Four-Point Function and $\rho' \to 2\pi\gamma$ Decay

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Radiative decay of the ρ meson has been studied within the framework of the four-point function of currents obeying $SU(2)\otimes SU(2)$ chiral algebra. A hard-pion calculation of the decay amplitude by means of the four-point function shows the correct gauge-invariant electromagnetic structure of the radiative process, while the same amplitude evaluated in the off-shell limits $q^2 \to 0$, $p^2 \to 0$ for the pions fails to account for the dominant dynamical features, such as the internal bremsstrahlung mechanism and gauge invariance. A comparative study of both on-shell and off-shell current-algebra predictions for decay rates and the photon energy spectrum have been presented, along with the results of the vector-meson pole-dominance model.

I. INTRODUCTION

THE three-body radiative decays of vector mesons are all allowed processes in nature. However, adequate experimental information is not available for these decay modes. Many theoretical investigations[1–7] have been made recently to estimate the decay rates and also to study the electromagnetic structure occurring in these rare processes. Some of these calculations are based on the usual vector-meson pole-dominance model.[1–3] With the advent of the current-algebra technique and its apparent success in a large number of processes,[8,9] it is plausible to investigate the radiative decays of vector mesons within the framework of equal-time current-commutation relations and the PCAC (partially conserved axial-vector current) assumption for the pseudoscalar meson. For the decay $V \to PP\gamma$, the matrix element consists of two distinct parts: a direct emission term and the internal bremsstrahlung term. In the latter mechanism one observes comparatively large branching ratios, and it is clear that the enhancement in the decay rate occurs because of the infrared divergence[2,3] of the photon energy spectrum. Current-algebra calculations[5,6] using the soft-pion assumption, on the other hand, suppress the entire dynamical details of the radiative processes and predict decay rates inadequately. In some cases, the soft-pion analysis[6] indicates complete forbiddenness of the decay modes. To overcome these difficulties associated with the soft-pion limit, we work with the hard-pion current-algebra technique that has been followed by many authors[10–13] with considerable success. The purpose of the present note is to investigate the radiative decay $\rho \to 2\pi\gamma$ by using four-point functions developed by Gerstein and Schnitzer[11] for the currents, which obey $SU(2)\otimes SU(2)$ chiral algebra. Assuming ρ dominance for the isovector part of the electromagnetic current and neglecting the "σ term," we obtain an on-mass-shell amplitude which is gauge-invariant for the internal bremsstrahlung part M_{ib} and also for the direct-emission amplitude M_d. The energy spectrum of the photon and the branching ratios to the main decay mode $\rho \to 2\pi$ are found to be in good agreement with the predictions of the pole model.[2]

Next we consider the same process in the off-shell limits for the pions and compare the results of the two different current-algebra approaches. Deviating from the usual soft-pion limits $q \to 0$, $p \to 0$, we follow the less stringent conditions $q^2 \to 0$, $p^2 \to 0$. The weak amplitudes which all survive under the present extrapolation are evaluated in a fashion similar to that followed in the earlier calculation[14] of the axial-vector form factors in K_{e4} decay. It is important to note that the bremsstrahlung mechanism is completely suppressed under the off-shell extrapolation and the whole amplitude has a structure analogous to the direct-emission amplitude. Finally, we present a comparative study of the present work with the pole-model calculation of Singer. We observe, in general, good agreement.

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II. DECAY AMPLITUDE IN FOUR-POINT FUNCTION

We consider the decay

$$\rho^d(Q,\epsilon')\rightarrow\pi^a(p)+\pi^b(q)+\gamma(k,\epsilon)\,, \qquad (1)$$

where ϵ and ϵ' are the polarization vectors of the photon and ρ meson, respectively, and the superscripts are the $SU(2)$ indices which run from 1 to 3.

