Application of Absorptive Corrections to Regge-Pole Exchanges*

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The example of the box amplitude in the high-energy limit suggests that the off-shell contribution to iterated scattering amplitudes is important at high energies. The relevance of this to the application of absorptive corrections to Regge-pole exchanges is discussed.

RECENTLY, absorptive corrections have been applied to Regge-pole exchanges by several authors.[1] Both the usual absorption model[2] and the eikonal formalism[3] have been employed to determine the form of the corrections. Whether it is appropriate to make such corrections to a Regge exchange at all seems to depend upon one's model (or view) of Regge behavior. In this paper, however, we are not concerned with such questions of "double counting," but rather with whether these approaches generate realistic corrections at high energies, assuming that some corrections should be made. We raise this question in the first place because it seems that in derivations of these formalisms one is essentially summing an iterated set of two-body scattering amplitudes in an approximation in which the contribution from intermediate states with the particles off the mass shell (energy-nonconserving intermediate states in a potential model) cancels to leading order in $1/p$. Thus the eikonal appears to be a function of the on-mass-shell Born amplitude only. Indeed, the usefulness of the absorption formulas derives from just the fact that they connect only amplitudes on the mass shell. However, the cancellation depends upon a particular extrapolation off the mass shell of the Born amplitude, and it is by no means obvious what extrapolation to use for a Regge exchange.[4] Furthermore, if one considers relativistic field theory rather than potential theory as a model for high-energy behavior, one again concludes that the off-shell contributions can be important. This is suggested by the particular example of the box amplitude (see Fig. 1) in the high-energy limit. Here one can explicitly isolate the contributions coming from (a) both intermediate-state particles being on-shell and (b) one or both of these particles being off-shell, and it turns out that it is the type (b) contribution which in fact gives the asymptotic behavior of the amplitude. There is no contradiction with the potential scattering result, since the kinematics for potential scattering and single-particle exchange are different at high energies.[5] The relation between potential scattering and field theoretic models will be discussed more fully elsewhere.

The invariant scattering amplitude for the box diagram is[6]

$$A(s,t)=i\frac{g^4}{(2\pi)^4}\int d^4k_1\frac{1}{(k_1^2+\mu^2-i\epsilon)(k_2^2+m^2-i\epsilon)(k_3^2+\mu^2-i\epsilon)(k_4^2+m^2-i\epsilon)},\tag{1}$$

where all particles are taken to be scalar mesons and g is the coupling constant. The invariant energy and momentum-transfer variables are defined by

$$s=-(p_1+p_2)^2 \quad\text{and}\quad t=-(p_1-p_3)^2,$$

so that for s-channel scattering $s\geq 4m^2$ and $t\leq 0$.

The high-energy form of the amplitude (1) has been obtained by many authors using the Feynman parameter technique to perform the integrations, and is[7]

$$A(s,t)=\frac{g^4}{8\pi^2}\frac{1}{[t(t-4\mu^2)]^{1/2}}\ln\left(\frac{-t+[t(t-4\mu^2)]^{1/2}}{t+[t(t-4\mu^2)]^{1/2}}\right)\frac{\ln s}{s}+\cdots,\tag{2}$$

where the remaining terms are of order $1/s$.

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[4] Specifically, a ladder model for Regge poles gives an off-mass-shell extrapolation which is quite different from that implied by a single-particle exchange. This difference is decisive for the question whether iterated Regge exchanges give rise to cuts or not. Cf., S. Mandelstam, Nuovo Cimento **30**, 1127 (1963).

[5] This point is well recognized by L. Domash [Princeton University thesis, 1967 (unpublished)].

[6] Viewing the box diagram in Fig. 1 as the first iteration, in the s channel, of a single-particle exchange amplitude, the particles labeled 2 and 4 are the intermediate-state particles.

[7] See R. J. Eden, P. V. Landshoff, D. I. Olive, and J. C. Polkinghorne, *The Analytic S Matrix* (Cambridge University Press, Cambridge, England, 1961), Chap. 3, Sec. 3. Note that these authors leave the t dependence of the amplitude in the form of an integral over some Feynman parameters. Doing the integrations one obtains the result (2).

