are completely crossing-symmetric. The ρ and f trajectories appear degenerate. These expressions show resonances in the $I=2$ channel, but they are not on the leading trajectories and can be pushed to higher energies by the usual trick of adding nonleading terms. So if the present evidence against the ρ choosing nonsense is confirmed, then the triple-product representation must be chosen to describe $\pi\pi$ scattering.

The author acknowledges very fruitful discussions with C. Goebel and B. Sakita.

PHYSICAL REVIEW VOLUME 177, NUMBER 5 25 JANUARY 1969

K^+-Nucleon Scattering in an Effective Chiral Lagrangian Model

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(Received 11 July 1968)

The S-wave K^+-nucleon scattering lengths are calculated in a phenomenological chiral model containing baryons and pseudoscalar mesons. Good agreement with experiment is found, in contrast to the usual current-algebra results which correspond to unphysical limits of our expressions. The sensitivity of the calculation to various modifications of the Langrangian is also discussed. In particular, the scattering lengths are observed to be rather dependent on the way $SU(3)$ symmetry breaking is introduced into the model.

INTRODUCTION

MANY authors[1] have recently discussed applications of chiral $SU(2)\times SU(2)$ and $SU(3)\times SU(3)$ effective Lagrangians wherein the pseudoscalar mesons are taken to transform nonlinearly under the group. In this way many of the so-called current-algebra (CA) results can be obtained when the appropriate extrapolations to zero meson momenta are made. In this note, we compute the K^+-nucleon S-wave scattering lengths with a chiral $SU(3)\times SU(3)$ Lagrangian consisting of baryons and pseudoscalar mesons. Our results reduce to the CA results[2] in the appropriate unphysical limit but are rather different for the physical processes. In this way, good agreement with experiment[3] is found and the reason that the CA results are so poor becomes understandable. We must point out, however, that the results depend on the way in which $SU(3)$ symmetry breaking is introduced. We do not consider here the case of $\bar{K}^+$-nucleon scattering which is complicated because inelastic channels are already open at threshold.

Our choice of effective Lagrangian is made so that the (partially) conserved axial-vector current (PCAC) computed in the canonical way has the experimental D/F ratio. However, the results for the scattering lengths turn out to be extremely insensitive to this ratio. The pion-nucleon scattering lengths are also computed and the same insensitivity is found.

The choice of chiral representation for the octet baryons is, of course, not unique. The $[(8,1),(1,8)]$ representation which does not contain any additional particles is used in the first part of our paper. We may also group a $\frac{1}{2}^-$ singlet together with the $\frac{1}{2}^+$ octet to form the $[(3,3^*),(3^*,3)]$ representation. A discussion of this case is also given and the results are also found to be in good agreement with experiment. Our Lagrangian gives the correct width for the decay $Y_0\rightarrow\Sigma\pi$.

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[1] K. Nishijima, Nuovo Cimento **11**, 698 (1959); F. Gürsey, Ann. Phys. (N. Y.) **12**, 91 (1961); T. K. Kuo and M. Sugawara, Phys. Rev. **151**, 1181 (1966); S. Weinberg, Phys. Rev. Letters **18**, 188 (1967); J. Schwinger, Phys. Letters **24B**, 473 (1967); J. A. Cronin, Phys. Rev. **161**, 1483 (1967); H. S. Mani, Y. Tomozawa, and Y. P. Yao, Phys. Rev. Letters **18**, 1084 (1967); L. S. Brown, Phys. Rev. **163**, 1802 (1967); J. Wess and B. Zumino, *ibid.* **163**, 1727 (1967); R. Arnowitt, M. H. Friedmann, and P. Nath, Phys. Rev. Letters **19**, 1085 (1967); W. A. Bardeen, L. S. Brown, B. W. Lee, and H. T. Nieh, *ibid.* **18**, 1170 (1967); P. Chang and F. Gürsey, Phys. Rev. **164**, 1752 (1967); **169**, 1397(E) (1968); T. Shiozaki, Progr. Theoret. Phys. (Kyoto) **39**, 195 (1968).