The decay amplitude may be defined as

$$M_{fi}=i(2\pi)^{-9/2}(16p_0q_0k_0Q_0)^{-1/2}(2\pi)^4 \times\delta^{(4)}(Q-p-q-k)T_{fi} \qquad (2)$$

with the invariant amplitude T_{fi} related to the four-point function[11]:

$$T_{fi}=e\epsilon_\lambda(k)\epsilon_\sigma'(Q)(m_\pi^2-p^2)(m_\pi^2-q^2)(m_\rho^2-k^2) \times(m_\rho^2-Q^2)\bar{T}_{\lambda\sigma}{}^{(2)abcd}(p,q,k)\,. \qquad (3)$$

$\bar{T}^{(2)}{}_{\lambda\sigma}{}^{abcd}(p,q,k)$ is the diagonalized four-point function of the currents. Following Gerstein and Schnitzer,[11] we may write

$$\bar{T}^{(2)}{}_{\lambda\sigma}{}^{abcd}(p,q,k)=\Delta_\pi(p)\Delta_\pi(q)\Delta^V{}_{\lambda\alpha}(k) \times\Delta^V{}_{\sigma\beta}(Q)M^{(2)}{}_{\alpha\beta}{}^{abcd}(p,q,k)\,, \qquad (4)$$

where Δ_π and Δ^V are the usual spin-0 and spin-1 propagators. Here c stands for the index 3 associated with the isovector component of electromagnetic current

$$j_\lambda=e(V_\lambda{}^3+\tfrac{1}{3}\sqrt{3}V_\lambda{}^8)\,.$$

The relevant structure for $M^{(2)}{}_{\lambda\sigma}{}^{abcd}$ is given below:

$$\begin{aligned}m_\pi{}^4F_\pi{}^4M^{(2)}{}_{\lambda\sigma}{}^{abcd}(p,q,k)={}&-C_A{}^2p_\mu q_\nu M_c{}^{(2)}{}_{\mu\nu\lambda\sigma}{}^{abcd}(p,q,k)+(m_\pi{}^2-p^2-q^2)m_\pi{}^{-2}\Gamma_\Sigma{}^{(2)}{}_{\lambda\sigma}{}^{abcd}(p+q,k)\\ &-\tfrac{1}{2}i(\epsilon^{ade}\epsilon^{bce}+\epsilon^{bde}\epsilon^{ace})[\Delta_{\sigma\lambda}{}^V(k)^{-1}+\Delta_{\sigma\lambda}{}^V(Q)^{-1}]+\Big(i\epsilon^{bde}\epsilon^{ace}\Big\{[C_Aq_\nu\Gamma^{(1)}{}_{\alpha\nu\sigma}(p+k,q)+\Delta_{\sigma\alpha}{}^V(Q)^{-1}]\Delta_{\alpha\alpha'}{}^A(p+k)\\ &\times[C_Ap_\mu\Gamma^{(1)}{}_{\alpha\mu\lambda}(q-Q,p)+\Delta^V{}_{\alpha\lambda}(k)^{-1}]+[C_A{}^2q_\nu(p+k)_\alpha\Gamma^{(1)}{}_{\nu\alpha\sigma}(q,p+k)-\tfrac{1}{2}(q-p-k)_\sigma-(C_A-\tfrac{1}{2}C_V)(p+k-q)_\alpha\\ &\times\Delta^V{}_{\alpha\sigma}(Q)^{-1}]\frac{F_\pi{}^{-2}}{[m_\pi{}^2-(p+k)^2]}[-C_A{}^2p_\mu(q-Q)_\alpha\Gamma^{(1)}{}_{\mu\alpha\lambda}(p,q-Q)+\tfrac{1}{2}(p-q+Q)_\lambda+(C_A-\tfrac{1}{2}C_V)(q-Q-p)_\alpha\\ &\times\Delta^V{}_{\alpha\lambda}(k)^{-1}]\Big\}+\{q\leftrightarrow p,\mu\leftrightarrow\nu,a\leftrightarrow b\}\Big)\,, \end{aligned} \qquad (5)$$

where the contact term $M_c{}^{(2)}$ has the expression

$$\begin{aligned}M_c{}^{(2)}{}_{\mu\nu\lambda\sigma}{}^{abcd}(p,q,k)={}&iC_V{}^{-2}C_A{}^{-1}m_A{}^{-2}[\epsilon^{abe}\epsilon^{dce}(\tfrac{3}{2}+\delta)(g_{\nu\lambda}g_{\mu\sigma}-g_{\lambda\mu}g_{\nu\sigma})\\ &+(\epsilon^{aed}\epsilon^{bce}-\epsilon^{bde}\epsilon^{ace})(\tfrac{1}{2}g_{\nu\lambda}g_{\mu\sigma}+\tfrac{1}{2}g_{\mu\lambda}g_{\nu\sigma}-g_{\mu\nu}g_{\lambda\sigma})]\,. \end{aligned} \qquad (6)$$

