Modified Extrapolation to Determine the $\pi\pi$ Cross Section*

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The dependence of the $\pi+N\rightarrow(2\pi)+N$ cross section for low invariant $\pi\pi$ mass and small $|t|$ is discussed with a view toward extrapolation to the pion pole and extraction of $\pi\pi$ phase shifts. There is a complicated t-dependent term in the cross section which makes extrapolation inaccurate. This term can be isolated by consideration of certain moments (in the distribution with respect to the pion angles θ, φ in the dipion rest frame), if nucleon helicity flip is assumed to dominate. Indirect empirical evidence and the absorption model with π exchange both support this assumption in the 5–10-BeV/c region. The assumption is weaker than the popular assumption that the Chew-Low pion-exchange formula holds, and can also be directly tested by examining the t dependence of certain moments at 0° (forward production of the dipion). For s- and p-wave $\pi\pi$ scattering, one considers the coefficients of 1, $\cos\theta$, and $\sin^2\theta$ in addition to the usual φ-dependent terms. The coefficient of $\sin^2\theta$ is the complicated term. It is reasonable to hope that the remaining moments are smoothly varying for $0\geq -t\gtrsim 4\mu^2$, and are suitable for extrapolation. They are constrained to a definite t dependence at 0° by angular momentum conservation. We feel that reliable determination of the $\pi\pi$ cross section in the ρ region and below is attainable by this method in high-statistics experiments currently under way.

INTRODUCTION

In 1959, Chew and Low[1] pointed out that the $\pi\pi$ elastic cross section could be obtained for an $\pi N\rightarrow\pi\pi N$ by extrapolating with respect to the nucleon momentum transfer to slightly unphysical values. This still appears to be one of the best hopes for obtaining the $\pi\pi$ amplitude; however, it has proven difficult to implement in practice. In this paper we will re-examine extrapolation to the pion pole and make a very simple suggestion for a modification of the extrapolation procedure at low $\pi\pi$ invariant mass. We do not attempt to analyze any $\pi\pi$ data nor do we review the current status of knowledge of the $\pi\pi$ amplitude.

Recently, high-statistics experiments have substantially improved the possibility of a good extrapolation. In the most thorough experiment of this type to date, Baton *et al.*[2] have investigated

$$\pi^-p\rightarrow\pi^-\pi^0p \tag{1}$$

at 2.77 GeV/c for $\pi\pi$ mass in the ρ region and below. More recently, a similar analysis has been made of data compiled by the University of Pennsylvania group.[3] Their work and the recent results of e^+e^- colliding beam experiments[4] confirms that fitting the $\pi\pi$ distributions in the physical region of $\pi N\rightarrow\pi\pi N$ directly in terms of free $\pi\pi$ scattering is a crude approximation. By comparison,[5] the more sophisticated methods yield a ρ resonance of much narrower width, albeit the same position. This result is reasonable since both final-state scattering effects and true background production of dipions should broaden the resonance (see Fig. 1). Background via diffraction dissociation has been studied[6] and is expected to be significant. It vanishes as $p_L\rightarrow\infty$ *for fixed* $m_{\pi\pi}$ at about the same rate as the π exchange process. The phase of the background amplitude has been examined experimentally by Walker,[6] confirming prediction. Apparent broadening of the ρ resonance by background follows because neither the resonance shape nor background shape in $m_{\pi\pi}$ is known precisely. In a many-parameter fit to data, where the background shape is assumed to be relatively simple, there is a long valley in χ^2 space. Proceeding down this, background decreases and width increases. The tendency of the fit is toward excessively large widths. The t

