Pion-Nucleon Scattering Lengths

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Pion-nucleon scattering lengths have been calculated using a model in which N, N^*, ρ, and ϵ exchanges are assumed to dominate the low-energy pion-nucleon interaction. This simple model yields remarkably good agreement with the experimental s- and p-wave scattering lengths. We find that the coupling of the ϵ meson to the nucleon is consistent with estimates made using chiral symmetry and current algebra.

THE problem of low-energy pion-nucleon scattering is one of the oldest and most fundamental problems of elementary-particle physics. While a variety of approaches have been used to study this problem, none have yielded a complete, simple understanding of the low-energy pion-nucleon interaction.

The Chew-Low model[1] provided a qualitative picture of low-energy p-wave scattering but could not yield information on s-wave scattering. On the other hand Sakurai's ρ-dominant approach[2] accounted for s-wave scattering but did not justify the neglect of other exchange contributions. Current-algebra calculations have provided good predictions for the s-wave[3] and p-wave[4] scattering lengths, but in order to obtain a clearer understanding of the low-energy pion-nucleon interaction a Lagrangian model is desirable. Such models based on chiral symmetry[5] have been used to study low-energy pion-nucleon scattering. However, these models, though reasonably adequate for calculating scattering lengths, do not directly yield the t-channel singularities required by the analysis of backward pion-nucleon scattering.[6] Lovelace,[6] on the basis of backward pion-nucleon dispersion relations, has discussed the need for exchange of a scalar dipion resonance in pion-nucleon scattering.

In a previous paper,[7] the authors have shown that a wide variety of phenomena involving the low-energy pion-pion interaction, including Adler's sum rule for pion-pion scattering,[8] the $K_1{}^0$-$K_2{}^0$ mass difference, constraints on s-wave pion-pion phase shifts at the K mass deduced from the two-pion decays of the neutral K meson, characteristics of K_{e4} decay, odd pion spectral shape for K decay into three pions, and Malamud and Schlein's "experimental" determinations of the pion-pion phase shifts,[9] can be understood on the basis of a simple model for the pion-pion interaction. This model assumes that the low-energy pion-pion scattering in the $I=J=0$ state is dominated by a broad resonance with a mass of $\sim$700 MeV and a width of $\sim$400 MeV, the ϵ meson.

In this paper we show that the s- and p-wave scattering lengths for pion-nucleon scattering may be obtained directly by taking into account the contributions from the exchanges of the lowest-lying meson and baryon states, i.e., the nucleon (N), the 3-3 resonance (N^*), the $I=J=1$ meson (ρ), and the $I=J=0$ resonance (ϵ). This simple model, while avoiding the introduction of pseudovector coupling of pions to nucleons which has been suggested by a number of authors,[5] gives very good agreement with the observed s- and p-wave pion-nucleon scattering lengths.

The effective Lagrangian of our model is given by

$$\begin{aligned}L_{\mathrm{int}}=&ig_r\bar{N}\gamma_5\boldsymbol{\tau}N\cdot\boldsymbol{\pi}+g_{\epsilon pp}\bar{N}N\epsilon+g_{\epsilon\pi\pi}\epsilon\boldsymbol{\pi}\cdot\boldsymbol{\pi}\\&+ig_{\rho NN}\bar{N}[\gamma_\mu+(i\kappa/2m_N)\sigma_{\mu\nu}\overleftrightarrow{\partial}_\nu]\boldsymbol{\tau}N\boldsymbol{\varrho}_\mu\\&+ig_{\rho\pi\pi}\boldsymbol{\varrho}_\mu\cdot\boldsymbol{\pi}\times\overleftrightarrow{\partial}_\mu\boldsymbol{\pi}+(g^*/m_\pi)\bar{N}_\mu{}^*N\partial_\mu\pi.\end{aligned}\quad(1)$$

The coupling constants are given as follows: $g_r{}^2/4\pi=14.64$, $g_{\rho\pi\pi}{}^2/4\pi=2.2$,[10] $g^{*2}/4\pi m_\pi{}^2=0.37$, and $\kappa=3.7$. We make use of the universality of the ρ coupling to the isotopic spin current to obtain $g_{\rho NN}$. Thus, the Lagrangian contains only one unknown parameter $g_{\epsilon pp}$. This parameter is fixed by the even s-wave scattering length. The other scattering lengths are predicted by the model.

