Current Algebra, Dispersion Relations, and the ρ^+-ρ^0 Mass Difference

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An analysis of the ρ^+-ρ^0 mass difference has been made using (a) on-mass-shell current algebra, and (b) dispersion theory. The former method gives a logarithmically divergent result similar to the one obtained by the phenomenological Lagrangian approach. In dispersion theory, if one writes unsubtracted dispersion relations for the form factors describing virtual-photon–ρ-meson scattering, then the mass difference is quadratically divergent. However, once-subtracted dispersion relations with the subtraction constant taken from current algebra, together with the assumption of ρ dominance of the dispersion integrals, gives a result similar to that obtained from (a).

1. INTRODUCTION

THE current-algebra calculation of electromagnetic (em) mass differences between the members of various isomultiplets has attracted a lot of attention in recent times. One calculates these up to first order in em interaction (then these are related to integrals over the forward virtual Compton scattering amplitudes for scattering with transverse and longitudinal photons) by taking into account a few low-lying states. Harari,[1] on the basis of a pure Regge-pole model (with no cuts[2]) has argued that $\Delta I=2$ mass differences (e.g., π^+-π^0, ρ^+-ρ^0, etc.), due to high-energy damping effects do not have a large Regge tail and are therefore dominated by low-lying states. However, if fixed poles in the angular momentum plane are present, then the pure Regge behavior of the virtual Compton amplitudes F_i (defined below) gets modified,[3] and F_1 and F_2, instead of satisfying unsubtracted dispersion relations as would be expected from Harari's argument, satisfy once-subtracted dispersion relations (see Sec. 3).

It may be mentioned that Das *et al.*[4] using chiral $SU(2)\times SU(2)$ current algebra and working with soft pions, have obtained finite π^+-π^0 mass difference in good agreement with experiment. Gerstein *et al.*[5] have shown that if one calculates the same mass difference by using hard-pion current algebra or the phenomenological chiral $SU(2)\times SU(2)$ Lagrangian approach,[6] this mass difference is logarithmically divergent. It so happens that the logarithmically divergent term is proportional to $(m_\pi/m_\rho)^2$, and therefore the mass difference obtained is not sensitive to the cutoff parameter. As a result of this, the mass difference is in good agreement with experiment. This is encouraging. We point out[7] here that this hard-pion current-algebra result could also be obtained by writing once-subtracted dispersion relations for the form factors F_1 and F_2 and by determining the subtraction constant from the soft-pion current algebra, provided that the dispersion integrals are saturated by π and A_1 single-particle states only. In the current-algebra calculation, a similar saturation scheme was used implicitly, hence the equivalence of this result is understandable. However, dispersion integrals also receive contributions from continuum and other single-particle states (e.g., ω and φ poles), which (in principle) are calculable. In this paper, we draw a similar conclusion[8] in the context of a calculation of the ρ^+-ρ^0 mass difference.

Assuming the validity of the first-order perturbation theory, the em mass difference is given by the expression[9]

$$m_{\rho^+}{}^2-m_{\rho^0}{}^2=-\frac{e^2}{4\pi i}\frac{1}{(2\pi)^3}\times\int\frac{d^4k}{k^2}\left(g_{\mu\nu}-\lambda\frac{k_\mu k_\nu}{k^2}\right)T_{\mu\nu}(q,k)\,,\quad(1.1)$$

where

$$T_{\mu\nu}(q,k)=i\int d^4x\, e^{-ik\cdot x}\{\langle\rho^+,q|\times T(V_\mu{}^{\rm em}(x),V_\nu{}^{\rm em}(0))|\rho^+,q\rangle-(\rho^+\to\rho^0)\}\,.\quad(1.2)$$

Here, $T_{\mu\nu}(q,k)$ is related to the forward virtual Compton-scattering amplitude of a photon of mass k^2 and a ρ meson (with its polarization states averaged) of momentum q with $q^2=-m_\rho{}^2$. $V_\mu{}^{\rm em}(x)$ is the em current. λ is a gauge-dependent parameter.

[1] H. Harari, Phys. Rev. Letters **17**, 1303 (1966). These arguments are based on the assumption $\alpha_2(0)<0$ for $I=2$ meson trajectories and have been used to calculate em mass differences in dispersion theory. See, e.g., S. N. Biswas, S. K. Bose, K. Datta, J. Dhar, Yu. V. Novozhilov, and R. P. Saxena, Phys. Rev. **164**, 1937 (1967).

