Regge Parametrization of Photoproduction Processes. I*†

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(Received 25 July 1968)

In this first of a series of articles on photoproduction processes, we write down the formulas for all the observables for π- and K-meson photoproduction processes in terms of the Regge parameters for the trajectories that can be exchanged in the t channel. Special consideration is given to the simplification of these formulas when polarized photon beams are used and special choices of the polarization angle are chosen. We are careful to take into account all the conspiracy relations and kinematic factors.

I. INTRODUCTION

IN the past two years there have been several attempts to fit the high-energy experimental data on π- and K-meson photoproduction in terms of the parameters of the dominant Regge trajectories that are exchanged in the t channel.[1–3] Reasonable fits for π^0 photoproduction were obtained by considering π and B exchange.[1] The forward peak in π^+ photoproduction was explained by introducing a conspirator trajectory which when combined with pion trajectory exchange fit the data reasonably well with only a few parameters.[2,3] A more ambiguous fit of forward K^+ photoproduction data was made by assuming a conspiracy between K and K_c in addition to K^{**} exchange.[2] Soon data will be available on K^0 photoproduction which in the Regge model gets contributions only from K^* exchange, making it the simplest of all the above processes to parametrize. The simple qualitative predictions of the Regge model—such as the vanishing of vector-meson exchange at the nonsense value of $\alpha_V=0$ which should occur in the physical region for the above reactions, and the expected increase in the rate of angular fall off of $d\sigma/dt$ with increasing s—have not yet been clearly substantiated in these experiments. These effects (if present) could be obscured in the former case by the existence of another trajectory with a nonnegligible contribution at that value of t (i.e., the B meson) and in the latter case by having several trajectories contribute in such a way as to obscure this effect. The above two qualitative predictions should become more striking when experiments are done using linearly polarized photons. The author has shown that using linearly polarized photons in the above reactions allows one to isolate the effects of trajectories of a given parity.[4,5] Thus in π^0 photoproduction the B contribution can be eliminated and the expected dip at the nonsense value $\alpha_\omega=0$ can be investigated. If the K^* trajectory is similar to the ω and ρ trajectories, as is indicated in charge-exchange scattering experiments, then K^0 photoproduction should also show an unambiguous dip in $d\sigma/dt$ at $\alpha_{K^*}=0$ even for unpolarized photons. The use of polarized photons should enable us to isolate the π and K trajectory contributions to π^+ and K^+ photoproduction unambiguously so that the simple qualitative $s^{2(\alpha-1)}$ behavior of $d\sigma/dt$ can be checked.

These qualitative predictions must be tested if we are to have any faith in the Regge parametrization. Given the number of adjustable parameters in a several Regge pole fit of the differential cross section, it is hard to have too much confidence in the parameters obtained in these previous fits. However, polarized photon cross sections at certain angles depend on fewer Regge parameters and outgoing nucleon polarization formulas are very sensitive to the relative importance of various trajectories and the explicit t dependence of the trajectories, especially where they cross the axis. Thus, we think it very useful at this time to give a complete Regge parametrization of all the observables, outgoing nucleon polarizations as well as the differential cross section, including the case when linearly polarized photons are used. We first develop a density matrix formalism for handling linearly polarized photons and then use the method of Hite and Jackson[6,7] for isolating the kinematic effects in the Regge parameterization. We explore the consequences of the proposed conspiracy in π^+ and K^+ photoproduction in considerable detail. When possible the value of the Regge parameters, as known from fits to previous experiments, pole fits, or symmetry-group arguments, are given.

II. DENSITY MATRIX FORMALISM

The problem of obtaining s-channel observables in terms of the t-channel helicity amplitudes that get contributions from the Regge trajectories was solved for the case of unpolarized initial particles by Gottfried and Jackson.[8] It turns out that an initially polarized

* Work supported in part by the U. S. Office of Naval Research.
† Paper based in part on material submitted to Harvard University in partial fulfillment of the requirements for the Doctor of Philosophy degree.
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[6] G. Hite, Doctoral thesis, University of Illinois, 1967 (unpublished).
[7] J. D. Jackson and G. Hite, Phys. Rev. 169, 1248 (1968).
[8] K. Gottfried and J. D. Jackson, Nuovo Cimento 33, 309 (1964).

photon beam causes little change in their approach (expecially if the photon is not crossed in going from the s to t channels). In the following we will briefly sketch how one obtains an expression for the density matrix of the outgoing nucleon in its rest frame in terms of the t-channel helicity amplitudes. The general transformation matrix relating s-channel center-of-mass helicity amplitudes to t-channel center-of-mass helicity amplitudes was derived by Trueman and Wick and is

$$f_{\lambda_c\lambda_d,\lambda_a\lambda_b}{}^s=\sum f_{\bar{\lambda}_b\lambda_{d'},\lambda_{a'}\bar{\lambda}_c}{}^t d_{\lambda_{a'}\lambda_a}{}^{s_a}(\psi_a)d_{\bar{\lambda}_b\lambda_b}{}^{s_b}(\psi_b) \times d_{\bar{\lambda}_c\lambda_c}{}^{s_c}(\psi_c)d_{\lambda_{d'}\lambda_d}{}^{s_d}(\psi_d)\,, \quad (1)$$

where λ_i corresponds to the helicity of the ith particle, s_i is the spin of particle i, and the $d_{\lambda\lambda'}{}^{s)}(\psi_i)$ are the elements of the $(2s+1)$-dimensional representation of the rotation group. For massive particles, the angle ψ_i corresponds to the angle between the s-channel spin quantization axis and the t-channel spin quantization axis in the rest frame of the particle. The derivation of this formula as well as the explicit expressions for ψ_i can be found in the literature.[9]

For the reactions we are considering we can define the t channel so that the photon does not get crossed. Since under the Lorentz transformation from the t-channel center-of-mass system to the s-channel center-of-mass system the photon helicity is unchanged, the rotation matrix for the photon is the identity matrix. Thus the s-channel center-of-mass helicity amplitude is given in terms of the t-channel helicity amplitudes as follows:

$$f_{0\lambda_d,\lambda_a\lambda_b}{}^s(s,t)=\sum f^t(s,t)_{\bar{\lambda}_b\lambda_{d'},\lambda_a 0}d_{\bar{\lambda}_b\lambda_b}{}^{\frac{1}{2}}(\psi_b) \times d_{\lambda_{d'}\lambda_d}{}^{\frac{1}{2}}(\psi_d) \quad (1')$$

(we have taken into account that the meson produced has zero spin), where λ_a is the photon helicity, λ_b is the target nucleon helicity, and λ_d is the outgoing baryon helicity.

A plane-polarized photon beam polarized at an angle φ_γ with respect to the scattering plane is described by the following density matrix:

$$\rho_{\lambda_a\lambda'_a}{}^{\text{initial}}=\frac{1}{4}\begin{pmatrix} 1 & -e^{-2i\varphi_\gamma} \\ -e^{2i\varphi_\gamma} & 1 \end{pmatrix}. \quad (2)$$

A factor of $\frac{1}{2}$ has been included to take into account averaging over the target nucleon spins.

All the observables can be obtained from the outgoing baryon density matrix. In the s-channel center-of-mass system this is

$$\rho_{\lambda\lambda'}(s,t)=\sum f_{0\lambda,\lambda_a\lambda_b}{}^s\rho_{\lambda_a\lambda_{a'}}{}^{\text{initial}}f_{0\lambda',\lambda_{a'}\lambda_b}{}^{s*} =\sum f_{\bar{\lambda}_b\lambda_{d'},\lambda_a 0}{}^t d_{\lambda_{d'}\lambda}{}^{\frac{1}{2}}(\psi_d)\rho_{\lambda_a\lambda_{a'}}{}^{\text{initial}} \times f_{\bar{\lambda}_b\lambda_{d''}\lambda_{a'}0}{}^{t*}d_{\lambda_{d''}\lambda'}{}^{\frac{1}{2}}(\psi_d). \quad (3)$$

This expression contains rotation matrices associated with rotations in the outgoing baryon rest frame. Making use of the fact that this density matrix is unaltered by a Lorentz transformation along the direction of motion of the outgoing baryon, we make such a Lorentz transformation into the outgoing baryon rest frame and choose a new quantization axis to eliminate the rotation matrices. We obtain

$$\rho_{mm'}=\sum f_{\bar{\lambda}_b m,\lambda_a 0}{}^t\rho_{\lambda_a\lambda_{a'}}{}^{\text{initial}}f_{\bar{\lambda}_b m',\lambda_{a'}0}{}^{t*}, \quad (3')$$

where m is the component of the outgoing baryon spin along the direction of the target nucleon in the outgoing baryon rest frame.

In a Regge description of reactions it is convenient to introduce the parity-conserving helicity amplitudes defined by[10]

$$f_{\lambda_c\lambda_d,\lambda_a\lambda_b}{}^{\pm}(s,t)=(1+z)^{-|\lambda+\mu|/2}(1-z)^{-|\lambda-\mu|/2}f_{\lambda_c\lambda_d,\lambda_a\lambda_b} \pm(-1)^{\lambda+\lambda_m}\eta_c\eta_d(-1)^{s_c+s_d}f_{-\lambda_c-\lambda_d,\lambda_a\lambda_b}(1+z)^{-|\lambda-\mu|/2} \times(1-z)^{-|\lambda+\mu|/2}, \quad (4)$$

where η_i are the intrinsic parities, $z=\cos\theta_t$, $\lambda=\lambda_a-\lambda_b$, $\mu=\lambda_c-\lambda_d$, and $\lambda_m=\max(|\lambda|,|\mu|)$. For the above reaction the t-channel helicity amplitudes are (in the following $\lambda_a=1$ and is suppressed)

$$f_{\frac{1}{2}\frac{1}{2}}{}^{\pm}=\frac{1}{\sin\theta_t}(f_{\frac{1}{2}\frac{1}{2}}\pm f_{-\frac{1}{2}-\frac{1}{2}})\,,$$
$$f_{\frac{1}{2}-\frac{1}{2}}{}^{\pm}=\frac{1}{2\cos^2(\frac{1}{2}\theta_t)}f_{\frac{1}{2}-\frac{1}{2}}\pm\frac{1}{2\sin^2(\frac{1}{2}\theta_t)}f_{-\frac{1}{2}\frac{1}{2}}\,, \quad (5)$$

where

$$\cos\theta_t=[2st+t^2-t\sum m_i^2+(m_d^2-m_b^2) \times(m_c^2-m_a^2)]/\tau_{ac}\tau_{bd}\,,$$
$$\tau_{ij}{}^2=[t-(m_i+m_j)^2][t-(m_i-m_j)^2].$$

These amplitudes are almost eigenstates of parity in the sense that for a trajectory of $P=(-1)^j$, $f_{\frac{1}{2}\frac{1}{2}}{}^{+}\sim s^{\alpha-1}$, $f_{\frac{1}{2}\frac{1}{2}}{}^{-}=0$, $f_{\frac{1}{2}-\frac{1}{2}}{}^{+}\sim s^{\alpha-1}$, $f_{\frac{1}{2}-\frac{1}{2}}{}^{-}\sim(1/s)f_{\frac{1}{2}-\frac{1}{2}}{}^{+}$, and inversely for $P-(-1)^j$.

A. Observables

We choose the following normalization for the helicity amplitudes in terms of the S matrix:

$$S_{\lambda_3\lambda_4,\lambda_1\lambda_2}-\delta_{\lambda_3\lambda_4,\lambda_1\lambda_2} =\frac{i(2\pi)^4\delta^4(p_3+p_4-p_1-p_2)}{(E_1E_2E_3E_4)^{1/2}}f_{\lambda_3\lambda_4,\lambda_1\lambda_2}. \quad (6)$$

Then we find that the differential cross section is given by

$$\frac{d\sigma}{dt}=\frac{1}{\pi(s-m^2)^2}\text{Tr}\rho_{\text{final}}. \quad (7)$$

[9] For example, H. Pilkuhn, *The Interaction of Hadrons* (North-Holland Publishing Company, Amsterdam, 1967).

[10] M. Gell-Mann, G. Goldberger, F. Low, E. Marx, and F. Zachariasen, Phys. Rev. 133, B145 (1964).

In terms of the parity-conserving helicity amplitudes we obtain

$$\frac{d\sigma}{dt}=\frac{1}{4\pi(s-m^2)^2}[|f_{\frac{1}{2}\frac{1}{2}}^-|^2|\sin\theta_t|^2(1+\cos2\varphi_\gamma)+|f_{\frac{1}{2}\frac{1}{2}}^+|^2|\sin\theta_t|^2(1-\cos2\varphi_\gamma)+|f_{\frac{1}{2}-\frac{1}{2}}^-|^2(1+\cos^2\theta_t-\cos2\varphi_\gamma\sin^2\theta_t)+|f_{\frac{1}{2}-\frac{1}{2}}^+|^2(1+\cos^2\theta_t+\cos2\varphi_\gamma\sin^2\theta_t)+4\cos\theta_t\,\mathrm{Re}f_{\frac{1}{2}-\frac{1}{2}}^+f_{\frac{1}{2}-\frac{1}{2}}^{-*}]. \quad (7')$$

For unpolarized photon beams the φ_γ terms are absent. In the forward direction $\sin\theta_t=0$ and $\cos\theta_t=-1$ so we find only $f_{\frac{1}{2}-\frac{1}{2}}^\pm$ contributes to the forward cross section. We notice that since at high energies f^+ and f^- get contributions from trajectories of $P=\pm(-1)^J$, respectively, for $\varphi_\gamma=0$ the leading contributions to $d\sigma/dt$ come from trajectories of parity $-(-1)^J$ and at $\varphi_\gamma=90°$ the leading contributions to $d\sigma/dt$ come from trajectories of parity $(-1)^J$.

We next want expressions for the various outgoing baryon polarizations in terms of the t-channel parity-conserving helicity amplitudes. The outgoing baryon polarization normal to the plane of scattering is defined by

$$\langle\mathbf{P}\cdot\hat{n}\rangle=(\mathrm{Tr}\rho\sigma_y)/\mathrm{Tr}\rho=-2\,\mathrm{Im}\rho_{\frac{1}{2}-\frac{1}{2}}/\mathrm{Tr}\rho;$$

$$\langle\mathbf{P}\cdot\hat{n}\rangle\frac{d\sigma}{dt}\pi(s-m^2)^2=\tfrac{1}{2}(1-\cos2\varphi_\gamma)|\sin\theta_t|(\cos\theta_t\,\mathrm{Im}f_{\frac{1}{2}\frac{1}{2}}^+f_{\frac{1}{2}-\frac{1}{2}}^{+*}+\mathrm{Im}f_{\frac{1}{2}\frac{1}{2}}^+f_{\frac{1}{2}-\frac{1}{2}}^{-*})+\tfrac{1}{2}(1+\cos2\varphi_\gamma)|\sin\theta_t|\times(\cos\theta_t\,\mathrm{Im}f_{\frac{1}{2}\frac{1}{2}}^-f_{\frac{1}{2}-\frac{1}{2}}^{*-}+\mathrm{Im}f_{\frac{1}{2}\frac{1}{2}}^-f_{\frac{1}{2}-\frac{1}{2}}^{+*}), \quad (8)$$

where again the $\cos2\varphi_\gamma$ term is simply absent for unpolarized photon beams. This polarization vanishes in the forward direction, since the t-channel non-spin-flip amplitude becomes equal to the s-channel spin-flip amplitude which vanishes in the forward direction by angular momentum conservation. We see that at $\varphi_\gamma=0$ we isolate the cross term $\mathrm{Im}f_{\frac{1}{2}\frac{1}{2}}^-f_{\frac{1}{2}-\frac{1}{2}}^{*-}$, whereas at $\varphi_\gamma=90°$ we isolate the cross term $\mathrm{Im}f_{\frac{1}{2}\frac{1}{2}}^+f_{\frac{1}{2}-\frac{1}{2}}^{+*}$. The polarization is zero if we exchange only one Regge trajectory because the interference terms then are purely real. In order to get large polarization perpendicular to the scattering plane in the Regge model, we need two relatively important same-parity trajectories which are out of phase. We choose the two polarizations in the plane to be along the z and x axes in the Jackson frame ($\hat{z}$ is the direction of the target nucleon in the outgoing baryon rest frame). $\langle\mathbf{P}\cdot\hat{z}\rangle$, which vanishes for unpolarized initial particles, is given by

$$\langle\mathbf{P}\cdot\hat{z}\rangle=\mathrm{Tr}\rho\sigma_z/\mathrm{Tr}\rho=\rho_{\frac{1}{2}\frac{1}{2}}-\rho_{-\frac{1}{2}-\frac{1}{2}}/\mathrm{Tr}\rho,$$

$$\frac{d\sigma}{dt}\pi(s-m^2)^2\langle\mathbf{P}\cdot\hat{z}\rangle=\tfrac{1}{2}\sin2\varphi_\gamma[|\sin\theta_t|^2\,\mathrm{Im}f_{\frac{1}{2}\frac{1}{2}}^+f_{\frac{1}{2}\frac{1}{2}}^{-*}-\sin^2\theta_t\,\mathrm{Im}f_{\frac{1}{2}-\frac{1}{2}}^+f_{\frac{1}{2}-\frac{1}{2}}^{*-}]. \quad (9)$$

This expression is large only for opposite-parity trajectories that are out of phase. It enables to determine the relative phase of opposite-parity contributions to the same helicity amplitude.