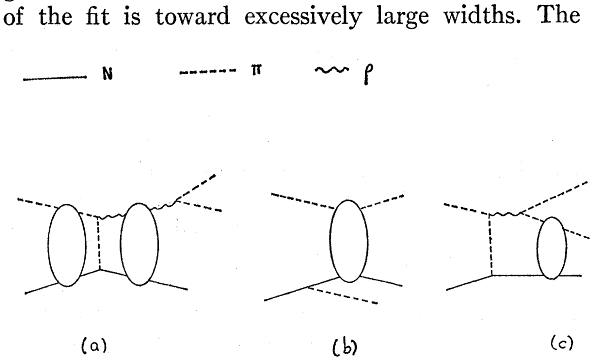

FIG. 1. Various $\pi+N\rightarrow\pi+\pi+N$ processes. (a) Typical π exchange process (via ρ production) with absorption in initial and final states. (b) Diffraction dissociation process. (c) Stimulated ρ-decay process.

* A preliminary version of a different flavor is contained in the Proceedings of the Informal Meeting on Experimental Meson Spectroscopy at the University of Pennsylvania, April 1968 (unpublished).

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[1] G. Chew and F. Low, Phys. Rev. **113**, 1640 (1959); or see M. Jacobs and G. Chew, *Strong-Interaction Physics* (W. A. Benjamin, Inc., New York, 1964), or G. Källén, *Elementary Particle Physics*, (Addison-Wesley Publishing Company, Inc., Reading, Mass., 1964).

[2] J. P. Baton, G. Laurens, and J. Regnier, Phys. Letters **25B**, 419 (1967).

[3] V. Hagopian (private communication).

[4] V. L. Auslander *et al.*, Phys. Letters **25B**, 433 (1967); P. Marin, in Proceedings 1967 International Symposium on Electron and Photon Interactions at High Energy (unpublished).

[5] J. Pisut and M. Roos, CERN Report No. TH 885 (unpublished).

[6] E. West, J. H. Boyet, A. R. Erwin, and W. D. Walhen, Phys. Rev. **149**, 1089 (1966); M. Ross and Y. Y. Yam, Phys. Rev. Letters **19**, 546 (1967); W. D. Walker, in Proceedings of the Informal Meeting on Experimental Meson Spectroscopy, University of Pennsylvania, April 1968 (unpublished).

dependence of the background is significantly less steep than the π exchange. Thus the cross section cannot be factored: $f(t)g(m_{\pi\pi})$, at least at small $-t$. The Saclay group[2] explicitly exhibit the strong variation of the dependence of the cross section on nucleon momentum transfer as a function of the dipion invariant mass. It is reasonable to expect that our information about the $\pi\pi$ s waves is even more uncertain than that on the ρ width.

Although it demonstrates the inadequacy of physical region fits, the extrapolation of the Saclay group cannot be considered satisfactory because it does not cope with the all-important very low momentum transfer dependence. The facts are that one cannot readily observe (and they did not efficiently observe) very low momentum transfers in reaction (1) [i.e., $-t\lesssim(2 \text{ or } 3)\mu^2$, where t is the four-momentum transfer to the nucleon, squared], while there is excellent evidence that the momentum transfer distribution is strongly structured in the region $-t<\mu^2$. The evidence for this structure is charged pion photoproduction which is strongly peaked within $-t<\mu^2$.[7] The well-confirmed vector-dominance model connects $\gamma N\to\pi^{\pm}N$ and $\pi N\to\rho N$ and predicts the same peaking in the latter for transverse ρ's. The available bubble chamber data are not sufficient to check this strong prediction.[8] The successful π-exchange absorption model, of Gottfried, Jackson, and others, also predicts this forward structure (see Fig. 1).[9] In other words, the Saclay group was forced to make a slowly varying extrapolation of their measurements through a region where further measurements almost certainly will show a rapid t dependence. They went further: There is a factor of t in the elementary pion-exchange formula and their data are consistent with this zero at $t=0$. The assumption that it is exactly valid places a constraint on the extrapolation enabling a vital reduction in statistical uncertainty. Unfortunately, the evidence cited above strongly indicates that the cross section is large at $t\to 0$.

