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Collisional-Radiative Recombination of Ions and Electrons in High-Pressure Plasmas in Which the Electron Temperature Exceeds the Gas Temperature*

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The recombination rate of electrons in a hypothetical helium-like plasma at high pressure is calculated for cases in which the gas temperature is fixed at 300°K, and the electron temperature is varied from 300 to 2000°K. Neutral collision-induced recombination and electron collision-induced recombination are considered for ranges of electron density from 10^{8} to 10^{13} cm^{-3} and gas densities from 10^{16} to 10^{19} cm^{-3}. An inverse three-halves dependence of recombination coefficient on electron temperature is found for certain densities.

INTRODUCTION

Theoretical considerations of collisional-radiative recombination of ions and electrons have been recently extended[1,2] to include neutral collision-induced recombination of the type proposed by Bates and Khare,[3] as well as electron collision-induced recombination processes,[4] for the case of thermal equilibrium between the free electrons and the heavy particles. In many experimental situations, the temperature of the electrons is higher than that of the heavy particles, either as a result of external heating of the plasma by some means such as microwave pulses, or by heating as a consequence of internal-energy sources. Of particular interest in such cases is the dependence of the recombination rate coefficient on electron temperature.

This paper reports a further extension of recombination rate calculations to plasmas having a neutral temperature of 300°K, and an electron temperature varying from 300 to 2000°K. Idealized ions of mass 4, with hydrogenic energy levels, were used in the calculations. Since the electron collision-induced recombination mechanisms are only slightly sensitive to the nature of the ions undergoing recombination, recombination rates for these idealized ions should reasonably approximate the recombination of He^{+} with electrons. Conversely, since the neutral collision part of the recombination process is mass dependent, only the qualitative aspects of the calculations would pertain to the recombination of other ions.

THEORY

The range of processes included in the calculations are essentially the same as those used by Deloche,[1,2]

$$X^{+}+2e \rightleftharpoons X^{*}(p)+e, \qquad (1)$$

$$X^{+}+e+X \rightleftharpoons X^{*}(p)+X, \qquad (2)$$

$$X^{+}+e \rightarrow X^{*}(p)+h\nu, \qquad (3)$$

$$X^{*}(p)+e \rightleftharpoons X^{*}(q)+e, \quad p>q, \qquad (4)$$

$$X^{*}(p)+X \rightleftharpoons X^{*}(q)+X, \quad p>q, \qquad (5)$$

$$X^{*}(p) \rightarrow X^{*}(q)+h\nu, \quad p>q, \qquad (6)$$

where $X^{*}(k)$ denotes the kth excited state of the neutral atom. Rate coefficients for processes (1) and (4) were calculated by the Gryzinski approximations,[5] rates for (3) were taken from Bates and Dalgarno,[6] and rates for (6) from Green *et al.*[7] Rates for (5) and the inverse of (2) were calculated according to the methods of Bates and Khare.[4] Process (2) presented the greatest difficulty in cases of unequal gas and electron temperature. These rate coefficients were approximated, as suggested by Collins,[8] by

$$A_{nc} = [X^+] \frac{g_n}{g^+ g_e} \left(\frac{h^2}{2\pi m K T_e} \right)^{3/2} e^{U_n/KT_g} A_{cn} \,, \quad (7)$$

where A_{nc} and A_{cn} are rate coefficients for process (2) and its inverse, respectively; $[X^+]$ is the ion concentration; g_n, g^+, and g_e are the degeneracies of the nth bound level, ion, and free electron, respectively; U_n is the ionization potential of the nth bound level; T_g and T_e are the heavy particle and electron temperatures, respectively; and m is the mass of the electron.

The actual calculation of recombination rate coefficients followed largely the original method of Bates *et al.*,[3] with minor modifications discussed elsewhere.[9] The final computations were performed on an IBM 360-50 in double-precision arithmetic.

