Current Algebra, Glashow's Model of *CP* Nonconservation, and $K\to 3\pi$

BARRY R. HOLSTEIN*
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
(Received 9 September 1968)

The Glashow model, which violates *CP* invariance by inserting two phase factors between the vector and axial-vector currents in the usual weak Hamiltonian, is applied to the decays $K\to 3\pi$, using standard techniques of current algebra and partially conserved axial-vector current. Experiments on $K\to 2\pi$ can determine one combination, S, of these phase factors, while possible *CP* violation in $K\to 3\pi$ can determine the other combination, D. As a result of calculated enhancement factors, it is shown that present experimental (~10% or more) crude limits on *CP* violation in $K\to 3\pi$ already limit D to less than 0.01, and that in possible future experiments on $K\to 3\pi$ one can hope to see a positive *CP*-violating effect if the Glashow model is a correct description of *CP* violation.

I. INTRODUCTION

SINCE the experimental discovery of $K_L^0\to 2\pi$,[1] indicating the breakdown of *CP* invariance, various theories have been advanced in order to account for this process.[2] One of the simplest and most interesting is that of Glashow,[3] who retains the standard current-current form of the weak Hamiltonian

$$\mathcal{H}_w(x)=(G_V/2\sqrt{2})\{\mathcal{J}^\nu(x),\mathcal{J}_\nu(x)\}$$

and merely inserts arbitrary phases on the axial-vector currents of the Cabibbo model[4]

$$\mathcal{J}^\nu(x)=\sqrt{2}\cos\theta[V_{\pi^-}{}^\nu(x)+A_{\pi^-}{}^\nu(x)e^{i\varphi}] +\sqrt{2}\sin\theta[V_{K^-}{}^\nu(x)+A_{K^-}{}^\nu(x)e^{i\psi}]+j^\nu{}_{\mathrm{leptonic}}(x).$$

This is the most general way to add phases, since the over-all phase, as well as the relative phase between $\Delta S=0$ and $\Delta S\neq 0$ parts, is unobservable.

In a recent paper,[5] hereafter referred to as A, this Glashow model was applied to the decay $K\to 2\pi$, using the techniques of current algebra and partially conserved axial-vector current (PCAC), and it was found to be consistent with present experimental results. In Sec. II we review these findings from the point of view of a "natural" phase convention associated with the freedom to combine with the *CP* operation a strangeness gauge transformation.[6] If we define $S\equiv\frac{1}{2}(\phi+\psi)$ and $D\equiv\frac{1}{2}(\phi-\psi)$, then in this "natural" phase convention the parity-violating (conserving) part of the strangeness-changing weak Hamiltonian depends only upon $S(D)$. We show that S may be determined directly from the value of the Wu-Yang parameter ϵ',[6] but that in order to determine the value of D, we must search for a manifestation of *CP* violation in a parity-conserving process such as $K\to 3\pi$.

In Sec. III, we discuss the *CP*-conserving decay $K\to 3\pi$, employing the soft-pion techniques developed originally by Nambu and Hara[7] and recently extended to include $\Delta I=\frac{3}{2}$ contributions by Bouchiat and Meyer.[8] In Sec. IV we use these techniques to treat the *CP*-nonconserving part of the decay $K\to 3\pi$ in Glashow's model.

Finally, Sec. V presents a discussion of possible experimental checks for *CP* violation and uses present results to give an upper limit on the phase angle $\phi-\psi$.

II. "NATURAL" PHASE CONVENTION OF GLASHOW'S MODEL

In A, the Glashow Hamiltonian was discussed from the point of view of the "standard" phase convention. Defining[9]

$$\begin{aligned}\tfrac{1}{2}M_c&\equiv\{A_{\pi^+}{}^\nu,A_{K^-\nu}\}+\{V_{\pi^+}{}^\nu,V_{K^-\nu}\},\\ \tfrac{1}{2}L_c&\equiv\{A_{\pi^+}{}^\nu,A_{K^-\nu}\}-\{V_{\pi^+}{}^\nu V_{K^-\nu}\},\\ \tfrac{1}{2}M_v&\equiv\{A_{\pi^+}{}^\nu,V_{K^-\nu}\}+\{V_{\pi^+}{}^\nu,A_{K^-\nu}\},\\ \tfrac{1}{2}L_v&\equiv\{A_{\pi^+}{}^\nu,V_{K^-\nu}\}-\{V_{\pi^+}{}^\nu,A_{K^-\nu}\},\end{aligned}\tag{1}$$

where $M_{c,v}$ ($L_{c,v}$) are even (odd) under interchange of V and A, we found in A the result that the $\Delta S=-1$ part of the Hamiltonian is of the form

* NSF Predoctoral Fellow.

[1] C. R. Christenson *et al.*, Phys. Rev. Letters **13**, 138 (1964).

[2] W. Alles, Phys. Letters **15**, 348 (1965); R. J. Oakes, Phys. Rev. Letters **20**, 1539 (1968); L. Wolfenstein, *ibid.* **13**, 562 (1964); F. Zachariasen and G. Zweig, *ibid.* **14**, 794 (1965); N. Cabibbo, Phys. Letters **12**, 137 (1964); L. Wolfenstein, *ibid.* **15**, 196 (1965).

[3] S. Glashow, Phys. Rev. Letters **14**, 35 (1964).

[4] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[5] B. R. Holstein, Phys. Rev. **171**, 1668 (1968). We note here that there are differences in phase convention and normalization between A and the present paper. In particular, in the present paper our octet states are identical with the states $|II_3Y\rangle$ in deSwart's phase convention (i.e., Condon-Shortley for I spin and V spin) except for an extra minus sign associated with $\bar{K}^0$ (i.e., $|\bar{K}^0\rangle=-|\frac{1}{2}\frac{1}{2}-1\rangle$). In A, they are the same as $|II_3Y\rangle$) except for extra minus signs associated with π^+, K^+, K^0, $\bar{K}^0$.

Also, in the present paper, we are using properly normalized spherical tensor operators; e.g., in the quark model

$$V_{\pi\pm}{}^\mu(x)=\mp\bar{\psi}(x)\sqrt{\tfrac{1}{2}}\tfrac{1}{2}(\lambda_1\pm i\lambda_2)\gamma^\mu\psi(x)$$

whereas in A, those current densities associated with the weak Hamiltonian are larger by a factor $\sqrt{2}$.

[6] T. T. Wu and C. N. Yang, Phys. Rev. Letters **13**, 380 (1964). See also T. D. Lee and C. S. Wu, Ann. Rev. Nuc. Sci. **16**, 471 (1966).

[7] Y. Nambu and Y. Hara, Phys. Rev. Letters **16**, 875 (1966).

[8] C. Bouchiat and Ph. Meyer, Phys. Letters **25B**, 282 (1967).

[9] In the following, the subscript c refers to a parity-conserving, v to a parity-violating quantity.

$$\mathcal{H}_w^+(\Delta S=-1)=-(G_V/2\sqrt{2})\cos\theta\sin\theta \times[(\cos S\cos D\, M_v-\sin S\sin D\, L_v)+(\cos^2 D\, M_c-\sin^2 D\, L_c)],$$
$$\mathcal{H}_w^-(\Delta S=-1)=+i(G_V/2\sqrt{2})\cos\theta\sin\theta \times[(\cos S\sin D\, M_v+\cos D\sin S\, L_v)+\cos D\sin D(M_c+L_c)], \tag{2}$$

where $D\equiv\frac{1}{2}(\varphi-\psi)$ and $S\equiv\frac{1}{2}(\varphi+\psi)$, and we have separated $\mathcal{H}_w$ into CP-conserving and CP-violating parts:

$$\mathcal{H}_w=\mathcal{H}_w^+ +\mathcal{H}_w^-,\quad \text{with}\quad CP\mathcal{H}_w^{\pm}(CP)^{-1}=\pm\mathcal{H}_w^{\pm}.$$

It has been noted that because of the conservation of strangeness by the strong and electromagnetic interactions, we are free to associate with the CP operation an arbitrary strangeness gauge transformation. This freedom has been used by Wu and Yang[6] to set

$$\langle 2\pi;I=0|\mathcal{H}_w|K^0\rangle=\text{Real},$$

where $\langle 2\pi;I=I'|$ represents a two-pion standing wave state of total isospin I'.