We find that the approach of the Saclay group can be modified slightly so that we still preserve an appropriate constraint at $t=0$. Rather than using elementary π exchange as a guide to this constraint, we use absorbed π exchange as a guide. This generalized model predicts that the amplitude is helicity flip at high energy. As we shall see, helicity-flip dominance means that all but one contribution vanish at $t=0$. This contribution (transverse ρ's) is just the one observed in photoproduction at 0°. It can be separated from the rest of the cross section. As far as we know, the remaining terms are subject to a simple extrapolation near $t=0$, for an energy region of, say, 5–10 GeV/c.

[7] A. M. Boyarski *et al.*, Phys. Rev. Letters **20**, 300 (1968).
[8] P. Schmusser (private communication); R. Diebold and J. Poirier, Phys. Rev. Letters **20**, 1532 (1968).
[9] Absorption model results referred to in this paper were performed by the authors using the simplified model of G. L. Kane, Phys. Rev. **163**, 1544 (1967).

We have examined all models proposed for $\pi N\to\pi\pi N$ at low $-t$ and high energy: (a) absorbed π exchange, (b) diffraction dissociation,[6] and (c) ω, A_2 exchange. The first two involve a πNN vertex and elastic scattering and will, according to sound theoretical arguments, be predominantly helicity flip near $t=0$ and, say, for $p_L=5$ to 10 GeV/c. Theoretically, the flip and nonflip amplitudes are comparable at 3 GeV/c for small t generally, while flip predominates at $t=0$. The nonflip amplitude falls $1/p_L$ faster than the flip. It may be risky, however, to go to much higher energy because vector and tensor exchange may take over from π exchange. Evidence at 6–8 GeV/c (Ref. 10) and meager evidence[11] at much higher energy are consistent with generalized π-exchange [in the sense of (a) and (b)] dominance.

Our proposal is to carry out extrapolation at very small momentum transfer separating out the term which is known to depend rapidly on t. The absorption model indicates that the analysis be confined to $-t\lesssim 4\mu^2$. At first it is likely for experimental reasons that only neutral dipion production

$$\pi^- p\to\pi^+\pi^- n \qquad (2)$$

will be amenable to this analysis.

HELICITY-FLIP FORMULA

The only assumption we will make in addition to terminating polynomials in the extrapolation functions is that the amplitude is predominantly helicity flip.

Our kinematics notation is given in Fig. 2. The analysis consists in detailed extrapolation of moments. Extraction of moments of the dipion-rest-frame $\pi\pi$ angular distribution has been emphasized by Schlein and collaborators.[12] We stress that each moment has a characteristic momentum-transfer behavior. The considerations below are valid in either the Gottfried-

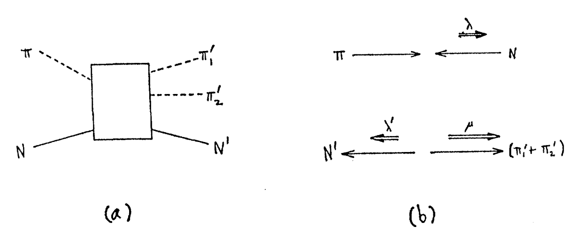

FIG. 2. Notation for $\pi+N\to\pi+\pi+N$: (a) $s=(p_\pi+p_N)^2$, $t=(p_N-p_{N'})^2$, $m_{\pi\pi}{}^2=(p_{\pi_1'}+p_{\pi_2'})^2$, θ and φ are angles of π_1' in the π_1', π_2' rest frame with respect to (1) $\mathbf{p}_\pi$ ("G-J frame") or to (2) $-\mathbf{p}_{N'}$ ("H frame"). p_L is the incident π lab three-momentum. (b) Typical helicity-flip process.

[10] M. Deutschmann *et al.*, Phys. Letters **18**, 351 (1965); J. A. Poirier *et al.*, Phys. Rev. **163**, 1462 (1967); W. D. Walker *et al.*, Phys. Rev. Letters **20**, 133 (1967).
[11] B. Y. Oh (private communication).
[12] Peter Schlein *et al.*, Phys. Rev. Letters **11**, 167 (1963); **19**, 1052 (1967).