$\langle\mathbf{P}\cdot\hat{x}\rangle$, which also vanishes for unpolarized initial particles, is given by

$$\pi(s-m^2)^2\frac{d\sigma}{dt}\langle\mathbf{P}\cdot\hat{x}\rangle=2\,\mathrm{Re}\rho_{\frac{1}{2}-\frac{1}{2}}=\tfrac{1}{2}\sin2\varphi_\gamma[|\sin\theta_t|\cos\theta_t\times\mathrm{Im}(f_{\frac{1}{2}\frac{1}{2}}^+f_{\frac{1}{2}-\frac{1}{2}}^{*-}-f_{\frac{1}{2}\frac{1}{2}}^-f_{\frac{1}{2}-\frac{1}{2}}^{+*})+|\sin\theta_t|\,\mathrm{Im}(f_{\frac{1}{2}\frac{1}{2}}^+f_{\frac{1}{2}-\frac{1}{2}}^{+*}-f_{\frac{1}{2}\frac{1}{2}}^-f_{\frac{1}{2}-\frac{1}{2}}^{*-})]. \quad (10)$$

This also is large only for opposite parity out of phase trajectories of relatively large importance. The longitudinal and transverse polarization in the scattering plane are linear combinations of $\langle\mathbf{P}\cdot\hat{x}\rangle$ and $\langle\mathbf{P}\cdot\hat{z}\rangle$ obtained by rotating from the target nucleon direction to the negative of the meson direction in the outgoing baryon rest frame.

III. EXPLICIT REGGE PARAMETRIZATION

We utilize a modification of the Hite[6] prescription for writing the contribution of a given Regge trajectory to the parity-conserving helicity amplitude. We factor out all the known t dependence, such as threshold behavior, from the Regge residue function and get an expression in terms of a reduced residue function which is analytic in t (and hopefully slowly varying in t), another function $F_\eta(t)$ which explicitly exhibits the kinematic singularities, and a third function $N_{\lambda\mu}(\alpha)$ which contains the assumed (or experimentally known) behavior of the residue functions in the nonsense region. Explicitly a trajectory of parity $\eta(-1)^J$ will contribute to f^η as follows:

$$f_{\lambda_c\lambda_d,\lambda_a\lambda_b}{}^{t\eta}=\frac{1+\sigma e^{-i\pi\alpha}}{2}\frac{N_{\lambda\mu}(\alpha)}{\sin\pi\alpha\,\Gamma(\alpha+1)}F_\eta(t)(\tau_{ab}\tau_{cd})^m\times\left(\frac{s-u}{s_0}\right)^{\alpha-m}\gamma_{\lambda_c\lambda_d\lambda_a\lambda_b}(t)t^n, \quad (11)$$

where $\eta=\pm1$, n is determined by looking at the conspiracy relations at $t=0$ and by factorization, σ is the signature of the trajectory. Explicit expressions for $F_\eta(t)$ as well as the motivation of this parametrization are left to an Appendix.

IV. π-MESON PHOTOPRODUCTION

A. Contributions of Trajectories to Various Charge States

If we assume in the matrix element $\langle N\bar{N}|J_\mu{}^e(0)|\pi\rangle$ that isospin is conserved and $J_\mu{}^e=J_\mu{}^{I=0}+J_\mu{}^{I=1}$ (that is, we break the photon into an isoscalar part and isovector part), we can then determine the quantum numbers of the t-channel particles allowed and their relative contributions to the various charge states by using the Wigner-Eckart theorem.

Explicitly, we find

$$\begin{aligned} T(\gamma+p\to n+\pi^+)&=-T(\rho)-T(B)-(1/\sqrt{2})\\ &\quad\times[T(\pi)+T(c)+T(A_2)+T(A_1)],\\ T(\gamma+n\to p+\pi^-)&=T(\rho)+T(B)-(1/\sqrt{2})\\ &\quad\times[T(\pi)+T(c)+T(A_2)+T(A_1)], \\ T(\gamma+p\to p+\pi^0)&=(1/\sqrt{2})[T(\rho)+T(B)\\ &\quad+T(\omega)+T(\varphi)],\\ T(\gamma+n\to n+\pi^0)&=(1/\sqrt{2})[-T(\rho)-T(B)\\ &\quad+T(\omega)+T(\varphi)]. \end{aligned} \tag{12}$$

We find that certain linear combinations of different charge-state cross sections will be relatively simple. That is,

$$\begin{aligned} \left.\frac{d\sigma}{dt}\right|_{\pi^0}+\left.\frac{d\sigma}{dt}\right|_{N\pi^0} &\sim|T(\rho)|^2+|T(B)|^2+|T(\omega)+T(\varphi)|^2,\\ \left.\frac{d\sigma}{dt}\right|_{\pi^0}-\left.\frac{d\sigma}{dt}\right|_{N\pi^0} &\sim \mathrm{Re}T(\rho)[T(\omega)+T(\varphi)]^*, \\ \left.\frac{d\sigma}{dt}\right|_{\pi^+}+\left.\frac{d\sigma}{dt}\right|_{\pi^-} &\sim 2|T(\rho)|^2+|T(B)|^2+|T(\pi)+T(A_1)|^2\\ &\quad+|T(A_2)+T(c)|^2,\\ \left.\frac{d\sigma}{dt}\right|_{\pi^+}-\left.\frac{d\sigma}{dt}\right|_{\pi^-} &\sim \mathrm{Re}T(\rho)[T(c)+T(A_2)]^*\\ &\quad+\mathrm{Re}T(B)[T(\pi)+T(A_1)]^*. \end{aligned} \tag{13}$$

B. G-Parity Restrictions

For $I=1$ trajectory exchange, the $N\bar{N}$ system has $I=1$ and is thus an eigenstate of G parity. We have

$$\begin{aligned} G|JM\lambda\lambda\rangle_\pm&=(-1)^{J+1}|JM\lambda\lambda\rangle_\pm,\\ G|JM\lambda\lambda'\rangle_\pm&=\pm(-1)^{J+1}|JM\lambda\lambda'\rangle_\pm,\quad \lambda\neq\lambda'. \end{aligned} \tag{14}$$

These equations show that the π can only couple to $f_{\frac12\frac12}{}^{J-}$, the B to $f_{\frac12\frac12}{}^{J-}$ and the A_1 to $f_{\frac12-\frac12}{}^{J-}$.

C. t=0 Constraint Equations

At $t=0$ analyticity requirements lead to the following relationship between the t-channel amplitudes[2,3]:

$$\lim_{t\to 0} if_{\frac12\frac12}{}^{-}+f_{\frac12\frac12}{}^{+}=O(t^{1/2}). \tag{15}$$

This relation can be satisfied by having each amplitude $f_{\frac12\frac12}{}^-$ and $f_{\frac12-\frac12}{}^+\sim t^{1/2}$ which is called evasion, or by having the two amplitudes related there, i.e.,

$$if_{\frac12\frac12}{}^-(t=0)=-f_{\frac12-\frac12}{}^+(t=0)=at^{-1/2}. \tag{15'}$$

In the Regge picture this requires the existence of two opposite-parity trajectories degenerate at $t=0$ and having their residues related $iB_{\frac12\frac12}{}^-(0)=-B_{\frac12-\frac12}{}^+(0)=dt^{-1/2}$; thus the term "conspiracy" is often used in the Regge picture. Whether or not a trajectory chooses to conspire affects the size of the cross section drastically in the foward direction, and thus is an experimentally determinable thing. In order to explain the forward peak in the π^+ photoproduction data, it is necessary to have a pion conspiring with an opposite trajectory, the c. In the following parametrization we assume only that the π conspires; the other trajectories are assumed to choose evasion at $t=0$.

D. Parametrization of Regge Pole Contributions

(1) π trajectory. If we assume that the π conspires and chooses sense in the nonsense region, realizing that parity and G parity limit the contribution to $f_{\frac12\frac12}{}^-$, we obtain

$$f_{\frac12\frac12}{}^{\pi-}=-\frac{(1+e^{-i\pi\alpha})t^{-1/2}(t-4m^2)^{1/2}\gamma_{\frac12\frac12}{}^{\pi}(s/s_0)^{\alpha-1}\alpha}{\Gamma(\alpha+1)\sin\pi\alpha}. \tag{16}$$

(2) Conspirator trajectory. In order to fit the forward data the conspirator must choose nonsense. Thus, we obtain

$$\begin{aligned} f_{\frac12\frac12}{}^{c+}&=-\frac{\alpha(1+e^{-i\pi\alpha})(t-\mu^2)\gamma_{\frac12\frac12}{}^{c}(s/s_0)^{\alpha-1}}{\Gamma(\alpha+1)\sin\pi\alpha},\\ f_{\frac12-\frac12}{}^{c+}&=\frac{\alpha(1+e^{-i\pi\alpha})t^{-1/2}(t-\mu^2)\gamma_{\frac12-\frac12}{}^{c}(s/s_0)^{\alpha-1}}{\Gamma(\alpha+1)\sin\pi\alpha},\\ f_{\frac12-\frac12}{}^{c-}&=-\frac{2(\alpha-1)(1+e^{-i\pi\alpha})t(t-4M^2)^{1/2}(t-\mu^2)^2\gamma_{\frac12-\frac12}{}^{c}(s/s_0)^{\alpha-2}}{s_0\Gamma(\alpha+1)\sin\pi\alpha}. \end{aligned} \tag{17}$$

(3) Vector-meson trajectories. There is strong evidence to believe that the ρ meson chooses sense.[11] Symmetry

[11] L. Bertocchi, Rapporteur's talk in *Proceedings of the Heidelberg International Conference on Elementary Particles*, edited by H. Filthuth (Interscience Publishers, Inc., New York, 1968).

arguments would suggest that ω and φ also choose sense, leading to

$$f_{\frac{1}{2}\frac{1}{2}}^{V+}=-\frac{\alpha(1-e^{-i\pi\alpha})(t-\mu^2)\gamma_{\frac{1}{2}\frac{1}{2}}^{V}(s/s_0)^{\alpha-1}}{\Gamma(\alpha+1)\sin\pi\alpha},$$

$$f_{\frac{1}{2}-\frac{1}{2}}^{V+}=\frac{\alpha^2(1-e^{-i\pi\alpha})t^{1/2}(t-\mu^2)\gamma_{\frac{1}{2}-\frac{1}{2}}^{V}(s/s_0)^{\alpha-1}}{\Gamma(\alpha+1)\sin\pi_\alpha},\tag{18}$$

$$f_{\frac{1}{2}-\frac{1}{2}}^{V-}=\frac{-2\alpha(\alpha-1)(1-e^{-i\pi\alpha})t(t-4M^2)^{1/2}(t-\mu^2)^2\gamma_{\frac{1}{2}-\frac{1}{2}}^{V}(s/s_0)^{\alpha-2}}{s_0\Gamma(\alpha+1)\sin\pi\alpha}.$$

A recent finite-energy sum-rule calculation[12] suggests that the ω may choose "nonsense." If this is the case, then

$$f_{\frac{1}{2}-\frac{1}{2}}^{\omega+}\sim\alpha,\qquad f_{\frac{1}{2}-\frac{1}{2}}^{\omega-}\sim(\alpha-1).\tag{18'}$$

(4) B trajectory. G parity and parity restrict the nucleon helicities to be the same in the t channel. Assuming that the B chooses "sense," we find

$$f_{\frac{1}{2}\frac{1}{2}}^{B-}=\frac{-\alpha(1-e^{-i\pi\alpha})t^{1/2}(t-4M^2)^{1/2}\gamma_{\frac{1}{2}\frac{1}{2}}^{B}(s/s_0)^{\alpha-1}}{\Gamma(\alpha+1)\sin\pi\alpha}.\tag{19}$$

(5) A_2 trajectory. Experimental evidence favors the A_2 choosing nonsense.[11] Thus, we find

$$f_{\frac{1}{2}\frac{1}{2}}^{+A_2}=\frac{-\alpha(1+e^{-i\pi\alpha})(t-\mu^2)\gamma_{\frac{1}{2}\frac{1}{2}}^{A_2}(s/s_0)^{\alpha-1}}{\Gamma(\alpha+1)\sin\pi\alpha},$$

$$f_{\frac{1}{2}-\frac{1}{2}}^{+A_2}=\frac{\alpha(1+e^{-i\pi\alpha})t^{1/2}(t-\mu^2)\gamma_{\frac{1}{2}-\frac{1}{2}}^{A_2}(s/s_0)^{\alpha-1}}{\Gamma(\alpha+1)\sin\pi\alpha},\tag{20}$$

$$f_{\frac{1}{2}-\frac{1}{2}}^{-A_2}=\frac{-2(\alpha-1)(1+e^{-i\pi\alpha})t(t-4M^2)^{1/2}(t-\mu^2)^2\gamma_{\frac{1}{2}-\frac{1}{2}}^{A_2}(s/s_0)^{\alpha-2}}{s_0\Gamma(\alpha+1)\sin\pi\alpha}.$$

(6) A_1 trajectory. Parity and G parity restrict the nucleon helicities to be opposite. Assuming that the A_1 chooses "sense," we obtain

$$f_{\frac{1}{2}-\frac{1}{2}}^{-A_1}=\frac{\alpha^2(1-e^{-i\pi\alpha})(t-4M^2)^{1/2}t\gamma_{\frac{1}{2}-\frac{1}{2}}^{A_1}(s/s_0)^{\alpha-1}}{\Gamma(\alpha+1)\sin\pi\alpha}.\tag{21}$$

E. Neutral-Pion Photoproduction

For neutral-pion photoproduction only the ω, ρ, φ, and B trajectories can be exchanged. If we assign the vector mesons to nonets, then $SU(3)$ yields $g_{\gamma\pi\omega}=-3g_{\gamma\pi\rho}$, $g_{\gamma\pi\varphi}\approx 0$. Thus, we can neglect the φ contribution. There are two VNN coupling constants, electric and anomalous magnetic moment. Because of the largeness of the ρ contribution to the isovector anomalous moment form factor,[13] it is not clear that we can neglect the ρ trajectory as is often done. Although symmetry arguments lead one to believe that the ρ and ω trajectories should be very similar and the residue functions should have the same behavior in the nonsense region, there is some indication from pp scattering that unlike the ρ, the ω trajectory may not cross the axis near $t=-0.6$.[14] This analysis and finite-energy sum rules[12] also suggest that the ω is choosing "nonsense" instead of sense. For these reasons we feel it is important to include both ω and ρ and to see what experimental observables will allow us to better determine the trajectories and nonsense behavior of these trajectories.

In the following parametrization of observables, we have assumed that the fixed pole[15] at $\alpha=0$, which is a wrong-signature nonsense point, is unimportant so that $f=\alpha C+D$ with D small. Thus, the only effect of this fixed pole (whose strength depends on the size of the third double spectral function) is to shift the possible zero at $\alpha=0$ to $\alpha=-D/C\approx 0$. We thus set $D=0$. If the fixed pole is important, then Regge cuts are also

[12] P. DiVecchia, F. Drago, and M. Paciello, University of Rome Internal Report No. N.150, 1968 (unpublished).

[13] A. Scotti and D. Wong, Phys. Rev. 138, B145 (1965).

[14] W. Rarita, R. J. Riddell, Jr., C. B. Chiu, and R. J. N. Phillips, Phys. Rev. 165, 1615 (1968).

[15] S. Mandelstam and L.-L. Wang, Phys. Rev. 160, 1490 (1967).

important and the Regge parametrization itself is of questionable value.