[2] The presence of Regge cuts might invalidate the assumption $\alpha_2(0)<0$. See, e.g., I. J. Muzinich, Phys. Rev. Letters **18**, 381 (1967).

[3] D. J. Gross and H. Pagels, Phys. Rev. **172**, 1381 (1968).

[4] T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low, and J. E. Young, Phys. Rev. Letters **18**, 759 (1967).

[5] I. S. Gerstein, B. W. Lee, H. T. Nieh, and H. J. Schnitzer, Phys. Rev. Letters **19**, 1064 (1967).

[6] See, e.g., B. W. Lee and H. T. Nieh, Phys. Rev. **166**, 1507 (1968), and G. C. Wick and B. Zumino, Phys. Letters **25B**, 479 (1967).

[7] K. C. Gupta and J. S. Vaishya, Phys. Rev. **176**, 2125 (1968).

[8] In the context of strong-interaction amplitudes, namely, π-ρ and π-A_1 scatterings, the question of subtractions for the invariant form factors involved will be discussed elsewhere.

[9] Riazuddin, Phys. Rev. **114**, 1138 (1959); V. Barger and E. Kazes, Nuovo Cimento **28**, 385 (1963).

In Sec. 2, we calculate $T_{\mu\nu}(q,k)$ using on-mass-shell current algebra.[10] In this model, $T_{\mu\nu}(q,k)$ is gauge-invariant and therefore λ is arbitrary. In Sec. 3, we calculate $T_{\mu\nu}(q,k)$ using fixed-k^2 unsubtracted and once-subtracted dispersion relations for F_i defined by

$$T_{\mu\nu}(q,k)=g_{\mu\nu}F_1+k_\mu k_\nu F_2+(q_\mu k_\nu+k_\mu q_\nu)F_3 \\ +i(q_\mu k_\nu-k_\mu q_\nu)F_4+q_\mu q_\nu F_5,$$

where

$$F_i\equiv F_i(\nu,k^2)\quad\text{with}\quad \nu=-q\cdot k.$$

The results are then utilized for calculating the ρ^+-ρ^0 mass difference.

2. CURRENT ALGEBRA AND ρ DOMINANCE

Using the field-current identity

$$V_\mu{}^a(x)=g_\rho\rho_\mu{}^a(x), \tag{2.1}$$

$T_{\mu\nu}(q,k)$ can be related to $M_{\mu\nu\lambda\sigma}(q,q,k,k)$ in an obvious manner, where we define[11]

$$\begin{aligned}M_{\mu\nu\lambda\sigma}&(q,p,k,k')\\ &=\int d^4xd^4yd^4z\, e^{iq\cdot x-ip\cdot y+ik\cdot z}\\ &\quad\times\langle T(V_\mu{}^a(x),V_\nu{}^b(y),V_\lambda{}^c(z),V_\sigma{}^d(0))\rangle_0\\ &=ig_\rho{}^{-4}\Delta_\rho{}^{\mu\mu'}(q)\Delta_\rho{}^{\nu\nu'}(p)\Delta_\rho{}^{\lambda\lambda'}(k)\Delta_\rho{}^{\sigma\sigma'}(k')\\ &\qquad\times F_{\mu'\nu'\lambda'\sigma'}{}^{abcd}(q,p,k).\end{aligned} \tag{2.2}$$

Here, $k'=q+k-p$, and $\Delta_\rho{}^{\mu\nu}(k)$ is the ρ-meson propagator given by

$$\Delta_\rho{}^{\mu\nu}(k)=g_\rho{}^2(k^2+m_\rho{}^2)^{-1}(g_{\mu\nu}+m_\rho{}^{-2}k_\mu k_\nu).$$

Assumption of the equal-time commutation relation

$$[V_0{}^a(x),V_\mu{}^b(y)]\delta(x_0-y_0)=2i\epsilon^{abc}V_\mu{}^c(x)\delta^4(x-y) \tag{2.3}$$

and ρ dominance gives[10]