[2] Y. Tomazawa, Nuovo Cimento **46**, 803 (1967); A. P. Balachandran, G. M. Gundzik, and F. Nicodemi, *ibid.* **44A**, 1257 (1966); P. Roy, Phys. Rev. **162**, 1644 (1967).

[3] S. Goldhaber *et al.*, Phys. Rev. Letters **9**, 135 (1962); V. J. Stenger *et al.*, Phys. Rev. **134**, B1111 (1964).

As a sidelight, we note that (under the usual kind of assumptions) the condition for the scattering lengths as computed from current algebra and from vector-meson exchange to coincide is that the (physical) ϕ meson decouple from the nucleons.

PHENOMENOLOGICAL LAGRANGIAN

Here, we follow the approach of Cronin.[1] Our notation is such that when we distinguish between the "left- and right-handed" subgroups of $SU(3)\times SU(3)$, unprimed tensor indices refer to the left one and primed tensor indices to the right one. Thus in a representation of the Dirac matrices where γ_5 is diagonal, the nucleon spinor may be written as

$$N_b{}^a=\begin{pmatrix} L_b{}^a \\ R_{b'}{}^{a'} \end{pmatrix},\quad \bar{N}_b{}^a\equiv(\bar{R}_{b'}{}^{a'},\bar{L}_b{}^a). \tag{1}$$

Equation (1) corresponds to assigning the baryons to the [(8,1),(1,8)] representation. We are using the two-component notation merely for convenience in constructing invariant and covariant $SU(3)\times SU(3)$ objects and do not mean to imply that the nucleons are massless. The meson matrices,[1] which satisfy

$$M_{b'}{}^a M_c{}^{b'}=\delta_c{}^a,\quad M_b{}^{a'}M_{c'}{}^b=\delta_{c'}{}^{a'}, \tag{2}$$

have the expansions

$$M_a{}^{b'}=\delta_a{}^b+2if\phi_a{}^b-2f^2\phi_a{}^c\phi_c{}^b+\cdots, \tag{3a}$$

$$M_{a'}{}^b=\delta_a{}^b-2if\phi_a{}^b-2f^2\phi_a{}^c\phi_c{}^b+\cdots, \tag{3b}$$

where $\phi_a{}^b$ stands for the usual octet of pseudoscalar mesons and f is the pion decay constant. We note, to avoid confusion, that after expanding M, for example, the distinction between primed and unprimed indices is dropped. This is because we are only interested in the *ordinary* $SU(3)$ transformation properties for the physical particles.

We write the phenomological Lagrangian density for meson-baryon scattering as the sum of several terms:

$$\mathcal{L}=\mathcal{L}_{\rm kin}+\mathcal{L}_{\rm viol}+\mathcal{L}_1+\mathcal{L}_2+\mathcal{L}_{\Delta M}, \tag{4a}$$

where the "kinematic" part is

$$\begin{aligned}\mathcal{L}_{\rm kin}=&-\bar{L}_b{}^a\sigma_\mu\partial_\mu L_a{}^b-\bar{R}_{b'}{}^{a'}\tilde{\sigma}_\mu\partial_\mu R_{a'}{}^{b'}\\ &-\frac{1}{8f^2}(\partial_\mu M_{b'}{}^a)(\partial_\mu M_a{}^{b'})\\ \simeq&-\bar{N}_b{}^a\gamma_\mu\partial_\mu N_a{}^b-\tfrac{1}{2}\partial_\mu\phi_b{}^a\partial_\mu\phi_a{}^b+\cdots,\end{aligned} \tag{4b}$$

with $\sigma_\mu=(-\boldsymbol{\sigma},i)\to\frac{1}{2}(1-\gamma_5)\gamma_\mu$ and $\tilde{\sigma}_\mu=(-\boldsymbol{\sigma},-i)\to\frac{1}{2}(1+\gamma_5)\gamma_\mu$. The chiral-symmetry-violating (PCAC) term is taken to lowest order to be