The differential cross section has the following Regge parametrization:

$$\frac{d\sigma}{dt}=\frac{1}{4\pi(s-m^2)^2}\Bigg[\frac{(1+\cos 2\varphi_\gamma)|\sin\theta_t|^2\alpha_B^2|t(t-4m^2)|\,|\gamma_{\frac{1}{2}\frac{1}{2}}{}^B|^2(s/s_0)^{2(\alpha_B-1)}}{|\cos\frac{1}{2}\pi\alpha_B\,\Gamma(\alpha_B+1)|^2}+(1-\cos^2\varphi_\gamma)|\sin\theta_t|^2(t-\mu^2)^2$$

$$\times\left(\frac{\alpha_\omega^2|\gamma_{\frac{1}{2}+\frac{1}{2}}{}^\omega|^2(s/s_0)^{2(\alpha_\omega-1)}+\alpha_\rho^2|\gamma_{\frac{1}{2}+\frac{1}{2}}{}^\rho|^2(s/s_0)^{2(\alpha_\rho-1)}}{|\cos\frac{1}{2}\pi\alpha_\omega\,\Gamma(\alpha_\omega+1)|^2|\cos\frac{1}{2}\pi\alpha_\rho\,\Gamma(\alpha_\rho+1)|^2}+\frac{2\alpha_\rho\alpha_\omega\cos\frac{1}{2}\pi(\alpha_\rho-\alpha_\omega)\,\gamma_{\frac{1}{2}\frac{1}{2}}{}^\omega\gamma_{\frac{1}{2}\frac{1}{2}}{}^\rho(s/s_0)^{\alpha_\rho+\alpha_\omega-2}}{|\cos\frac{1}{2}\pi\alpha_\omega\cos\frac{1}{2}\pi\alpha_\rho\,\Gamma(\alpha_\rho+1)\Gamma(\alpha_\omega+1)|}\right)$$

$$+(1+\cos^2\theta_t-\cos 2\varphi_\gamma\sin^2\theta_t)t^2(t-4m^2)^2(t-\mu^2)^4\left(\frac{4\beta^2(\alpha_\omega-1)^2|\gamma_{\frac{1}{2}-\frac{1}{2}}{}^\omega|^2(s/s_0)^{2(\alpha_\omega-2)}}{s_0^2|\cos\frac{1}{2}\pi\alpha_\omega\,\Gamma(\alpha_\omega+1)|^2}\right.$$

$$\left.+\frac{4\alpha_\rho^2(\alpha_\rho-1)^2|\gamma_{\frac{1}{2}-\frac{1}{2}}{}^\rho|^2(s/s_0)^{2(\alpha_\rho-2)}}{s_0^2|\cos\frac{1}{2}\pi\alpha_\rho\,\Gamma(\alpha_\rho+1)|^2}+\frac{4\alpha_\rho\beta(\alpha_\rho-1)(\alpha_\omega-1)\gamma_{\frac{1}{2}-\frac{1}{2}}{}^\rho\gamma_{\frac{1}{2}-\frac{1}{2}}{}^\omega(s/s_0)^{\alpha_\rho+\alpha_\omega-4}\cos\frac{1}{2}\pi(\alpha_\rho-\alpha_\omega)}{s_0^2|\cos\frac{1}{2}\pi\alpha_\rho\cos\frac{1}{2}\pi\alpha_\omega\,\Gamma(\alpha_\omega+1)\Gamma(\alpha_\rho+1)|}\right)$$

$$+(1+\cos^2\theta_t+\cos^2\varphi_\gamma\sin^2\theta_t)|t|(t-\mu^2)^2\left(\frac{\beta^2\alpha_\omega^2|\gamma_{\frac{1}{2}-\frac{1}{2}}{}^\omega|^2(s/s_0)^{2(\alpha_\omega-1)}}{|\cos\frac{1}{2}\pi\alpha_\omega\,\Gamma(\alpha_\omega+1)|^2}+\frac{\alpha_\rho^4|\gamma_{\frac{1}{2}-\frac{1}{2}}{}^\rho|^2(s/s_0)^{2(\alpha_\rho-1)}}{|\cos\frac{1}{2}\pi\alpha_\rho\,\Gamma(\alpha_\rho+1)|^2}\right.$$

$$\left.+\frac{2\alpha_\rho^2\beta\alpha_\omega\gamma_{\frac{1}{2}-\frac{1}{2}}{}^\rho\gamma_{\frac{1}{2}-\frac{1}{2}}{}^\omega(s/s_0)^{\alpha_\rho+\alpha_\omega-2}\cos\frac{1}{2}\pi(\alpha_\rho-\alpha_\omega)}{|\cos\frac{1}{2}\pi\alpha_\rho\cos\frac{1}{2}\pi\alpha_\omega\,\Gamma(\alpha_\rho+1)\Gamma(\alpha_\omega+1)|}\right)-8\cos\theta_t(t-\mu^2)^3|t|^{3/2}\frac{|t-4m^2|}{s_0}$$

$$\times\left(\frac{\beta^2\alpha_\omega(\alpha_\omega-1)|\gamma_{\frac{1}{2}-\frac{1}{2}}{}^\omega|^2(s/s_0)^{2\alpha_\omega-3}}{|\cos\frac{1}{2}\pi\alpha_\omega\,\Gamma(\alpha_\omega+1)|^2}+\frac{\alpha_\rho^3(\alpha_\rho-1)|\gamma_{\frac{1}{2}-\frac{1}{2}}{}^\rho|^2(s/s_0)^{2\alpha_\rho-3}}{|\cos\frac{1}{2}\pi\alpha_\rho\,\Gamma(\alpha_\rho+1)|^2}\right.$$

$$\left.+\frac{\alpha_\rho\beta[\alpha_\rho(\alpha_\omega-1)+\alpha_\omega(\alpha_\rho-1)]\gamma_{\frac{1}{2}-\frac{1}{2}}{}^\rho\gamma_{\frac{1}{2}-\frac{1}{2}}{}^\omega\cos\frac{1}{2}\pi(\alpha_\rho-\alpha_\omega)]}{|\cos\frac{1}{2}\pi\alpha_\rho\cos\frac{1}{2}\pi\alpha_\omega\,\Gamma(\alpha_\rho+1)\Gamma(\alpha_\omega+1)|}\right)\Bigg], \quad (22)$$

where $\beta=\alpha_\omega$ for the ω choosing sense and $\beta=1$ for the ω choosing nonsense. For unpolarized photons the terms containing φ_γ are not present. We notice that if we use a linearly polarized photon beam with $\varphi_\gamma=0$ (polarized in the plane), then the B is not contributing. If the ω meson then behaves similarly to the ρ meson, there must be prominent dip at $\alpha_\omega=0$ (which might be slightly obscured by the presence of the ρ). If the ρ and ω meson trajectories are reasonably degenerate and we lump the coupling constants of the ρ and ω together into an effective vector-meson coupling constant, then the energy and dependence of the cross section can be written as

$$s^2\frac{d\sigma}{dt}=a(t)\sin^2\varphi_\gamma s^{2\alpha^V}+b(t)\cos^2\varphi_\gamma s^{2\alpha^B}+c(t)s^{2\alpha^V-2}, \quad (22')$$

where $a(t)$ contains known t factors and $\gamma_{\frac{1}{2}\frac{1}{2}}{}^V$, $\gamma_{\frac{1}{2}-\frac{1}{2}}{}^V$, $b(t)$ contains known t factors and $\gamma_{\frac{1}{2}\frac{1}{2}}{}^B$, $c(t)$ contains known t factors and $\gamma_{\frac{1}{2}-\frac{1}{2}}{}^V$. Thus, by looking at 3 different angles φ_γ for a given s, we can determine the explicit t dependence of the reduced residue functions and compare them with their pole values.

In order to determine the relative importance of the ω and ρ, as well as to determine the ω trajectory function better and see if the ω residue functions choose sense (as expected by symmetry arguments) or nonsense as suggested by FESR, (finite-energy sum rules) we look at the expression for the difference

$$(d\sigma/dt)(\gamma+p\to p+\pi^0)-(d\sigma/dt)(\gamma+n\to n+\pi^0)$$

in terms of the residues for $\gamma+p\to p+\pi^0$.

$$\Delta=\frac{d\sigma}{dt}(\pi^0)-\frac{d\sigma}{dt}(n\pi^0)$$

$$=\frac{(1-\cos 2\varphi_\gamma)4\alpha_\rho\alpha_\omega\cos\frac{1}{2}\pi(\alpha_\rho-\alpha_\omega)\,\gamma_{\frac{1}{2}\frac{1}{2}}{}^\omega\gamma_{\frac{1}{2}\frac{1}{2}}{}^\rho(s/s_0)^{\alpha_\rho+\alpha_\omega-2}|\sin\theta_t|^2(t-\mu^2)^2}{|\cos\frac{1}{2}\pi\alpha_\omega\cos\frac{1}{2}\pi\alpha_\rho\,\Gamma(\alpha_\omega+1)\Gamma(\alpha_\rho+1)|}$$

$$+\frac{(1+\cos^2\theta_t+\cos 2\varphi_\gamma\sin^2\theta_t)|t|(t-\mu^2)^2 4\alpha_\rho^2\beta\alpha_\omega\gamma_{\frac{1}{2}-\frac{1}{2}}{}^\rho\gamma_{\frac{1}{2}-\frac{1}{2}}{}^\omega(s/s_0)^{\alpha_\rho+\alpha_\omega-2}\cos\frac{1}{2}\pi(\alpha_\rho-\alpha_\omega)}{|\cos\frac{1}{2}\pi\alpha_\omega\cos\frac{1}{2}\pi\alpha_\rho\,\Gamma(\alpha_\rho+1)\Gamma(\alpha_\omega+1)|}$$

$$+\text{terms of } O(1/s^2)\times(d\sigma/dt). \quad (23)$$

(The φ_γ terms are not present if we use unpolarized photons. Polarized photons yield no extra information here.) We first notice that regardless of whether the ω chooses sense or nonsense, Δ must vanish at $\alpha_\rho=0$ and at $\alpha_\omega=0$. If it vanishes only at $\alpha_\rho=0$, i.e., $t=-0.6$, then this is evidence that the ω trajectory does not cross the axis in the t region under study. If the ω trajectory does behave similarly to the ρ [FESR calculations are in agreement with $\alpha_\omega(t)=0.61+0.75t$, which implies a zero near $t=-0.8$ BeV], then we notice that if the ω chooses nonsense, then Δ will change sign at $\alpha_\omega(t)=0$. If ω chooses sense, Δ will not change sign at $\alpha_\omega(t)=0$ unless $\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{\rho}\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{\omega}$ is negligible with respect to $\gamma_{\frac{1}{2}\frac{1}{2}}{}^{\rho}\gamma_{\frac{1}{2}\frac{1}{2}}{}^{\omega}$. The πp, pp, and $p\bar{p}$ scattering data could be fitted[14] with the following trajectories:

$$\begin{aligned}\alpha_\rho(t)&=0.58\pm0.01+0.97\pm0.04t \text{ (all fits)},\\ \alpha_\omega(t)&=0.45+0.31t \text{ (fit 1)}\\ &=0.21+1.66t+1.65t^2 \text{ (fit 2)}\\ &=0.36+0.32t \text{ (fit 3)}.\end{aligned} \tag{23'}$$

We see that the ω trajectory is not well known from experiment. We can evaluate the ρ, ω, and B coupling constants at the respective poles by comparing the Regge expressions with the elementary-particle-exchange expressions near the pole. For vector-meson exchange, the photon vertex is given by

$$i\lambda_V\epsilon_{\mu\nu\rho\sigma}\epsilon_\mu K_\nu q_\rho$$

and the nucleon vertex by

$$i\bar{u}(P_2)[g_{V1}\gamma_\mu+g_{V2}\sigma_{\mu\nu}(P_2-P_1)_\nu]u(P_1).$$

This yields

$$\begin{aligned}f_{\frac{1}{2}\frac{1}{2}}{}^{+}&=\tfrac{1}{4}(t-\mu^2)\lambda_V\frac{2mg_{V1}+g_{V2}t}{t-m_V{}^2},\\ f_{\frac{1}{2}-\frac{1}{2}}{}^{+}&=-\frac{t-\mu^2}{4\sqrt{t}}\lambda_V\frac{tg_{V1}+2mtg_{V2}}{t-m_V{}^2}.\end{aligned} \tag{18'}$$

Comparing with the Regge expression, Eq. (18), near the pole, we find for $t\approx m_V{}^2$

$$\begin{aligned}\gamma_{\frac{1}{2}\frac{1}{2}}{}^{V}&=\tfrac{1}{8}\pi\alpha'(t=m_V{}^2)(2mg_{V1}+g_{V2}t)\lambda_V,\\ \gamma_{\frac{1}{2}-\frac{1}{2}}{}^{V}&=\tfrac{1}{8}\pi\alpha'(t=m_V{}^2)(g_{V1}+2mg_{V2})\lambda_V.\end{aligned} \tag{24}$$

For the B meson there is only one coupling constant and elementary B exchange leads to

$$f_{\frac{1}{2}\frac{1}{2}}{}^{-}=\tfrac{1}{8}(t-\mu^2)(t-4m^2)^{1/2}[t^{1/2}\lambda_Bg_B/(t-m_B{}^2)], \tag{19'}$$

which, when compared to the Regge expression near the pole [Eq. (19)], gives

$$\gamma_{\frac{1}{2}\frac{1}{2}}{}^{B}(t=m_B{}^2)=\tfrac{1}{16}\pi(t-\mu^2)\lambda_Bg_B\alpha'(t=m_B{}^2). \tag{25}$$

By analyzing low-energy π photoproduction data using dispersion relations, Berends *et al.*[16] found the following values for the ω and B coupling constants:

$$\begin{aligned}\lambda_\omega g_{\omega1}&=-22.5\pm5\ \mu\text{b}^{1/2}\ (\text{GeV}/c)^{-3},\\ \lambda_\omega g_{\omega2}&=-1.0\pm1.0\ \mu\text{b}^{1/2}\ (\text{GeV}/c)^{-2},\\ \lambda_Bg_B&\approx40.5\ \mu\text{b}^{1/2}\ (\text{GeV}/c)^{-3}.\end{aligned} \tag{26}$$

If we then use the ratios $g_{\omega1}{}^2/g_{\rho1}{}^2$ and $g_{\rho2}{}^2/g_{\rho1}{}^2$, found by Scotti and Wong in their fit of nucleon-nucleon scattering[13] and the $SU(3)$ prediction $\lambda_\rho=\frac{1}{3}\lambda_\omega$ and their value $g_{\omega1}{}^2/4\pi=16.7$, we find for the ρ meson

$$\begin{aligned}\lambda_\rho g_{\rho1}&\approx-3.2\ \mu\text{b}^{1/2}(\text{GeV}/c)^{-3},\\ \lambda_\rho g_{\rho2}&\approx-9.6\ \mu\text{b}^{1/2}(\text{GeV}/c)^{-2}.\end{aligned} \tag{27}$$

Although we should take the ρ values as crude estimates with $\lambda_\rho g_\rho$ probably too large, it appears we should not neglect the ρ contribution. In their fit to the differential cross-section data using a Regge model, Ader *et al.*[1] assumed that the t dependence of $\gamma(t)$ was exactly that found at the pole and found that

$$\begin{aligned}\lambda_\omega g_{\omega1}&=-22.8\ \mu\text{b}^{1/2}(\text{GeV}/c)^{-3},\\ \lambda_\omega g_{\omega2}&=-4.56\ \mu\text{b}^{1/2}(\text{GeV}/c)^{-2},\\ \lambda_Bg_B&=34\ \mu\text{b}^{1/2}(\text{GeV}/c)^{-3},\end{aligned} \tag{26a}$$

not in disagreement with the dispersion-theory result, if we assume that most of $\lambda_\omega g_{\omega2}$ was really coming from the ρ. It still remains to be seen if in fitting more complicated data the t dependence suggested by the pole fit is correct. The nucleon polarization perpendicular to the plane of scattering is due largely to ρ, ω interference. Its size depends explicitly on the difference between the ρ and ω trajectories, it should become very small at $\alpha_\rho=0$ and $\alpha_\omega=0$, and the detailed behavior depends critically on the relative size of the ρ and ω residue functions and whether or not the ω chooses sense. If we use polarized photons beams, then it is also possible, by eliminating the first-order vector-meson contribution, by choosing $\varphi_\gamma=0$, to look at the contribution to the polarization coming from B-V interference. The magnitude depends on the difference between V and B trajectories, and the details again are sensitive to the nonsense properties of the ω meson. Explicitly

$$\begin{aligned}-2\langle\mathbf{P}\cdot\hat{n}\rangle\frac{d\sigma}{dt}\pi(s-m^2)^2=&(1-\cos2\varphi_\gamma)\frac{|\sin\theta_t|\cos\theta_t\sin\frac{1}{2}\pi(\alpha_\rho-\alpha_\omega)|t|^{1/2}(t-\mu^2)^2}{|\Gamma(\alpha_\rho+1)\Gamma(\alpha_\omega+1)\cos\frac{1}{2}\pi\alpha_\rho\cos\frac{1}{2}\pi\alpha_\omega|}\alpha_\omega\alpha_\rho\left(\begin{bmatrix}\alpha_\omega\\1\end{bmatrix}\gamma_{\frac{1}{2}\frac{1}{2}}\gamma_{\frac{1}{2}-\frac{1}{2}}-\alpha_\rho\gamma_{\frac{1}{2}\frac{1}{2}}\gamma_{\frac{1}{2}-\frac{1}{2}}\right)\\ &\times(s/s_0)^{\alpha_\rho+\alpha_\omega-2}+\sum_{i=1}^{2}(1+\cos2\varphi_\gamma)|\sin\theta_t|\frac{(\sin\frac{1}{2}\pi(\alpha_B-\alpha_{V_i}))\alpha_B\gamma_{\frac{1}{2}\frac{1}{2}}{}^{B}\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{V_i}(s/s_0)^{\alpha_B+\alpha_{V_i}-3}}{|\Gamma(\alpha_B+1)\Gamma(\alpha_{V_i}+1)\cos\frac{1}{2}\pi\alpha_{V_i}\cos\frac{1}{2}\pi\alpha_B|}\\ &\times\left(2\frac{\cos\theta_t}{s_0}\begin{bmatrix}\alpha_{V_i}\\1\end{bmatrix}|t|^{3/2}|t-4m^2|(t-\mu^2)^2(\alpha_{V_i}+1)-\begin{bmatrix}\alpha_{V_i}{}^2\\\alpha_{V_i}\end{bmatrix}|t(t-\mu^2)|t-4m^2|^{1/2}(s/s_0)\right),\end{aligned} \tag{27}$$

[16] F. Berends, A. Donnachie, and D. Weaver, Nucl. Phys. **B4**, 105 (1968).

where $i=1$, 2 corresponds to the ρ, ω, respectively, and in the

$$\begin{bmatrix} a \\ b \end{bmatrix}$$

a is choosing "sense" and b corresponds to choosing "nonsense." The two polarizations in the plane of scattering vanish for unpolarized photons. They measure the $V-B$ interference. Explicitly, for polarized photon beams,

$$\pi(s-m^2)^2\frac{d\sigma}{dt}\langle\mathbf{P}\cdot\hat{x}\rangle=\tfrac{1}{2}\sin^2\varphi_\gamma|\sin\theta_t|\cos\theta_t\sum_{i=1}^{2}\sin\tfrac{1}{2}\pi(\alpha_B-\alpha_{V_i})$$
$$\times\frac{\gamma_{\frac{1}{2}\frac{1}{2}}{}^B\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{V_i}(s/s_0)^{\alpha_B-\alpha_{V_i}-2}\alpha_B\begin{bmatrix}\alpha_{V_i}{}^2\\ \alpha_{V_i}\end{bmatrix}|t(t-\mu^2)||t-4m^2|^{1/2}}{|\Gamma(\alpha_B+1)\Gamma(\alpha_{V_i}+1)\cos\frac{1}{2}\pi\alpha_B\cos\frac{1}{2}\pi\alpha_{V_i}|}, \quad (28)$$

$$\pi(s-m^2)^2\frac{d\sigma}{dt}\langle\mathbf{P}\cdot\hat{z}\rangle=\tfrac{1}{2}\sin2\varphi_\gamma|\sin\theta_t|^2\sum_{i=1}^{2}\sin\tfrac{1}{2}\pi(\alpha_B-\alpha_{V_i})\frac{\alpha_B\alpha_{V_i}|t(t-4m^2)|^{1/2}|t-\mu^2|\gamma_{\frac{1}{2}\frac{1}{2}}{}^B\gamma_{\frac{1}{2}\frac{1}{2}}{}^{V_i}(s/s_0)^{\alpha_B+\alpha_{V_i}-2}}{|\Gamma(\alpha_B+1)\Gamma(\alpha_{V_i}+1)\cos\frac{1}{2}\pi\alpha_{V_i}\cos\frac{1}{2}\pi\alpha_B|}. \quad (29)$$

We notice that the ratio of these two polarizations tells us the relative size of the two vector-meson residue functions.