$$\begin{aligned}F_{\mu\nu\lambda\sigma}{}^{abcd}(q,p,k)&=\bar F_{\mu\nu\lambda\sigma}{}^{abcd}(q,p,k)+\epsilon^{abe}\epsilon^{ecd}g_\rho{}^{-2}\\ &\times\Gamma_{\mu\nu'\nu}(-q,\,p)\Delta_\rho{}^{\nu'\nu''}(p-q)\Gamma_{\nu''\lambda\sigma}(p-q,\,k')\\ &+\epsilon^{ace}\epsilon^{bed}g_\rho{}^{-2}\Gamma_{\mu\lambda'\lambda}(-q,\,-k)\Delta_\rho{}^{\lambda'\lambda''}(k-q)\Gamma_{\nu\lambda''\sigma}(p,k')\\ &+\epsilon^{ade}\epsilon^{bce}g_\rho{}^{-2}\Gamma_{\mu\sigma'\sigma}(-q,k')\Delta_\rho{}^{\sigma'\sigma''}(k-p)\Gamma_{\nu\lambda\sigma''}(p,k-p),\end{aligned} \tag{2.4}$$

where

$$\int d^4xd^4y\, e^{-iq\cdot x-ik\cdot y}\langle T(V_\mu{}^a(x),V_\nu{}^b(0),V_\lambda{}^c(y))\rangle_0 \\ =i\epsilon^{abc}g_\rho{}^{-3}\Delta_\rho{}^{\mu\mu'}(q)\Delta_\rho{}^{\nu\nu'}(q-k)\Delta_\rho{}^{\lambda\lambda'}(k)\Gamma_{\mu'\nu'\lambda'}(q,k). \tag{2.5}$$

$\Gamma_{\mu\nu\lambda}(q,k)$ is the three-point proper vertex and $\bar F_{\mu\nu\rho\sigma}{}^{abcd}(q,p,k)$ is the four-point vertex, called the "contact" or "seagull" term. These vertices are assumed to be smooth functions of momenta:

$$\Gamma_{\mu\nu\lambda}(q,k)=2g_\rho{}^{-1}m_\rho{}^2[g_{\mu\nu}(2q+k)_\lambda \\ +g_{\mu\lambda}(k-q)_\nu-g_{\nu\lambda}(2k-q)_\mu], \tag{2.6}$$

$$\bar F_{\mu\nu\lambda\sigma}{}^{abcd}(q,p,k)=2g_\rho{}^{-1}m_\rho{}^4[3\epsilon^{abe}\epsilon^{ecd}(g_{\nu\lambda}g_{\mu\sigma}-g_{\nu\sigma}g_{\mu\lambda}) \\ +(\epsilon^{ace}\epsilon^{bde}+\epsilon^{ade}\epsilon^{bce})(g_{\mu\sigma}g_{\nu\lambda}+g_{\nu\sigma}g_{\mu\lambda}-2g_{\lambda\sigma}g_{\mu\nu})]. \tag{2.7}$$

Using Eqs. (2.1–2.7) with $p=q$ and $k=k'$, straightforward algebra gives

$$F_1(\nu,k^2)=-\frac{m_\rho{}^4}{3(m_\rho{}^2+k^2)^2}\left[3+\frac{k^2}{m_\rho{}^2}-\frac{8k^2+2k^4m_\rho{}^{-2}}{(k^2-2\nu)}\right] \\ +(\nu\rightarrow-\nu), \tag{2.8a}$$

$$F_2(\nu,k^2)=-\frac{m_\rho{}^4}{3(m_\rho{}^2+k^2)^2}\left[\frac{k^2}{4m_\rho{}^4}-\frac{-5+k^4/4m_\rho{}^4-k^2/m_\rho{}^2}{k^2-2\nu}\right] \\ +(\nu\rightarrow-\nu). \tag{2.8b}$$

The other form factors are related to F_1 and F_2 through the following relations[12]:

$$F_1+k^2F_2=\nu F_3,\quad k^2F_3=\nu F_5, \tag{2.9}$$

and

$$F_4=0.$$

These relations follow from the conservation of the vector current and the assumption that the Schwinger terms are c numbers or at least that the equal-time commutator $[V_0{}^a(x),V_\mu{}^b(y)]\delta(x_0-y_0)$ does not have any $I=2$ compoennt. Equations (2.9) ensure the gauge invariance of $T_{\mu\nu}(q,k)$.