RESULTS

It was found that consideration of neutral collision-induced transitions between 70 excited-state levels and electron collision-induced transitions between 50 excited-state levels were required for the populations to converge to values which were independent of the number of levels used in the calculation. As in earlier work,[9,10] no assumptions were made concerning Saha equilibrium in levels above some arbitrary quantum number. Consequently equilibrium generally did not occur until somewhat larger values of quantum number than the 30 assumed by Deloche.[1] Nevertheless results at thermalized values of the electron temperature are in excellent agreement with those of Deloche.[2]

Figure 1 presents the values of recombination rate coefficient at $T_e = T_g = 300°\text{K}$ as a function of electron density and parametrically as a function of gas density. Of interest is the result that for these parameters the recombination rate is simply additive

$$\alpha = \alpha_e + \alpha_n \,, \quad (8)$$

where α_e and α_n are the recombination-rate coefficients for purely electron collision-induced and neutral collision-induced recombination, respectively, and α is the rate coefficient for the process including both mechanisms. Such is essentially the case also for an electron temperature of 500°K, as shown in Fig. 2.

Figures 3 and 4 illustrate nonadditive behavior of the two-constituent recombination processes at electron temperatures of 1000 and 2000°K, respectively. In these cases

$$\alpha \leqslant \alpha_e + \alpha_n \,. \quad (9)$$

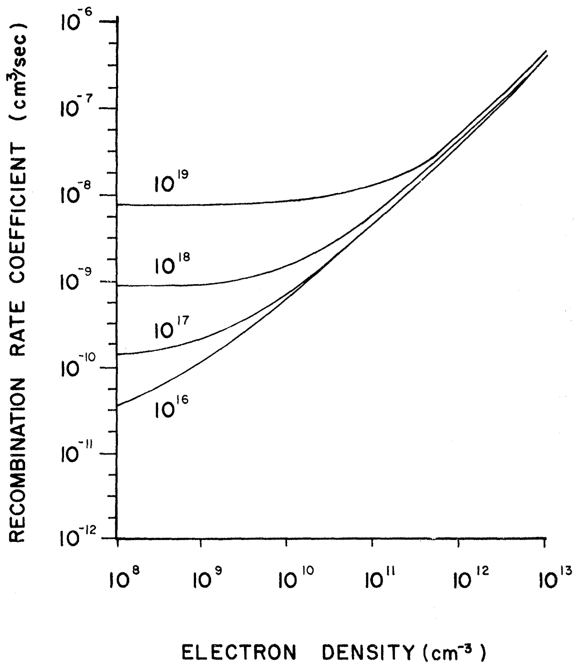

FIG. 1. Calculated recombination rate coefficients as a function of electron density and parametrically as a function of neutral number density for $T_g = 300°\text{K}$ and $T_e = 300°\text{K}$.

For increasing values of electron density, the contribution to the recombination-rate coefficient α from neutral-assisted processes is found to progressively decrease. This apparently reflects an increasing depopulation, as a result of ionizing collisions with the increasingly abundant electrons at the higher temperature, of the Saha group[9] of energy levels which at low electron densities had been in equilibrium at the lower gas temperature.

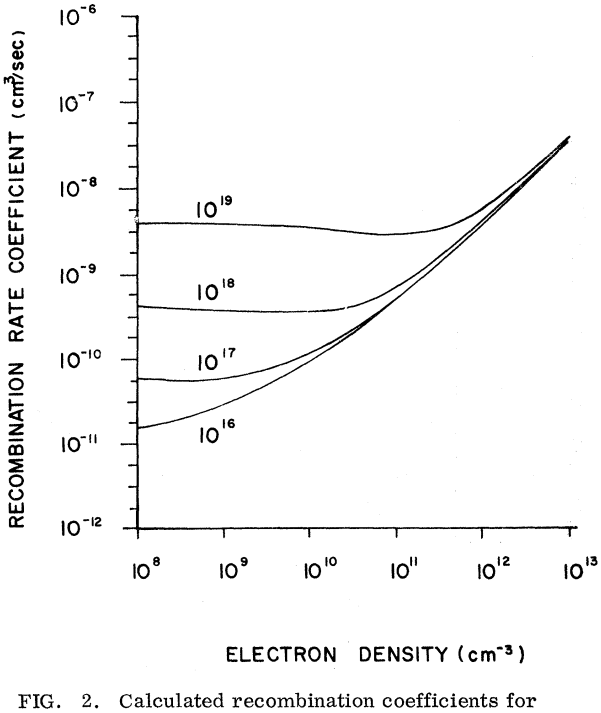

FIG. 2. Calculated recombination coefficients for $T_g = 300°\text{K}$ and $T_e = 500°\text{K}$.