The Feynman parameter technique gives no insight into whether the leading behavior of amplitude (2) is an on-shell or off-shell contribution. However, by performing the integrations in (1) directly to obtain the high-energy form of the amplitude, one can explicitly show that the leading term in (2) is an off-shell contribution. First we make the change of variables $d^4k_1 \rightarrow dt_1 dk_2^2 dt_3 dk_4^2$, where $t_1 = -k_1^2$ and $t_3 = -k_3^2$. The Jacobian for this transformation is[8]

$$J = \theta(-D)/2\sqrt{(-D)}\,, \tag{3}$$

where θ denotes the unit step function and D is symmetric 4×4 determinant

$$D = \det|2k_i \cdot k_j|\,. \tag{4}$$

In terms of s, t, and the new variables of integration, one finds

$$D = -s^2\lambda(t,t_1,t_3) - 4stk_2^2k_4^2 + \cdots, \tag{5}$$

where

$$\lambda(t,t_1,t_3) = 2tt_1 + 2tt_3 + 2t_1t_3 - t^2 - t_1^2 - t_3^2\,, \tag{6}$$

and the remaining terms in (5) can be neglected at high energy ($s \gg 4m^2$).

Making this change of variables and using the high-energy form of the Jacobian, (1) becomes

$$A(s,t) = i\frac{g^4}{32\pi^4}\int \frac{dt_1 dt_3 dk_2^2 dk_4^2 \theta[s^2\lambda(t,t_1,t_3) + 4stk_2^2k_4^2]}{(t_1-\mu^2+i\epsilon)(k_2^2+m^2-i\epsilon)(t_3-\mu^2+i\epsilon)(k_4^2+m^2-i\epsilon)(s^2\lambda+4stk_2^2k_4^2)^{1/2}} + \cdots. \tag{7}$$

The step function in the integrand of (7) determines the kinematically allowed regions over which the integrations are to be performed.[9,10] One finds that k_2^2 and k_4^2 are allowed to vary over the region of the k_2^2, k_4^2 plane which includes the origin and is bounded by the hyperbolas

$$k_2^2k_4^2 = -s\lambda(t,t_1,t_3)/(4t)\,, \tag{8}$$

and that t_1 and t_3 are both negative and lie in the region of the t_1, t_3 plane that is bounded by the parabola

$$(t_1-t_3)^2 = t[2(t_1+t_3)-t]\,. \tag{9}$$

One might note that for large s the hyperbolas (8) lie quite far from the origin of the k_2^2, k_4^2 plane, indicating that particles 2 and 4 can get far off their mass shells ($k_2^2 = k_4^2 = -m^2$).

Performing the k_2^2 and k_4^2 integrations gives

$$A(s,t) = i(g^4/16\pi^4)[-(\pi i/s)\ln(-s\lambda/m^4t) + (\pi i)^2/s]\phi(t)\,, \tag{10}$$

where

$$\phi(t) = \int \frac{dt_1 dt_3 \theta(\lambda(t,t_1,t_3))}{(t_1-\mu^2)(t_3-\mu^2)}\,. \tag{11}$$

The amplitude has been left in the form (10) in order to isolate the on-shell and off-shell contributions. Separating the propagators for particles 2 and 4 into off-shell and on-shell parts by the formal substitutions

$$(k_j^2+m^2-i\epsilon)^{-1} = P(k_j^2+m^2)^{-1} + i\pi\delta(k_j^2+m^2) \tag{12}$$

in (7), one sees that the contribution which results from constraining both intermediate particles to be on the mass shell is

$$i(g^4/32\pi^4)(i\pi)^2 s^{-1}\phi(t)\,, \tag{13}$$

which is just one-half of the second term in (10). The remaining contribution to (10) is off-shell.[11] Since it is

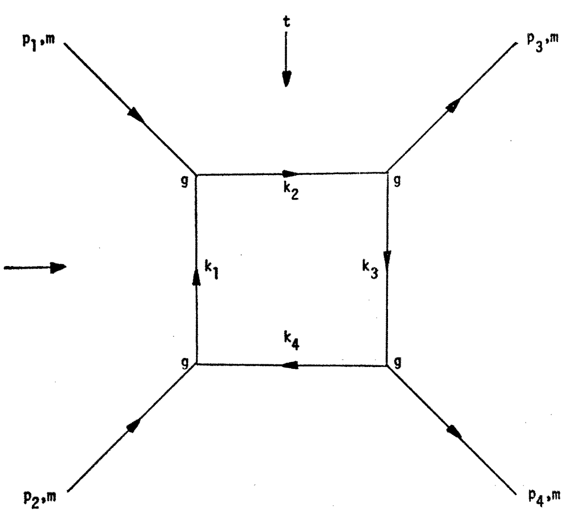

FIG. 1. The box diagram.

[8] Since the transformation of variables is not one-to-one an extra factor of 2 must be supplied. See T. W. B. Kibble, Phys. Rev. **117**, 1159 (1959).