$$\mathcal{L}_{\rm viol}=-\tfrac{1}{2}(\pi^0\pi^0+2\pi^+\pi^-)\mu^2-(\bar{K}^+K^++\bar{K}^0K^0)\mu_K{}^2+\cdots, \tag{4c}$$

where μ and μ_K are, respectively, the π- and K-meson masses. The chiral invariant coupling which generates the baryon mass and the interaction with mesons has the form

$$\begin{aligned}\mathcal{L}_1=&-m(\bar{L}_c{}^aM_{b'}{}^cR_{d'}{}^{b'}M_a{}^{d'}+\bar{R}_{c'}{}^{a'}M_b{}^{c'}L_d{}^bM_{a'}{}^d)\\ \simeq&-m\bar{N}_b{}^aN_a{}^b+2imf(\bar{N}_c{}^a\gamma_5N_b{}^c-\bar{N}_b{}^c\gamma_5N_c{}^a)\phi_a{}^b\\ &+2mf^2(\bar{N}_c{}^aN_b{}^c+\bar{N}_b{}^cN_c{}^a)\phi_d{}^b\phi_a{}^d-4mf^2\bar{N}_c{}^aN_d{}^b\\ &\times\phi_b{}^c\phi_a{}^d+\cdots,\end{aligned} \tag{4d}$$

where m will be taken to be the nucleon mass. We must add another interaction term to make the D/F ratio of the baryon axial-vector current come out right. This term is

$$\begin{aligned}\mathcal{L}_2=&(\alpha\bar{L}_d{}^c\sigma_\mu L_b{}^d+\beta\bar{L}_b{}^d\sigma_\mu L_d{}^c)(M_{e'}{}^b\overleftrightarrow{\partial}_\mu M_c{}^{e'})\\ &+(\alpha\bar{R}_{d'}{}^{c'}\tilde{\sigma}_\mu R_{b'}{}^{d'}+\beta\bar{R}_{b'}{}^{d'}\tilde{\sigma}_\mu R_{d'}{}^{c'})(M_e{}^{b'}\overleftrightarrow{\partial}_\mu M_{c'}{}^e)\\ \simeq&4if(\alpha\bar{N}_e{}^b\gamma_\mu\gamma_5N_a{}^e+\beta\bar{N}_a{}^e\gamma_\mu\gamma_5N_e{}^b)\partial_\mu\phi_b{}^a\\ &+4f^2(\alpha\bar{N}_e{}^b\gamma_\mu N_a{}^e+\beta\bar{N}_a{}^e\gamma_\mu N_e{}^a)(\phi_c{}^a\overleftrightarrow{\partial}_\mu\phi_b{}^c)+\cdots.\end{aligned} \tag{4e}$$

In Eq. (4e) the parameters α and β can be computed by comparing the partially conserved axial-vector current of our Lagrangian with the experimental[4] one. We compute the axial current, $P_{\mu b}{}^a$, as

$$iP_{\mu a}{}^b\epsilon_b{}^a=\frac{\partial\mathcal{L}}{\partial(\partial_\mu N_b{}^a)}\delta_AN_b{}^a+\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_b{}^a)}\delta_A\phi_b{}^a, \tag{5a}$$

where

$$\delta_AN_b{}^a=\gamma_5(\epsilon_b{}^cN_c{}^a-\epsilon_c{}^aN_b{}^c), \tag{5b}$$

$$\delta_A\phi_b{}^a=\frac{1}{if}\epsilon_b{}^a+\cdots, \tag{5c}$$

and

$$\epsilon_b{}^a=-(\epsilon_a{}^b)^*.$$

This then leads to

$$P_{\mu a}{}^b=\frac{1}{f}\partial_\mu\phi_a{}^b+i(1-4\alpha)\bar{N}_e{}^b\gamma_\mu\gamma_5N_a{}^e-i(1+4\beta)\bar{N}_a{}^e\gamma_\mu\gamma_5N_e{}^b, \tag{6}$$

which, on comparison with the experimental current, gives

$$\alpha=\tfrac{1}{4}(1-D-F)\simeq-0.050, \tag{7a}$$

$$\beta=\tfrac{1}{4}(-1-D+F)\simeq-0.33, \tag{7b}$$

corresponding to the usual value $D/F\simeq1.7$ $(D+F=g_A)$.