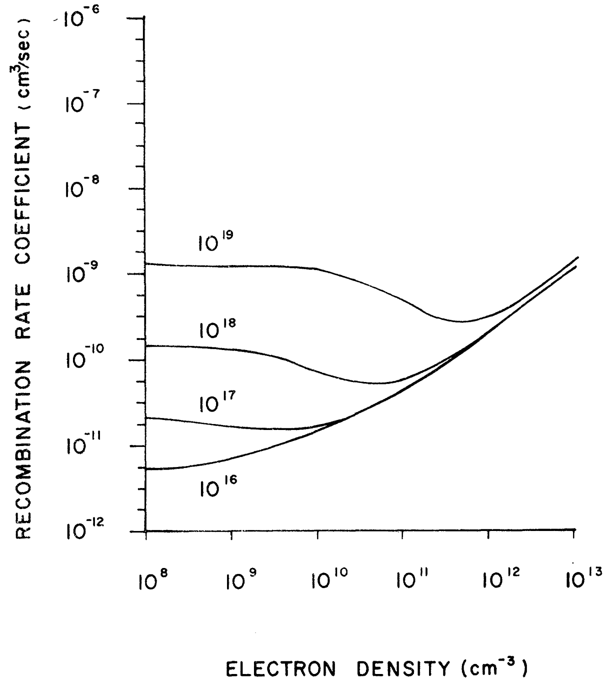

FIG. 3. Calculated recombination coefficients for $T_g = 300°\mathrm{K}$ and $T_e = 1000°\mathrm{K}$.

Since the main contribution to the net rate of neutrally assisted recombination results from Process (5) with $X^*(p)$ representing a level in the Saha group, a depopulation of this group reduces the rate at which process (5) can transfer electrons to levels of sufficiently low quantum number to be permanently bound, consequently reducing α_n.

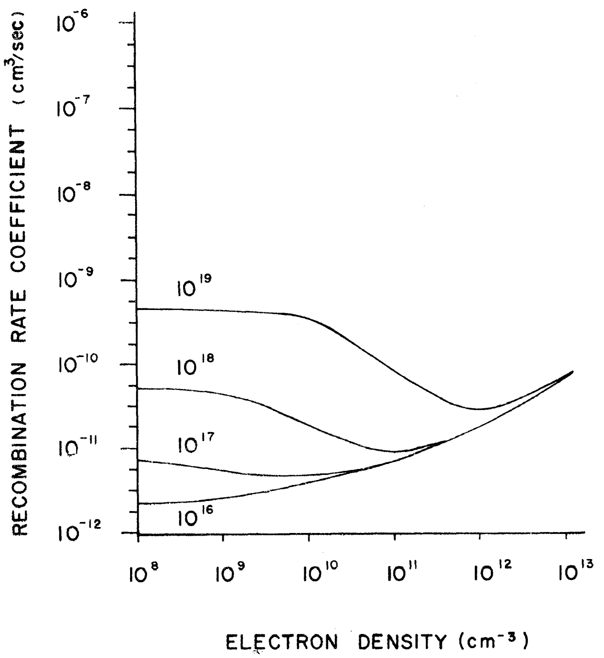

FIG. 4. Calculated recombination coefficients for $T_g = 300°\mathrm{K}$ and $T_e = 2000°\mathrm{K}$.

Of considerable interest is the electron temperature dependence of the total recombination-rate coefficient. For sufficiently low electron densities, a credible inverse three-halves dependence is found. Figure 5 presents values of plasma parameters for which

$$\alpha(T) = \alpha(T_0)(T/T_0)^{-R}, \qquad (10)$$

where $\alpha(T)$ is the total recombination rate coefficient at the electron temperature T, T_0 is a reference electron temperature, less than T. For plasmas having characteristic parameters which plot above and to the left of the line representing the greatest reference temperature, T_0, less than the electron temperature, an inverse 1.5 ± 0.15 power reasonably approximates the expected temperature dependence for changes to temperatures remaining greater than T_0. Conversely, for plasmas mapping to the right and below the appropriate line, a greater inverse dependence is expected.