A "natural" phase convention for the Glashow model may be defined by applying the gauge transformation $\mathcal{H}'=e^{-iDS}\mathcal{H}e^{+iDS}$ to the original Hamiltonian,[2] where S is the strangeness. If we divide the resultant Hamiltonian into CP even and odd parts, we find

$$\mathcal{H}_w^+(\Delta S=-1)=-(G_V/2\sqrt{2})\cos\theta\sin\theta \times[\cos S\, M_v+\cos D\, M_c],$$
$$\mathcal{H}_w^-(\Delta S=-1)=+i(G_V/2\sqrt{2})\cos\theta\sin\theta \times[\sin S\, L_v+\sin D\, L_c]. \tag{3}$$

Thus, in this "natural" phase convention the parity-conserving (-violating) portion of the weak Hamiltonian is dependent only upon the phase angle $D(S)$, and the CP-conserving and CP-violating parts are dynamically dissimilar in that the CP-conserving (-violating) part of the weak Hamiltonian is symmetric (antisymmetric) under interchange of V and A.

With this in mind we now consider the results of A. There we defined

$$\begin{aligned}\langle 2\pi;I=0|\mathcal{H}_w^+|K_+\rangle&=A_0,\\ \langle 2\pi;I=2|\mathcal{H}_w^+|K_+\rangle&=\beta A_0,\\ \langle 2\pi;I=0|\mathcal{H}_w^-|K_-\rangle&=i\alpha A_0,\\ \langle 2\pi;I=2|\mathcal{H}_w^-|K_-\rangle&=i\alpha\chi A_0,\end{aligned} \tag{4}$$

where A_0, β, α, χ are real by CPT and $K_\pm$ are linear combinations of K^0 and $\bar{K}^0$ such that

$$CP|K_\pm\rangle=\pm|K_\pm\rangle.$$

In Table I are listed the soft-pion values for these parameters obtained by application of the methods of A to both the "standard" and "natural" phase conventions. We note that A_0, β, α, χ are not physical observables (i.e., not directly measurable); their values may therefore depend on the particular phase convention being employed, as is clear from Table I. On the other hand, physical observables such as

$$\eta_{+-}=\frac{\langle\pi^+\pi^-|\mathcal{H}_w|K_L\rangle}{\langle\pi^+\pi^-|\mathcal{H}_w|K_S\rangle},\quad \eta_{00}=\frac{\langle\pi^0\pi^0|\mathcal{H}_w|K_L\rangle}{\langle\pi^0\pi^0|\mathcal{H}_w|K_S\rangle},$$

or the Wu-Yang parameters ϵ,ϵ' are directly measurable and must have the same form in all phase conventions. Thus, for example, since in the "natural" phase convention $|\epsilon'|=(\sqrt{\tfrac{1}{2}})\alpha\chi$ (Ref. 10) depends only on S, this must be true also in the "standard" phase convention. This feature is evident from Table I, where in both cases we find[11]

$$|\epsilon'|=\tfrac{4}{3}t\tan S, \tag{5}$$

where we have defined

$$\Gamma_+\equiv\langle 0|M_v|K^0\rangle,$$
$$\Gamma_-\equiv\langle 0|L_v|K^0\rangle,\quad t\equiv\Gamma_-/\Gamma_+.$$

For the remainder of this text, we shall employ the "natural" phase convention because of the simplicity afforded by its elegant form.

Equation (5) indicates that S may be determined directly from the experimental value of $|\epsilon'|$. Present experimental results indicate on this basis that S is less than 10^{-3} and may possibly equal zero. In any case, this gives us no direct method to evaluate D, although a nonzero value of D affects the decay $K\to 2\pi$ via virtual transitions that contribute to the mass matrix.

TABLE I. A comparison of the parameters A_0, β, α, χ evaluated by soft-pion techniques in both the "natural" and "standard" phase conventions.

	"Natural" phase conv.	"Standard" phase conv.[a]
A_0	$+\dfrac{G_V}{2\sqrt{2}}\cos\theta\sin\theta\dfrac{(\sqrt{6})\Gamma_+}{4F_\pi^2}\cos S$	$\sim+\dfrac{G_V}{2\sqrt{2}}\cos\theta\sin\theta\dfrac{(\sqrt{6})\Gamma_+}{4F_\pi^2}$
β	0	~ 0
α	$-(19/3)t\tan S$	$\sim-\tan D-(19/3)t\tan S$
χ	$-(\sqrt{\tfrac{1}{2}})8/19$	$-\dfrac{4\sqrt{2}}{3}\dfrac{t\tan S}{\tan D+(19/3)t\tan S}$

[a] In the "standard"-phase-convention results, we have neglected certain small terms of order $\tan S\tan D$.

III. $K\to 3\pi$: CP-CONSERVING CONTRIBUTION

Since our procedures are basically the same, it is useful to recapitulate the work of Bouchiat and Meyer[8] in the case of the CP-conserving weak Hamiltonian. We begin by making a complete isospin analysis of the $K\to 3\pi$ amplitude in the linear approximation, meaning

[10] See Appendix I of A.

[11] Similar independence of convention is found for ϵ, but the discussion is more involved, as it must include the mixing parameter $m'=i\langle K_-|M|K_+\rangle$, M being the conventional mass matrix.

we retain terms only of zeroth and first order in the energy.[12]

$$\begin{aligned}
&\mathrm{Amp}^{(+)}(K^+\to\pi^+\pi^0\pi^0)\\
&\quad=-\tfrac{1}{3}\sqrt{2}[(\alpha_1-(\sqrt{\tfrac{1}{2}})\alpha_3)+(\beta_1-(\sqrt{\tfrac{1}{2}})\beta_3)E_+]\\
&\qquad\qquad-(\tfrac{3}{2}\sqrt{\tfrac{1}{5}})\gamma_3(E_+-\tfrac{1}{3}M_K)\,,\\
&\mathrm{Amp}^{(+)}(K^+\to\pi^+\pi^+\pi^-)\\
&\quad=\tfrac{1}{3}\sqrt{2}[(2\alpha_1-\sqrt{2}\alpha_3)+(\beta_1-(\sqrt{\tfrac{1}{2}})\beta_3)(M_K-E_-)]\\
&\qquad\qquad+(\tfrac{3}{2}\sqrt{\tfrac{1}{5}})\gamma_3(E_--\tfrac{1}{3}M_K)\,, \qquad (6)\\
&\mathrm{Amp}^{(+)}(K_-\to\pi^+\pi^-\pi^0)\\
&\quad=\tfrac{1}{3}\sqrt{2}[(\alpha_1+\sqrt{2}\alpha_3)+(\beta_1+\sqrt{2}\beta_3)E_0]\,,\\
&\mathrm{Amp}^{(+)}(K_+\to\pi^+\pi^-\pi^0)=(\sqrt{\tfrac{1}{5}})\gamma_3(E_--E_+)\,,\\
&\mathrm{Amp}^{(+)}(K_-\to\pi^0\pi^0\pi^0)\\
&\quad=-\tfrac{1}{3}\sqrt{2}[(3\alpha_1+3\sqrt{2}\alpha_3)+(\beta_1+\sqrt{2}\beta_3)M_K]\,,
\end{aligned}$$

where α_i, β_i, γ_i are arbitrary real constants, and $i=1$ (3) refers to the $\Delta I=\frac{1}{2}$ ($\frac{3}{2}$) part of $\mathcal{H}_w^+$.[13] By means of current algebra and PCAC, we may go to the unphysical limit in which one pion has vanishing four-momentum. In contrast to the case of $K\to 2\pi$ discussed in A, we may still retain over-all energy-momentum conservation with the remaining particles each on their mass shells. In this way, we are able to relate an unphysical $K\to 3\pi$ amplitude to a physical $K\to 2\pi$ matrix element, which we may obtain from experiment.