If we now assume the exact validity[15] of Weinberg's sum rules[16]

$$C_Vm_\rho{}^2=C_Am_A{}^2\,,\qquad m_A{}^2=2m_\rho{}^2\,, \qquad (7)$$

we obtain $C_A=\frac{1}{2}C_V$. Using this relation, the decay matrix element takes a very simplified form which may be obtained from Eqs. (3) to (6):

$$\begin{aligned}T_{fi}={}&\frac{1}{4F_\pi{}^2}\bigg[[\tfrac{1}{4}m_\rho{}^2(3-\delta)(\epsilon^{ade}\epsilon^{bce}+\epsilon^{bde}\epsilon^{ace})\epsilon\cdot\epsilon'-\tfrac{1}{2}\epsilon^{abe}\epsilon^{cde}(1+\delta)(p+q)\cdot\epsilon(p-q)\cdot\epsilon']+\frac{\epsilon^{abe}\epsilon^{cde}}{2(Q\cdot k)}[2q\cdot\epsilon p\cdot\epsilon'-2p\cdot\epsilon q\cdot\epsilon'\\ &-k\cdot(p-q)\epsilon\cdot\epsilon'][2m_\rho{}^2-(1+\delta)p\cdot(p+q)]+\bigg(\frac{m_\rho{}^2(3-\delta)\epsilon^{bde}\epsilon^{ace}p\cdot\epsilon q\cdot\epsilon'}{2(p\cdot k)}+(a\leftrightarrow b,p\leftrightarrow q)\bigg)-\tfrac{1}{4}\epsilon^{abe}\epsilon^{cde}\\ &\times[q\cdot\epsilon p\cdot\epsilon'-p\cdot\epsilon q\cdot\epsilon'-2(1+\delta)(p\cdot\epsilon p\cdot\epsilon'-q\cdot\epsilon q\cdot\epsilon')]-\tfrac{1}{4}(\epsilon^{ade}\epsilon^{bce}+\epsilon^{bde}\epsilon^{ace})[q\cdot\epsilon p\cdot\epsilon'+p\cdot\epsilon q\cdot\epsilon'-2p\cdot q\epsilon\cdot\epsilon']\\ &-\{\tfrac{1}{2}\epsilon^{bde}\epsilon^{ace}[q\cdot\epsilon q\cdot\epsilon'+\epsilon\cdot\epsilon' q\cdot(p+k)-(1+\delta)(p\cdot k\epsilon\cdot\epsilon'+q\cdot\epsilon'(p+q)\cdot\epsilon)]+(a\leftrightarrow b,p\leftrightarrow q)\}\\ &-\bigg(\frac{\epsilon^{bde}\epsilon^{ace}}{2(m_A{}^2-m_\pi{}^2-2p\cdot k)}[p\cdot qp\cdot\epsilon q\cdot\epsilon'+q\cdot(p+k)p\cdot\epsilon p\cdot\epsilon'+p\cdot kq\cdot\epsilon q\cdot\epsilon'+p\cdot kq\cdot(p+k)\epsilon\cdot\epsilon'+(1+\delta)(m_\pi{}^2p\cdot\epsilon p\cdot\epsilon'\\ &+m_\pi{}^2p\cdot k\epsilon\cdot\epsilon'+m_\pi{}^2p\cdot\epsilon q\cdot\epsilon'+2q\cdot kp\cdot\epsilon q\cdot\epsilon'-2p\cdot kq\cdot\epsilon q\cdot\epsilon')+(1+\delta)^2(p\cdot kq\cdot\epsilon q\cdot\epsilon'-p\cdot kq\cdot Q\epsilon\cdot\epsilon'-q\cdot kq\cdot\epsilon'p\cdot\epsilon\\ &-q\cdot Qp\cdot\epsilon(p+q)\cdot\epsilon')]+(a\leftrightarrow b,p\leftrightarrow q)\bigg)\bigg]\,. \end{aligned} \qquad (8)$$

[15] The exact validity of Weinberg's sum rules has been questioned by Gerstein and Schnitzer. Though the sum rules do not exactly follow when various high-energy constraints are imposed on the vertex functions [Eq. (31b)] in Ref. 11, they are assumed to be true for simplicity in numerical calculations.