Jackson (G-J) frame ($\hat{z}$=incident direction) or helicity (H) frame ($\hat{z}$=minus final nucleon direction) in the dipion rest frame, since the two frames are identical for 0° production.

Let $A_{\lambda\lambda'}$ be the amplitude for $\pi N\rightarrow\pi\pi N$ with λ, λ' the initial and final nucleon helicities in the over-all rest frame. Conservation of angular momentum in the beam direction requires

$$A_{(\pm\mp)}=\sum_{\mu}(t_0-t)^{n/2}\sum_{l} m^{\mu}{}_{l(\pm\mp)}Y_l^{\mu}(\Omega)\,, \tag{3}$$

where Ω corresponds to the dipion decay angles (θ,ϕ),

$$\begin{aligned} t_0-t &\propto \theta_{\mathrm{PROD}}{}^2\,,\\ t_0 &\approx -[(m_{\pi\pi-\mu}{}^2 2)/2p_L]^2\,,\\ n &= |\lambda+\mu-\lambda'|\,, \end{aligned} \tag{4}$$

and l is the angular momentum of the dipion in its rest frame and μ is the dipion helicity (see Fig. 2). Since the difference between t_0-t and $-t$ is too small to measure at the energies of interest $[t_0\lesssim(\frac{1}{4}p_L)^2\lesssim 0.003\ \mathrm{GeV}^2]$, we will use $-t$ as the production angle variable.

In the following we confine ourselves to the ρ region and below and only consider s- and p-wave dipions. There are well-known relations due to parity conservation among the $m_l{}^{\mu}$'s for $l=1$, which can be written

$$m_{+-}{}^{\pm1}=-m_{-+}{}^{\mp1}\,,\quad m_{++}{}^{\pm1}=m_{--}{}^{\mp1}\,,\quad m_{+-}{}^{0}=m_{-+}{}^{0}\,,$$
$$m_{++}{}^{0}=-m_{--}{}^{0}\,.$$

Using these, we obtain from (3), dropping the helicity subscripts $+-$,

$$\begin{aligned} s^2(t-\mu^2)^2\frac{d^3\sigma}{dtdm_{\pi\pi}{}^2d\Omega}=&\tfrac{1}{2}[|m_1{}^{-1}Y_1{}^{-1}+(-t)^{1/2}\\ &\times(m_1{}^0Y_1{}^0+m_0Y_0)-tm_1{}^1Y_1{}^1|^2+|-m_1{}^{-1}Y_1{}^1\\ &+(-t)^{1/2}(m_1{}^0Y_1{}^0+m_0Y_0{}^0)+tm_1{}^1Y_1{}^{-1}|^2]\,. \end{aligned} \tag{5}$$

The π-exchange power-law dependence s^{-2} and pole at $t=\mu^2$ are explicitly exhibited for convenience. Rewriting (5), we have

$$\begin{aligned} s^2(t-\mu^2)^2d^3\sigma/dtdm_{\pi\pi}{}^2d\Omega=&-t|m_0|^2\\ &-t2\sqrt{3}\,\mathrm{Re}m_1{}^{0*}m_0\cos\theta-t|m_1{}^0|^2 3\cos^2\theta+(|m_1{}^{-1}|^2\\ &+t^2|m_1{}^1|^2)\times\tfrac{3}{2}\sin^2\theta+(-t)^{1/2}\,\mathrm{Re}(m_1{}^{-1}+tm_1{}^1)^*\\ &\times m_0(\sqrt{}6)\sin\theta\cos\varphi+(-t)^{1/2}\,\mathrm{Re}(m_1{}^{-1}-tm_1{}^1)^*\\ &\times m_1{}^0 3\sqrt{2}\sin\theta\cos\theta\cos\varphi\\ &+t\,\mathrm{Re}m_1{}^{1*}m_1{}^{-1}3\sin^2\theta\cos2\varphi\,. \end{aligned} \tag{6}$$