By means of the usual methods and the definition $\phi_a(x)=(1/iF_\pi m_\pi^2)\partial^\mu A_\mu{}^a(x)$,[14] we find

$$\langle\pi_q{}^a\pi_p{}^b\pi_s{}^c|\mathcal{H}_w{}^+(0)|K_k{}^n\rangle \underset{q\to 0}{\longrightarrow} -\frac{1}{F_\pi}\langle\pi_p{}^b\pi_s{}^c|[F_a{}^5(0),\mathcal{H}_w{}^+(0)]|K_k{}^n\rangle\,, \qquad (7)$$

where

$$F_a{}^5(t)=\int d^3x\, A_a{}^0(\mathbf{x},t)\,.$$

Now, as discussed in Appendix I, because of the symmetry of $\mathcal{H}_w^+$ under the interchange of V and A, we may replace $F_a{}^5(0)$ by

$$F_a(0)=\int d^3x\, V_a{}^0(\mathbf{x},0)\,,$$

which by the conserved vector current (CVC) hypothesis is just the isotopic spin operator I_a, if we simultaneously replace $\{A_1,A_2\}+\{V_1,V_2\}$ by $\{V_1,A_2\}+\{A_1,V_2\}$ and vice versa.[15] Now the commutation of an isospin operator with an isotensor operator of rank I cannot change the value of the rank. Therefore,the $\Delta I=\frac{1}{2}(\frac{3}{2})$ contribution to the $K\to 3\pi$ amplitude is related only to the $\Delta I=\frac{1}{2}$ ($\frac{3}{2}$) part of the $K\to 2\pi$ matrix element. We can parametrize the $K\to 2\pi$ amplitude as follows:

$$\begin{aligned}
\mathrm{Amp}(K^+\to\pi^+\pi^0)&=(\sqrt{\tfrac{3}{10}})\mathfrak{f}_3\,,\\
\mathrm{Amp}(K_+\to\pi^+\pi^-)&=(\sqrt{\tfrac{2}{3}})\mathfrak{f}_1+\sqrt{(2/15)}\mathfrak{f}_3\,, \qquad (8)\\
\mathrm{Amp}(K_+\to\pi^0\pi^0)&=-(\sqrt{\tfrac{2}{3}})\mathfrak{f}_1+2\sqrt{(2/15)}\mathfrak{f}_3\,,
\end{aligned}$$

where the $\mathfrak{f}_i$ are contributions of the $\Delta I=\frac{1}{2}i$ part of $\mathcal{H}_w^+$ to this matrix element.[16]

Now in evaluating a matrix element such as

$$\langle\pi_p{}^b\pi_s{}^c|[I_a,\mathcal{H}_w(0)]|K_k{}^n\rangle$$

it is useful, instead of carrying out the commutation, to let I_a operate, respectively, to the left and right:

$$\begin{aligned}
&\langle\pi_p{}^b\pi_s{}^c|[I_a,\mathcal{H}_w(0)]|K_k{}^n\rangle\\
&\qquad=(\langle\pi_p{}^b\pi_s{}^c|I_a)\mathcal{H}_w(0)|K_k{}^n\rangle\\
&\qquad\qquad-\langle\pi_p{}^b\pi_s{}^c|\mathcal{H}_w(0)(I_a|K_k{}^n\rangle)\,.
\end{aligned}$$

In this way, we find (where $\mathcal{H}_w{}^i$ represents the $\Delta I=\frac{1}{2}i$ part of $\mathcal{H}_w$)

$$\begin{aligned}
&\langle\pi_q{}^+\pi_p{}^-\pi_s{}^0|\mathcal{H}_w{}^i(0)|K_k{}^0\rangle\\
&\underset{q\to 0}{\longrightarrow}\frac{\cos D}{F_\pi\cos S}[\langle\pi^+\pi^-|\mathcal{H}_w{}^i(0)|K^0\rangle+\langle\pi^0\pi^0|\mathcal{H}_w{}^i(0)|K^0\rangle]\\
&\underset{\to 0s}{\longrightarrow}-\frac{\cos D}{2F_\pi\cos S}\langle\pi^+\pi^-|\mathcal{H}_w{}^i(0)|K^0\rangle\\
&\underset{p\to 0}{\longrightarrow}-\frac{\cos D}{F_\pi\cos S}[\langle\pi^+\pi^-|\mathcal{H}_w{}^i|K^0\rangle+\langle\pi^0\pi^0|\mathcal{H}_w{}^i|K^0\rangle\\
&\qquad\qquad-(\sqrt{\tfrac{1}{2}})\langle\pi^+\pi^0|\mathcal{H}_w{}^i(0)|K^+\rangle]. \qquad (9)
\end{aligned}$$

In our isospin expansion, the unphysical limit is reached by merely replacing each energy by its value in this limit, taking account of energy-momentum conservation in $K\to 2\pi$.[17] Thus we find

$$\begin{aligned}
&\langle\pi_q{}^+\pi_p{}^-\pi_s{}^0|\mathcal{H}_w(0)|K_k{}^0\rangle\\
&\underset{q\to 0}{\longrightarrow}\tfrac{1}{3}[(\alpha_1+\sqrt{2}\alpha_3)+\tfrac{1}{2}(\beta_1+\sqrt{2}\beta_3)M_K]+\tfrac{1}{2}(\sqrt{\tfrac{1}{10}})\gamma_3M_K\\
&\underset{p\to 0}{\longrightarrow}\tfrac{1}{3}[(\alpha_1+\sqrt{2}\alpha_3)+\tfrac{1}{2}(\beta_1+\sqrt{2}\beta_3)M_K]-\tfrac{1}{2}(\sqrt{\tfrac{1}{10}})\gamma_3M_K\\
&\underset{s\to 0}{\longrightarrow}\tfrac{1}{3}(\alpha_1+\sqrt{2}\alpha_3).
\end{aligned}$$

Equating these two expressions, we find for the $\Delta I=\frac{1}{2}$ contribution

$$\tfrac{1}{3}(\alpha_1+\tfrac{1}{2}\beta_1M_K)=0\,,\quad \tfrac{1}{3}\alpha_1=-\frac{\cos D}{2F_\pi\cos S}(\sqrt{\tfrac{1}{3}})\mathfrak{f}_1 \qquad (10)$$

[12] We have no $I=0$ contribution, as such a state would have to be completely antisymmetric in space and is thus outside the linear approximation. We have no $I=3$ contribution since Glashow's Hamiltonian has no $\Delta I=\frac{5}{2}$ or $\Delta I=\frac{7}{2}$ part.

[13] In general we should allow these parameters to be complex in order to be consistent with the phases demanded by unitarity. However, we shall perform the current algebraic manipulations assuming them to be real and patch on the strong-interaction phase shifts when they are needed at a later stage.

[14] In the quark model we use $A_i{}^\mu(x)=\bar{\psi}(x)\frac{1}{2}\lambda_i\gamma^\mu\gamma^5\psi(x)$ so that our Goldberger-Treiman relation is $F_\pi=-iMg_A/g_r$.

[15] Note that such a replacement does not alter the isospin properties.

[16] We again delete the strong-interaction phase shifts demanded by unitarity.

[17] We use the kaon rest frame and, for the purpose of this extrapolation, assume that all pions of equal mass so that when the four-momentum of one pion vanishes, the other two pions share the kaon energy equally, each taking away energy $\frac{1}{2}M_K$.

which yields the result

$$\alpha_1 = -\frac{\sqrt{3}\mathfrak{f}_1}{2F_\pi}\frac{\cos D}{\cos S}, \quad M_K\beta_1 = \frac{\sqrt{3}\mathfrak{f}_1}{F_\pi}\frac{\cos D}{\cos S}. \qquad (11)$$

For the $\Delta I = \frac{3}{2}$ contribution, we need all three results:

$$\frac{\cos D}{\cos S}(\sqrt{\tfrac{3}{5}})\frac{\mathfrak{f}_3}{F_\pi} = \tfrac{1}{3}\sqrt{2}(\alpha_3 + \tfrac{1}{2}\beta_3 M_K) + \tfrac{1}{2}(\sqrt{\tfrac{1}{10}})\gamma_3 M_K,$$

$$-\frac{\cos D}{\cos S}(\sqrt{\tfrac{3}{5}})\frac{\mathfrak{f}_3}{2F_\pi} = \tfrac{1}{3}\sqrt{2}(\alpha_3 + \tfrac{1}{2}\beta_3 M_K) - \tfrac{1}{2}(\sqrt{\tfrac{1}{10}})\gamma_3 M_K,$$

$$-\frac{\cos D}{\cos S}(\sqrt{\tfrac{3}{5}})\frac{\mathfrak{f}_3}{6F_\pi} = \tfrac{1}{3}\sqrt{2}\alpha_3,$$

which yields

$$\alpha_3 = -\tfrac{1}{2}(\sqrt{\tfrac{3}{10}})\frac{\mathfrak{f}_3}{F_\pi}\frac{\cos D}{\cos S},$$

$$M_K\beta_3 = \tfrac{5}{2}(\sqrt{\tfrac{3}{10}})\frac{\mathfrak{f}_3}{F_\pi}\frac{\cos D}{\cos S}, \qquad (12)$$

$$M_K\gamma_3 = \frac{3}{2}\frac{(\sqrt{6})\mathfrak{f}_3}{F_\pi}\frac{\cos D}{\cos S}.$$

Bouchiat and Meyer compare these values with experiment and find generally good agreement, assuming that $(\cos D/\cos S) \simeq 1$.