Finally, we must add a term $\mathcal{L}_{\Delta M}$ to split the octet baryon masses in the observed way. This term must have a $T_3{}^3$ transformation property with respect to the ordinary $SU(3)$ group in order for the Gell-Mann–Okubo relation to hold. However, its transformation property with respect to chiral $SU(3)\times SU(3)$ is *a priori* unknown and can only be determined by trial and error. We shall first adopt the simplest possibility:

$$\begin{aligned}\mathcal{L}_{\Delta M}=&(m-m_\Sigma)[\bar{N}_b{}^aN_a{}^b-\bar{N}_3{}^bN_b{}^3]+(m_\Sigma-m_\Xi)\bar{N}_b{}^3N_3{}^b\\ =&-(m_\Sigma-m)\bar{\Sigma}\Sigma-(m_\Lambda-m)\bar{\Lambda}\Lambda-(m_\Xi-m)\bar{\Xi}\Xi.\end{aligned} \tag{8}$$

[4] W. Willis *et al.*, Phys. Rev. Letters **13**, 291 (1964); H. Courant *et al.*, Phys. Rev. **136**, B1791 (1964).

Substituting Eq. (1) into Eq. (8) shows that $\mathcal{L}_{\Delta M}$ transforms under the chiral group like (8,8). It breaks chiral $SU(2)\times SU(2)$ invariance for the strange baryons but retains it for the part of the Lagrangian containing the nucleons. Other forms of $\mathcal{L}_{\Delta M}$ with different chiral transformation properties will be discussed later and and will be found to lead to worse predictions for the K^+N scattering lengths.

It is important to note that in the presence of $\mathcal{L}_{\Delta M}$ the divergence of the strangeness-changing axial-vector current will pick up an additional term so that kaon PCAC will no longer be exact.

The Lagrangian, Eq. (4a), which we have just constructed contains only one arbitrary parameter. This is the value of the "degenerate" baryon mass, in Eq. (4d). If we were to choose any other value than the nucleon mass we could compensate for it in Eq. (8). The choice of the nucleon mass gives the best results and corresponds to the most natural reduction (without $\mathcal{L}_{\Delta M}$) to chiral $SU(2)\times SU(2)$ for the nucleon.

K^+-NUCLEON SCATTERING

We shall compute the K^+-nucleon and K^+-proton scattering amplitudes. These are related to the isospin amplitudes by

$$T(K^+n)=\tfrac{1}{2}(T_0+T_1)\,,$$
$$T(K^+p)=T_1.$$

The interaction Hamiltonian is taken to be the negative of the relevant terms of Eq. (4a). Thus,

$$\begin{aligned}\mathcal{H}_I=&-i(g_{p\Lambda K}\bar{p}\gamma_5\Lambda K^+ + f_{p\Lambda K}\bar{p}\gamma_\mu\gamma_5\Lambda\partial_\mu K^+\\&+g_{n\Sigma K}\bar{n}\gamma_5\Sigma^- K^+ + f_{n\Sigma K}\bar{n}\gamma_\mu\gamma_5\Sigma^-\partial_\mu K^+ + g_{p\Sigma K}\bar{p}\gamma_5\Sigma^0 K^+\\&+f_{p\Sigma K}\bar{p}\gamma_\mu\gamma_5\Sigma^0\partial_\mu K^+ + \text{H.c.}) - 2mf^2(2\bar{p}p\bar{K}^+K^+\\&+\bar{n}n\bar{K}^+K^+) - 4f^2[(\alpha-\beta)\bar{p}\gamma_\mu p - \beta\bar{n}\gamma_\mu n]\\&\times\bar{K}^+\overleftrightarrow{\partial}_\mu K^+,\end{aligned}\qquad(9)$$

with

$$g_{p\Lambda K}=-(\sqrt{6})mf,\quad f_{p\Lambda K}=(4f/\sqrt{6})(\beta-2\alpha)\,,$$
$$g_{n\Sigma K}=-2mf,\qquad f_{n\Sigma K}=4f\beta\,,$$
$$g_{p\Sigma K}=-\sqrt{2}mf,\qquad f_{p\Sigma K}=2\sqrt{2}f\beta.$$