[9] It is important to retain the term involving $k_2^2k_4^2$ in the determinant (5). If this term is dropped then k_2^2 and k_4^2 can assume all values from $-\infty$ to $+\infty$, and the integrals over these variables are logarithmically divergent. If performed according to the principal value prescription each integration will give $i\pi$. In any case, the resultant amplitude will have a leading term of order $1/s$, which is incorrect.

[10] In a recent article H. Rothe [Phys. Rev. **159**, 1471 (1967)] neglects the $k_2^2k_4^2$ term in (5) when he considers the high-energy form of box diagrams with one or two of the single-particle exchanges replaced by Regge-pole exchanges. This causes him to neglect branch cuts in the k_2^2 and k_4^2 complex planes, which should not be neglected, and therefore apparently invalidates the argument that he gives to explain the cancellation of Regge cuts in these amplitudes. While we do believe in the cancellation of the Regge cuts in these amplitudes, we do not believe that it occurs by the mechanism that Rothe gives.

[11] For completeness we add that the $\ln(-s\lambda/tm^4)/s$ term comes from having one intermediate-state particle on-shell and the other off-shell, while having both off-shell leads to a $1/s$ contribution which is identical to (13).

the $\ln s/s$ term which gives the dominant behavior at large s, one sees that the box amplitude provides a concrete example of a situation where a contribution of type (b) is important at high energy. In other words, if only the on-shell contribution had been retained one would have obtained an incorrect high-energy form for the box amplitude.

To check that the first term in (10) gives the correct leading term, the t_1 and t_3 integrations can be done to give

$$\phi(t)=\frac{2\pi}{[t(t-4\mu^2)]^{1/2}}\ln\left(\frac{-t+[t(t-4\mu^2)]^{1/2}}{t+[t(t-4\mu^2)]^{1/2}}\right), \quad (14)$$

and therefore

$$A(s,t)=\frac{g^4}{8\pi^2}\frac{1}{[t(t-4\mu^2)]^{1/2}}\ln\left(\frac{-t+[t(t-4\mu^2)]^{1/2}}{t+[t(t-4\mu^2)]^{1/2}}\right)\frac{\ln s}{s}+\cdots, \quad (15)$$

which is the same as (2).

In conclusion we feel that this example suggests that the off-shell contribution to iterated scattering amplitudes is important at high energy (if one believes field theory). Consequently, any iterative scheme which does not include this contribution may make a serious omission. Since the absorption and eikonal formalisms apparently do just this, their ability to generate believable corrections to Regge-pole exchanges is questionable.

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ρ Bootstrap in the Unitarized Strip Approximation

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We present a method of improving the new strip approximation of Chew and Jones by calculating parts of the elastic double spectral functions using the Mandelstam iteration procedure. These double spectral functions are used to obtain additional contributions to the left-hand cuts of the partial-wave amplitudes, and also to estimate the inelasticity within the strip region. The inelastic N/D equations are solved in the way proposed by Frye and Warnock. The method is applied to the problem of bootstrapping the ρ trajectory in π-π scattering, and some preliminary results are presented. We find that it is possible to obtain a self-consistent trajectory with the correct physical mass, width, and intercept $\alpha(0)$, but that the solution is by no means unique, since self-consistency can be achieved with trajectories having intercepts anywhere from $\alpha(0)=1$ to $\alpha(0)\approx 0.2$. Also, the trajectories have a large curvature, and large residue, which result in a violation of crossed-channel unitarity for low l.

I. INTRODUCTION

OVER the past few years many attempts have been made to demonstrate that the ρ meson approximately "bootstraps" itself in π-π scattering.[1–10] The zero external spins and equal-mass kinematics make this one of the most attractive bootstrap problems, but as more sophisticated methods have been applied to the problem, it has become clear that while many qualitative features support the bootstrap hypothesis, the quantitative details of the solutions to the various models are very unsatisfactory.[8,10]

Probably the most comprehensive approach has been the so-called "new form" of the strip approximation devised by Chew and Jones,[11,12] which parametrizes the amplitude in terms of the Regge poles in each channel, and then uses the N/D equations to impose unitarity and so determine the Regge parameters. However, it has been found that this approximation is inadequate,[10] one of its principal deficiencies being that it includes the forces only in the first Born approximation. It has been shown recently that the N/D equations for nonrelativistic potential scattering give much more satisfactory results if the forces are included up to at least the third Born approximation,[13] and we can expect that this will also be true in relativistic calculations.

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