F. Charged-Pion Photoproduction

Charged-pion photoproduction is much more complicated than neutral-pion photoproduction because the number of trajectories that can be exchanged is large and because the angular distribution exhibits a sharp peak in the forward direction. The data for small $|t|<m_\pi{}^2$ resemble remarkably those predicted by gauge-invariant elementary pion exchange (including the electric Born term), and at higher energy the data can be fit by an *ad hoc* form-factor model[17]

$$\frac{d\sigma}{dt}=\left(\frac{d\sigma}{dt}\right)_{\text{Born}}e^{3t}. \quad (30)$$

In the Regge pole model we have to postulate the existence of a new trajectory, the c, conspiring with the π to obtain a forward peak with the correct properties. That is because in the forward direction only $f_{\frac{1}{2}-\frac{1}{2}}{}^t$ is nonzero by angular momentum conservation. However, in the Regge picture, in order to satisfy the $t=0$ constraint equation in the case of nondegenerate trajectories, $f_{\frac{1}{2}-\frac{1}{2}}{}^+\sim t^{1/2}$ which is proportional to $1/s$ in the forward direction. The extra factor of $1/s$ would lead to a suppression of the forward cross section. Thus, to salvage the model, a new trajectory degenerate with the π at $t=0$ of opposite parity must be postulated. We will show that the conspiracy hypothesis does exactly what we want—it reproduces almost exactly the contributions of the gauge-invariant diagrams near the forward direction, and the slope of the Regge trajectory provides a natural exponential falloff of the form $\exp[2\alpha' t\ln(s/s_0)]$. We will then give some reasons why the higher trajectories such as ρ, A_2 are not expected to play an important role for $|t|<0.1$.

If we look at π^+ photoproduction, then there are two diagrams contributing to the gauge-invariant Born approximation. (See Fig. 1.)

The pion-exchange diagram contributes to the kinematic-singularity-free amplitudes B_i of Ball[18] as follows:

$$B_3=eg/(t-\mu^2) \quad (31a)$$

and the nucleon pole diagram yields

$$B_1=eg/(s-m^2),\quad B_2=eg/(s-m^2),$$
$$B_3=eg/2(s-m^2). \quad (31b)$$

Thus, the two diagrams satisfy the gauge-invariance condition on the B_i:

$$(s-u)B_2=2(t-\mu^2)B_3. \quad (32)$$

Thus, B_2 and B_3 are inextricably related by gauge invariance. The contribution of both diagrams to CGLN (Chew, Goldberger, Low, and Nambu) invariant amplitudes A_i is[18]

$$A_1=B_1-MB_6=eg/(s-m^2),$$
$$A_2=2B_2/(t-\mu^2)=2eg/(s-m^2)(t-\mu^2). \quad (33)$$

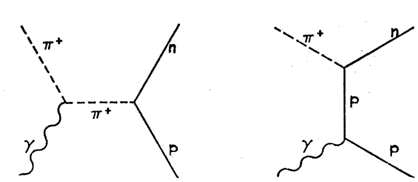

FIG. 1. Born terms for π^+ photoproduction.

[17] A. M. Boyarski *et al.*, Phys. Rev. Letters **20**, 300 (1968).

[18] J. S. Ball, Phys. Rev. **124**, 2014 (1961).

We thus call the "gauge-invariant" elementary pion the joint contribution of the π and N pole terms to B_2 and B_3, i.e., to A_2 (notice the kinematic pion pole) and call the gauge-invariant contribution of the nucleon pole to A_1 the "gauge-invariant" nucleon contribution. Making use of the simple relation between the parity-conserving helicity amplitudes and the A_i, we find that "gauge-invariant" elementary pion exchange leads to

$$f_{\frac{1}{2}\frac{1}{2}}^{-}=\frac{t^{1/2}(t-4m^2)^{1/2}eg}{2(s-m^2)}. \tag{34}$$

Evaluating the Reggeized pion contribution at the pion pole yields the relationship

$$\gamma_{\frac{1}{2}\frac{1}{2}}^{\pi}(t=\mu^2)=-\tfrac{1}{4}\pi\frac{eg}{s_0}\mu^2\frac{d\alpha_\pi}{dt}\bigg|_{t=\mu^2}. \tag{35}$$

The "gauge-invariant" nucleon pole term contributes as follows:

$$f_{\frac{1}{2}\frac{1}{2}}^{+}=\frac{-eg}{4(s-m^2)}(t-\mu^2),$$
$$f_{\frac{1}{2}\frac{1}{2}}^{-}=\frac{(t-\mu^2)(t-4m^2)^{1/2}t^{-1/2}eg}{4(s-m^2)}, \tag{36}$$
$$f_{\frac{1}{2}-\frac{1}{2}}^{+}=\frac{(t-\mu^2)t^{-1/2}meg}{2(s-m)^2}.$$

We notice that the nucleon pole contributions obey the constraint equation by "conspiring" with themselves, i.e., as $t\to 0$, we have $f_{\frac{1}{2}-\frac{1}{2}}^{+}=-if_{\frac{1}{2}\frac{1}{2}}^{-}=Ct^{-1/2}$. Notice that away from $t=0$ the two amplitudes are quite different. We also see from the above that the elementary pion evades (i.e., $f_{\frac{1}{2}\frac{1}{2}}^{-}\sim t^{1/2}$).

For a conspiring Regge pion we enforce externally the requirement that

$$f_{\frac{1}{2}-\frac{1}{2}}^{+c}=-f_{\frac{1}{2}\frac{1}{2}}^{-\pi}=Ct^{-1/2} \tag{37}$$

as $t\Rightarrow 0$. This leads to

$$\alpha_\pi(t=0)=\alpha_c(t=0) \tag{37$'$}$$

and

$$f_{\frac{1}{2}\frac{1}{2}}^{c}(t=0)=t^{-1/2}(t-\mu^2)[\gamma_{\frac{1}{2}\frac{1}{2}}^{\pi}(0)/\gamma_{\frac{1}{2}\frac{1}{2}}^{\pi}(\mu^2)]\times(eg/s)m(s/s_0)^{\alpha(0)}. \tag{37$''$}$$

We thus see that near $t=0$ (and thus near the forward direction for high s) the contribution of the conspirator to $f_{\frac{1}{2}-\frac{1}{2}}^{+}$ will be the same as the nucleon pole contribution to $f_{\frac{1}{2}-\frac{1}{2}}^{+}$ if

$$\gamma^\pi(0)/\gamma^\pi(\mu^2)\approx\tfrac{1}{2}. \tag{38}$$

Now $O(4)$ symmetry suggests that the pion would not couple to $N\bar{N}$ at $t=0$ if $\mu=0$. This may suggest the following one-parameter t dependence of the pion residue function:

$$\gamma_{\frac{1}{2}\frac{1}{2}}^{\pi}(t)=-\tfrac{1}{4}\pi\frac{d\alpha}{dt}\bigg|_{\mu^2}\frac{eg}{s_0}\mu^2\left(1+\frac{\lambda(t-\mu^2)}{\mu^2}\right). \tag{39}$$

If we determine λ by having the conspirator give the same contribution to $f_{\frac{1}{2}-\frac{1}{2}}^{+}$ as the nucleon pole in the forward direction (i.e., reproduce the forward peak), then $\lambda\approx 0.5$, and $\gamma^\pi(t=-\mu^2)=0$. It is interesting that this extra t dependence added to $f_{\frac{1}{2}\frac{1}{2}}$ with $\lambda\approx 0.5$ exactly reproduces the contribution of the nucleon pole term to $f_{\frac{1}{2}\frac{1}{2}}^{-}$.

If we approximate the denominator of the Reggeized pion contribution by its value near the pion pole, we obtain

$$f_{\frac{1}{2}\frac{1}{2}}^{-\,\mathrm{Regge}\,\pi}=\tfrac{1}{2}t^{-1/2}[(t-4m^2)^{1/2}/s]eg\times[\mu^2+\tfrac{1}{2}(t-\mu^2)](s/s_0)^{\alpha\pi}, \tag{40}$$

whereas the contribution of the gauge-invariant elementary pion plus the gauge-invariant electric Born term to $f_{\frac{1}{2}\frac{1}{2}}^{-}$ is

$$f_{\frac{1}{2}\frac{1}{2}}^{-}=\tfrac{1}{2}t^{-1/2}[(t-4m^2)/(s-m^2)]eg[t+\tfrac{1}{2}(t-\mu^2)]. \tag{40$'$}$$

The two expressions are identical at the pion pole, are equal in magnitude at $t=0$, and for large t the Regge pion contribution to $f_{\frac{1}{2}\frac{1}{2}}^{-}$ is $\frac{1}{3}$ that of the Born diagram and include a form factor $\exp[2\alpha' t\ln(s/s_0)]$. The reason that the Reggeized conspiring pion cannot be exactly similar to the Born terms is that we want the Reggeized pion residues to vanish in the limit $m_\pi\to 0$, $t\to 0$ as is necessary in the $O(4)$ scheme.

Now that we have shown that we can reproduce the forward peak with the conspiracy hypothesis, we must be able to get the correct form factor, explain the experimental fact that $s^2(d\sigma/dt)$ seems to be independent of s, and discuss the contribution of the other trajectories.

In an investigation of 6 reactions where the π, ρ, and A_2 (A_1 was neglected) could be exchanged, Haber *et al.* found[19] that as long as $|t|<0.25$, the contributions from $P=-(-1)^J$ trajectory exchange dominate $d\sigma/dt$, i.e.,

$$\sum_{P=-(-1)^J}|f_i|^2/\sum_{\mathrm{all}}|f_i|^2>0.75, \tag{41}$$

When they include all the data available, they obtain a best fit for the pion trajectory

$$\alpha_\pi(t)=1.5\pm 0.5(t-m_\pi^2). \tag{42}$$

Their fit deteriorated badly for $t<-0.3$. This is not in disagreement with the data of Boyarski *et al.*,[17] which is consistent with $\alpha(t)$ in $s^2(d\sigma/dt)=a(t)s^{2\alpha}$ having a slope of about 1.0 for $|t|<0.3$. For larger t, the t dependence of $s^2d\sigma/dt$ is small and oscillatory. One explanation is that the ρ and A_2 start becoming important for large t values. We can get an upper bound on the ρ contribu-

[19] B. Haber, U. Maor, G. Yekutieli, and E. Gotsman, Phys. Rev. **168**, 1773 (1968).

tion by using π^0 photoproduction data and $SU(2)$. Our previous estimations on the relative size of the ω and ρ coupling constants leads us to guess that the ρ contributes at most 2/11 of the total neutral-pion cross section. Its contribution to the charged-pion cross section (neglecting interference) is [by $SU(2)$] twice this value. At $E\gamma=5$ GeV, the ρ contribution to π^+ photoproduction is for given t at most the following percentage: $t=0$, 2%; $t=-0.1$, 33%; $t=-0.2$, 26%; $t=-0.3$, 15%; $t=-0.4$, 12%; $t=-0.60$, 12%; $t=-1.0$, 25%. Thus, we see the ρ is most important near its maximum at $t=-0.1$ and for $t<-1.0$ BeV. These crude generous estimates show that the ρ should not be neglected near $t\approx-0.1$ in detailed fits, and that the $\pi+c$ contributes at least $\frac{2}{3}$ of the magnitude of the differential cross section (neglecting A_2 and A_1). This is not in disagreement with Haber *et al.* For $|t|<-0.1$ the kinematic enhancement $[(t-m_\rho^2)/(t-m_\pi^2)]^2$ of the π more than compensates the difference in heights of the two trajectories.

In order to explain the form factor $\exp(3t)$ we must compare it to the Regge form factor $\exp[2\alpha' t\ln(s/s_0)]$. Choosing $s_0=1$ leads to values of α_c' from 0.45–0.6 for E_γ from 16 to 6 GeV. Thus, we see the conspirator has a small slope. If we assume $\alpha_\pi'=1$, then the form factor associated with the pion contribution is around e^{6t} for $E_\gamma\approx 10$ GeV.

At present, the conspiracy hypothesis is little more than a way of getting around the forward peak problem. We have said that the conspirator reproduces the forward peak by construction; the relatively t-independent behavior of $s^2 d\sigma/dt$ we have attributed to the combination of having many trajectories present and a small slope for the c. If we are away from the forward direction, then linearly polarized photons enable us to separate contributions from opposite-parity trajectories. At $\varphi_\gamma=0°$ we can eliminate the contributions from the c, ρ, and A_2. Thus, at this angle of polarization the data should resemble pure Reggeized pion exchange and we should be able to measure $\alpha_\pi(t)$. If $\alpha_\pi'(t)\approx 1.0$ as indicated in other experiments, the falloff of the cross section should be like e^{6t} for 10-GeV photons. At $\varphi_\gamma=90°$ we can completely eliminate the contribution of the π and B. Thus near $t=-0.6$, where the ρ goes through a nonsense zero, only the c and A_2 are left, so we can probably check to see of the c normalization and exponential falloff with $a_c'\approx 0.5$ is a valid parametrization. Explicitly we get the following parametrization of $d\sigma/dt$:

$$
\begin{aligned}
\frac{d\sigma}{dt}=\frac{1}{4\pi(s-m^2)^2}\Bigg[&|\sin\theta_t|^2(1+\cos2\varphi_\gamma)|t-4m^2|\Bigg(\frac{\alpha_\pi^2|\gamma_{\frac12\frac12}{}^{\pi}|^2|t|^{-1}(s/s_0)^{2(\alpha_\pi-1)}}{|\Gamma(\alpha_\pi+1)\sin\frac12\pi\alpha_\pi|}+\frac{\alpha_B^2|t||\gamma_{\frac12\frac12}{}^{B}|^2(s/s_0)^{2(\alpha_B-1)}}{|\Gamma(\alpha_B+1)\cos\frac12\pi\alpha_B|^2}\\
&+\frac{2\sin\frac12\pi(\alpha_B-\alpha_\pi)\,\alpha_B\alpha_\pi\gamma_{\frac12\frac12}{}^{\pi}\gamma_{\frac12\frac12}{}^{B}(s/s_0)^{\alpha_B+\alpha_\pi-2}}{|\Gamma(\alpha_\pi+1)\Gamma(\alpha_B+1)\sin\frac12\pi\alpha_\pi\cos\frac12\pi\alpha_B|}\Bigg)+|\sin\theta_t|^2(1-\cos2\varphi_\gamma)(t-\mu^2)^2\Bigg(\frac{\alpha_c^2|\gamma_{\frac12\frac12}{}^{c}|^2(s/s_0)^{2(\alpha_c-1)}}{|\Gamma(\alpha_c+1)\sin\frac12\pi\alpha_c|^2}\\
&+\frac{\alpha_\rho^2|\gamma_{\frac12\frac12}{}^{\rho}|^2(s/s_0)^{2(\alpha_\rho-1)}}{{}^{+}\Gamma(\alpha_\rho+1)\cos\frac12\pi\alpha_\rho|^2}+\frac{\alpha_{A_2}{}^2|\gamma_{\frac12\frac12}{}^{A2}|(s/s_0)^{2(\alpha_{A2}-1)}}{|\Gamma(\alpha_{A_2}+1)\sin\frac12\pi\alpha_{A_2}|^2}+\frac{2\alpha_\rho\alpha_c\sin\frac12\pi(\alpha_\rho-\alpha_c)\gamma_{\frac12\frac12}{}^{\rho}\gamma_{\frac12\frac12}{}^{c}(s/s_0)^{\alpha_\rho+\alpha_c-2}}{|\Gamma(\alpha_\rho+1)\Gamma(\alpha_c+1)\sin\frac12\pi\alpha_c\cos\frac12\pi\alpha_\rho|}\\
&+\frac{2\alpha_c\alpha_{A_2}\cos\frac12\pi(\alpha_{A_2}-\alpha_c)\,\gamma_{\frac12\frac12}{}^{A2}\gamma_{\frac12\frac12}{}^{c}(s/s_0)^{\alpha_{A2}+\alpha_c-2}}{|\Gamma(\alpha_{A_2}+1)\Gamma(\alpha_c+1)\sin\frac12\pi\alpha_c\sin\frac12\pi\alpha_{A_2}|}\Bigg)+(1+\cos^2\theta_t+\cos2\varphi_\gamma\sin^2\theta_t)(t-\mu^2)^2\\
&+\Bigg(\frac{\alpha_c^2|t|^{-1}|\gamma_{\frac12-\frac12}{}^{c}|^2(s/s_0)^{2(\alpha_c-1)}}{|\Gamma(\alpha_c+1)\sin\frac12\pi\alpha_c|^2}+\frac{\alpha_\rho^4|t||\gamma_{\frac12-\frac12}{}^{\rho}|^2(s/s_0)^{2(\alpha_\rho-1)}}{|\Gamma(\alpha_\rho+1)\cos\frac12\pi\alpha_\rho|^2}+\frac{\alpha_{A_2}{}^2|t||\gamma_{\frac12-\frac12}{}^{A2}|^2(s/s_0)^{2(\alpha_{A2}-1)}}{|\Gamma(\alpha_{A_2}+1)\sin\frac12\pi\alpha_{A_2}|^2}\\
&+\frac{2\alpha_\rho^2\alpha_c\sin\frac12\pi(\alpha_\rho-\alpha_c)\,\gamma_{\frac12-\frac12}{}^{\rho}\gamma_{\frac12-\frac12}{}^{c}(s/s_0)^{\alpha_\rho+\alpha_c-2}}{|\Gamma(\alpha_\rho+1)\Gamma(\alpha_c+1)\sin\frac12\pi\alpha_c\cos\frac12\pi\alpha_\rho|}+\frac{2\alpha_{A_2}\alpha_c\cos\frac12\pi(\alpha_{A_2}-\alpha_c)\,\gamma_{\frac12-\frac12}{}^{A2}\gamma_{\frac12-\frac12}{}^{c}(s/s_0)^{\alpha_{A2}+\alpha_c-2}}{|\Gamma(\alpha_{A_2}+1)\Gamma(\alpha_c+1)\sin\frac12\pi\alpha_c\sin\frac12\pi\alpha_{A_2}|}\Bigg)\\
&+\text{terms down by } O(1/s)\Bigg]. \qquad (43)
\end{aligned}
$$

This expression is not as dreadful as it first appears and lends itself to a sequence of more refined experiments. If we have unpolarized photon beams, we use the values of the B parameters known from π^0 photoproduction, and the pole fit + conspiracy determination of the c and π parameters. Extrapolating the coupling constant from the pole value (or doing a more refined π^0 experiment to get the magnitude of the ρ residues), we can then estimate the size of the A_2 contribution. Once we have polarized photons available, then we notice that at $\varphi_\gamma=90°$ the expression for $d\sigma/dt$ simplifies to

$$\left.\frac{d\sigma}{dt}\right|_{\varphi_\gamma=90°}\approx\frac{1}{2\pi(s-m^2)^2}|\sin\theta_t{}^2|t-4m^2|\left[\frac{\alpha_\pi{}^2|\gamma_{\frac12\frac12}{}^\pi|^2|t|^{-1}(s/s_0)^{2(\alpha_\pi-1)}}{|\Gamma(\alpha_\pi+1)\sin\frac12\pi\alpha_\pi|}+\frac{\alpha_B{}^2|t||\gamma_{\frac12+\frac12}{}^B|(s/s_0)^{2(\alpha_B-1)}}{|\Gamma(\alpha_B+1)\cos\frac12\pi\alpha_B|^2}\right.$$
$$\left.+\frac{2\sin\frac12\pi(\alpha_B-\alpha_\pi)\alpha_B\alpha_\pi\gamma_{\frac12\frac12}{}^\pi\gamma_{\frac12\frac12}{}^B(s/s_0)^{\alpha_B+\alpha_\pi-2}}{|\Gamma(\alpha_\pi+1)\Gamma(\alpha_B+1)\sin\frac12\pi\alpha_\pi\cos\frac12\pi\alpha_B|}\right].\qquad(43')$$

The second and third terms are fairly small and thus could easily be replaced by the values for the B and π parameters determined from π^0 photoproduction, and a pole fit, respectively. Thus, away from the forward direction, we should have no trouble measuring $\alpha_\pi(t)$, and we can see whether or not our simple parametrization of $\gamma^\pi(t)$ suggested by $O(4)$ and the pole fit is valid for a reasonable range of t.

Similarly at $\varphi_\gamma=0$, the first two lines of Eq. (43) vanish and we are left with the contributions of the ρ, c, and A_2. The c is most important for $|t|<-0.1$. The ρ contribution vanishes around $t=-0.6$ GeV. Using the pole fit for c, we should be able to get an upper estimate on the size of the A_2 contribution near $t=-0.6$. Previous experiments suggest that the ρ trajectory is given by

$$\alpha_\rho(t)=0.58+0.96t,\qquad(23')$$

the A_2 by[11]

$$\alpha_{A_2}=0.30+0.69t,\qquad(44a)$$

and the B by[1]

$$\alpha_B=-0.3+0.96t.\qquad(44b)$$

In order to get the correct exponential falloff (e^{3t}), we have seen $\alpha_c'\approx0.45$ implying

$$\alpha_c=0.45t-\mu^2.\qquad(44c)$$

If we consider the difference between the π^+ and π^- photoproduction differential cross sections, we isolate the π-B and ρ-c interference terms. In terms of the π^+ parameters, we obtain

$$\frac{d\sigma^+}{dt}-\frac{d\sigma^-}{dt}=\frac{1}{4\pi(s-m^2)^2}\left[|\sin\theta_t|^2(1+\cos2\varphi_\gamma)|t-4m^2|\frac{4\sin\frac12\pi(\alpha_B-\alpha_\pi)\,\alpha_B\alpha_\pi\gamma_{\frac12\frac12}{}^\pi\gamma_{\frac12\frac12}{}^B(s/s_0)^{\alpha_B+\alpha_\pi-2}}{|\Gamma(\alpha_\pi+1)\Gamma(\alpha_B+1)\sin\frac12\pi\alpha_\pi\cos\frac12\pi\alpha_B|}\right.$$
$$+|\sin\theta_t|^2(1-\cos2\varphi_\gamma)(t-\mu^2)^2\frac{4\alpha_\rho\alpha_c\sin\frac12\pi(\alpha_\rho-\alpha_c)\,\gamma_{\frac12\frac12}{}^\rho\gamma_{\frac12\frac12}{}^c(s/s_0)^{\alpha_\rho+\alpha_c-2}}{|\Gamma(\alpha_\rho+1)\Gamma(\alpha_c+1)\sin\frac12\pi\alpha_c\cos\frac12\pi\alpha_\rho|}$$
$$\left.+(1+\cos^2\theta_t+\cos2\varphi_\gamma\sin2\theta_t)(t-\mu^2)^2\frac{4\alpha_\rho{}^2\alpha_c\sin\frac12\pi(\alpha_\rho-\alpha_c)\,\gamma_{\frac12-\frac12}{}^\rho\gamma_{\frac12-\frac12}{}^c(s/s_0)^{\alpha_\rho+\alpha_c-2}}{|\Gamma(\alpha_\rho+1)\Gamma(\alpha_c+1)\sin\frac12\pi\alpha_c\cos\frac12\pi\alpha_\rho|}\right].\qquad(45)$$

We notice that if we could do polarized photon π^- as well as π^+ photoproduction, then we could isolate the individual interference terms.

We next turn our attention to the outgoing nucleon polarization. As we have said before, this is primarily from same-parity trajectories interfering. Including $c-\rho$ and $c-A_2$ interference, we obtain for $\langle\mathbf{P}\cdot\hat{n}\rangle$

$$-\langle\mathbf{P}\cdot\hat{n}\rangle\frac{d\sigma}{dt}\pi(s-m^2)^2=\tfrac12(1-\cos2\varphi_\gamma)\cos\theta_t|\sin\theta_t|(t-\mu^2)^2$$
$$\times\left[\frac{\cos\frac12\pi(\alpha_c-\alpha_\rho)\,\alpha_c\alpha_\rho[\alpha_\rho|t|^{1/2}\gamma_{\frac12\frac12}{}^c\gamma_{\frac12-\frac12}{}^\rho-|t|^{-1/2}\gamma_{\frac12\frac12}{}^\rho\gamma_{\frac12-\frac12}{}^c](s/s_0)^{\alpha_\rho+\alpha_c-2}}{|\Gamma(\alpha_c+1)\Gamma(\alpha_\rho+1)\sin\frac12\pi\alpha_c\cos\frac12\pi\alpha_\rho|}\right.$$
$$\left.+\frac{\sin\frac12\pi(\alpha_c-\alpha_{A_2})\,\alpha_c\alpha_{A_2}[|t|^{1/2}\gamma_{\frac12\frac12}{}^c\gamma_{\frac12-\frac12}{}^{A_2}-|t|^{-1/2}\gamma_{\frac12\frac12}{}^{A_2}\gamma_{\frac12-\frac12}{}^c](s/s_0)^{\alpha_\rho+\alpha_{A_2}-2}}{|\Gamma(\alpha_c+1)\Gamma(\alpha_{A_2}+1)\sin\frac12\pi\alpha_c\sin\frac12\pi\alpha_{A_2}|}\right].\qquad(46)$$

We notice the following: The $\rho-c$ interference term vanishes at $\alpha_\rho=0$ ($t=-0.6$) and the A_2-c interference term vanishes at $\alpha_A=0$. If we use polarized photon beams, the polarization should behave like $\sin^2\varphi_\gamma$. Any $\cos^2\varphi_\gamma$ behavior is an indication that the A_1 contribution (which we have neglected) is not negligible and that the contribution is coming from $A_1-\pi$ interference. If the outgoing nucleon polarization is measured for both π^+ and π^- photoproduction, we can make use of the fact that $\gamma_{\pi^+}{}^c=\gamma_{\pi^-}{}^c$, $\gamma_{\pi^+}{}^\rho=-\gamma_{\pi^-}{}^\rho$, $\gamma_{\pi^+}{}^{A_2}=\gamma_{\pi^-}{}^{A_2}$ and take the sum and difference of the two polarization measurements to isolate the $\rho-c$ interference term from the A_2-c interference term. Then we can explicitly measure the magnitude of the ρ and A_2 contributions. The other two outgoing nucleon polarizations vanish if the initial particles are unpolarized. For polarized photons then depend primarily on $\pi-c$, $\pi-\rho$, and $\pi-A_2$ interference. For completeness, they are given by

$$\pi\langle\mathbf{P}\cdot\hat{x}\rangle(s-m^2)^2\frac{d\sigma}{dt}=\tfrac{1}{2}\sin 2\varphi_\gamma|\sin\theta_t|\cos\theta_t(t-\mu^2)|t-4m^2|^{1/2}\Bigg[\frac{\sin\frac{1}{2}\pi(\alpha_c-\alpha_\pi)|t|^{-1}\alpha_\pi\alpha_c\gamma_{\frac{1}{2}\frac{1}{2}}{}^{\pi}\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{c}(s/s_0)^{\alpha_\pi+\alpha_c-2}}{|\Gamma(\alpha_c+1)\Gamma(\alpha_\pi+1)\sin\frac{1}{2}\pi\alpha_c\sin\frac{1}{2}\pi\alpha_\pi|}$$
$$+\frac{\cos\frac{1}{2}\pi(\alpha_\rho-\alpha_\pi)\,\alpha_\pi\alpha_\rho{}^2\gamma_{\frac{1}{2}\frac{1}{2}}{}^{\pi}\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{\rho}(s/s_0)^{\alpha_\pi+\alpha_\rho-2}}{|\Gamma(\alpha_\pi+1)\Gamma(\alpha_\rho+1)\sin\frac{1}{2}\pi\alpha_\pi\cos\frac{1}{2}\pi\alpha_c|}+\frac{\sin\frac{1}{2}\pi(\alpha_{A_2}-\alpha_\pi)\alpha_\pi\alpha_{A_2}\gamma_{\frac{1}{2}\frac{1}{2}}{}^{\pi}\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{A_2}(s/s_0)^{\alpha_\pi+\alpha_{A2}-2}}{|\Gamma(\alpha_\pi+1)\Gamma(\alpha_{A_2}+1)\sin\frac{1}{2}\pi\alpha_\pi\sin\frac{1}{2}\pi\alpha_{A_2}|}\Bigg],\quad(47)$$

$$\pi\langle\mathbf{P}\cdot\hat{z}\rangle(s-m^2)^2\frac{d\sigma}{dt}=\tfrac{1}{2}\sin 2\varphi_\gamma|\sin\theta_t|^2(t-\mu^2)|t-4m^2|^{1/2}|t|^{-1/2}\Bigg[\frac{\sin\frac{1}{2}\pi(\alpha_\pi-\alpha_c)\,\gamma_{\frac{1}{2}\frac{1}{2}}{}^{\pi}\gamma_{\frac{1}{2}+\frac{1}{2}}{}^{c}(s/s_0)^{\alpha_\pi+\alpha_c-2}\alpha_\pi\alpha_c}{|\Gamma(\alpha_c+1)\Gamma(\alpha_\pi+1)\sin\frac{1}{2}\pi\alpha_c\sin\frac{1}{2}\pi\alpha_\pi|}$$
$$+\frac{\cos\frac{1}{2}\pi(\alpha_\pi-\alpha_\rho)\,\gamma_{\frac{1}{2}\frac{1}{2}}{}^{\pi}\gamma_{\frac{1}{2}\frac{1}{2}}{}^{\rho}(s/s_0)^{\alpha_\pi+\alpha_\rho-2}\alpha_\pi\alpha_\rho}{|\Gamma(\alpha_\pi+1)\Gamma(\alpha_\rho+1)\sin\frac{1}{2}\pi\alpha_\pi\cos\frac{1}{2}\pi\alpha_\rho|}+\frac{\sin\frac{1}{2}\pi(\alpha_\pi-\alpha_{A_2})\gamma_{\frac{1}{2}\frac{1}{2}}{}^{\pi}\gamma_{\frac{1}{2}\frac{1}{2}}{}^{A_2}(s/s_0)^{\alpha_\pi+\alpha_{A2}-2}\alpha_{A_2}\alpha_\pi}{|\Gamma(\alpha_\pi+1)\Gamma(\alpha_{A_2}+1)\sin\frac{1}{2}\pi\alpha_\pi\sin\frac{1}{2}\pi\alpha_{A_2}|}\Bigg].\quad(48)$$

V. K-MESON PHOTOPRODUCTION

A. Contributions of Trajectories to Various Charge States

For K-meson photoproduction we can exchange $K(490)$, $K^*(890)$, $K^{**}(1420)$, $K_A(1320)$. If the K conspires in an analogous fashion to the pion, there is also a K_c. For notational convenience we will denote K^* by V, K^{**} by T, K_A, by A, and K_c by c. In order to find the relative contributions of these trajectories to the possible final charge states and to relate the residue functions of this reaction to those found in pion photoproduction, we must appeal to $SU(3)$.

Photon Vertex

In $SU(3)$ the electromagnetic current is assumed to be a pure octet current of the form

$$J_\mu{}^e=F_\mu{}^3+(1/\sqrt{3})F_\mu{}^8.\quad(49)$$

The pseudoscalar mesons are assigned to an octet and the vector and tensor mesons are assigned to a nonet in the fashion of Okubo[20] which takes into account mixing of the isoscalar mesons. The $SU(3)$ matrix element for the photon vertex $\langle P|J_\mu|X\rangle$, where P stands for the pseudoscalar octet and X is any octet or nonet, will in general have both F- and D-type couplings. If $X=P$, then generalized Bose statistics allows only F-type couplings. If $X=V$ (vector-meson nonet), then if we use the vector-dominance relation

$$J_\mu{}^e=\sqrt{3}\rho_\mu+\omega_{8\mu},\quad(50)$$

generalized Bose statistics allows only D-type coupling. If $X=T$ (tensor-meson nonet), then we find that $A_2{}^0\to\rho^0+\gamma$ is pure D-type coupling. But this vertex violates charge-conjugation invariance, therefore only F coupling is allowed. Thus, $SU(3)$ implies that only the $K^{*0}\equiv V^0$ can be exchanged for neutral K photoproduction. For the $\gamma\pi$ and $K\gamma$ vertices, the only coupling constants that are nonzero are

$$g_{\gamma K^+K^+}=g_{\gamma\pi^+\pi^+}=-g_{\gamma K^-K^-}=-g_{\gamma\pi^-\pi^-}=e,$$
$$g_{\gamma K^+V^+}=g_{\gamma\pi\rho}=-\tfrac{1}{3}g_{\gamma\pi\omega}=-\tfrac{1}{2}g_{\gamma K^0V^0},$$
$$g_{\gamma K^+T^+}=g_{\gamma\pi^+A_2^+}=-g_{\gamma K^-T^-}=-g_{\gamma\pi^-A_2^-},\quad(51)$$
$$(V=K^*,\ T=K^{**}).$$

2. Baryon Vertex

The general form of the BBX vertex (X=octet or nonet) is given by

$$\sqrt{2}\gamma(f\langle\bar{B}[V,B]_-\rangle+d\langle\bar{B}\{V,B\}_+\rangle+\beta\langle V\rangle\langle\bar{B}B\rangle),\quad(52)$$

where $\langle\ \ \rangle$ means trace. One often introduces the $SU(3)$ mixing parameter α by $f=1-\alpha$, $d=\alpha$.