The amplitude for pion-nucleon scattering $\pi(k)+N(p)\rightarrow\pi(k')+N(p')$ is given by[11]

$$M=(m_N/4\pi W)\bar{U}(p')[-A+\tfrac{1}{2}i(\boldsymbol{k}+\boldsymbol{k}')\cdot\gamma B]U(p),\quad(2)$$

where W is the center-of-mass energy. The contributions from the various exchanges are

$$A_N{}^{(+)}=A_N{}^{(-)}=0,\quad(3a)$$

$$B_N{}^{(\pm)}=g_r{}^2\left[\frac{1}{(m_N{}^2-s)}\mp\frac{1}{(m_N{}^2-u)}\right],\quad(3b)$$

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$$A_{N^*}{}^{(\pm)}=\begin{pmatrix}1\\1/2\end{pmatrix}\left(\frac{1}{m^{*2}-s}\pm\frac{1}{m^{*2}-u}\right)(\alpha_1 t+\alpha_2)\,, \tag{4a}$$

$$B_{N^*}{}^{(\pm)}=\begin{pmatrix}1\\-1/2\end{pmatrix}\left(\frac{1}{m^{*2}-s}\mp\frac{1}{m^{*2}-u}\right)(\beta_1 t+\beta_2)\,, \tag{4b}$$

where

$$\alpha_1=\tfrac{2}{3}(g^{*2}/m_\pi{}^2)[(m_N+m^*)/2]=\beta_1(m_N+m^*)\,, \tag{4c}$$

$$\alpha_2=\tfrac{2}{3}(g^{*2}/m_\pi{}^2)q^{*2}[m_N+m^* +\tfrac{1}{3}(m^*-m_N/E^*+m_N)(E^*-m_N)]\,, \tag{4d}$$

$$\beta_2=\tfrac{2}{3}(g^{*2}/m_\pi{}^2)q^{*2}[1-\tfrac{1}{3}(E^*+m_N/E^*-m_N)]\,; \tag{4e}$$

q^* and E^* are the center-of-mass momentum and energy of the nucleon when $W=m^*$;

$$A_\rho{}^{(+)}=B_\rho{}^{(+)}=0\,, \tag{5a}$$

$$A_\rho{}^{(-)}=-g_{\rho\pi\pi}g_{\rho pp}(\kappa/2m_N)[(s-u)/(m_\rho{}^2-t)]\,, \tag{5b}$$

$$B_\rho{}^{(-)}=g_{\rho\pi\pi}g_{\rho pp}[2(1+\kappa)/(m_\rho{}^2-t)]\,; \tag{5c}$$

and

$$A_\epsilon{}^{(-)}=B_\epsilon{}^{(-)}=B_\epsilon{}^{(+)}=0\,, \tag{6a}$$

$$A_\epsilon{}^{(+)}=g_{\epsilon\pi\pi}g_{\epsilon pp}/(m_\epsilon{}^2-t)\,; \tag{6b}$$

s, t, u are the Mandelstam variables, and the superscripts designate the (even, odd) amplitudes. The advantage of using the odd and even amplitudes rather than the isotopic spin amplitudes is that it enables us to isolate the ρ and ϵ contributions, respectively.

The ϵ-N coupling constant is determined from the even s-wave scattering length. There are two determinations of the experimental value of the even s-wave scattering length, namely $a_s{}^{(+)}=-0.001$ [12] or $a_s{}^{(+)}=-0.013$.[13] However, the value of $g_{\epsilon pp}$ is relatively insensitive to this variation in the scattering length, and we obtain $g_{\epsilon pp}{}^2/4\pi=11$.