3. UNSUBTRACTED AND ONCE-SUBTRACTED DISPERSION RELATIONS: ρ^+-ρ^0 MASS DIFFERENCE

The question of subtractions in the dispersion relations for $F_i(\nu,k^2)$ is related to their behavior for large ν and fixed k^2. If a pure Regge-pole model is applicable for virtual-photon scattering, then one expects, for $\nu\rightarrow\infty$ and fixed k^2,

$$F_j(\nu,k^2)\sim\sum_i\gamma_{ji}(k^2)\nu^{\alpha_i(0)},\quad j=1,\,2 \tag{3.1}$$

where $\gamma_{ji}(k^2)$ are the residue functions at $t=0$ (forward direction). Following de Alfaro *et al.*,[13] if we assume $\alpha_i(0)<0$ (as we are considering the $\Delta I=2$ mass shift), then the Regge behavior (3.1) will imply that $F_j(\nu,k^2)$ satisfy the unsubtracted dispersion relation[1]:

$$F_j(\nu,k^2)=\frac{1}{\pi}\int\frac{\text{Im}F_j(\nu',k^2)}{(\nu'-\nu)}d\nu'. \tag{3.2}$$

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[10] For three-point functions, see S. Weinberg and H. J. Schnitzer, Phys. Rev. 164, 1828 (1967); for four-point functions see I. S. Gerstein and H. J. Schnitzer, *ibid.*, 170, 1638 (1968).

[11] Here, we follow notations similar to that of the first paper in Ref. 10.

[12] The crossing symmetry requires $F_i(\nu,k^2)=F_i(-\nu,k^2)$ for $i=1,2,4,5$ and $F_3(\nu,k^2)=-F_3(-\nu,k^2)$.

[13] V. de Alfrao, S. Fubini, G. Furlan, and C. Rossetti, Phys. Letters 21, 576 (1966).

The right-hand side can be calculated in the standard way, starting from

$$\mathrm{Im}T_{\mu\nu}(q,k)=-(2\pi)^4\sum_n \{[\langle\rho^+,q|V_\mu{}^{\mathrm{em}}(0)|n\rangle \times\langle n|V_\nu{}^{\mathrm{em}}(0)|\rho^+,q\rangle\delta(p_n-q-k) -\langle\rho^+,q|V_\nu{}^{\mathrm{em}}(0)|n\rangle\langle n|V_\mu{}^{\mathrm{em}}(0)|\rho^+,q\rangle\delta(-p_n+q-k)] -(\rho^+\to\rho^0)\}. \quad (3.3)$$

Now, we make the assumption that the imaginary part of $T_{\mu\nu}(q,k)$ is dominated by low-lying single-particle states and the inelastic contribution is small. It is obvious that for an isovector photon, only the ρ^+ term contributes with a ρ intermediate state.[14] For isoscalar photon, π and A_1 states contribute equally to the ρ^+ and ρ^0 terms and hence do not contribute to the mass difference. From Eq. (2.5), we have

$$\langle\rho^+,p|V_\sigma{}^3(0)|\rho^+,k\rangle=\frac{2m_\rho{}^2\epsilon_\lambda(p)\epsilon_\nu(k)}{[m_\rho{}^2+(p-k)^2]} \times[-g_{\nu\lambda}(p+k)_\sigma+2p_\nu g_{\lambda\sigma}+2k_\lambda g_{\sigma\nu}]. \quad (3.4)$$

Using this in Eq. (3.3), we get[15]

$$\mathrm{Im}F_1(\nu,k^2)=\frac{8\pi}{3}\left(k^2+\frac{k^4}{4m_\rho{}^2}\right)\frac{m_\rho{}^4}{(m_\rho{}^2+k^2)^2} \times\delta(k^2-2\nu)-(\nu\to-\nu), \quad (3.5\text{a})$$

$$\mathrm{Im}F_2(\nu,k^2)=\tfrac{1}{3}\pi\left(-5-\frac{k^2}{m_\rho{}^2}+\frac{k^4}{4m_\rho{}^4}\right)\frac{m_\rho{}^4}{(m_\rho{}^2+k^2)^2} \times\delta(k^2-2\nu)-(\nu\to-\nu). \quad (3.5\text{b})$$

Using Eqs. (3.5), we can calculate F_i ($i=1$, 2, 3, 5) from unsubtracted dispersion relations. If these are used to calculate (1.1),[16] we find the mass difference to be quadratically divergent[17]:

$$|m_{\rho^+}{}^2-m_{\rho^0}{}^2|\simeq(e^2/4\pi)(\Lambda^2/16\pi). \quad (3.6)$$

Here, Λ is a cutoff parameter. If, in contrast, we assume once-subtracted dispersion relations for the form factors F_j ($j=1$, 2), we have

$$F_j(\nu,k^2)-F_j(\nu_0,k^2)=\frac{(\nu-\nu_0)}{\pi}\int\frac{\mathrm{Im}F_j(\nu',k^2)}{(\nu'-\nu_0)(\nu'-\nu)}d\nu'. \quad (3.7)$$