IV. $K \to 3\pi$—CP-VIOLATING CONTRIBUTION

Things are not quite as simple in the case of the CP-violating Hamiltonian. We can make an isotopic-spin analysis as before:

$$\begin{aligned}
&\mathrm{Amp}^{(-)}(K^+ \to \pi^+\pi^0\pi^0) \\
&= -\tfrac{1}{3}\sqrt{2}i[(\alpha_1' - (\sqrt{\tfrac{1}{2}})\alpha_3') + (\beta_1' - (\sqrt{\tfrac{1}{2}})\beta_3')E_+] \\
&\qquad - i(\sqrt{\tfrac{1}{5}})\tfrac{3}{2}\gamma_3'(E_+ - \tfrac{1}{3}M_K), \\
&\mathrm{Amp}^{(-)}(K^+ \to \pi^+\pi^+\pi^-) \\
&= \tfrac{1}{3}\sqrt{2}i[(2\alpha_1' - \sqrt{2}\alpha_3') + (\beta_1' - (\sqrt{\tfrac{1}{2}})\beta_3')(M_K - E_-)] \\
&\qquad + i(\sqrt{\tfrac{1}{5}})\tfrac{3}{2}\gamma_3'(E_- - \tfrac{1}{3}M_K), \qquad (13) \\
&\mathrm{Amp}^{(-)}(K_+ \to \pi^+\pi^-\pi^0) \\
&= \tfrac{1}{3}\sqrt{2}i[(\alpha_1' + \sqrt{2}\alpha_3') + (\beta_1' + \sqrt{2}\beta_3')E_0], \\
&\mathrm{Amp}^{(-)}(K_- \to \pi^+\pi^-\pi^0) = i(\sqrt{\tfrac{1}{5}})\gamma_3'(E_- - E_+), \\
&\mathrm{Amp}^{(-)}(K_+ \to \pi^0\pi^0\pi^0) \\
&= -\tfrac{1}{3}\sqrt{2}i[(3\alpha_1' + 3\sqrt{2}\alpha_3') + (\beta_1' + \sqrt{2}\beta_3')M_K],
\end{aligned}$$

where α_i', β_i', γ_i' are again arbitrary real constants and the subscript i denotes that they are produced by the $\Delta I = \frac{1}{2}i$ part of $\mathcal{H}_w^-$. We may also, as before, take the zero-momentum limit for one of the final-state pions:

$$\langle \pi_q{}^a \pi_p{}^b \pi_s{}^c | \mathcal{H}_w^-(0) | K_k{}^n \rangle \underset{q\to 0}{\longrightarrow} -\frac{1}{F_\pi}\langle \pi_p{}^b \pi_s{}^c | [F_a{}^5(0), \mathcal{H}_w^-(0)] | K_k{}^n \rangle.$$

This time, however, $\mathcal{H}_w^-$ is antisymmetric under the interchange $V \leftrightarrow A$, so that replacement of $F_a{}^5(0)$ by $F_a(0)$ is not permissible. For this case, then, we must actually carry out the commutation, yielding some new current-current structure which is, in general, *not* the same as $\mathcal{H}_w^-$ (P-violating, $\Delta S = -1$). Thus we are unable to relate the unphysical $K \to 3\pi$ amplitude to the experimental CP-violating $K \to 2\pi$ amplitudes. Instead, we evaluate these $K \to 2\pi$ structures by means of the current algebraic techniques developed in A:

$$\begin{aligned}
&\langle \pi_p{}^b \pi_s{}^c | [F_a{}^5(0), \mathcal{H}_w^-(0)] | K_k{}^n \rangle \\
&\cong \frac{1}{2F_\pi{}^2}\langle 0 | \{[F_b{}^5(0), [F_c{}^5(0), [F_a{}^5(0), \mathcal{H}_w^-]]] \\
&\qquad + [F_c{}^5(0), [F_b{}^5(0), [F_a{}^5(0), \mathcal{H}_w^-(0)]]]\} | K_k{}^n \rangle. \qquad (14)
\end{aligned}$$

Since there are five parameters to be determined, we must take five different zero-momentum limits. This is done in Appendix II and it is shown that:

$$\begin{aligned}
&\alpha_1' = 7\zeta\Gamma_-, \qquad M_K\beta_1' = -(10/3)\zeta\Gamma_-, \\
&\alpha_3' = 4\sqrt{2}\zeta\Gamma_-, \quad M_K\beta_3' = -(22\sqrt{2}/3)\zeta\Gamma_-, \\
&\qquad\qquad\qquad M_K\gamma_3' = (\tfrac{4}{3}\sqrt{10})\zeta\Gamma_-, \qquad (15)
\end{aligned}$$

where

$$\zeta = -\frac{G_V}{2\sqrt{2}}\cos\theta\,\sin\theta\frac{3}{8F_\pi{}^3}\sin D,$$

$$\Gamma_- = \langle 0 | L_v | K^0 \rangle.$$

It is of interest to compare the relative magnitudes of corresponding CP-conserving and CP-violating quantities. We find

$$\begin{aligned}
&\frac{\alpha_1'}{\alpha_1} = -\frac{14}{\sqrt{3}}B, \qquad \frac{\beta_1'}{\beta_1} = -\frac{10}{3\sqrt{3}}B, \\
&\frac{\alpha_3'}{\alpha_3} = -\frac{16}{\sqrt{3}}\frac{1}{r}B, \quad \frac{\beta_3'}{\beta_3} = -\frac{88}{15\sqrt{3}}\frac{1}{r}B, \quad \frac{\gamma_3'}{\gamma_3} = \frac{8}{9\sqrt{3}}\frac{1}{r}B,
\end{aligned} \qquad (16)$$

where $B = (\zeta\Gamma_- F_\pi/\mathfrak{f}_1)\cos S/\cos D$ and $r = (\sqrt{\frac{1}{5}})\mathfrak{f}_3/\mathfrak{f}_1$ is a measure of the violation of the $\Delta I = \frac{1}{2}$ rule in $K \to 2\pi$ decay. If we treat the CP-conserving $K \to 3\pi$ decay in the same way as the CP-violating, using the soft-pion results of Table I, we must have $r = 0$. Instead, however, we employ the experimental $K \to 2\pi$ results, which yield[18]

$$1/40 \lesssim r \lesssim 1/30.$$

We note that, because of the smallness of r, the ratio of CP-violating to CP-conserving parameters is at least an order of magnitude greater for the $\Delta I = \frac{3}{2}$ quantities compared to those with $\Delta I = \frac{1}{2}$.

---

[18] We note that $r = a_3/a_1$ in the notation of Ref. 8. These authors suggest that the difference between $\mathrm{Re}\,r$ and $|r|$ gives a measure of uncertainty in the theoretical predictions, since it is connected with the unitarity phases previously neglected. They give $\mathrm{Re}\,r_{\mathrm{exp}} = 0.025 \pm 0.01$ and $|r|_{\mathrm{exp}} = 0.031 \pm 0.001$. The values given in the text are just the mean values cited above.

In order to evaluate these quantities numerically, we need to know the value of B. If we make the approximation that $\mathfrak{f}_1$ is replaced by its soft-pion value[19]

$$\mathfrak{f}_1=(\surd\tfrac{1}{2})A_0=+\tfrac{1}{4}\sqrt{3}\frac{\Gamma_+}{F_\pi^2}\frac{G_V}{2\sqrt{2}}\cos\theta\sin\theta\cos S$$

then

$$B=-\tfrac{1}{2}\sqrt{3}t\tan D. \qquad (17a)$$

If we wish to go still further, we may employ the $SU(3)$ sum rules[20] and the convergent intermediate vector-boson model of Glashow, Schnitzer, and Weinberg[21] for t, which yields

$$t=-2.0. \qquad (17b)$$

These numerical results will be of interest in Sec. V.