Computing the "tree" graphs from Eq. (9) gives

$$A(K^+p)=4mf^2+\sum_Y\left\{f_{pYK}^2(m+m_Y)-2f_{pYK}g_{pYK}-\frac{m_Y-m}{m_Y^2-u}[g_{pYK}-(m+m_Y)f_{pYK}]^2\right\},\qquad(10a)$$

$$B(K^+p)=-8f^2(\alpha-\beta)+\sum_Y\left\{f_{pYK}^2-\frac{1}{m_Y^2-u}\times[g_{pYK}-(m+m_Y)f_{pYK}]^2\right\},\qquad(10b)$$

$$A(K^+n)=2mf^2+f_{n\Sigma K}^2(m+m_\Sigma)-2f_{n\Sigma K}g_{n\Sigma K}-\frac{(m_\Sigma-m)}{m_\Sigma^2-u}[g_{n\Sigma K}-(m+m_\Sigma)f_{n\Sigma K}]^2,\qquad(11a)$$

$$B(K^+n)=8f^2\beta+f_{n\Sigma K}^2-\frac{1}{m_\Sigma^2-u}[g_{n\Sigma K}-(m+m_\Sigma)f_{n\Sigma K}]^2,\qquad(11b)$$

where A and B correspond to the Chew-Goldberger-Low-Nambu (CGLN) decomposition[5] of the scattering amplitude:

$$T=-A+(i\gamma\cdot q_1)B.\qquad(12)$$

Furthermore s, t, and u are the usual Mandelstam variables, while m is the nucleon mass and m_Y is the mass of hyperon Y. From Eqs. (10) and (11) we calculate the s-wave scattering lengths as

$$a_s(K^+p)=\frac{m}{(m+\mu_K)(4\pi)}\left[4mf^2-8\mu_K f^2(\alpha-\beta)-\sum_{Y=\Lambda,\Sigma}\frac{(f_{pYK}\mu_K-g_{pYK})^2}{m_Y+m-\mu_K}\right],\qquad(13a)$$

$$a_s(K^+n)=\frac{m}{(m+\mu_K)(4\pi)}\left[2mf^2+8\mu_K f^2\beta-\frac{(f_{n\Sigma K}\mu_K-g_{n\Sigma K})^2}{m_\Sigma+m-\mu_K}\right].\qquad(13b)$$

The numerical results, taking $f=\mu^{-1}$, are $a_s(K^+p)=-0.25\mu^{-1}$ and $a_s(K^+n)=-0.09\mu^{-1}$. The corresponding experimental values[3] are $a_s(K^+p)=-0.22\mu^{-1}$ and $a_s(K^+n)=-0.097\mu^{-1}$, so the agreement is quite good. If we were to set $\alpha=\beta=0$ in Eqs. (13), corresponding to the neglect of the term in the Lagrangian $\mathcal{L}_2$, we would have $a_s(K^+p)=-0.27\mu^{-1}$ and $a_s(K^+n)=-0.10\mu^{-1}$. This case corresponds to $D/F=0$ rather than $D/F=1.7$. It is remarkable that such a large change in the axial-vector current D/F ratio produces such a small change in the scattering lengths.