Table I gives the predictions of our model as compared with the experimentally determined[11–14] scattering lengths. Except for $a_{p-3/2}{}^{(+)}$, the predictions are in excellent agreement. Because $g_{\epsilon pp}$ is insensitive to the exact value of $a_s{}^{(+)}$ from which it is determined, the predictions for $a_{p-1/2}{}^{(+)}$ and $a_{p-3/2}{}^{(+)}$ are not affected by the choice of $a_s{}^{(+)}$.

Table I. Scattering lengths.

	Theory	Experiment[a]	
$a_s{}^{(+)}$	-0.001[b]	-0.001[c]	
	-0.013[b]		-0.013[d]
$a_s{}^{(-)}$	$+0.094$	$+0.097$[c]	$+0.097$[c]
$a_{p1/2}{}^{(+)}$	-0.071	-0.059[e]	-0.069[f]
$a_{p1/2}{}^{(-)}$	-0.015	-0.021[e]	-0.016[f]
$a_{p3/2}{}^{(+)}$	$+0.111$	$+0.134$[e]	$+0.137$[f]
$a_{p3/2}{}^{(-)}$	-0.087	-0.081[e]	-0.081[f]

[a] Errors are $\sim\pm0.005$.
[b] $a_s{}^{(+)}$ is used to fix the ϵ-N coupling constant. This coupling constant and the predicted values of the other scattering lengths are insensitive to the choice of the value of $a_s{}^{(+)}$.
[c] See Ref. 12.
[d] See Ref. 13.
[e] See Ref. 11.
[f] See Ref. 14.

The model which we have used incorporates the s-wave dipion resonance required by backward pion-nucleon dispersion relations and is remarkably simple. We have been able to predict the s- and p-wave scattering lengths without introducing pseudovector coupling, "contact" interactions, or other schemes which may be found in the literature.

The value of $g_{\epsilon pp}{}^2/4\pi=11$ is insensitive to the large difference in the value of $a_s{}^{(+)}$ from which it is determined. Furthermore, this value is quite reasonable and is in qualitative agreement with predictions of the chiral symmetric σ model $(g_{\epsilon pp}{}^2/4\pi=g_r{}^2/4\pi=14.64)$ and the current-algebra prediction $g_{\epsilon pp}{}^2/4\pi=18$.[15]

An estimate of $g_{\epsilon pp}{}^2/4\pi$ may also be obtained from nucleon-nucleon scattering. It had been found[16] necessary to exchange a scalar meson to explain the characteristic of the nucleon-nucleon interaction, and the value of the coupling constant found is $\sim$10 to 20 for masses of the scalar meson of the order of 700 MeV.

It is of interest to note that Adler[17] has obtained a consistency condition for pion-nucleon scattering which predicts that $A^{(+)}/4\pi=g_r{}^2/4\pi m_N=2.2/m_\pi$ at the unphysical "threshold." We find

$$\begin{aligned}A^{(+)}/4\pi&=\alpha_2/4\pi[1/(m^{*2}-s_0)+1/(m^{*2}-u_0)] +(g_{\epsilon\pi\pi}g_{\epsilon pp}/4\pi m_\epsilon{}^2)\\&=(3.2-0.6)/m_\pi=2.6/m_\pi\,,\end{aligned} \tag{7}$$

where $s_0=(m_N+m_\pi)^2$ and $u_0=(m_N-m_\pi)^2$.

Adler's consistency condition refers to the value of $A^{(+)}$ at an unphysical point. Hamilton and Woolcock[10] obtain a value 2.6, while Roper, Wright, and Feld[13] obtain a value 2.8 for $A^{(+)}/4\pi$ at the physical threshold. Thus our result for $A^{(+)}$ at the physical threshold is in excellent agreement with the experimental data and is approximately equal to Adler's theoretical result. In our model the contributions to $A^{(+)}$ come from the N^* and ϵ alone, since the nucleon contribution vanishes as seen from Eq. (3a). This is in contrast to models using pseudovector coupling of pions and nucleons where the nucleon contribution itself exactly satisfies Adler's consistency condition.

Having obtained the correct scattering lengths, the low-energy phase shifts for pion-nucleon scattering may then be determined. The results of these calculations will be presented elsewhere.

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