V. EXPERIMENTAL CHECKS OF *CP* NONCONSERVATION

We now consider various experimental checks of possible CP noninvariance in $K\to3\pi$ decay in order to decide which of these have the best chance of finding an effect and to set an upper limit on the phase angle D from present experimental results. In order to do so, however, since several of these tests involve interference, we need now to patch onto our framework the strong-interaction phase shifts demanded by unitarity.[22] We follow the procedure of Barrett and Truong.[23] These authors separate the $K\to3\pi$ amplitude in the linear approximation into three portions—an $I=1$ component completely symmetric under permutation of the space-isospin indices of the pions, an $I=1$ component of mixed symmetry under interchange, and an $I=2$ component of mixed symmetry under interchange.[24] Assuming the dominant effect in the final-state interaction to be π-π s-wave scattering in the $I=0$ state, they argue that only the average strong-interaction phase shift δ associated with the completely symmetric $I=1$ state should be appreciable. Using the Khuri-Treiman equations, they estimate

$$\tan\delta\simeq\sin\delta\simeq a_0m_\pi,$$

where a_0 is the $I=0$ π-π scatteringlength. In accordance with their approach, then, we propose to append the phase factor $\exp(i\delta)$ to the symmetric $I=1$ part of the decay amplitude, but leave the remainder unchanged.

We define (in agreement with the notation of Barrett and Truong)

$$\begin{aligned}
\lambda e^{i\varphi_\lambda}&=\tfrac{1}{3}\sqrt{2}[(\alpha_{1c}-(\surd\tfrac{1}{2})\alpha_{3c})+\tfrac{1}{3}M_K(\beta_{1c}-(\surd\tfrac{1}{2})\beta_{3c})],\\
be^{i\varphi_b}&=\tfrac{1}{6}\sqrt{2}\frac{m_\pi^2}{M_K}(\beta_{1c}-(\surd\tfrac{1}{2})\beta_{3c}),\\
ce^{i\varphi_c}&=-\tfrac{3}{4}(\surd\tfrac{1}{5})\frac{m_\pi^2}{M_K}\gamma_{3c},\\
\tilde{\lambda}e^{i\tilde{\varphi}_\lambda}&=\tfrac{1}{3}\sqrt{2}[(\alpha_{1c}+\sqrt{2}\alpha_{3c})+\tfrac{1}{3}M_K(\beta_{1c}+\sqrt{2}\beta_{3c})],\\
\tilde{b}e^{i\tilde{\varphi}_b}&=\tfrac{1}{6}\sqrt{2}\frac{m_\pi^2}{M_K}(\beta_{1c}+\sqrt{2}\beta_{3c}),\\
\tilde{c}e^{i\tilde{\varphi}_c}&=-\tfrac{1}{2}(\surd\tfrac{3}{5})\frac{m_\pi^2}{M_K}\gamma_{3c},
\end{aligned} \qquad (18)$$

where $\alpha_{1c}\equiv\alpha_1+i\alpha_1'$, etc.

One test for possible CP nonconservation is to look for a difference in the rates of $K^\pm\to\pi^\pm\pi^+\pi^-$ ($\tau^\pm$) and $K^\pm\to\pi^\pm\pi^0\pi^0$ ($\tau'^\pm$) decay. CPT invariance demands (apart from small electromagnetic effects) that[25]

$$\Gamma(\tau^+)+\Gamma(\tau'^+)=\Gamma(\tau^-)+\Gamma(\tau'^-).$$

However, CPT is not sufficient to ensure the equalities

$$\Gamma(\tau^+)=\Gamma(\tau^-),\quad \Gamma(\tau'^+)=\Gamma(\tau'^-),$$

unless the $\tau^\pm$ and $\tau'^\pm$ states are not connected to each other via the strong interaction S matrix;[22] these equalities follow from CP invariance, and thus provide a possible test. We define the CP-violating quantities

$$\Delta\Gamma(\tau)=\frac{\Gamma(\tau^+)-\Gamma(\tau^-)}{\Gamma(\tau^+)+\Gamma(\tau^-)},\quad \Delta\Gamma(\tau')=\frac{\Gamma(\tau'^+)-\Gamma(\tau'^-)}{\Gamma(\tau'^+)+\Gamma(\tau'^-)}. \qquad (19)$$

In Glashow's model these must vanish, even though CP is violated, since a nonzero value requires interference between the symmetric part and mixed symmetry part of the decay amplitude.[26] But these terms vanish in the linear approximation when integrated over the entire Dalitz plot.[27] Within the linear approximation, nonzero $\Delta\Gamma(\tau)$ or $\Delta\Gamma(\tau')$ are produced by a model of CP violation which includes a $\Delta I=\frac{5}{2}$ or $\frac{7}{2}$ portion leading to an

[19] As mentioned before, this approximation is apparently inconsistent with a nonzero value for r. However, any numerical inaccuracy which arises at this point is probably small compared to, and may be absorbed into, the uncertainty involved in the subsequent calculation of t, which is done in the soft-kaon limit.

[20] S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 139 (1967).

[21] S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev Letters **19**, 205 (1967).

[22] Y. Ueda and S. Okubo, Phys. Rev. **139**, B1591 (1965).

[23] B. Barrett and T. Truong, Phys. Rev. Letters **17**, 880 (1966). We emphasize that we are using the procedure of Barrett and Truong purely for illustrative purposes. It appears to us that even if $I=0$ s-wave scattering does dominate, the phase shift δ_M could be of the same order of magnitude as δ_s. [See, for example, B. Ya'Zeldovich, Soviet J. Nucl. Phys. **6**, 611 (1968)]. Thus, in our theoretical expressions involving δ (Eqs. 20,25) the δ's are not necessarily the same, but should be of the same order of magnitude.

[24] Barrett and Truong also included a completely symmetric $I=3$ state, which we neglect since the Glashow model contains only $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ components.

[25] T. D. Lee and C. S. Wu, Ann. Rev. Nuc. Sci. **16**, 471 (1966).

[26] Or between the mixed-symmetry components. But this term is small and also vanishes when integrated over the Dalitz plot.

[27] This is not quite true for $\Delta\Gamma(\tau')$ in certain parametrizations. We are using the $SU(2)$ limit in the amplitude (not in the phase space, however) so that $(S_0-S_3)\propto(E_3-\frac{1}{3}M_K)$ which vanishes when integrated over the Dalitz plot. If one breaks this symmetry by giving the pions their physical masses, then in τ' decay an integral of S_0-S_3 over the Dalitz plot is nonzero, as discussed by T. Devlin, Phys. Rev. Letters **20**, 683 (1968).

$I=3$ final state.[22] Recent experiments have indeed shown $\Delta\Gamma(\tau)$ to be zero to within 1 part in 1000. They give

$\Delta\Gamma(\tau)=-0.0002\pm0.001$, Ford *et al.* (Ref. 28)

$\Delta\Gamma(\tau)=-0.0025\pm0.005$, Fletcher *et al.* (Ref. 29).

A practical test for charged K decays in our model is a comparison of slopes.[30] We may define the CP-violating quantities

$$\Delta(\tau)=\frac{a(\tau^+)-a(\tau^-)}{a(\tau^+)+a(\tau^-)}=\tan\delta\frac{b\sin(\varphi_b-\varphi_\lambda)+c\sin(\varphi_c-\varphi_\lambda)}{b\cos(\varphi_b-\varphi_\lambda)+c\cos(\varphi_c-\varphi_\lambda)},$$
$$\Delta(\tau')=\frac{a(\tau'^+)-a(\tau'^-)}{a(\tau'^+)+a(\tau'^-)}=\tan\delta\frac{b\sin(\varphi_b-\varphi_\lambda)-c\sin(\varphi_c-\varphi_\lambda)}{b\cos(\varphi_b-\varphi_\lambda)-c\cos(\varphi_c-\varphi_\lambda)}. \tag{20}$$

In terms of our parameters, we find

$$\Delta(\tau)=-2(\sqrt{\tfrac{1}{3}})B\left(\frac{1}{1-8r}-\frac{13}{1+r}\right)\tan\delta,$$
$$\Delta(\tau')=2(\sqrt{\tfrac{1}{3}})B\left(\frac{5}{1+\frac{11}{2}r}+\frac{13}{1+r}\right)\tan\delta. \tag{21}$$

If we use the value for B given in Eq. (17) and the experimental result $r=1/30$, we have

$$\Delta(\tau)=+22.5\tan\delta\tan D,$$
$$\Delta(\tau')=-33.6\tan\delta\tan D. \tag{22}$$

Experimentally the situation has been summarized by Bell and Steinberger, who give[31]

$$2Fa(\tau^-)=0.23\pm0.04,\quad 2Fa(\tau^+)=0.22\pm0.03,$$

where $F=2M_KQ/3m_\pi^2$. From this we find

$$\Delta(\tau)_{\text{exp}}=-0.02\pm0.11. \tag{23}$$

Comparing this with the theoretical expression, we see that

$$\tan\delta\tan D=0.001\pm0.005.$$

Of interest are the factors -22.4, 33.6 multiplying $\tan\delta\tan D$ in Eq. (22). These rather large "enhancement" factors are the reason why a 10% experiment in the slope can limit $\tan\delta\tan D$ to the 1% level. Such factors result from the relatively large values of ϕ_λ, ϕ_b, ϕ_c

$$\tan\varphi_\lambda=-\frac{26}{\sqrt{3}}B\frac{1}{1+r},$$
$$\tan\varphi_b=\frac{4}{\sqrt{3}}B\frac{1}{1-(5/4)r}, \tag{24}$$
$$\tan\varphi_c=\frac{8}{9\sqrt{3}}\frac{1}{r}B,$$

which in turn are due to the sizable coefficients (e.g., 7, $22\sqrt{2}/3$) appearing in the expressions for the CP-violating parameters of Eq. (13).