Now it is interesting to consider the limit of Eqs. (13) where all baryon masses are taken to be degenerate and where the meson mass μ_K is neglected compared to the baryon mass m. Then we find

$$a_s(K^+p)\to\frac{-\mu_K f^2 m}{(2\pi)(m+\mu_K)},\qquad(14a)$$

$$a_s(K^+n)\to\tfrac{1}{2}a_s(K^+p).\qquad(14b)$$

These are precisely the CA results[2] and lead, for $f=\mu^{-1}$, to the rather large values $a_s(K^+p)=-0.37\mu^{-1}$, $a_s(K^+n)$

[5] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1957).

$=-0.185\mu^{-1}$. The reason it is necessary to consider the octet baryon masses degenerate to reproduce the CA results is that it is only in this limit (exact chiral symmetry except for the K-mass terms) that the usual kaon PCAC holds.

Thus we see that the failure of the current-algebra results to agree well with experiment just indicates that the amplitude at the zero kaon four-momentum limit differs sizably from the amplitude at the physical threshold.

We should emphasize that what we mean by the CA result is the one derived with the usual kaon PCAC and the limit of zero kaon four-momentum. In our Lagrangian model the divergence of the strangeness-changing axial-vector current is proportional to the kaon field only when there is no $SU(3)$ symmetry breaking. The CA treatments quoted[2] do not proceed in this way but consider the kaon field to be defined by a numerical coefficient times the divergence. Presumably, this effect would lead to an undetermined modification of this coefficient from its value in the limit where the axial-vector matrix elements are related by $SU(3)$. We are not labelling the so-called "σ terms" as part of the CA result.

K^+-NUCLEON SCATTERING WITH VECTOR-MESON EXCHANGE

Sakurai[6] has pointed out that the CA results for the s-wave pion-nucleon scattering lengths can be gotten from the vector-meson exchange model together with the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation.[7] Here we investigate the extension of this result to the K^+-nucleon scattering lengths. The object is to derive in this way Eqs. (14) which we have just quoted for comparison with the zero-energy limits of our results.

We choose as the interaction Hamiltonian of baryons, pseudoscalar mesons, and vector mesons the following:

$$\mathcal{H}_I=-\frac{f_V}{\sqrt{2}}i(\bar{N}_c{}^a\gamma_\mu N_b{}^c-\bar{N}_b{}^c\gamma_\mu N_c{}^a)\rho_{\mu a}{}^b-f_V(\sqrt{\tfrac{3}{2}})\times i\bar{N}_b{}^a\gamma_\mu N_a{}^b\phi_\mu+if_V\frac{1}{\sqrt{2}}\rho_{\mu b}{}^a(\phi_c{}^b\overleftrightarrow{\partial}_\mu\phi_a{}^c)\,, \tag{15}$$

where $\rho_{\mu b}{}^a$ is the vector-meson octet, ϕ_μ is the (unmixed) vector-meson singlet, and $f_V{}^2/4\pi\simeq2.8$ from pion-nucleon scattering.[8] We shall adopt the canonical mixing scheme[9] wherein the unmixed particles are related to the physical (tilde) particles by

$$\begin{aligned}\phi_\mu&=(\sqrt{\tfrac{2}{3}})\tilde{\omega}_\mu-(\sqrt{\tfrac{1}{3}})\tilde{\phi}_\mu\,,\\ \omega_{8\mu}&=(\sqrt{\tfrac{1}{3}})\tilde{\omega}_\mu+(\sqrt{\tfrac{2}{3}})\tilde{\phi}_\mu\,.\end{aligned} \tag{16}$$

In Eq. (15) the relative coefficient between the singlet and octet vector-meson coupling to baryons was chosen so that with the mixing scheme of Eq. (16) $\tilde{\phi}_\mu$ decouples from the nucleon.[10] This leads to the scattering amplitudes

$$A(K^+p)=A(K^+n)=0\,, \tag{17a}$$

$$B(K^+p)=-\tfrac{1}{2}f_V{}^2\left(\frac{1}{M_\rho{}^2-t}+\frac{3}{\tilde{M}_\omega{}^2-t}\right), \tag{17b}$$

$$B(K^+n)=-\tfrac{1}{2}f_V{}^2\left(\frac{-1}{M_\rho{}^2-t}+\frac{3}{\tilde{M}_\omega{}^2-t}\right). \tag{17c}$$