For purposes of comparison with other work,[13] the density matrix elements are

$$\begin{aligned} \rho_s&=-t(m_0)^2/N\,,\\ \rho_{s0}&=-t\,\mathrm{Re}m_1{}^{0*}m_0/N\,,\\ \rho_{s1}&=-(-t)^{1/2}\,\mathrm{Re}(m_1{}^{-1}-tm_1{}^1)^*m_1/2N\,,\\ \rho_{00}&=-t|m_1{}^0|^2/N\,,\\ \rho_{11}&=(|m_1{}^{-1}|^2+t^2|m_1{}^1|^2)/2N\,,\\ \rho_{01}&=-(-t)^{1/2}\,\mathrm{Re}(m_1{}^{-1}-tm_1{}^1)^*m_1{}^0/2N\,,\\ \rho_{1-1}&=-t\,\mathrm{Re}m_1{}^{1*}m_1{}^{-1}/N\,, \end{aligned}$$

where

$$\rho_s+\rho_{00}+2\rho_{11}=1$$

or

$$\begin{aligned} N=&-t|m_0|^2-t|m_1{}^0|^2+|m_1{}^{-1}|^2+t^2|m_1{}^1|^2\\ &=s^2(t-\mu^2)^2d^2\sigma/dtdm_{\pi\pi}{}^2\,. \end{aligned} \tag{7}$$

In the above, the cross section was constructed from helicity flip alone. We welcome small helicity flip contributions to dipions of helicity μ if the appropriate terms cancel at $t=0$ as in π exchange.

We now make the essential approximation: *The bilinear combinations of m's can be approximated by linear functions of t.* Absorption model calculations suggest that this approximation is likely to be valid to better than 10% for the range $-\mu^2\geq -t\geq 4\mu^2$ but is likely to be very poor if $-t$ ranges up to $10\mu^2$.

Thus we write

$$\begin{aligned} -\alpha_1t+\alpha_2t^2&=-t(|m_0|^2+3|m_1{}^0|^2)=(\rho_s+3\rho_{00})N\,,\\ -\beta_1t+\beta_2t^2&=-t2\sqrt{3}\,\mathrm{Re}m_1{}^{0*}=2\sqrt{3}\rho_{s0}N\,,\\ \gamma_0-\gamma_1t+\gamma_2t^2-\gamma_3t^3&=|m_1{}^{-1}|^2-2t|m_1{}^0|^2+t^2|m_1{}^1|^2\\ &=(3\rho_{11}-3\rho_{00})N\,, \end{aligned} \tag{8}$$

so that the cross section becomes

$$\begin{aligned} s^2(t-\mu^2)^2d^3t/dtdm_{\pi\pi}{}^2d\Omega=&(-\alpha_1t+\alpha_2t^2)+(-\beta_1t+\beta_2t^2)\\ &\times\cos\theta+(\gamma_0-\gamma_1t+\gamma_2t^2-\gamma_3t^3)\times\tfrac{3}{2}\sin^2\theta\\ &+\varphi\text{-dependent terms.} \end{aligned} \tag{9}$$

For simplicity of exposition, we do not exhibit the φ-dependent terms in detail. The essential point of our argument is now clear: We retain the $t=0$ constraint on two of three moments. The third, $\sin^2\theta$, term is associated with $m_{+-}{}^1$, which vanishes for elementary or evasive Reggeized pion exchange, but need not and does not for absorbed pion exchange. The $\sin^2\theta$ term, because it varies rapidly, can probably not be accurately determined in the near future.