For the neutral K mesons, K_L and K_S, additional types of tests for CP noninvariance become available. One possibility is a charge-asymmetry experiment, wherein one compares the number of events in the left- and right-hand sides of the Dalitz plot.[32] Barrett and Truong defined

$$\Delta N(K_L)=\frac{N_R-N_L}{N_R+N_L}=\frac{8}{3\pi}F\frac{\bar{c}\sin\bar{\varphi}_c}{\bar{\lambda}\cos\bar{\varphi}_\lambda}\sin\delta, \tag{25}$$

where F is the same as defined above and N_R (N_L) denotes the total number of events in the right- (left-) hand side of the Dalitz plot. In terms of our parameters, we find

$$\Delta N(K_L)=+\frac{8}{3\pi}F\times12\frac{m_\pi^2}{M_K^2}\frac{B}{1-2r}\sin\delta. \tag{26}$$

Using the value for B given by Eq. (17) and $r=1/30$ yields

$$\Delta N(K_L)=2.1\sin\delta\tan D. \tag{27}$$

Experimentally, the situation was recently summarized by Rubbia and Okun[33] who report

$N_R/N_L-1=(0\pm5)\times10^{-2}$, Nefkens *et al.* (Ref. 34)

$N_R/N_L-1=(-4\pm1.5)\times10^{-2}$, Hopkins *et al.* (Ref. 35).

Because of the experimental uncertainty mentioned in Ref. 35, we use Nefkens's result, whereby

$$\sin\delta\tan D=0.00\pm0.01. \tag{28}$$

For the neutral K complex, one might also contemplate interference experiments, employing either the

[28] W. T. Ford *et al.*, Phys. Rev. Letters **18**, 1214 (1967).
[29] C. R. Fletcher *et al.*, Phys. Rev. Letters **19**, 98 (1967).
[30] To be definite, we define our slopes by

$$|M|^2\propto1-a(2/m_\pi^2)(S_3-S_0)+O((S_3-S_0)^2),$$

where $S_3=(k-q_3)^2$ and $S_0=\frac{1}{3}(S_1+S_2+S_3)$.
[31] J. S. Bell and J. Steinberger, *Proceedings of the Oxford International Conference on Elementary Particles, 1965* (Rutherford High-Energy Laboratory, Chilton, Berkshire, England, 1966), p. 195.
[32] Our Dalitz plot is oriented so that on the right-hand side $T_{\pi^+}>T_{\pi^-}$, while on the left-hand side $T_{\pi^-}>T_{\pi^+}$.
[33] C. Rubbia and L. Okun, *Proceedings of the Heidelberg Conference on Elementary Particles, 1967*, edited by H. Filthuth (Interscience Publishers, Inc., New York, 1968). p. 301.
[34] B. M. K. Nefkens *et al.*, Phys. Rev. **157**, 1233 (1967).
[35] H. W. K. Hopkins *et al.*, Phys. Rev. Letters **19**, 185 (1967). Although there seems to be a positive effect here, the authors do not propose to see any CP violation and seem to believe rather that there exists some experimental bias.

$\pi^+\pi^-\pi^0$ or the $3\pi^0$ mode. It has been shown by Sehgal and Wolfenstein[36] that observation of such an interference effect between K_L and K_S decay is clear evidence for CP violation. These authors show that if one starts with the coherent mixture of K_L and K_S

$$N(|K_L\rangle + R|K_S\rangle),$$

then the partial decay rate into a set a of final states is given by

$$I_a(t) = N^2[\gamma_{La}e^{-\gamma_L t} + \gamma_{Sa}e^{-\gamma_S t}|R|^2 + 2(\gamma_{La}\gamma_{Sa})^{1/2}\,\mathrm{Re}(RV_a e^{-i\Delta m t})e^{-\frac{1}{2}(\gamma_L+\gamma_S)t}],$$

where $\Delta m = m_S - m_L$ is the mass difference, γ_L and γ_S are the widths of K_L and K_S, and

$$\gamma_{La} = \sum_\alpha |\langle\alpha|T|K_L\rangle|^2, \quad \gamma_{Sa} = \sum_\alpha |\langle\alpha|T|K_S\rangle|^2,$$

$$V_a = \sum_\alpha \langle\alpha|T|K_L\rangle^* \langle\alpha|T|K_S\rangle/(\gamma_{La}\gamma_{Sa})^{1/2},$$

where $|\alpha\rangle$ denotes a final state, completely specified by, say, all polarizations and momenta, while the sum is over all states in the set a.

V_a defines the possible interference effect in that if $|V_a| = 1$, as in the decay $K \to 2\pi$, complete interference is possible and can be obtained at $t=0$ by using a beam with $|R|^2 = \gamma_{La}/\gamma_{Sa}$. Now $|R|^2 \ll 1$ may be produced regeneratively, while $|R| \lesssim 1$ can be obtained near the source of a pure K^0 or $\bar{K}^0$ beam. Thus, in order to produce a sizable interference effect, we demand that

$$\gamma_{Sa}/\gamma_{La} > 1.$$

Now intuitively we suspect that $\gamma_S/\gamma_L \ll 1$ for the $K \to 3\pi$ decay mode, so that a sizable interference would not be seen. The experiment has actually been done, and on the basis of less than a hundred events, a Carnegie-Tech–Brookhaven collaboration[37] indeed sees no evidence for interference, but their rather large statistical limits actually are quite restrictive as to the allowed value for D. They define

$$A(K_S \to \pi^+\pi^-\pi^0)/A(K_L \to \pi^+\pi^-\pi^0) = x + iy$$

and find

$$x = 0.14 \pm 0.32, \quad y = 0.33_{-0.67}^{+0.55}.$$

In our notation, approximating the amplitudes by their value at the center of the Dalitz, plot we have

$$\frac{A(K_S \to \pi^+\pi^-\pi^0)}{A(K_L \to \pi^+\pi^-\pi^0)} \cong \frac{i\bar{\lambda}\sin\bar{\varphi}_\lambda + \Upsilon\bar{\lambda}\cos\bar{\varphi}_\lambda}{\bar{\lambda}\cos\bar{\varphi}_\lambda + i\Upsilon\bar{\lambda}\sin\bar{\varphi}_\lambda},$$
$$= \frac{i\tan\bar{\varphi}_\lambda + \Upsilon}{1 + i\Upsilon\tan\bar{\varphi}_\lambda}, \tag{29}$$

where Υ is the mixing parameter in our phase convention such that

$$|K_L\rangle \cong |K_-\rangle + \Upsilon|K_+\rangle,$$
$$|K_S\rangle \cong |K_+\rangle + \Upsilon|K_-\rangle.$$

We have[38]

$$\Upsilon = -\frac{\Gamma_{+-} + i2M_{+-}}{\Delta\gamma + i2\Delta m} \cong -(\sqrt{\tfrac{1}{2}})e^{i\delta_S}\left(\frac{m'}{\Delta m} + i\alpha\right)$$
$$\text{with} \quad \delta_S = \tan^{-1}\left(-\frac{2\Delta m}{\Delta\gamma}\right) \cong \tfrac{1}{4}\pi, \tag{30}$$

where M, Γ are the usual Hermitian matrices describing the K^0, $\bar{K}^0$ system, $\Delta m = m_S - m_L$ is the mass difference, $\Delta\gamma = \gamma_S - \gamma_L$ is the difference in widths, α is the parameter defined in Eq, 4. and $m' = i\langle K_-|M|K_+\rangle = iM_{-+}$. Values for α and m' were deduced from the published results for $K_L \to 2\pi$ and are listed in Table I of A.[39] We see that, using these values $|\Upsilon| \sim 10^{-3}$.