For the PBB vertex there is only one coupling constant and we find

$$g_{\bar{p}K^+\Lambda}=g_{\bar{n}K^0\Lambda}=[(3-2\alpha)/\sqrt{3}]g_{NN\pi},$$
$$g_{\bar{p}K^+\Sigma^0}=-g_{\bar{n}K^0\Sigma^0}=(1/\sqrt{2})g_{\bar{p}K^0\Sigma^+}=(1/\sqrt{2})g_{\bar{n}K^+\Sigma^-}$$
$$=-(1-2\alpha)g_{NN\pi}.\quad(53)$$

There have been several recent dispersion-theory calculations of these coupling constants. Kim finds[21] that

$$g_{KN\Lambda}{}^2=13.5\pm2.2,\quad g_{\pi N\Sigma}{}^2=0.2\pm0.4.\quad(54a)$$

This is consistent with the work of Chan and Meire[22] who found that

$$g_{KN\Lambda}{}^2=13.0\pm3,\quad g_{KN\Sigma}{}^2=0.0\pm1.\quad(54b)$$

Both these results are consistent with $SU(3)$ and yield ($g_{\pi NN}{}^2=14.5$) $\alpha\approx0.6$. If K exchange were the dominant

[20] S. Okubo, Phys. Letters 5, 165 (1963).

[21] J. Kim, Phys. Rev. Letters 19, 1079 (1967).

[22] C. Chan and F. Meiere, Phys. Rev. Letters 20, 568 (1968).

process in K^+ photoproduction, then

$$\frac{d\sigma}{dt}(\Sigma_0)\Big/\frac{d\sigma}{dt}(\Lambda_0)=3\left(\frac{1-2\alpha}{3-2\alpha}\right)^2\approx\frac{1}{27}. \tag{55}$$

This in violent disagreement with the data[23] which suggest the ratio is near one except perhaps in the extreme forward direction. Since the K pole is not much nearer $t=0$ than the K^* pole, the kinematic enhancement of the K relative to the other trajectories resulting from the pole offsets the lowness of the K trajectory only in the extreme forward direction where the other trajectories become kinematically suppressed because they must evade.

For the VBB vertex there are two coupling constants corresponding to electric coupling and anomalous magnetic moment coupling (or alternatively to helicity-flip and helicity-nonflip coupling). Each of these two types of coupling has its own α. We find that

$$g_{\bar{p}V^+\Sigma^0}=g_{\bar{n}V^-\Sigma^0}=[-(3-2\alpha)/\sqrt{3}]g_{\rho NN}\,,$$

$$g_{\bar{p}V^+\Sigma^0}=-g_{\bar{n}V^-\Sigma^0}=(1/\sqrt{2})g_{\bar{p}V^0\Sigma^+}=(1/\sqrt{2})g_{\bar{n}V^0\Sigma^-}=-(1-2\alpha)g_{\rho NN}. \tag{56}$$

We will discuss what is known about α_{flip} and α_{nonflip} later.

For the TBB vertex we again have two coupling constants corresponding to spin-flip and spin-nonflip for the Baryons. There are again two f/d ratios and we find that

$$g_{\bar{p}T^+\Lambda}=g_{\bar{n}T^0\Lambda}=[-(3-2\alpha)/\sqrt{3}]g_{A_2\bar{N}N}\,,$$

$$g_{\bar{p}\Sigma^0T^+}=-g_{\bar{n}T^-\Sigma^0}=(1/\sqrt{2})g_{\bar{p}T^0\Sigma^+}=(1/\sqrt{2})g_{\bar{n}T^0\Sigma^-}=-(1-2\alpha)g_{\bar{N}NA_2}. \tag{57}$$

We leave the discussion of the experimental values found for α to later.

B. Conspiracy Relations

The nature of the constraint equations for K photoproduction is slightly different from those for π photoproduction, since we are now dealing with the unequal-mass case. In the unequal-mass case, at $t=0$, there is a conspiracy of the type discussed by Jones.[24] Since at $t=0$, $\cos\theta_t=-1+O(t)$, the maximum singularity of $f_{\frac{1}{2}-\frac{1}{2}}{}^{+}+f_{\frac{1}{2}-\frac{1}{2}}{}^{-}$ is $1/t$ and that of $f_{\frac{1}{2}-\frac{1}{2}}{}^{+}-f_{\frac{1}{2}-\frac{1}{2}}{}^{-}$ is t^0. The $1/t$ singularity which behaves as s in the forward direction is necessary if a trajectory is not to be kinematically suppressed. Thus, to prevent suppression and to satisfy the above two relations, one must have the following "conspiracy" at $t=0$:

$$f_{\frac{1}{2}-\frac{1}{2}}{}^{+}=f_{\frac{1}{2}-\frac{1}{2}}{}^{-}=c/t. \tag{58}$$

This result can also be obtained directly by looking at the connection between the invariant amplitudes and the parity-conserving helicity amplitudes as found in Ref. 1, for example.

Owing to the small difference in mass between the two baryons involved, there is now a pseudothreshold near the physical region $t=(m-Y)^2$, where Y is the Σ or Λ mass. If we now look at the relationship between the parity-conserving helicity amplitudes and the invariant amplitudes, we find that near this pseudothreshold analyticity requires that the following relationships must be satisfied:

$$\begin{aligned} f_{\frac{1}{2}\frac{1}{2}}{}^{+}+\frac{Y+m}{Y-m}f_{\frac{1}{2}-\frac{1}{2}}{}^{+}&=O([t-(m-Y)^2]^{1/2})\,,\\ f_{\frac{1}{2}\frac{1}{2}}{}^{-}-f_{\frac{1}{2}-\frac{1}{2}}{}^{-}&=O(t-(m-Y)^2)\,. \end{aligned} \tag{59}$$

For similar analyticity reasons, the first of these two relations must be satisfied by evasion. The second relation can be satisfied by conspiracy:

$$f_{\frac{1}{2}\frac{1}{2}}{}^{-}=f_{\frac{1}{2}-\frac{1}{2}}{}^{-}\quad\text{at}\quad t=(m-Y)^2\,, \tag{60a}$$

or evasion:

$$f_{\frac{1}{2}\frac{1}{2}}{}^{-}\quad\text{and}\quad f_{\frac{1}{2}-\frac{1}{2}}{}^{-}\sim t-(m-Y)^2\,. \tag{60b}$$

We notice that the two nonvacuous conspiracy relations go over to the single conspiracy relation found for pion photoproduction as we let $Y\to m$.

The situation for K exchange is the following: The Reggeized K contributes to both $f_{\frac{1}{2}\frac{1}{2}}{}^{-}$ and $f_{\frac{1}{2}-\frac{1}{2}}{}^{-}$, the two being related by the conspiracy relation at $t=(m-Y)^2$. The conspiracy at $t=0$ allows $f_{\frac{1}{2}-\frac{1}{2}}{}^{-}$ to behave as t^{-1} if a conspirator c exists. This allows both the K and the c to contribute in the forward direction.

C. Parametrization of Regge-Pole Contributions

(1) K trajectory. We assume that the K is conspiring and chooses sense in the nonsense region and find that

$$\begin{aligned} f_{\frac{1}{2}\frac{1}{2}}{}^{K-}&=\frac{-\alpha(1+e^{-i\pi\alpha})[t-(Y+m)^2]^{1/2}t^{-1/2}\gamma_{\frac{1}{2}\frac{1}{2}}{}^{K}(s/s_0)^{\alpha_K-1}}{\Gamma(\alpha+1)\sin\pi\alpha}\,,\\ f_{\frac{1}{2}-\frac{1}{2}}{}^{K-}&=\frac{\alpha^2(1+e^{-i\pi\alpha})[t-(m+Y)^2]^{1/2}t^{-1}\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{K}(s/s_0)^{\alpha-1}}{\Gamma(\alpha+1)\sin\pi\alpha}\,,\\ f_{\frac{1}{2}-\frac{1}{2}}{}^{K+}&=\frac{-2\alpha(\alpha-1)(1+e^{-i\pi\alpha})[t-(m+Y)^2][t-(m-Y)^2]^{1/2}(s/s_0)^{\alpha-2}(t-m_K{}^2)\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{K}}{s_0\Gamma(\alpha+1)\sin\pi\alpha}. \end{aligned} \tag{61}$$

[23] B. Richter, Stanford Linear Accelerator Center Report No. SLAC 353, 1967 (unpublished).
[24] S. Frautschi and L. Jones, Phys. Rev. 169, 1335 (1968).

(2) C trajectory. We assume that the K_c chooses nonsense as does the π_c and assume that it is conspiring with the K.

$$f_{\frac{1}{2}\frac{1}{2}}{}^{c+}=\frac{-\alpha(1+e^{-i\pi\alpha})(t-m_K{}^2)[t-(Y-m)^2]^{1/2}t^{-1/2}\gamma_{\frac{1}{2}\frac{1}{2}}{}^c(s/s_0)^{\alpha-1}}{\Gamma(\alpha+1)\sin\pi\alpha},$$

$$f_{\frac{1}{2}-\frac{1}{2}}{}^{c+}=\frac{\alpha(1+e^{-i\pi\alpha})(t-m_K{}^2)[t-(m-Y)^2]^{1/2}t^{-1}\gamma_{\frac{1}{2}-\frac{1}{2}}{}^c(s/s_0)^{\alpha-1}}{\Gamma(\alpha+1)\sin\pi\alpha},\tag{62}$$

$$f_{\frac{1}{2}-\frac{1}{2}}{}^{c-}=\frac{-2(\alpha-1)(1+e^{-i\pi\alpha})[t-(m+Y)^2]^{1/2}(t-m_K{}^2)^2[t-(m-Y)^2]\gamma_{\frac{1}{2}-\frac{1}{2}}{}^c(s/s_0)^{\alpha-2}}{s_0\Gamma(\alpha+1)\sin\pi\alpha}.$$

(3) V [$K^*(890)$] trajectory. We assume that the V is similar to the ρ trajectory and chooses sense.

$$f_{\frac{1}{2}\frac{1}{2}}{}^{V+}=\frac{-\alpha(1-e^{-i\pi\alpha})(t-m_K{}^2)[t-(m-Y)^2]^{1/2}t^{-1/2}\gamma_{\frac{1}{2}\frac{1}{2}}{}^V(s/s_0)^{\alpha-1}}{\Gamma(\alpha+1)\sin\pi\alpha},$$

$$f_{\frac{1}{2}-\frac{1}{2}}{}^{V+}=\frac{\alpha^2(1-e^{-i\pi\alpha})(t-m_K{}^2)[t-(m-Y)^2]^{1/2}\gamma_{\frac{1}{2}-\frac{1}{2}}{}^V(s/s_0)^{\alpha-1}}{\Gamma(\alpha+1)\sin\pi\alpha},\tag{63}$$

$$f_{\frac{1}{2}-\frac{1}{2}}{}^{V-}=\frac{-2(\alpha-1)\alpha[t-(m+Y)^2]^{1/2}[t-(m-Y)^2](t-m_K{}^2)^2\gamma_{\frac{1}{2}-\frac{1}{2}}{}^V(s/s_0)^{\alpha-2}}{s_0\Gamma(\alpha+1)\sin\pi\alpha}.$$

(4) T (K^{**}) trajectory. We assume that the T trajectory is similar to the A_2 trajectory and thus chooses nonsense.

$$f_{\frac{1}{2}\frac{1}{2}}{}^{T+}=\frac{-\alpha(1+e^{-i\pi\alpha})(t-m_K{}^2)[t-(m-Y)^2]^{1/2}t^{-1/2}(s/s_0)^{\alpha-1}\gamma_{\frac{1}{2}\frac{1}{2}}{}^T}{\Gamma(\alpha+1)\sin\pi\alpha},$$

$$f_{\frac{1}{2}-\frac{1}{2}}{}^{T+}=\frac{\alpha(1+e^{-i\pi\alpha})(t-m_K{}^2)[t-(m-Y)^2]^{1/2}(s/s_0)^{\alpha-1}\gamma_{\frac{1}{2}-\frac{1}{2}}{}^T}{\Gamma(\alpha+1)\sin\pi\alpha},\tag{64}$$

$$f_{\frac{1}{2}-\frac{1}{2}}{}^{T-}=\frac{-2(\alpha-1)(1+e^{-i\pi\alpha})[t-(m+Y)^2]^{1/2}[t-(m-Y)^2](t-m_K{}^2)^2\gamma_{\frac{1}{2}-\frac{1}{2}}{}^T(s/s_0)^{\alpha-2}}{s_0\Gamma(\alpha+1)\sin\pi\alpha}.$$

(5) A (K_A) trajectory. We assume that this octet companion of the A_1 chooses sense.

$$f_{\frac{1}{2}\frac{1}{2}}{}^{A-}=\frac{-\alpha(1-e^{-i\pi\alpha})[t-(m+Y)^2]^{1/2}t^{-1/2}[t-(m-Y)^2]\gamma_{\frac{1}{2}\frac{1}{2}}{}^A(s/s_0)^{\alpha-1}}{\Gamma(\alpha+1)\sin\pi\alpha},$$

$$f_{\frac{1}{2}-\frac{1}{2}}{}^{A-}=\frac{\alpha^2(1-e^{-i\pi\alpha})[t-(m+Y)^2]^{1/2}[t-(m-Y)^2]\gamma_{\frac{1}{2}-\frac{1}{2}}{}^A(s/s_0)^{\alpha-1}}{\Gamma(\alpha+1)\sin\pi\alpha},\tag{65}$$

$$f_{\frac{1}{2}-\frac{1}{2}}{}^{A+}=\frac{-2\alpha(\alpha-1)(1-e^{-i\pi\alpha})[t-(m+Y)^2][t-(m-Y)^2]^{1/2}\gamma_{\frac{1}{2}-\frac{1}{2}}{}^A(s/s_0)^{\alpha-2}}{s_0\Gamma(\alpha+1)\sin\pi\alpha}.$$

D. K^0 Photoproduction

K^0 photoproduction is an ideal reaction to test the Regge-pole model since $SU(3)$ allows only the K^{*0} to be exchanged. In a parametrization of four reactions where only K^* and K^{**} could be exchanged, Sarma and Reeder[25] found that the K^* trajectory was similar to the ρ and was given by

$$\alpha_{K^*}=0.35+0.96t+0.16t^2.\tag{66}$$

They found that the dip in the cross section as well as the zero in the polarization near $t=-0.4$ could be explained by having the K^* and K^{**} residues at the nonsense value $\alpha=0$. Thus in K^0 photoproduction there should be a

[25] K. Sarma and D. Reeder, Nuovo Cimento 53A, 808 (1968).

large minimum (if not zero) near $t=-0.4$, and we should unambiguously be able to determine the trajectory function. Since there is no polarization if only one trajectory can be exchanged, we write down only the parametrization of the differential cross section.

$$\frac{d\sigma}{dt}=\frac{1}{4\pi(s-m^2)^2}\frac{1}{|\cos\frac{1}{2}\pi\alpha\Gamma(\alpha+1)|^2}\Big[|\sin\theta_t|^2(1-\cos 2\varphi_\gamma)(t-m_K^2)^2[t-(m-Y)^2]t^{-1}\alpha^2|\gamma_{\frac{1}{2}\frac{1}{2}}{}^V|^2\left(\frac{s}{s_0}\right)^{2(\alpha-1)}$$
$$+(1+\cos^2\theta_t+\cos 2\varphi_\gamma\sin^2\theta_t)\alpha^4(t-m_K^2)^2|t-(m-Y)^2|\gamma_{\frac{1}{2}-\frac{1}{2}}{}^V|^2\left(\frac{s}{s_0}\right)^{2(\alpha-1)}+(1+\cos^2\theta_t-\cos 2\varphi_\gamma\sin^2\theta_t)4\alpha^2(\alpha-1)^2$$
$$\times(t-m_K^2)^4[t-(m-Y)^2]^2|t-(m+Y)^2|\gamma_{\frac{1}{2}-\frac{1}{2}}{}^V|^2\frac{1}{s_0^2}\left(\frac{s}{s_0}\right)^{2(\alpha-2)}-8\cos\theta_t\alpha^3(\alpha-1)(t-m_K^2)^3|t-(m-Y)^2|^{3/2}$$
$$\times|t-(m+Y)^2|^{1/2}\frac{1}{s_0}\left(\frac{s}{s_0}\right)^{2\alpha-3}|\gamma_{\frac{1}{2}-\frac{1}{2}}{}^V|^2\Big]. \quad (67)$$

We notice that the use of polarized photons gives no added information unless opposite-parity $[P=-(-1)^J]$ trajectories or cuts are present. If we use polarized photons and find deviations from $\sin^2\varphi_\gamma$ behavior, this would indicate the presence of a reasonably high opposite-parity trajectory or the presence of Regge cuts.[4]