It may be mentioned that if fixed poles in the angular momentum plane are present, then the pure Regge behavior (3.1) gets modified to[3]

$$F_j(\nu,k^2)\sim R_j(k^2)+\sum_i\gamma_{ji}(k^2)\nu^{\alpha_i(0)}, \quad (3.8)$$

where $R_j(k^2)$ are related to the residue of the fixed poles. The fixed poles in the angular momentum plane have been shown to occur in the current-algebra calculation for nonstrong processes at nonsense points with right signature.[18,19] It is now easily seen that the assumption of the once-subtracted dispersion relation (3.7) is consistent with the modified Regge representation (3.8).

Evaluating the right-hand side of Eq. (3.7) with the help of Eqs. (3.5), we note that it is identical to that evaluated from current algebra [see Eqs. (2.8)].[20,21]

However, dispersion theory alone cannot determine the subtraction constant $F_i(0,k^2)$, so we utilize the current-algebra result (2.8). Evaluating (1.1), we find the mass difference to be logarithmically divergent[22]:

$$(m_{\rho^+}{}^2-m_{\rho^0}{}^2)_{\mathrm{div}}=-\frac{e^2}{4\pi}\frac{m_\rho{}^2}{4\pi}\times\tfrac{3}{4}\ln\frac{\Lambda^2}{m_\rho{}^2}.$$

This result is similar to that obtained by Schwinger[23] and Lee and Nieh[6] from the phenomenological-Lagrangian approach. Note that our result for the mass difference is not very insensitive to the cutoff parameter Λ, as was the case with π^+-π^0 mass difference. For this reason, we refrain from comparing it with experiment.

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[14] Note that the π and A_1 single-particle states do not contribute because of G-parity considerations.

[15] Other ImF_i's are related to Im$F_{1,2}$ through the relations: $\mathrm{Im}F_1+k^2\,\mathrm{Im}F_2=\nu\,\mathrm{Im}F_3$, $k^2\,\mathrm{Im}F_3=\nu\,\mathrm{Im}F_5$, and $\mathrm{Im}F_4=0$.

[16] Here, we work in the gauge $\lambda=4$. This ensures that contact diagrams having the covariant structure $g_{\mu\nu}$ do not contribute to the mass difference. In contrast, Biswas *et al.*, [Ref. 1] insist on gauge invariance of the amplitudes for virtual Compton scattering in analogy with the physical case. This may explain the difference between the results quoted there and our calculations in dispersion theory.

[17] In Ref. 1, an attempt has been made to argue the convergence of em mass shifts on the basis of large-ν behavior of the amplitudes $F_i(\nu,k^2)$ for fixed k^2. W. N. Cottingham and J. Gibb [Phys. Rev. Letters **18**, 883 (1967)] have argued that the logarithmic divergence in the em mass shifts is reflected in requiring one subtraction in the form factors $F_{1,2}$. We feel that the convergence of the mass shifts is also related to the asymptotic behavior of the form factors for large k^2. See also J. D. Bjorken, Phys. Rev. **148**, 1467 (1966), D. G. Boulware and S. Deser, *ibid.* **175**, 1912 (1968), and Ref. 3.

[18] Virendra Singh, Phys. Rev. Letters **18**, 36 (1967); J. B. Bronzan, I. S. Gerstein, B. W. Lee, and F. E. Low, *ibid.* **18**, 32 (1967); Phys. Rev. **157**, 1448 (1967).

[19] D. J. Gross and H. Pagels, Phys. Rev. Letters **20**, 961 (1968).

[20] Unsubtracted dispersion relations, along with Eqs. (3.5) [ρ dominance] for the form factors F_3 and F_5, reproduce the result (2.8) for current-algebra four-point vertex functions.

[21] For the sake of simplicity, one can choose $\nu_0=0$ (see e.g., Ref. 7).

[22] It may be pointed out that M. Halpern and G. Segrè [Phys. Rev. Letters **19**, 611 (1967)] have presented general arguments that the field algebra, in general, gives logarithmically divergent em mass shifts. See also P. Olesen, *ibid.* **20**, 525 (1968).

[23] J. Schwinger, in *Proceedings of the International Conference on Particles and Fields, Rochester, 1967* (Interscience Publishers, Inc., New York, 1967).