This gives the scattering lengths

$$a_s(K^+p)=\frac{-f_V{}^2}{4\pi}\frac{\mu_K m}{2(m+\mu_K)}\left(\frac{1}{M_\rho{}^2}+\frac{3}{\tilde{M}_\omega{}^2}\right), \tag{18a}$$

$$a_s(K^+n)=\frac{-f_V{}^2}{4\pi}\frac{\mu_K m}{2(m+\mu_K)}\left(-\frac{1}{M_\rho{}^2}+\frac{3}{\tilde{M}_\omega{}^2}\right). \tag{18b}$$

Setting $M_\rho=\tilde{M}_\omega$ and using the KSRF result that $f_V{}^2/M_\rho{}^2=f^2$ turns Eqs. (18) into the CA results given in Eqs. (14). Now suppose that we had not chosen the coefficients in Eq. (15) so that $\tilde{\phi}_\mu$ decoupled from the nucleons. This is easily accomplished by adding a term $-gi(\bar{N}_b{}^a\gamma_\mu N_a{}^b)\phi_\mu$ to $\mathcal{H}_I$, where g is an unknown coupling constant. This term gives an additional contribution to $a_s(K^+p)$ of

$$-(\tfrac{2}{3})^{1/2}\frac{f_Vg}{4\pi}\frac{\mu_K m}{m+\mu_K}\left(\frac{1}{\tilde{M}_\omega{}^2}-\frac{1}{\tilde{M}_\phi{}^2}\right).$$

Since $\tilde{M}_\omega$ is quite different from $\tilde{M}_\phi$, this term can only be zero if g is zero. Thus the simultaneous validity of the CA result and the vector-meson result requires the singlet vector meson to decouple from the nucleons.

PION-NUCLEON SCATTERING

Using the Lagrangian of Eq. (4a) and calculating as previously explained, leads to the following pion-nucleon scattering amplitudes:

$$A^{1/2}=A^{3/2}=2mf^2g_A{}^2\,, \tag{19a}$$

$$B^{1/2}=(mfg_A)^2\left[\frac{6}{m^2-s}+\frac{2}{m^2-u}\right]-2f^2(g_A{}^2-1)\,, \tag{19b}$$

$$B^{3/2}=-\frac{(2mfg_A)^2}{m^2-\mu}+f^2(g_A{}^2-1)\,. \tag{19c}$$

In Eqs. (19), $g_A=D+F$ and is the usual axial-vector coupling constant. Setting $\mathfrak{L}_2$ equal to zero would mean $g_A=1$, so it is this term which accomplishes the axial-

[6] J. J. Sakurai, Phys. Rev. Letters **17**, 552 (1966).

[7] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255, 384(E) (1966); Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966).

[8] J. J. Sakurai, Phys. Rev. Letters **17**, 1021 (1966).

[9] S. Okubo, Phys. Letters **5**, 165 (1963).

[10] H. Sugawara and F. Von Hippel, Phys. Rev. **145**, 1331 (1966).

vector renormalization in some sense. The amplitudes of Eqs. (19) give the s-wave scattering lengths,

$$a_1=\frac{m\mu f^2}{(2\pi)(m+\mu)}\left[1-\frac{g_A{}^2(m-\mu)\mu}{4m^2-\mu^2}\right], \tag{20a}$$

$$a_3=\frac{-\mu f^2 m}{(4\pi)(m+\mu)}\left[1+\frac{\mu g_A{}^2}{2m-\mu}\right], \tag{20b}$$

and the p-wave scattering lengths,

$$a_{33}=\frac{2}{3}\frac{m^3f^2}{\pi(m+\mu)}\frac{g_A{}^2}{\mu(2m-\mu)^2}, \tag{21a}$$

$$a_{13}=-\tfrac{1}{2}a_{33}, \tag{21b}$$

$$a_{31}=a_{33}-\frac{f^2}{16m\pi(m+\mu)}\times\left[\frac{8m^3+4m^2\mu-2m\mu^2+\mu^3}{(2m-\mu)\mu}g_A{}^2+(2m+\mu)\right], \tag{21c}$$

$$a_{11}=a_{13}+\frac{f^2}{8m\pi(m+\mu)}\times\left[-\frac{(4m^3+2m^2\mu-m\mu^2-\mu^3)}{(2m-\mu)\mu}g_A{}^2+(2m+\mu)\right]. \tag{21d}$$

TABLE I. Numerical results.