[16] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967); K. Kawarabayashi and M. Suzuki, *ibid.* **16**, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966); T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1967).

In deriving Eq. (8), we have neglected the "σ term" [second term in Eq. (5)]. The second and the third terms in Eq. (8) correspond to the ρ-meson and pion bremsstrahlung diagrams in the pole model (Figs. 1 to 3 in Ref. 2).

If we now combine the internal bremsstrahlung terms with the first term in Eq. (8), we get a gauge-invariant amplitude and we denote this amplitude by M_{ib}. The remaining terms in Eq. (8) are analogous to the direct emission amplitudes in the pole model, and their combination which is also gauge-invariant is denoted by M_d. It is important to note that the last term in Eq. (8) has no analogous counterpart in the vector-dominance model[2] since A_1-meson dominance in the intermediate state has not been considered in the earlier pole-model calculation.

III. OFF-SHELL AMPLITUDE

In this section we shall present the interesting aspects of the investigation of the same decay process in the off-shell limits $q^2 \to 0$, $p^2 \to 0$. The invariant amplitude T_{fi} in Eq. (2) under present off-shell limits may be given by

$$T_{fi}=\frac{e\epsilon_\lambda}{F_\pi^2}\int d^4x d^4y\; e^{-ip\cdot x-iq\cdot y}\langle 0|T\{\partial_\mu A_\mu{}^a(x)\partial_\nu A_\nu{}^b(y)V_\lambda{}^c(0)\}|\rho^d(Q)\rangle. \tag{9}$$

Decomposing the T product by the standard method[17] and assuming conserved vector current (CVC) and neglecting the so-called "σ term," we may simplify Eq. (9) as

$$T_{fi}=\frac{e\epsilon_\lambda}{F_\pi^2}\Big[-p_\mu q_\nu\int d^4x d^4y\; e^{-iq\cdot x-ip\cdot y}\langle 0|T\{A_\mu{}^a(x),A_\nu{}^b(y),V_\lambda{}^c(0)\}|\rho^d(Q)\rangle$$
$$+\epsilon^{abe}(q-p)_\nu\int d^4x\; e^{-i(p+q)\cdot x}\langle 0|T\{V_\nu{}^e(x),V_\lambda{}^c(0)\}|\rho^d(Q)\rangle+g_\rho(\epsilon^{ace}\epsilon^{bde}+\epsilon^{bce}\epsilon^{ade})\epsilon\cdot\epsilon'$$
$$+\Big\{\epsilon^{bce}p_\mu\int d^4x\; e^{-ip\cdot x}\langle 0|T\{A_\mu{}^a(x),A_\lambda{}^e(0)\}|\rho^d(Q)\rangle+(\mu\leftrightarrow\nu,p\leftrightarrow q,a\leftrightarrow b)\Big\}\Big]. \tag{10}$$

We calculate the weak amplitude terms in a manner similar to that in our calculation[14] of the axial-vector form factors in K_{e4} decay. The final expression of the off-shell decay amplitude in the limit $q^2\to 0$, $p^2\to 0$ may be written down:

$$T_{fi}=-\frac{e}{8F_\pi m_\rho}\Big[(3+\delta)\epsilon^{abe}\epsilon^{cde}(q\cdot\epsilon'p\cdot\epsilon-p\cdot\epsilon'q\cdot\epsilon)-(\epsilon^{ace}\epsilon^{bde}+\epsilon^{bce}\epsilon^{ade})(q\cdot\epsilon'p\cdot\epsilon+p\cdot\epsilon'q\cdot\epsilon-p\cdot q\epsilon\cdot\epsilon')$$
$$-\frac{4\epsilon^{abe}\epsilon^{cde}(p\cdot q)[2q\cdot\epsilon p\cdot\epsilon'-2p\cdot\epsilon q\cdot\epsilon'-\epsilon\cdot\epsilon'k\cdot(p-q)]}{(m_\rho^2-2p\cdot q)}+\Big(\frac{\epsilon^{ace}\epsilon^{bde}}{(m_\rho^2-p\cdot k)}\{[p\cdot kq\cdot\epsilon q\cdot\epsilon'-p\cdot qq\cdot\epsilon'p\cdot\epsilon+q\cdot(p+k)p\cdot k\epsilon\cdot\epsilon'$$
$$-q\cdot(p+k)p\cdot\epsilon p\cdot\epsilon']+(1+\delta)[p\cdot kq\cdot\epsilon q\cdot\epsilon'+p\cdot\epsilon q\cdot\epsilon'(2p\cdot q+p\cdot k)+2q\cdot(p+k)p\cdot k\epsilon\cdot\epsilon']$$
$$+(1+\delta)^2[p\cdot kq\cdot(p+k)\epsilon\cdot\epsilon'+p\cdot\epsilon p\cdot\epsilon'q\cdot(p+k)-p\cdot kq\cdot\epsilon q\cdot\epsilon'+p\cdot\epsilon q\cdot\epsilon'q\cdot(2k+p)]\}+(a\leftrightarrow b,p\leftrightarrow q)\Big)\Big]$$
$$-\frac{2e\epsilon^{abe}\epsilon^{cde}m_\rho[2q\cdot\epsilon p\cdot\epsilon'-2p\cdot\epsilon q\cdot\epsilon'-k\cdot(p-q)\epsilon\cdot\epsilon']}{F_\pi(m_\rho^2-2p\cdot q)}+\frac{\sqrt{2}em_\rho}{F_\pi}(\epsilon^{ace}\epsilon^{bde}+\epsilon^{bce}\epsilon^{ade})\epsilon\cdot\epsilon'+\Big[\frac{e\epsilon^{bce}\epsilon^{ade}}{4F_\pi m_\rho}\Big(\frac{m_\rho^2(1+\delta)q\cdot\epsilon p\cdot\epsilon'}{m_\pi^2-2q\cdot k}$$
$$-\frac{1}{m_\rho^2+2p\cdot(q+k)}\{2m_\rho^2[p\cdot\epsilon'q\cdot\epsilon-2p\cdot(q+k)\epsilon\cdot\epsilon'+\delta(p\cdot\epsilon p\cdot\epsilon'-\epsilon\cdot\epsilon'p\cdot(q+k))]$$
$$-[2p\cdot\epsilon'q\cdot\epsilon(p\cdot k+q\cdot k+p\cdot q)+\delta m_\rho^2q\cdot\epsilon p\cdot\epsilon']\}\Big)+(a\leftrightarrow b,p\leftrightarrow q)\Big]. \tag{11}$$

[17] S. Weinberg, Phys. Rev. Letters 17, 336 (1966); 18, 1178 (1967).

TABLE I. Partial decay widths $\Gamma(\rho \to 2\pi\gamma)/\Gamma(\rho \to 2\pi)$.

Decay modes	δ	Present calculation On-shell	Off-shell	Vector-dominance model[a]	Soft-pion current-algebra result
$\rho^0 \to \pi^+\pi^-\gamma$	$-\frac{1}{2}$	3.41×10^{-3}	6.48×10^{-3}	1.67×10^{-3}	2.4×10^{-3}
	0	2.91×10^{-3}	5.70×10^{-3}		
$\rho^{+,-} \to \pi^{+,-}\pi^0\gamma$	$-\frac{1}{2}$	9.10×10^{-4}	1.58×10^{-3}	6.89×10^{-4}	6.0×10^{-4}
	0	7.24×10^{-4}	1.36×10^{-3}		

[a] Reference 2.

Equation (11) does not contain any bremsstrahlung amplitude. The bremsstrahlung part which is contained in the weak amplitude [first term in Eq. (10)] drops out in the off-shell limits $q^2 \to 0$, $p^2 \to 0$.