On the other hand, we find

$$\tan\bar{\varphi}_\lambda = -\frac{54}{\sqrt{3}}\frac{B}{1-2r}. \tag{31}$$

Using the value of B from Eq. (17) and $r = 1/30$, we find

$$\tan\bar{\varphi}_\lambda \cong -58\tan D. \tag{32}$$

We see once more a sizable enhancement factor 57.5, and it is this which enables us to place a fairly strong limit on $\tan D$. We have

$$x \cong \mathrm{Re}\Upsilon, \quad y \cong \mathrm{Im}\Upsilon + \tan\bar{\varphi}_\lambda.$$

Since $|\Upsilon| \sim 10^{-3}$ we neglect it compared to $\tan\bar{\varphi}_\lambda$, which yields, using the quoted experimental results,

$$\tan D = 0.006_{-0.009}^{+0.012}. \tag{33}$$

We have discussed measurement of possible CP violation employing the $\tau^\pm, \tau'^\pm$ modes alone and the neutral decay modes alone. We may also gain such information by combining both sets of data. In this approach one makes an isospin analysis and compares predicted total decay rates with experiment. This method was first employed by Cabibbo[40] and Gaillard,[41] who, however, assumed the $\Delta I = \frac{1}{2}$ rule. This has been recently extended by Barshay and Devlin[42] and Gaillard[43] to include $\Delta I > \frac{1}{2}$. One *can* compare τ rates with τ' rates or $K_L \to \pi^+\pi^-\pi^0$ with $K_L \to \pi^0\pi^0\pi^0$. From

[36] L. M. Sehgal and L. Wolfenstein, Phys. Rev. **162**, 1362 (1967).

[37] D. G. Hill *et al.*, in *Thirteenth International Conference on High Energy Physics, 1966* (University of California Press, Berkeley, 1967), paper 4a.12, p. 63.

[38] This is derived in the $K^0, \bar{K}^0$ representation by T. D. Lee and C. S. Wu, Ann. Rev. Nuc. Sci., **16**, 471 (1966).

[39] Table I in A was derived for the case $\phi = \psi$ in the "standard" phase convention, in which case, with respect to parity violating decays, the two conventions are identical.

[40] N. Cabibbo, in *Symmetries in Elementary Particle Physics*, edited by A. Zichichi (Academic Press Inc., New York, 1965).

[41] M. Gaillard, Nuovo Cimento **35**, 1225 (1965).

[42] S. Barshay and T. Devlin, Phys. Rev. Letters **19**, 881 (1967).

[43] M. Gaillard, Nuovo Cimento **52**, A359 (1968).

TABLE II. A summary of *CP*-violating predictions and results.

CP-violating quantity	Theoretical expression	Experimental limit	Limit on tan*D*
$\Delta(\tau)$	$+22.5 \tan\delta \tan D$	-0.02 ± 0.11	$-(0.001\pm0.005)\cot\delta$
$\Delta(\tau')$	$-33.6 \tan\delta \tan D$	...	...
$\Delta N(K_L)$	$2.1 \sin\delta \tan D$	0.00 ± 0.025	$(0.00 \pm0.01)\csc\delta$
$\tan\tilde{\phi}_\lambda$	$-58.0 \tan D$	$0.33_{-0.67}^{+0.55}$	$-0.006_{-0.009}^{+0.012}$
$0.856(2\Gamma_{+00}/\Gamma_{+-0})-1$	$640 \tan^2 D$	<0.09	<0.012
Phase angle in $n\,\beta$ decay	$D+S$	<0.03	$\sim<0.03$

Eq. (6) and Eq. (13), we see that[44]

$$\tfrac{1}{4}\Gamma_{++-}/\Gamma_{+00}=1\,,\quad \tfrac{3}{2}\Gamma_{+-0}/\Gamma_{000}=1\,, \tag{34}$$

if there is no contribution from an $I=3$ final state. But the absence of such effects has already been indicated by measurement of τ^+,τ^- decay rates (unless the final state interactions are very weak). And indeed experimentally the agreement is quite good[45]:

$$\tfrac{1}{4}\Gamma_{++-}/\Gamma_{+00}=1.00\pm0.03\,,\quad \tfrac{3}{2}\Gamma_{+-0}/\Gamma_{000}=0.97\pm0.10\,.$$

When, however, we compare charged and neutral K decays, we find[42]

$$\frac{(\bar{\lambda}\cos\tilde{\phi}_\lambda)^2}{|\lambda|^2}=\frac{\Gamma_{+-0}+\Gamma_{000}}{\Gamma_{++-}+\Gamma_{+00}}=\frac{1}{2}\frac{\Gamma_{+-0}}{\Gamma_{+00}}\,. \tag{35}$$

These ratios would be unity in the absence of CP violation and if the $\Delta I=\frac{1}{2}$ rule were valid. Instead, experimentally, Barshay and Devlin give[46]

$$\tfrac{1}{2}\Gamma_{+-0}/\Gamma_{+00}=0.816\pm0.034\,,$$
$$(\Gamma_{+-0}+\Gamma_{000})/(\Gamma_{++-}+\Gamma_{+00})=0.825\pm0.031\,.$$

In terms of our parameters, we have

$$\frac{(\bar{\lambda}\cos\tilde{\phi}_\lambda)^2}{|\lambda|^2}=\frac{(1-2r)^2}{(1+r)^2+[(26/\sqrt{3})B]^2}\,. \tag{36}$$

Using the numerical value for B in Eq. (17) and $r=1/30$ yields

$$\frac{(\bar{\lambda}\cos\tilde{\phi}_\lambda)^2}{|\lambda|^2}=0.815\frac{1}{1+630\tan^2 D}\,. \tag{37}$$

If, referring to the experimental results, we demand that this quantity be >0.785, then we find

$$|\tan D|<0.008\,. \tag{38}$$

A somewhat larger value may be tolerated because of the uncertainty in r mentioned in Ref. 18. With $r=1/40$ we have

$$\frac{(\bar{\lambda}\cos\tilde{\phi}_\lambda)^2}{|\lambda|^2}=0.856\frac{1}{1+690\tan^2 D}\,. \tag{39}$$

This case is still consistent with no CP violation, but now

$$|\tan D|<0.012\,. \tag{40}$$

We again note the large enhancement factors (630 and 640) which enable us to restrict D to the 1% level or less.

In Table II we have summarized the *CP*-violating quantities and the limits present experiments have placed on D. We see that all upper limits are at about the level of one part in a hundred.[47] An interesting feature is the large enhancement factors present which enables the rather statistically crude (~10%) experiments discussed above to place the same upper limit on D as done by the elegant nuclear β-decay experiments[48] which have given an upper limit of $|\phi|<0.03$. And, from another point of view, these large factors give hope that if and when the $K\to3\pi$ experiments are improved either CP violation will be found or a rather strong limit on D at around the 0.001 level will be given. In fact, there exist theoretical reasons, based on rough calculations of the mixing parameter m', to suggest that $\tan D$ may be not greater than several parts in a thousand,[49] a small value to be sure, but, with the enhancement factors given, possibly large enough for future generation experiments to reveal.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge many very helpful discussions, especially with respect to the "natural" phase convention, with Professor L. Wolfenstein.

[44] These predictions are of course with the standard phase-space corrections already made. In the following we estimate the rate by taking the absolute square of the symmetric $I=1$ part only, which is correct up to possible quadratic terms.

[45] We use the values given in Ref. 42 and standard phase space factors. With corrected phase space factors given by T. Devlin, Phys. Rev. Letters **20**, 683 (1968) the neutral K decay result becomes $\frac{3}{2}(\Gamma_{+-0}/\Gamma_{000})=0.91\pm0.10$, which is still in agreement with the no $\Delta I>\frac{3}{2}$ rule.

[46] The first of these quantities is actually a corrected value given by T. Devlin, Phys. Rev. Letters **20**, 683 (1968).

[47] Actually $\Delta(\tau)$ and $\Delta N(K_L)$ yield limits on $\tan\delta\tan D$ and $\sin\delta\tan D$. If, however, we use the result of Barrett and Truong that $\tan\delta\simeq\sin\delta\simeq a_0 m_\pi$, then it is probable that a_0 lies somewhere between $0.2/m_\pi$ which is Weinberg's value and $0.5/m_\pi$ which was assumed by Barrett and Truong. In either case the upper limit for $\tan D$ is still at the 1% level.

[48] M. Burgy *et al.*, Phys. Rev. Letters **1**, 324 (1958); F. Calaprice *et al.*, *ibid.* **18**, 918 (1967).