We can estimate the size of the Regge residue functions using experimental f/d ratios and a pole fit. The elementary-particle K^* exchange can be put in a form similar to (18″) giving

$$\begin{aligned} f_{\frac{1}{2}\frac{1}{2}}{}^+ &= \tfrac{1}{4}(t-m_K^2)[t-(m-Y)^2]^{1/2}t^{-1/2} \\ &\quad\times[(m+Y)g_1+g_2t]g_{\gamma K^0K^{0*}}, \\ f_{\frac{1}{2}-\frac{1}{2}}{}^+ &= -\tfrac{1}{4}(t-m_K^2)[t-(m-Y)^2]^{1/2} \\ &\quad\times[g_1+(m+Y)g_2]g_{\gamma K^0K^{0*}}. \end{aligned} \quad (68)$$

Thus, the pole fit to the residue functions yields for $t\approx m_V^2$

$$\begin{aligned} \gamma_{\frac{1}{2}\frac{1}{2}}{}^V &= \tfrac{1}{8}\pi\alpha'(t=m_V^2)[(m+Y)g_1+g_2t]g_{\gamma K^0K^{0*}}, \\ \gamma_{\frac{1}{2}-\frac{1}{2}}{}^V &= \pi\alpha'(t=m_V^2)[g_1+(m+Y)g_2]g_{\gamma K^0K^{0*}}. \end{aligned} \quad (69)$$

$SU(3)$ tells us that

$$g_{\gamma K^0K^{0*}}=\tfrac{2}{3}g_{\gamma\pi\omega}=-2g_{\gamma\pi\rho}$$
and (70)
$$g_{\bar{p}V^0\Sigma^+}=-\sqrt{2}(1-2\alpha)g_{\rho NN}.$$

There are conflicting values for the f/d ratios. Sarma and Reeder[25] find for the vector mesons, using a t-independent $\gamma_{\frac{1}{2}\frac{1}{2}}{}^V$,

$$(f/d)_{\text{nonflip}}=-6.5\,,\quad (f/d)_{\text{flip}}=0.36$$
$$(\text{i.e., }\alpha_{\text{flip}}=-0.18\,,\quad \alpha_{\text{nonflip}}=1.52).$$

This leads to $\gamma_{\frac{1}{2}\frac{1}{2}}{}^V\approx 4\gamma_{\frac{1}{2}\frac{1}{2}}{}^\rho$ near $t=m_V^2$ for our t-dependent fit

$$\gamma_{\frac{1}{2}-\frac{1}{2}}{}^V\approx -6\gamma_{\frac{1}{2}-\frac{1}{2}}{}^\rho. \quad (71)$$

In a different sort of fit, Sarma and Renninger found that the total f/d ratio found from charge exchange scattering at $t=0$ was consistent with $f/d=-1.5\pm 0.9$. The errors are too large to allow us to relate the ρ and V coupling constants. Barger and Olsson found[26] for the nonflip amplitudes $f/d=-2.0$. This would give $\alpha=-1$ and

$$\gamma_{\frac{1}{2}\frac{1}{2}}{}^V\approx 8.5\gamma_{\frac{1}{2}\frac{1}{2}}{}^\rho. \quad (72)$$

The ρ contribution to π^0 photoproduction is probably in the vicinity of 10–20%. These above guesses to the f/d ratios, then, say that very crudely the K^0 cross section is, within one order of magnitude, the same as the π^0 cross section. What we do learn, however, is that if the ρ contribution to π^0 photoproduction is known fairly accurately, then this cross section would be a good way of determining the f/d ratios for the vector mesons. An even more sensitive test would be to compare the following cross sections (which are extremely difficult to measure):

$$\begin{aligned} \frac{(d\sigma/dt)(\gamma+N\to K^0+\Sigma^0)}{(d\sigma/dt)(\gamma+N\to K^0+\Lambda^0)} &= \frac{3(1-2\alpha)^2}{(3-2\alpha)^2}, \\ \frac{(d\sigma/dt)(\gamma+P\to K^0+\Sigma^+)}{(d\sigma/dt)(\gamma+N\to K^0+\Lambda^0)} &= \frac{6(1-2\alpha)^2}{(3-2\alpha)^2}. \end{aligned} \quad (73)$$

E. K^+ Photoproduction

K^+ photoproduction appears to qualitatively different from π^+ photoproduction in that the data[23] do not show any indication of a peak in the forward direction. Symmetry arguments would lead one to suspect a great similarity, but the kinematical fact that the pion pole is very near the physical region causes the pion to predominate in the forward direction in spite of its relatively low height. In K^+ photoproduction the K pole does not lead to such a large kinematic enhancement of the K-exchange contribution, making V and T exchange more important. The fact that K exchange is not the dominating process can be seen from the fact

[26] V. Barger and M. Olsson, Phys. Rev. Letters **18**, 294 (1967).

that pure K exchange predicts that

$$\frac{(d\sigma/dt)|_{\Sigma^0}}{(d\sigma/dt)|_{\Lambda^0}} \approx \frac{1}{27}, \tag{55}$$

whereas experimentally the ratio is near to one for t not in the extreme forward direction. In the Regge picture with conspiring K and c, the photoproduction cross-section ratios extrapolated to $t=0$ should depend only on K and c exchange and should give the above ratio, since the other trajectories do not contribute at $t=0$.[2] There are indications that this is indeed the case. Once we know the size of K^0 photoproduction, we will know how large the V contribution to K^+ photoproduction since the $SU(3)$ relations $g_{\gamma K^+K^+} = -\frac{1}{2} g_{\gamma K^0 K^0}$, $g_{K^+N\Sigma} = (1/\sqrt{2}) g_{K^0 N\Sigma}$ imply that the V contribution to K^+ production is $\frac{1}{8}$ its contribution to K^0 production. In our parametrization of the differential cross section, we will neglect the K_A trajectory, since nothing is known about it and neglect the V-T interference since the trajectories are nearly degenerate. Sarma and Reeder[25] have fitted their data with

$$\begin{aligned} \alpha_{K^*} &= 0.35 + 0.96t + 0.16t^2, \\ \alpha_{K^{**}} &= 0.24 + 0.69t. \end{aligned} \tag{74}$$

We obtain the following parametrization of $d\sigma/at$:

$$\begin{aligned}
\frac{d\sigma}{dt} = \frac{1}{4\pi(s-m^2)^2} \Bigg[& \frac{|\sin\theta_t|^2 (1+\cos 2\varphi_\gamma) |t-(Y+m)^2| |t|^{-1} |\gamma_{\frac12\frac12}{}^K|^2 (s/s_0)^{2(\alpha_K-1)} \alpha_K{}^2}{|\Gamma(\alpha+1) \sin\frac12\pi\alpha_K|^2} + (1-\cos 2\varphi_\gamma) \\
& \times |\sin\theta_t|^2 (t-m_K{}^2)^2 |t-(m-Y)^2| \Bigg(\frac{\alpha_c{}^2 |t|^{-1} |\gamma_{\frac12\frac12}{}^c|^2 (s/s_0)^{2(\alpha_c-1)}}{|\Gamma(\alpha_c+1)\sin\frac12\pi\alpha_c|^2} + \frac{\alpha_V{}^2 |t|^{-1} |\gamma_{\frac12\frac12}{}^V|^2 (s/s_0)^{2(\alpha_V-1)}}{|\Gamma(\alpha_V+1)\cos\frac12\pi\alpha_V|^2} \\
& + \frac{\alpha_T{}^2 |t|^{-1} |\gamma_{\frac12\frac12}{}^T|^2 (s/s_0)^{2(\alpha_T-1)}}{|\Gamma(\alpha_T+1)\sin\frac12\pi\alpha_T|^2} + \frac{2\alpha_c\alpha_V \sin\frac12\pi(\alpha_V-\alpha_c) \gamma_{\frac12\frac12}{}^c \gamma_{\frac12\frac12}{}^V (s/s_0)^{\alpha_c+\alpha_V-2}}{|\Gamma(\alpha_c+1)\Gamma(\alpha_V+1)\sin\frac12\pi\alpha_c \cos\frac12\pi\alpha_V|} \\
& + \frac{2\alpha_c\alpha_T \gamma_{\frac12\frac12}{}^c \gamma_{\frac12\frac12}{}^T \cos\frac12\pi(\alpha_T-\alpha_c) (s/s_0)^{\alpha_c+\alpha_T-2}}{|\Gamma(\alpha_c+1)\Gamma(\alpha_T+1)\sin\frac12\pi\alpha_c \sin\frac12\pi\alpha_T|} \Bigg) + (1+\cos^2\theta_t - \cos 2\varphi_\gamma \sin^2\theta_t) \\
& \times \frac{\alpha_K{}^4 |t-(m+Y)^2| t^{-2} |\gamma_{\frac12-\frac12}{}^K|^2 (s/s_0)^{2(\alpha_K-2)}}{|\Gamma(\alpha_K+1)\sin\frac12\pi\alpha_K|^2} + (1+\cos^2\theta_t + \cos 2\varphi_\gamma \sin^2\theta_t)(t-m_K{}^2)^2 \\
& \times |t-(m-Y)|^2 \Bigg(\frac{|\gamma_{\frac12-\frac12}{}^c|^2 (s/s_0)^{2(\alpha_c-1)} \alpha_c{}^2 |t|^{-2}}{|\Gamma(\alpha_c+1)\sin\frac12\pi\alpha_c|} + \frac{\alpha_V{}^4 |\gamma_{\frac12-\frac12}{}^V|^2 (s/s_0)^{2(\alpha_V-1)}}{|\Gamma(\alpha_V+1)\cos\frac12\pi\alpha_V|} + \frac{\alpha_T{}^2 |\gamma_{\frac12-\frac12}{}^T|^2 (s/s_0)^{2(\alpha_T-1)}}{|\Gamma(\alpha_T+1)\sin\frac12\pi\alpha_T|^2} + 2\sin\tfrac12\pi \\
& \times (\alpha_V-\alpha_c)|t|^{-1} \frac{\gamma_{\frac12-\frac12}{}^c \gamma_{\frac12-\frac12}{}^V (s/s_0)^{\alpha_c+\alpha_V-2}}{|\Gamma(\alpha_c+1)\Gamma(\alpha_V+1)\sin\frac12\pi\alpha_c\cos\frac12\pi\alpha_V|} + \frac{2\cos\frac12\pi(\alpha_T-\alpha_c)\, \gamma_{\frac12-\frac12}{}^c \gamma_{\frac12-\frac12}{}^T (s/s_0)^{\alpha_c+\alpha_T-2}}{|\Gamma(\alpha_c+1)\Gamma(\alpha_T+1)\sin\frac12\pi\alpha_c\cos\frac12\pi\alpha_V|} \Bigg) + 4\cos\theta_t (t-m_K{}^2) \\
& \times [(t-(m+Y)^2][t-(m-Y)^2]|^{1/2} \gamma_{\frac12-\frac12}{}^K (s/s_0)^{\alpha_K-1} \alpha_K{}^2 \Bigg(\frac{t^{-2} \gamma_{\frac12-\frac12}{}^c \alpha_c \cos\frac12\pi(\alpha_K-\alpha_c)(s/s_0)^{\alpha_c-1}}{|\sin\frac12\pi\alpha_c \Gamma(\alpha_c+1)\Gamma(\alpha_K+1)\sin\frac12\pi\alpha_K|} \\
& + \frac{t^{-1}\sin\frac12\pi(\alpha_V-\alpha_\pi)\alpha_V{}^2 \gamma_{\frac12-\frac12}{}^V (s/s_0)^{\alpha_V-1}}{|\Gamma(\alpha_K+1)\Gamma(\alpha_V+1)\sin\frac12\pi\alpha_K\cos\frac12\pi\alpha_V|} + \frac{t^{-1}\alpha_T \gamma_{\frac12-\frac12}{}^T \cos\frac12\pi(\alpha_K-\alpha_T)(s/s_0)^{\alpha_T-1}}{|\Gamma(\alpha_T+1)\Gamma(\alpha_K+1)\sin\frac12\pi\alpha_K\sin\frac12\pi\alpha_V|} \Bigg) \Bigg].
\end{aligned} \tag{75}$$

We notice that if we have a polarized photon beam and set $\varphi_\gamma = 0$ and we are not in the forward direction, we can isolate the contribution of the K trajectory exchange and obtain

$$\begin{aligned}
\left.\frac{d\sigma}{dt}\right|_{\gamma=0} \sim & \frac{(s/s_0)^{2(\alpha_K-1)}}{2\pi(s-m^2)^2} \frac{1}{|\Gamma(\alpha+1)\sin\frac12\pi\alpha_K|^2} \\
& \times [|\sin\theta_t|^2 |t-(m+Y)^2| \, |t|^{-1} |\gamma_{\frac12\frac12}{}^K|^2 \alpha_K{}^2 \\
& \times \cos^2\theta_t \, \alpha_K{}^4 |t-(m+Y)^2| t^{-2} |\gamma_{\frac12-\frac12}{}^K|^2].
\end{aligned} \tag{75'}$$

If we take the conspiracy hypothesis seriously, then $\gamma_{\frac12\frac12}{}^K$, $\gamma_{\frac12-\frac12}{}^K$ and $\gamma_{\frac12-\frac12}{}^c$ are known from the known pole value of the coupling constant $\gamma_{\frac12\frac12}{}^K(t=m_K{}^2)$ and the conspiracy relations relating $\gamma_{\frac12\frac12}{}^K$, $\gamma_{\frac12-\frac12}{}^K$ and $\gamma_{\frac12-\frac12}{}^K$, $\gamma_{\frac12-\frac12}{}^c$. The contribution of elementary K exchange to $f_{\frac12\frac12}$ is

$$f_{\frac12\frac12}{}^- = \frac{[t-(m-Y)^2][t-(m+Y)^2]^{1/2} t^{-1/2} e g_{KNY}}{2(s-m^2)}. \tag{76}$$

Thus, near $t=m_k{}^2$ we obtain for large s

$$\gamma_{\frac12\frac12}{}^K = -\tfrac14 \pi\alpha'(m_K{}^2)(eg/s_0)[m_K{}^2-(m-Y)^2]. \tag{77}$$

At $t=(m-Y)^2$ we have the conspiracy condition $f_{\frac{1}{2}\frac{1}{2}}{}^{-}=f_{\frac{1}{2}-\frac{1}{2}}{}^{-}$ which yields

$$\gamma_{\frac{1}{2}\frac{1}{2}}{}^{K}[(m-Y)^2]=\frac{\alpha}{m-Y}\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{K}\approx-\tfrac{1}{4}(m-Y)\pi\alpha'(m_K{}^2)\times\frac{eg}{s_0}\frac{\gamma_{\frac{1}{2}\frac{1}{2}}[(m-Y)^2]}{\gamma_{\frac{1}{2}\frac{1}{2}}(m_K{}^2)}. \quad (78)$$

At $t=0$ the conspiracy relation $f_{\frac{1}{2}-\frac{1}{2}}{}^{-}=f_{\frac{1}{2}-\frac{1}{2}}{}^{+}$ leads to

$$\alpha_K(0)=\alpha_c(0)\approx-m_K{}^2$$

and

$$\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{c}(0)=\tfrac{1}{4}(Y+m)\frac{\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{K}(0)}{\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{K}[(m-Y)^2]}\times\frac{\gamma_{\frac{1}{2}\frac{1}{2}}{}^{K}[(m-Y)^2]}{\gamma_{\frac{1}{2}\frac{1}{2}}{}^{K}(m_K{}^2)}\pi\alpha'(m_K{}^2)\frac{eg}{s_0}. \quad (79)$$

As in the case of the pion conspiracy, we assume a one-parameter t dependence for $\gamma_{\frac{1}{2}\frac{1}{2}}{}^{K}$

$$\gamma_{\frac{1}{2}\frac{1}{2}}{}^{K}=-\tfrac{1}{4}\pi\alpha'(m_K{}^2)\frac{eg}{s_0}[m_K{}^2-(m-Y)^2]\times\left[1+\frac{\lambda(t-m_K{}^2)}{m_K{}^2}\right]. \quad (80)$$

In their fit to forward K^+ photoproduction, Ball *et al.*[2] found that $\lambda\approx\frac{3}{4}$. Assuming that

$$\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{K}(0)/\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{K}[(m-Y)^2]=1,$$

we obtain

$$\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{c}(0)=\tfrac{1}{4}\pi\alpha'(m_K{}^2)\frac{eg}{s_0}(Y+m)\times\left[(1-\lambda)+\frac{\lambda(m-Y)^2}{m_K{}^2}\right]. \quad (81)$$

As has been stated before, the coupling constants are known from dispersion-theory analysis:

$$g_{KN\Lambda}{}^2=13.5\pm2.2, \quad g_{KN\Sigma}{}^2=0.2\pm0.4, \quad g_{\pi NN}{}^2=14.5.$$

Thus, if we can do the polarized-photon experiment, we can check to see if the K trajectory is as expected, i.e., $\alpha_K\approx t-m_K{}^2$, and see if the t dependence of the residue functions is that suggested by $O(4)$ and $SU(3)$ breaking. The V coupling constants will be known, once a K^0 photoproduction experiment is done, which leaves only the T coupling constants unknown. The T coupling constants are related to the A_2 contribution to π^+ photoproduction by $SU(3)$ if we know the tensor-meson f/d ratios. The analysis of Sarma and Reeder yielded[25]

$$(f/d)_{\text{nonflip}}=-1.7, \quad (f/d)=_{\text{flip}}-1.8.$$

Unfortunately, we do not as yet know the A_2 contribution to π^+ photoproduction. However, the above f/d ratios tell us that for T exchange alone

$$\frac{d\sigma}{dt}(\Sigma_0)\Big/\frac{d\sigma}{dt}(\Lambda_0)\approx1.9. \quad (82)$$

If we really believe the above f/d ratios, then this suggests that K^{**} exchange is playing an important part in K^+ photoproduction, since experimentally this is close to the ratio found. There is lack of agreement on the f/d ratios for the vector-meson trajectory. The values in the literature[25–27] are not inconsistent with the experimental f/d ratio.