CONCLUSION

It appears that Figs. 1–4 offer reasonable approximations to the recombination rate coefficients expected in recombining helium afterglows in which both neutral-assisted and electron-assisted collisional processes must be considered. Of significance is the appearance of a $T_e^{-1.5}$ dependence on electron temperature for certain values of the experimental parameters, for this value of temperature dependence has been reported in helium.[11,12] However, these measurements were made in a system for which the experimental parameters

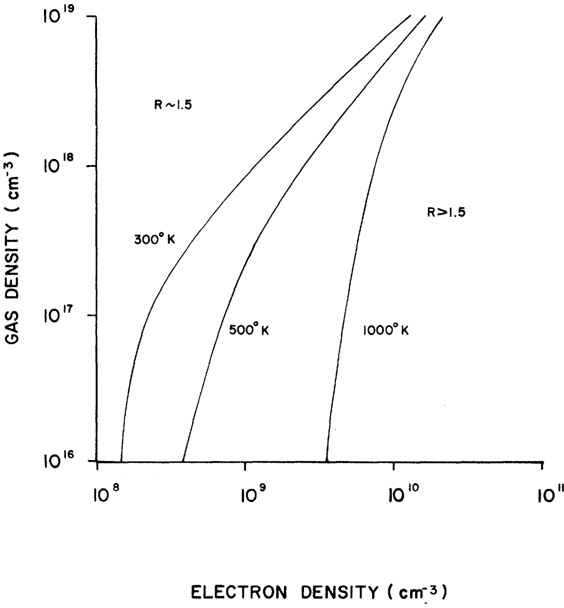

FIG. 5. Graph of plasma parameters for which the recombination rate coefficient varies as T_e^{-R}. Plasmas plotting above and to the left of the line denoting the greatest temperature not exceeding the electron temperature are expected to vary as the inverse $\frac{3}{2}$ power of the electron temperature.

appear to lie in the bordering region of Fig. 5 and therefore cannot offer concrete support for the theory. Chen *et al.*,[11] for example, report a $T_e^{-1.5}$ dependence for a helium plasma with $T_e = 1250°\text{K}$, $N_g = 10^{18}\ \text{cm}^{-3}$, and $N_e = 10^{10}$ to $10^{11}\ \text{cm}^{-3}$. Nevertheless, this temperature dependence is considerably weaker than is indicated by the theory of collisional-radiative recombination in the absence of neutrally assisted processes. The above calculations of the effect of such collisional processes for heated afterglows indicate a change toward increasing agreement with these experimentally determined dependences of the collisional-radiative recombination coefficient on electron temperature.

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Late-Time Source of Atomic Light in the Helium Afterglow*

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The time-dependent behavior of the light emitted from a pulsed helium afterglow at 3 Torr has been investigated with an improved spectrometric system of sufficient sensitivity-bandwidth ratio to resolve the behavior of the atomic light over a 50-msec time interval. The relationship between the atomic light and the time-resolved electron density, as measured with a 36-GHz free-space microwave interferometer, was found to indicate the importance of a persistent source of ionization at late times. Absorption measurements of time-dependent behavior of both the atomic 2^3S and molecular $2^3\Sigma$ metastable species were found to strongly favor mutual ionization of two metastable molecules as being the most important source of late-time ionization at 3 Torr.

INTRODUCTION

Recent measurements[1-3] of the time-dependent behavior of the atomic light emitted from low-pressure helium afterglows have presented compelling evidence that the principal source of this light is two-electron collisional-radiative recombination[4] of He^+ with electrons. However, other results have persisted in suggesting a possible secondary source of excited atomic systems at late afterglow times. In particular, the measurements of Kerr[2] and Oskam[3] at pressures in excess of 3 Torr show relatively increasing apparent lifetimes of the populations of radiating atomic states with increasing time. These lifetimes are in excess of that expected on the basis of recombination of an He^+ population decaying solely by the normal-loss processes associated with this ion and in the absence of late-time sources of ionization. Apparently either a continuing late-time source of ionization exists, such as metastable-metastable ionization, or some minor source of excited atomic