[49] L. Wolfenstein, Proceedings of the Erice Summer School, 1968 (to be published).

APPENDIX A.

We have

$$[F_a{}^5,(\{A_1,A_2\}\pm\{V_1,V_2\})]$$
$$=(\{[F_a{}^5,A_1],A_2\}+\{A_1,[F_a{}^5,A_2]\})$$
$$\pm(\{[F_a{}^5,V_1],V_2\}+\{V_1,[F_a{}^5,V_2]\})$$
$$=(\{[F_a,V_1],A_2\}\pm\{V_1,[F_a,A_2]\})$$
$$\pm(\{[F_a,A_1],V_2\}\pm\{A_1,[F_a,V_2]\}), \quad (41)$$

where we have used the result that[50]

$$[F_a{}^5,V_i]=[F_a,A_i] \quad \text{and} \quad [F_a{}^5,A_i]=[F_a,V_i].$$

If we choose the plus sign in the above, we have

$$[F_a{}^5(\{A_1,A_2\}+\{V_1,V_2\})]=[F_a,(\{V_1,A_2\}+\{A_1,V_2\})].$$

We note that the two operators in parentheses have the same symmetry under the interchange $V\leftrightarrow A$. Continuing this operation, we find

$$\langle\pi_q{}^a\pi_p{}^b\pi_s{}^c|M(0)|K_k{}^n\rangle$$
$$\underset{q\to 0}{\longrightarrow} -\frac{1}{2F_\pi{}^3}\langle 0|\{[F_b(0),[F_c(0),[F_a(0),M'(0)]]]$$
$$+[F_c(0),[F_b(0),[F_a(0),M'(0)]]]\}|K_k{}^n\rangle, \quad (42)$$

where

$$M=\{A_1,A_2\}+\{V_1,V_2\} \quad \text{and} \quad M'=\{V_1,A_2\}+\{A_1,V_2\}.$$

Now allowing the F's to operate to the left and right respectively, as done in Sec. II, yields

$$\langle\pi_q{}^a\pi_p{}^b\pi_s{}^c|M(0)|K_k{}^n\rangle$$
$$\underset{q\to 0}{\longrightarrow}\frac{1}{16F_\pi{}^3}(\tau^a\tau^b\tau^c+\tau^a\tau^c\tau^b)_{mn}\langle 0|M'(0)|K_k{}^m\rangle$$
$$=\frac{1}{8F_\pi{}^3}[\tau^a\delta^{bc}]_{mn}\langle 0|M'(0)|K_k{}^m\rangle. \quad (43)$$

Thus, for M, the isotopic spin structure of the $K\to 3\pi$ decay amplitude is clearly seen to consist only of $I=1$. It is also seen that one cannot carry through this argument for $L=\{A_1,A_2\}-\{V_1,V_2\}$.

APPENDIX B.

Suppose we define

$$\tfrac{1}{2}\Gamma_\pm=\langle 0|\{A_{\pi^{+\nu}},V_{K^{-\nu}}\}\pm\{V_{\pi^{+\nu}},A_{K^{-\nu}}\}|K^0\rangle.$$

Then by using the result

$$\langle\pi_q{}^a\pi_p{}^b\pi_s{}^c|\mathcal{H}_w{}^-(0)|K_k{}^n\rangle$$
$$\underset{q\to 0}{\longrightarrow} -\frac{1}{2F_\pi{}^3}\langle 0|[F_b{}^5(0),[F_c{}^5(0),[F_a{}^5(0),\mathcal{H}_w{}^-(0)]]]$$
$$+[F_c{}^5(0),[F_b{}^5(0),[F_a{}^5(0),\mathcal{H}_w{}^-(0)]]]|K^n\rangle \quad (44)$$

[50] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

we have the result

$$\text{Amp}^{(-)}(K^0\to\pi_q{}^+\pi_p{}^-\pi_s{}^0)\underset{q\to 0}{\longrightarrow} -i\frac{G_V}{2\sqrt{2}}\cos\theta\sin\theta\frac{\Gamma_-}{F_\pi{}^3}\sin D$$
$$=i\tfrac{1}{3}[(\alpha_1'+\sqrt{2}\alpha_3')+\tfrac{1}{2}(\beta_1'+\sqrt{2}\beta_3')M_K]$$
$$+\tfrac{1}{2}i(\surd\tfrac{1}{10})\gamma_3'M_K$$
$$\underset{p\to 0}{\longrightarrow} -i\frac{G_V}{2\sqrt{2}}\cos\theta\sin\theta\frac{\Gamma_-}{2F_\pi{}^3}\sin D$$
$$=i\tfrac{1}{3}[(\alpha_1'+\sqrt{2}\alpha_3')+\tfrac{1}{2}(\beta_1'+\sqrt{2}\beta_3')M_K]$$
$$-\tfrac{1}{2}i(\surd\tfrac{1}{10})\gamma_3'M_K$$
$$\underset{s\to 0}{\longrightarrow} -i\frac{G_V}{2\sqrt{2}}\cos\theta\sin\theta\frac{15}{8}\frac{\Gamma_-}{F_\pi{}^3}\sin D$$
$$=i\tfrac{1}{3}(\alpha_1'+\sqrt{2}\alpha_3'). \quad (45)$$

From this we find

$$\alpha_1'+\sqrt{2}\alpha_3'=-\frac{G_V}{2\sqrt{2}}\cos\theta\sin\theta\frac{45}{8}\frac{\Gamma_-}{F_\pi{}^3}\sin D,$$
$$M_K\gamma_3'=-\frac{G_V}{2\sqrt{2}}\cos\theta\sin\theta(\surd\tfrac{5}{2})\frac{\Gamma_-}{F_\pi{}^3}\sin D,$$
$$M_K(\beta_1'+\sqrt{2}\beta_3')=+\frac{G_V}{2\sqrt{2}}\cos\theta\sin\theta\frac{27}{4}\frac{\Gamma_-}{F_\pi{}^3}\sin D. \quad (46)$$

To proceed further, we must extract information about the charged kaon decays. Considering $K^+\to\pi^+\pi^0\pi^0$ we have

$$\text{Amp}^{(-)}(K^+\to\pi_q{}^+\pi_p{}^0\pi_s{}^0)\underset{q\to 0}{\longrightarrow} i\frac{G_V}{2\sqrt{2}}\cos\theta\sin\theta\frac{1}{4\sqrt{2}}\frac{\Gamma_-}{F_\pi{}^3}\sin D$$
$$=-\tfrac{1}{3}i\sqrt{2}(\alpha_1'-(\surd\tfrac{1}{2})\alpha_3')+i(\tfrac{1}{2}\surd\tfrac{1}{5})\gamma_3'M_K$$
$$\underset{p\to 0}{\longrightarrow} i\frac{G_V}{2\sqrt{2}}\cos\theta\sin\theta\frac{3}{2\sqrt{2}}\frac{\Gamma_-}{F_\pi{}^3}\sin D$$
$$=-\tfrac{1}{3}i\sqrt{2}[(\alpha_1'-(\surd\tfrac{1}{2})\alpha_3')+\tfrac{1}{2}(\beta_1'-(\surd\tfrac{1}{2})\beta_3')M_K]$$
$$-i(\surd\tfrac{1}{5})\tfrac{1}{4}\gamma_3'M_K. \quad (47)$$

From this we find

$$(\alpha_1'-\surd\tfrac{1}{2}\alpha_3')=-\frac{G_V}{2\sqrt{2}}\cos\theta\sin\theta\frac{9}{8}\frac{\Gamma_-}{F_\pi{}^3}\sin D,$$
$$M_K(\beta_1'-\surd\tfrac{1}{2}\beta_3')=-\frac{G_V}{2\sqrt{2}}\cos\theta\sin\theta\frac{3}{2}\frac{\Gamma_-}{F_\pi{}^3}\sin D. \quad (48)$$

Combining the two sets of results, we find

$$\alpha_1'=7\zeta\Gamma_-, \qquad M_K\beta_1'=-(10/3)\zeta\Gamma_-,$$
$$\alpha_3'=4\sqrt{2}\zeta\Gamma_-, \qquad M_K\beta_3'=-(22\sqrt{2}/3)\zeta\Gamma_-,$$
$$M_K\gamma_3'=(\tfrac{4}{3}\surd 10)\zeta\Gamma_-, \quad (49)$$

where

$$\zeta=-\frac{G_V}{2\sqrt{2}}\cos\theta\sin\theta\frac{3}{8}\frac{\sin D}{F_\pi{}^3}.$$