}-M_{\rho^0}\geqslant 0$ and $M_{\rho^\pm}-M_{\rho^0}<0$ separately.

Since the variation of $g^2/4\pi$ with E_P and E_V is fairly small, we have plotted $g^2/4\pi$ against M_ρ for $\eta=0.5$ and $E_P=0.010$ BeV^2 in Fig. 1.[19] It is clear that the results for $g^2/4\pi$ strongly favor a small value for $M_{\rho^\pm}-M_{\rho^0}$. If we arbitrarily take the point of view that any value for $g^2/4\pi$ smaller than 0.4 would be in poor agreement with experiment and that the calculation is meaningful (a conclusion supported by the results of Ref. 1), then we can conclude that it is highly probable that $M_{\rho^\pm}-M_{\rho^0}\leqslant 3.7$ MeV if $M_{K^{*0}}-M_{K^\pm}=8$ MeV, $M_{\rho^\pm}-M_{\rho^0}\leqslant 2.3$ MeV if $M_{K^{*0}}-M_{K^{*\pm}}=6.3$ MeV, and $M_{\rho^\pm}-M_{\rho^0}\leqslant 0.7$ MeV if $M_{K^{*0}}-M_{K^{*\pm}}=4$ MeV.

The possible negative values of M_ρ are influenced on the other hand by the results for $h^2/4\pi$ as is shown in the plot of $h^2/4\pi$ against M_ρ in Fig. 2. Here we have chosen the values $E_P=E_V=0$. Again, a value for $M_{\rho^\pm}$ close to M_{ρ^0} is strongly favored. Thus if we conclude that a

[19] It should be noted that the value $\eta=0.392$ is close to the minimum necessary to satisfy the consistency condition Eq. (10). For certain values of M_ρ it turns out to be too small and the value $\eta=0.410$ is used instead. Further, an exploratory calculation with Eq. (9a) will show that $g^2/4\pi$ cannot be increased very much by increasing E_P.

value for $h^2/4\pi$ less than 1.5 is in poor agreement with experiment, it is highly likely that $M_{\rho^\pm}-M_{\rho^0}\gtrsim -1.2$ MeV for $M_{K^{*0}}-M_{K^{*\pm}}=8$ MeV, $M_{\rho^\pm}-M_{\rho^0}\gtrsim -0.8$ MeV for $M_{K^{*0}}-M_{K^{*\pm}}=6.3$ MeV, and $M_{\rho^\pm}-M_{\rho^0}\gtrsim -0.6$ MeV for $M_{K^{*0}}-M_{K^{*\pm}}=4$ MeV.

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Sum Rules in $\pi\Sigma$ Scattering and the $\pi\Lambda\Sigma$ and $\pi\Sigma\Sigma$ Coupling Constants

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Following a procedure proposed recently by Atkinson, we derive sum rules for the A and B amplitudes for the reaction $\pi^-+\Sigma^+\to\pi^++\Sigma^-$ by equating unsubtracted dispersion relations at $t=0$ and at fixed u or fixed backward angle. Combining these with the already known superconvergence relation for the B amplitude, and assuming the sum rules can be saturated with known resonances, we obtain three equations for two unknown coupling constants $g_{\pi\Lambda\Sigma}^2$ and $g_{\pi\Sigma\Sigma}^2$. Choosing to fix $u=0$, one obtains values of the coupling constants an order of magnitude larger than expected on the basis of, say, $SU(3)$. We argue that this is probably because of the large extrapolation to unphysical values of $\cos\theta$ required in evaluating the fixed-u dispersion relation for $u=0$. Taking u to be positive in such a way as to minimize the required extrapolations in angle, or choosing fixed $\cos\theta=-1$, one obtains results that are reasonably consistent with one another and with $SU(3)$, to within estimated uncertainties of 50% or more, resulting from experimental error in the resonance widths, large cancellations between the contributions of different resonances, and unknown nonresonant-background and high-energy contributions.

RECENTLY Atkinson has shown that one can obtain sum rules in certain cases, by equating two different unsubtracted dispersion relations, with different variables held fixed, for the same scattering amplitude.[1] To summarize the procedure briefly, he considers an amplitude that satisfies an unsubtracted dispersion relation with either the Mandelstam variable t or u held fixed. (We will work in the channel where s is the total center-of-mass energy squared.) One can then write

$$A(s,t,u)=\int\frac{\mathrm{Im}A(s',t,\tau-s'-t)ds'}{s'-s}=\int\frac{\mathrm{Im}A(s',\tau-s'-u,u)ds'}{s'-s},\quad(1)$$

where

$$\tau=\sum_{i=1}^{4}m_i^2=s+t+u\quad(2)$$

and m_i is the mass of one of the two incident or two outgoing particles. Provided no subtractions are needed in either dispersion relation, the second equality in Eq. (1) then yields a sum rule for ImA.

In this paper, we are going to apply this technique to both the A and B amplitudes for the reaction

$$\pi^-+\Sigma^+\to\Sigma^-+\pi^+.\quad(3)$$

(We use the standard notation, as given, for example, in the paper of Frautschi and Walecka,[2] for the invariant amplitudes in pion-baryon scattering.) This reaction corresponds to isospin 2 in the t ($\pi\pi\to\Sigma\bar{\Sigma}$) channel. Since at large s and fixed t one has[3]

$$\mathrm{Im}A(s,t)\sim s^{\alpha(t)},\quad \mathrm{Im}B(s,t)\sim s^{\alpha(t)-1},\quad(4)$$

where $\alpha(t)$ is the leading Regge trajectory in the t channel, the usual assumption that no $I=2$ trajectory reaches $j=0$ for $t<0$ implies that B is actually superconvergent, while A satisfies a dispersion relation with no subtractions, provided t is held fixed at a value less than or equal to zero. The superconvergence relation for B has been studied previously.[4,5] If one assumes that the superconvergent sum rule is saturated by the Λ and Σ poles and the known resonances in the $\Sigma\pi$ system, one obtains a relationship between the $\pi\Lambda\Sigma$ and $\pi\Sigma\Sigma$ coupling constants and the experimentally measurable masses and widths of the resonances. Since there is only one equation, the superconvergence relation by itself is not sufficient to determine the values of the coupling constants, unless one invokes $SU(3)$ and a specific value for the d/f ratio. The two additional Atkinson-type sum rules we obtain here, combined with the

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