We next turn our attention to the outgoing baryon polarization. The polarization normal to the plane of scattering will get contributions from cV, cT, VT, and KA interference. Explicitly,

$$\begin{aligned}-\langle\mathbf{P}\cdot\hat{n}\rangle\frac{d\sigma}{dt}\pi(s-m^2)&=\tfrac{1}{2}(1-\cos^2\varphi_\gamma)\cos\theta_t(t-m_K{}^2)^2|t-(m-Y)^2||\sin\theta_t|\\
&\times\Bigg[\alpha_c\alpha_V\cos\tfrac{1}{2}\pi(\alpha_c-\alpha_V)\frac{[\alpha_V|t|^{-1/2}\gamma_{\frac{1}{2}\frac{1}{2}}{}^{c}\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{V}-|t|^{-3/2}\gamma_{\frac{1}{2}\frac{1}{2}}{}^{V}\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{c}]}{|\Gamma(\alpha_c+1)\Gamma(K_V+1)\sin\frac{1}{2}\pi\alpha_c\cos\frac{1}{2}\pi\alpha_V|}\left(\frac{s}{s_0}\right)^{\alpha_c+\alpha_V-2}\\
&+\alpha_c\alpha_T\sin\tfrac{1}{2}\pi(\alpha_c-\alpha_T)\frac{[|t|^{-1/2}\gamma_{\frac{1}{2}\frac{1}{2}}{}^{c}\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{T}-|t|^{-3/2}\gamma_{\frac{1}{2}\frac{1}{2}}{}^{T}\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{c}]}{|\Gamma(\alpha_c+1)\Gamma(\alpha_T+1)\sin\frac{1}{2}\pi\alpha_c\sin\frac{1}{2}\pi\alpha_T|}\left(\frac{s}{s_0}\right)^{\alpha_c+\alpha_T-2}\\
&+\alpha_V\alpha_T\cos\tfrac{1}{2}\pi(\alpha_V-\alpha_c)|t|^{-1/2}\frac{[\alpha_V\gamma_{\frac{1}{2}\frac{1}{2}}{}^{T}\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{V}-\gamma_{\frac{1}{2}\frac{1}{2}}{}^{V}\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{T}]}{|\Gamma(\alpha_V+1)\Gamma(\alpha_T+1)\sin\frac{1}{2}\pi\alpha_T\cos\frac{1}{2}\pi\alpha_V|}\left(\frac{s}{s_0}\right)^{\alpha_V+\alpha_T-2}\Bigg]\\
&+\tfrac{1}{2}(1+\cos^2\varphi_\gamma)|\sin\theta_t\cos\theta_t||t-(m+Y)^2|[t-(m-Y^2)]\cos\tfrac{1}{2}\pi(\alpha_K-\alpha_K)\alpha_K\alpha_A\\
&\times\frac{[\alpha_A|t|^{-1/2}\gamma_{\frac{1}{2}\frac{1}{2}}{}^{K}\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{A}-\alpha_K|t|^{-3/2}\gamma_{\frac{1}{2}\frac{1}{2}}{}^{A}\gamma_{\frac{1}{2}-\frac{1}{2}}{}^{K}]}{|\Gamma(\alpha_K+1)\Gamma(\alpha_A+1)\sin\frac{1}{2}\pi\alpha_K\cos\frac{1}{2}\pi\alpha_A|}\left(\frac{s}{s_0}\right)^{\alpha_K+\alpha_A-2}. \quad (83)\end{aligned}$$

This polarization should be large. If we used polarized photon beams, then the extent of $\cos^2\varphi_\gamma$ behavior in the

[27] K. Sarma and G. Renninger, Phys. Rev. Letters **20**, 399 (1968).

polarization gives us a measurement of the size of the K_A contribution to this process. If the parametrization of Sarma and Reeder is correct, then this polarization should have a large dip (if not a zero) near $t=-0.4$.

The other two polarizations depend on KV, KT, and KC interference and are present only for polarized initial particles. We explicitly obtain

$$\langle \mathbf{P}\cdot\hat{z}\rangle \frac{d\sigma}{dt}\pi(s-m^2)^2=\tfrac{1}{2}\sin^2\varphi_\gamma|\sin\theta_t|^2(t-m_K{}^2)|t-(m-Y)^2|t-(m+Y)^2|^{1/2}$$
$$\times\Bigg[\sin\tfrac{1}{2}\pi(\alpha_K-\alpha_c)\left(\frac{s}{s_0}\right)^{\alpha_K+\alpha_c-2}\alpha_K\alpha_c\frac{(|t|^{-1}\gamma_{\frac{1}{2}\frac{1}{2}}{}^K\gamma_{\frac{1}{2}\frac{1}{2}}{}^c+|t|^{-2}\alpha_K\gamma_{\frac{1}{2}-\frac{1}{2}}{}^K\gamma_{\frac{1}{2}-\frac{1}{2}}{}^c)}{|\Gamma(\alpha_K+1)\Gamma(\alpha_c+1)\sin\frac{1}{2}\pi\alpha_K\sin\frac{1}{2}\pi\alpha_c|}$$
$$+\cos\tfrac{1}{2}\pi(\alpha_K-\alpha_V)\left(\frac{s}{s_0}\right)^{\alpha_K+\alpha_V-2}\alpha_V\alpha_K\frac{(|t|^{-1}\gamma_{\frac{1}{2}\frac{1}{2}}{}^K\gamma_{\frac{1}{2}\frac{1}{2}}{}^V+|t|^{-1}\alpha_V\gamma_{\frac{1}{2}-\frac{1}{2}}{}^K\gamma_{\frac{1}{2}-\frac{1}{2}}{}^V)}{|\Gamma(\alpha_K+1)\Gamma(\alpha_V+1)\sin\frac{1}{2}\pi\alpha_K\cos\frac{1}{2}\pi\alpha_V|}$$
$$+\sin\tfrac{1}{2}\pi(\alpha_K-\alpha_T)\left(\frac{s}{s_0}\right)^{\alpha_K+\alpha_T-2}\alpha_K\alpha_T\frac{(|t|^{-1}\gamma_{\frac{1}{2}\frac{1}{2}}{}^K\gamma_{\frac{1}{2}\frac{1}{2}}{}^T+|t|^{-1}\alpha_T\gamma_{\frac{1}{2}-\frac{1}{2}}{}^K\gamma_{\frac{1}{2}-\frac{1}{2}}{}^T)}{|\Gamma(\alpha_K+1)\Gamma(\alpha_T+1)\sin\frac{1}{2}\pi\alpha_K\sin\frac{1}{2}\pi\alpha_T|}\Bigg], \quad (84)$$

$$\langle \mathbf{P}\cdot\hat{x}\rangle \frac{d\sigma}{dt}(s-m^2)^2=\tfrac{1}{2}\sin^2\varphi_\gamma|\sin\theta_t|\cos\theta_t(t-m_K{}^2)|t-(m+Y)^2|^{1/2}|t-(m-Y)^2|^{1/2}$$
$$\times\Bigg[\alpha_c\alpha_K\sin\tfrac{1}{2}\pi(\alpha_c-\alpha_K)\frac{|t|^{-3/2}(\alpha_K\gamma_{\frac{1}{2}\frac{1}{2}}{}^c\gamma_{\frac{1}{2}-\frac{1}{2}}{}^K+\gamma_{\frac{1}{2}\frac{1}{2}}{}^K\gamma_{\frac{1}{2}-\frac{1}{2}}{}^c)}{|\Gamma(\alpha_K+1)\Gamma(\alpha_c+1)\sin\frac{1}{2}\pi\alpha_K\sin\frac{1}{2}\pi\alpha_c|}\left(\frac{s}{s_0}\right)^{\alpha_K+\alpha_c-2}$$
$$+\alpha_K\alpha_V\cos\tfrac{1}{2}\pi(\alpha_K-\alpha_V)\frac{(|t|^{-1/2}\gamma_{\frac{1}{2}\frac{1}{2}}{}^K\gamma_{\frac{1}{2}-\frac{1}{2}}{}^V-\alpha_K|t|^{-3/2}\gamma_{\frac{1}{2}\frac{1}{2}}{}^V\gamma_{\frac{1}{2}-\frac{1}{2}}{}^K)}{|\Gamma(\alpha_K+1)\Gamma(\alpha_V+1)\sin\frac{1}{2}\pi\alpha_K\cos\frac{1}{2}\pi\alpha_V|}\left(\frac{s}{s_0}\right)^{\alpha_K+\alpha_V-2}$$
$$+\alpha_K\alpha_T\sin\tfrac{1}{2}\pi(\alpha_T-\alpha_K)\frac{(\alpha_K|t|^{-3/2}\gamma_{\frac{1}{2}\frac{1}{2}}{}^T\gamma_{\frac{1}{2}-\frac{1}{2}}{}^K+|t|^{-1/2}\gamma_{\frac{1}{2}\frac{1}{2}}{}^K\gamma_{\frac{1}{2}-\frac{1}{2}}{}^T)}{|\Gamma(\alpha_K+1)\Gamma(\alpha_T+1)\sin\frac{1}{2}\pi\alpha_K\sin\frac{1}{2}\pi\alpha_T|}\left(\frac{s}{s_0}\right)^{\alpha_K+\alpha_T-2}\Bigg]. \quad (85)$$

ACKNOWLEDGMENT

The author would like to thank M. Moravcsik for suggesting this study and encouraging its completion.

APPENDIX: KINEMATIC SINGULARITIES AND NONSENSE BEHAVIOR OF AMPLITUDES

In this paper the kinematic singularities of the amplitudes were determined from the threshold behavior of the $f^{\pm}$ according to the prescription of Hite.[6] We have also taken into account the experimentally known or assumed behavior of the residue functions at the nonsense points to define a reduced residue function $\gamma(t)$ which is analytic in t and hopefully slowly varying. When there are constraints among the amplitudes, such as $t=0$, we have used the experimental knowledge of the presence or absence of dips to determine the powers of t present at $t=0$ in the parity-conserving amplitudes. We now discuss the parametrization in explicit detail.

The partial-wave expansion for the parity-conserving amplitudes is given by

$$f_{\lambda_3\lambda_4,\lambda_1\lambda_2}{}^{\eta}=\sum_J(2J+1)\times[e_{\lambda\mu}{}^{J+}F_{\lambda_3\lambda_4,\lambda_1\lambda_2}{}^{J\eta}+e_{\lambda\mu}{}^{J-}F_{\lambda_3\lambda_4,\lambda_1\lambda_2}{}^{J(-\eta)}]. \quad (A1)$$

Hite has shown that the $F^{J\eta}$ have the following threshold behavior:

$$F_{\lambda_3\lambda_4,\lambda_1\lambda_2}{}^{J\eta}\sim[t-(m_1+m_2)^2]^{\frac{1}{2}L+}[t-(m_1-m_2)^2]^{\frac{1}{2}L-}\times[t-(m_3+m_4)^2]^{\frac{1}{2}L'+}[t-(m_3-m_4)^2]^{\frac{1}{2}L'-}, \quad (A2)$$

where $L^{\pm}$ are the minimum values of angular momentum at the normal and pseudonormal thresholds consistent with parity conservation. Since

$$e_{\lambda\mu}{}^J\sim(z_t)^{J-M},$$
$$z_t=\frac{2t(s-u)+(m_1{}^2-m_2{}^2)(m_3{}^2-m_4{}^2)}{\tau_{12}\tau_{34}}, \quad (A3)$$

we define

$$F_{\pm}(t)\equiv[t-(m_1+m_2)^2]^{-\frac{1}{2}n+}[t-(m_1-m_2)^2]^{-\frac{1}{2}n-}\times[t-(m_3+m_4)^2]^{-\frac{1}{2}n'+}[t-(m_3-m_4)^2]^{-\frac{1}{2}n'-} \quad (A4)$$

with

$$n_{\pm}=J-L_{\pm},$$
$$n_+=s_a+s_b-\tfrac{1}{2}[1\mp n_an_b(-1)^{s_a+s_b}],$$
$$n_-=\begin{cases}n_+ \text{ for a two-boson vertex}\\ s_a+s_b-\frac{1}{2}[1\pm n_an_b(-1)^{s_a+s_b}] \text{ for a two-fermion vertex},\end{cases}$$

n_a the intrinsic parity of particle a, and we recognize the following: a trajectory of $P=(-1)^J$ contributes only to F^{J+} and its kinematic t dependence is for

$$f^+, \quad F_+(t)(\tau_{12}\tau_{34})^m$$

and for

$$f^-, \quad F_+(t)(\tau_{12}\tau_{34})^{m+1}. \tag{A5}$$

Similarly for a trajectory of $P=-(-1)^J$, the kinematic t dependence is for

$$f^-, \quad F_-(t)(\tau_{12}\tau_{34})^m,$$

and for

$$f^+, \quad F_-(t)(\tau_{12}\tau_{34})^{m+1}. \tag{A6}$$

At $t=0$ the maximum t singularity is $t^{-1/2(|\lambda|+|\mu|)}$. However, assumptions to the types of Regge poles that are exchanged, constraint equations, and factorization requirements often do not allow this maximum singularity. This was discussed earlier.

We introduce the "kinematic-singularity-free" $\gamma(t)$ by the equation [B is the residue of F^J at $J=\alpha(t)$]

$$\begin{aligned} f_{\lambda_3\lambda_4,\lambda_1\lambda_2}{}^+ &= \left(\frac{1+\sigma_i e^{-i\pi\alpha}}{2}\right)\frac{(2\alpha+1)}{\sin\pi\alpha} B_{\lambda_3\lambda_4,\lambda_1\lambda_2}{}^+ E_{\lambda\mu}{}^{\alpha+} \\ &= \left(\frac{1+\sigma_i e^{-i\pi\alpha}}{2}\right)\frac{N_{\lambda\mu}(\alpha)F_+(t)(\tau_{12}\tau_{34})^m}{\sin\pi\alpha\,\Gamma(\alpha+1)} \\ &\quad \times\left(\frac{s-u}{s_0}\right)^{\alpha-m} \gamma_{\lambda_3\lambda_4,\lambda_1\lambda_2}{}^+(t)t^n, \end{aligned} \tag{A7}$$

where n is determined from the way we satisfy the conspiracy relations and factorization, and $N_{\lambda\mu}$ takes into account the assumed or experimentally known behavior of B in the nonsense region.

There are various possible behaviors of B in the nonsense region which are compatible with analyticity. For right-signature nonsense points, analyticity requires[10] $B_{SN}\sim\sqrt{\alpha}$ in order to cancel a $1/\sqrt{\alpha}$ behavior in $E_{SN}{}^+$. Factorization then requires

$$B_{SN}{}^2 = B_{SS}B_{NN}\sim\alpha.$$

There are several ways of satisfying this relationship. The four experimentally distinguishable ways are

(a) "choosing sense"—

$$B_{SS}=1, \quad B_{SN}\sim\sqrt{\alpha}, \quad B_{NN}\sim\alpha;$$

(b) "choosing nonsense"—

$$B_{SS}\sim\alpha, \quad B_{SN}\sim\sqrt{\alpha}, \quad B_{NN}\sim 1;$$

(c) Chew mechanism—

$$B_{SS}\sim\alpha, \quad B_{SN}\sim\alpha\sqrt{\alpha}, \quad B_{NN}\sim\alpha^2;$$

(d) no-compensation mechanism—

$$B_{SS}\sim\alpha^2, \quad B_{SN}\sim\alpha\sqrt{\alpha}, \quad B_{NN}\sim\alpha.$$

For wrong-signature nonsense points, Mandelstam and Wang[15] have proven that B_{SN} has a one-over-square-root singularity and B_{NN} has a fixed pole due to the presence of the third double spectral function (which also gives Regge cuts if large). In this paper, we assume that the effects of this fixed pole is small so that the above choices of choosing sense, etc., are still experimentally valid in the sense that if $B_{NN}\sim c/\alpha - d\alpha$, then the Regge residue will vanish at $\alpha=(c/d)^{1/2}$ which is almost 0, since $c\ll d$.

For pion photoproduction,

$$\begin{aligned} F_+(t) &= (t-4m^2)^{-1/2}, \\ F_-(t) &= (t-\mu^2)^{-1}t^{1/2}, \end{aligned} \tag{A8}$$

whereas for kaon photoproduction,

$$\begin{aligned} F_+(t) &= [t-(m+Y)^2]^{-1/2}, \\ F_-(t) &= (t-m_K{}^2)^{-1}[t-(m-Y)^2]^{-1/2}. \end{aligned} \tag{A9}$$

The present experimental evidence is that the ρ chooses sense,[28] and A_2 nonsense.[29] There are some theoretical arguments[12] that the ω chooses nonsense.

[28] G. Höhler, J. Baacke, and G. Eisenbeiss, Phys. Letters **22**, 203 (1966).

[29] M. Krammer and U. Maor, CERN Report No. CERN 67-22, 1967 (unpublished).