Consider Legendre moments of this cross section:

$$a_L=\int\frac{d\Omega}{4\pi}P_Ls^2(t-\mu^2)^2(d^3\sigma/dtdm_{\pi\pi}{}^2d\Omega)\,.$$

Thus $a_0=(-\alpha_1t+\alpha_2t^2)+(\gamma_0-\gamma_1t+\gamma_2t^2+\gamma_3t^3)$, etc. We see that to determine the α's, β's, γ's efficiently we should

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[13] K. Gottfried and J. D. Jackson, Nuovo Cimento **33**, 309 (1964); L. Durand, III, and Y. T. Chiu, in *Lectures in Theoretical Physics* (University of Colorado Press, Boulder, Colo., 1965).

minimize in given $m_{\pi\pi}$ bins the expressions

$$\sum_{\text{events}} [s^2(t-\mu^2)^2(1+5P_2)W-(-\alpha_1 t+\alpha_2 t^2)]^2, \tag{10}$$

$$\sum_{\text{events}} [s^2(t-\mu^2)^2 3P_1 W-(-\beta_1 t+\beta_2 t^2)]^2, \tag{11}$$

$$\sum_{\text{events}} [s^2(t-\mu^2)^2(-\tfrac{5}{2}P_2)W -(\gamma_0-\gamma_1 t+\gamma_2 t^2-\gamma_3 t^3)]^2, \tag{12}$$

where $W=W_0/4\pi\Delta t\Delta m_{\pi\pi}^2$ with W_0 the cross section per event characteristic of the experiment, Δt the full range of t from which all events are chosen, and $\Delta m_{\pi\pi}^2$ the bin size for $m_{\pi\pi}^2$. We note that this form is, of course, more efficient than grouping events in t bins; all the t resolution available should be made use of.

EXTRAPOLATION

At $t=\mu^2$ Chew and Low found

$$s(t-\mu^2)^2 d^3\sigma/dtdm_{\pi\pi}^2 d\Omega|_{t=\mu^2} =\frac{(g^2/4\pi)\mu^2 m_{\pi\pi}k_{\pi\pi}}{2\pi(4M^2p_L^2/s^2)}\frac{d\sigma_{\pi\pi}}{d\Omega}, \tag{13}$$

where $d\sigma_{\pi\pi}/d\Omega$ is the $\pi\pi$ differential cross section, p_L is the π lab three-momentum, and $g^2/4\pi\approx 14$ is the πNN pseudoscalar coupling constant. With reference to the incident pion direction (Gottfried-Jackson frame)

$$\frac{d\sigma_{\pi\pi}}{d\Omega}=\frac{4\pi}{k^2}|\sum_l (2l+1)^{1/2}a_l Y_l^0|^2,$$

$$a_l=\sum_I C_I e^{i\delta_{lI}}\sin\delta_{lI},$$

where δ_{lI} are the phase shifts for isospin I and angular momentum l, the C_I's the isospin coefficients (given below).

$$\frac{k^2}{4\pi}\frac{d\sigma_{\pi\pi}}{d\Omega}=|a_0|^2+6\text{Re}a_0{}^*a_1\cos\theta+9|a_1|^2\cos^2\theta. \tag{14}$$

Meanwhile, in the helicity frame

$$\frac{k^2}{4\pi}\frac{d\sigma_{\pi\pi}}{d\Omega}=|\sum_{l,\mu}(2l+1)^{1/2}a_l d_{\mu 0}{}^{(l)}(\psi)Y_l^\mu|^2,$$

where $d_{00}{}^1=\cos\psi$, $d_{-10}{}^1=-d_{10}{}^1=-\sin\psi/\sqrt{2}$, and where ψ is the angle between incident π and negative direction of the final nucleon. This satisfies

$$\cos\psi=\frac{2m^2(s+t-m^2-M^2)+(m^2+\mu^2-t)(m^2+M^2-s)}{\{[t-(m-\mu)^2][t-(m+\mu)^2][s-(m-M)^2][s-(m+M)^2]\}^{1/2}} \tag{15}$$

so that

$$d_{00}{}^1\approx\frac{1+t/m^2}{1-t/m^2} \quad\text{and}\quad -\sqrt{2}d_{10}{}^1\approx\frac{2m(-t)^{1/2}}{1-t/m^2}.$$

Note that these are rather rapid functions of t (m is the $\pi\pi$ mass). In the physical region of $\pi N\to\pi\pi N$, ψ is, of course, real. At the extrapolation point one should consider the continuation in t of (15) from the physical region.