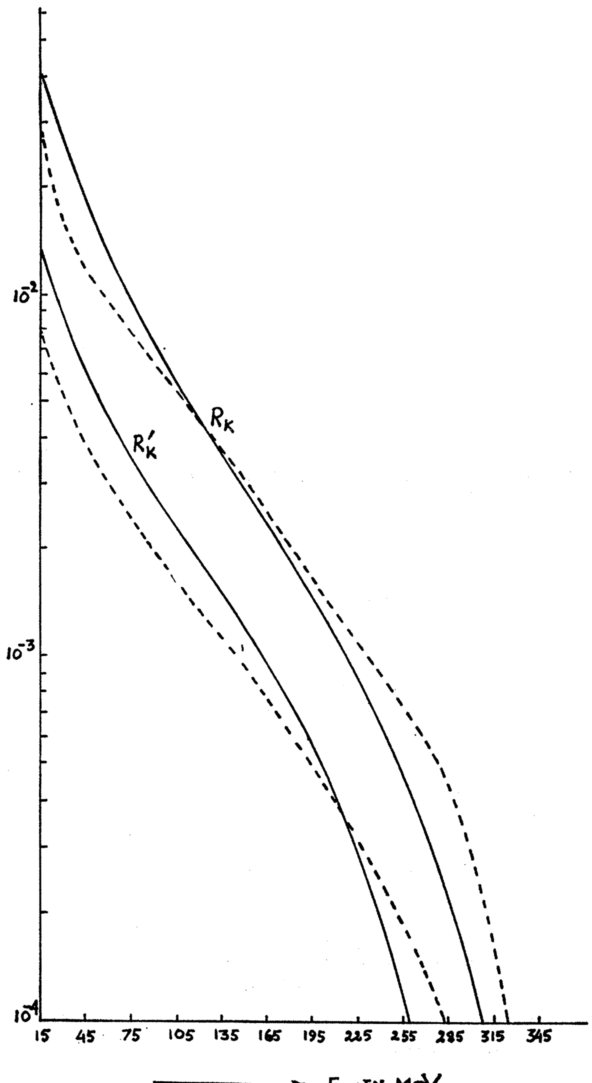

FIG. 1. Relative probabilities R_K and R_K' for photon emission with energy exceeding E_γ versus E_γ in MeV. The continuous curves correspond to the present calculation with parameter $\delta=-\frac{1}{2}$; the dashed curves represent results of the pole model (Ref. 2) of Singer.

IV. RESULTS AND DISCUSSION

With the on-shell and the off-shell direct amplitudes[18] derived in the earlier sections, we compute the branching ratios of both neutral and charged ρ-meson decay to the dominant mode $\rho \to 2\pi$ and compare with the results obtained in the vector-dominance model of Singer.[2] For the numerical part we have taken the width $\Gamma(\rho \to 2\pi)$ to be 100 MeV.[19] Other constants like masses are taken from Ref. 20. With two different values[21] of the parameter δ, the partial decay widths[22] are computed and tabulated in Table I. The agreement, in general, is good, particularly for the on-shell calculation. It has been pointed out recently by Singer[23] that the weak amplitude terms in radiative decay processes may have a contribution to the decay rates that is significant compared to the contribution obtained in the soft-pion limit. Actually, we observe that the dynamical structure of the matrix element improves the result considerably. In the present calculation, in addition to the contributions which correspond to the amplitude in the pole-dominance model, we have also included contributions coming from the chiral partner of the ρ meson, namely, the A_1 meson. Since the mass of the A_1 meson is $\sqrt{2}$ times the mass of the ρ meson, our numerical results are not much different from those of Singer.

For the energy spectrum of the photon, we treat the direct and the internal bremsstrahlung amplitudes separately. In the case of photon emission in the bremsstrahlung mechanism, we calculate the relative probabilities $R_K=[\Gamma_{\rho^0}(\pi^+\pi^-\gamma)/\Gamma_{\rho^0}(\pi^+\pi^-)]$ and

$$R_K'=[\Gamma_{\rho^{+,-}}(\pi^{+,-}\pi^0\gamma)/\Gamma_{\rho^{+,-}}(\pi^{+,-}\pi^0)]$$

for emission of photons with energy exceeding E_γ and for infrared divergence in the phase-space integration we use the same cutoff, $E_\gamma=15$ MeV, as that used in

[18] Here we mean for the on-shell direct amplitude the sum of all terms in Eq. (8) except the two bremsstrahlung terms.

[19] V. L. Auslander *et al.*, Phys. Letters **25B**, 433 (1967).

[20] A. H. Rosenfeld *et al.*, University of California Lawrence Radiation Laboratory Report No. UCRL-8030, 1967 (unpublished).