With s and p waves, then

$$\frac{k^2}{4\pi}\frac{d\sigma_{\pi\pi}}{d\Omega}=|a_0|^2+6(a_0a_1)d_{00}{}^1\cos\theta+9|a_1|^2 \times[d_{00}{}^{12}\cos^2\theta+(d_{10}{}^1)^2\sin^2\theta]+6(\text{Re}a_0{}^*a_1) \times(-\sqrt{2}d_{10}{}^1)\sin\theta\cos\varphi+10(-\sqrt{2}d_{10}{}^1d_{00}{}^1) \times|a_1|^2\sin\theta\cos\theta\cos\varphi+9|a_1|^2(d_{10}{}^1)^2 \times\sin\theta\cos 2\varphi. \tag{16}$$

Depending on whether the moments are evaluated in the G-J frame or H frame we compare (14) or (16) with the extrapolation of (9) and (13).

We obtain, using the G-J frame:

$$x\equiv K(-\alpha_1\mu^2+\alpha_2\mu^4)=|a_0|^2+9|a_1|^2, \tag{17}$$

$$y\equiv K(-\beta_1\mu^2+\beta_2\mu^4)=6\ \text{Re}a_0a_1{}^*, \tag{18}$$

$$z\equiv K(\gamma_0-\gamma_1\mu^2+\gamma_2\mu^4-\gamma_3\mu^6)=-9|a_1|^2, \tag{19}$$

where

$$K=\left[\frac{1}{2\pi k_{\pi\pi}}\frac{g^2}{4\pi}\frac{\mu^2 m_{\pi\pi}}{(2Mp_L/s)^2}\right]^{-1},$$

while for the H frame

$$x=|a_0|^2+9|a_1|^2(d_{00}{}^1)^2, \tag{17'}$$

$$y=6\ \text{Re}(a_0a_1{}^*)d_{00}{}^1, \tag{18'}$$

$$z=9|a_1|^2[(d_{10}{}^1)^2-(d_{00}{}^1)^2]. \tag{19'}$$

The $d_{\mu\lambda}{}^1$'s are sufficiently rapidly varying that although the H and G-J frames are identical for 0° production, there is a substantial difference at the extrapolation point:

$$d_{00}{}^1\approx\frac{1+\mu^2/m^2}{1-\mu^2/m^2},$$

$$d_{10}{}^1\approx\frac{2m(-\mu^2)^{1/2}}{1-\mu^2/m^2}.$$

For charged dipions [reaction (1)] on the H frame, for example (using notation δ_I), we have

$$x=\sin^2\delta_0+9(d_{00}{}^1)^2\sin^2\delta, \qquad y=6d_{00}{}^1\sin\delta_0\sin\delta_1\cos(\delta_0-\delta_1), \tag{20}$$

while, correspondingly for neutral dipions [reaction (2)],

$$x=\tfrac{1}{9}[\sin^2\delta_2+4\sin^2\delta_0+4\sin\delta_0\sin\delta_2\cos(\delta_0-\delta_2)]+9(d_{00}{}^1)^2\sin^2\delta_1,$$
$$y=2d_{00}{}^1[\sin\delta_2\sin\delta_1\cos(\delta_0-\delta_1)+2\sin\delta_0\sin\delta_1\times\cos(\delta_2-\delta_1)]. \quad (21)$$

Referring back to Eqs. (10) and (11) and recalling the nature of the data, we see that both x and y are extrapolations of large moments in the cross section for reaction (2) and can be accurately determined with the statistics of experiments now underway. The extrapolated moment z- and the φ-dependent terms will be difficult to determine but can, in principle, be determined and serve as a test of the procedure. Using just x and y for reaction (2), we see there are two equations and three unknowns. With both charged and neutral dipion experiments, using just these two moments, all three phase shifts can be determined.

CONCLUSIONS

Extrapolation in $\pi N\rightarrow\pi\pi N$ is a hopeful technique for extracting the $\pi\pi$ amplitude in the ρ region and below. Very low $-t$ ($\lesssim 4\mu^2$) should be investigated. Experiments in the 5-10-GeV/c region look most promising. Rapid t dependences can be hoped to be absent in certain moments. Interesting results will be obtained, extrapolating two moments with a few thousand events. It would be nice if other moments could be handled to check the procedure. Of course, reasonable behavior of δ_1 is an important check. The moment analysis can be carried out in both G-J and H frames as a check. The behavior in the two frames is rather different, but theoretical models do not make it clear that one frame is more promising than the other for extrapolation.

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Diffraction Dissociation and the Production of Baryon Resonances

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A model incorporating virtual-meson diffraction scattering followed by a final-state resonant interaction of the meson with an emergent nucleon or isobar is proposed for the production of $I=\frac{1}{2}$ baryon resonances in the reaction $pp\rightarrow pN_{1/2}{}^*$. Absolute predictions are compared with the experimental data relating to enhancements observed in missing-mass experiments. Agreement is favorable in the case of the $P_{11}(1470)$, but otherwise the many resonances predicted by phase-shift analyses complicate the issue and prevent a clear-cut test of the model. The proposed $\Delta P=(-1)^{\Delta J}$ selection rule is discussed in relation to the large cross section predicted by the model for the "forbidden" $S_{11}(1550)$.

I. INTRODUCTION

EXPERIMENTAL studies[1–5] of the reaction $pp\rightarrow pN^*$ at high energies have recently attracted considerable interest as possible examples of the reaction mechanism known as diffraction dissociation,[6] which is thought to possess the following characteristic features:

1. Constant total cross section at high energies.
2. A very low momentum-transfer elastic scattering between the incident particles leads to the materialization of the dissociated system. The dissociation products (in this case the decay products of the N^*) therefore have a diffractive distribution.

[1] G. Belletini *et al.*, Phys. Letters **18**, 167 (1967).
[2] E. W. Anderson *et al.*, Phys. Rev. Letters **16**, 855 (1966).
[3] I. M. Blair *et al.*, Phys. Rev. Letters **17**, 789 (1966).
[4] K. J. Foley *et al.*, Phys. Rev. Letters **19**, 397 (1967).
[5] H. L. Anderson *et al.*, Phys. Rev. Letters **18**, 90 (1967).
[6] M. L. Good and W. D. Walker, Phys. Rev. **120**, 1857 (1960).

3. The larger the mass change in the dissociation process the less the probability and the higher the energy required.
4. Zero transfer of quantum numbers such as B, Q, S, I, G, C in the process, which is sometimes visualized[7] as "vacuon" or "Pomeranchukon" exchange.
5. Only orbital angular momentum (not internal-symmetry quantum numbers) may be transferred to the dissociating system. Thus only "natural parity" changes are allowed, $\Delta P=(-1)^{\Delta L}$. With incident pseudoscalar particles, the transition $0^-\rightarrow 0^-, 1^+, 2^-\cdots$, can occur, but not $0^-\rightarrow 0^+, 1^-, 2^+, 3^-\cdots$. With incident baryons no final states are disallowed, and a stronger rule $\Delta P=(-1)^{\Delta J}$ has been suggested,[8] requiring $\frac{1}{2}^+\rightarrow\frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+, \frac{7}{2}^-, \cdots$, only. However, the experimental evidence concerning this rule is inconclusive, and it has

[7] M. Ross and L. Stodolsky, Phys. Rev. **149**, 1172 (1966).
[8] D. R. O. Morrison, Phys. Rev. **165**, 1699 (1968).