[21] From the analysis of Schnitzer and Weinberg (Ref. 10) we find that the consistent choice of the parameter δ for the $\rho\pi$ system should be $-\frac{1}{2}$.

[22] For $[\Gamma_{\rho^{+,-}}(\pi^{+,-}\pi^0\gamma)/\Gamma_{\rho^{+,-}}(\pi^{+,-}\pi^0)]$ in the pole model, we tabulate the value which correspond to $f_{\rho\omega\pi}^2/4\pi=0.68$ which is the present accepted value for this constant (see Ref. 3).

[23] P. Singer, Northwestern University Report (unpublished).

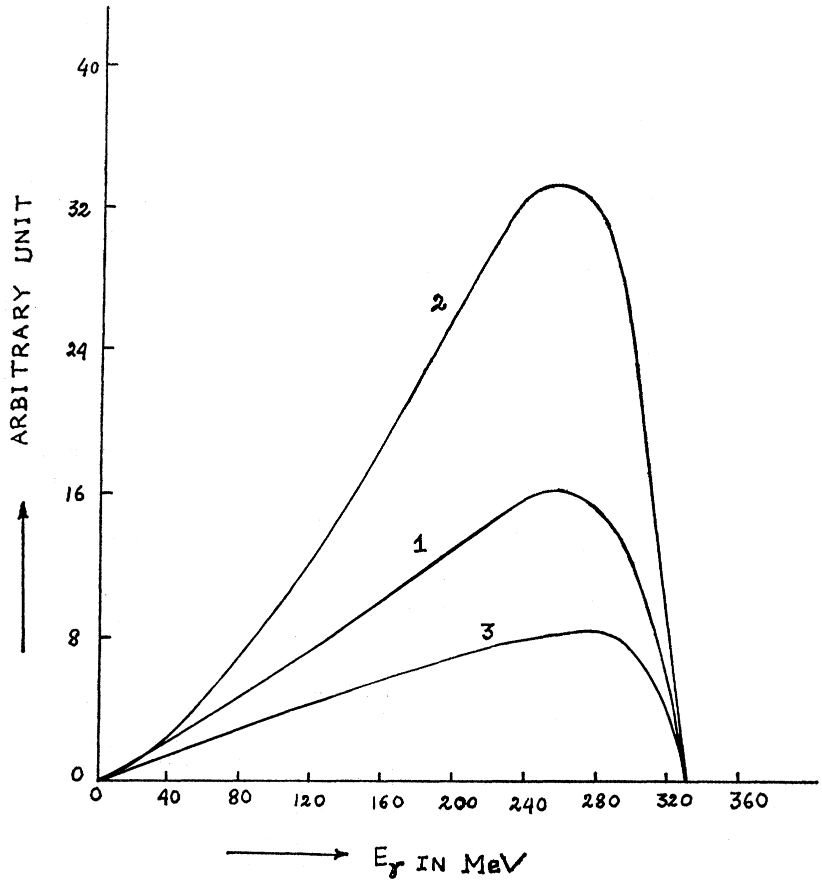

FIG. 2. Photon energy spectrum for the direct emission in $\rho^0 \rightarrow \pi^+\pi^-\gamma$ for $\delta = -\frac{1}{2}$. The curves correspond to (1) on-shell amplitude, (2) off-shell amplitude, and (3) pole model (Ref. 2).

Ref. 2. With $\delta = -\frac{1}{2}$, we find that the curves showing the variation of R_K and R_K' with E_γ (Fig. 1) are in excellent agreement with those obtained in the pole model. The direct-emission spectrum (Fig. 2) for both the on-shell and the off-shell amplitudes shows a little shift of the peak. The reason may be due to the inclusion of A_1-meson poles. Following Singer, we have not taken into account any effect of the anomalous magnetic moment of the ρ meson and the final-state $\pi\pi$ interaction. Also we have not considered any radiative corrections[24] which, in principle, should eliminate the divergence difficulties associated in soft-photon emission in the bremsstrahlung process. We conclude that the hard-pion method is more consistent than the off-shell analysis in explaining the important features of radiative processes. Radiative decays of other vector mesons like ω, ϕ, and K^* are under investigation.

[24] J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley Publishing Co., Inc., Reading, Massachusetts, 1955), Chap. 16.

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