	$g_A\approx1.2$	$g_A=1$	Expt.
a_1	0.133	0.144	$(\ 0.171\pm0.005)\mu^{-1}$
a_3	-0.0775	-0.08	$(-0.088\pm0.004)\mu^{-1}$
a_{33}	0.075	0.057	$(\ 0.215\pm0.005)\mu^{-3}$
a_{13}	-0.037	-0.029	$(-0.029\pm0.005)\mu^{-3}$
a_{31}	-0.042	-0.034	$(-0.038\pm0.005)\mu^{-3}$
a_{11}	-0.137	-0.10	$(-0.101\pm0.007)\mu^{-3}$

The numerical results are given in Table I. In this case the effect of the term $\mathcal{L}_2$ is somewhat larger than for the K^+-nucleon case but it is still not appreciable. Also, the agreement with experiment for the s-wave scattering lengths is not quite so good. The results for $g_A=1$ are identical to those of Cronin[11] where the baryons were assigned to the $[(3,3^*),(3^*,3)]$ chiral representation.

$SU(3)$ SYMMETRY BREAKING

In this section we discuss models with $SU(3)$ symmetry-breaking terms for the baryons which have different chiral transformation properties from Eq. (8).

If we assume the symmetry breaking to transform as $T_3{}^3+T_{3'}{}^{3'}$, a natural procedure is to replace $(\mathcal{L}_1+\mathcal{L}_{\Delta M})$ in Eq. (4a) by

$$\begin{aligned}
\mathcal{L}_1+\mathcal{L}_{\Delta M}\rightarrow\ &-m_\Sigma[\bar{L}_c{}^aM_{b'}{}^cR_{d'}{}^{b'}M_a{}^{d'}+\bar{R}_{c'}{}^{a'}M_b{}^{c'}L_d{}^bM_{a'}{}^d]\\
&+\tfrac{1}{2}(m_\Sigma-m_\Xi)[\bar{L}_c{}^3M_{b'}{}^cR_{d'}{}^{b'}M_3{}^{d'}+\bar{R}_{c'}{}^{3'}M_b{}^{c'}L_d{}^bM_{3'}{}^d+\bar{L}_b{}^dM_{c'}{}^bR_{3'}{}^{c'}M_d{}^{3'}+\bar{R}_{b'}{}^{d'}M_c{}^{b'}L_3{}^cM_{d'}{}^3]\\
&+\tfrac{1}{2}(m_\Sigma-m)[\bar{L}_3{}^aM_{b'}{}^3R_{d'}{}^{b'}M_a{}^{d'}+\bar{R}_{3'}{}^{a'}M_b{}^{3'}L_d{}^bM_{a'}{}^d+\bar{L}_b{}^dM_{3'}{}^bR_{a'}{}^{3'}M_d{}^{a'}+\bar{R}_{b'}{}^{d'}M_3{}^{b'}L_a{}^3M_{d'}{}^a]\\
\simeq&-m_\Sigma\bar{N}_b{}^aN_a{}^b+2im_\Sigma f(\bar{N}_c{}^a\gamma_5N_b{}^c-\bar{N}_b{}^c\gamma_5N_c{}^a)\phi_a{}^b+2m_\Sigma f^2(\bar{N}_c{}^aN_b{}^c+\bar{N}_b{}^cN_c{}^a)\phi_d{}^b\phi_a{}^d\\
&-4m_\Sigma f^2\bar{N}_c{}^aN_d{}^b\phi_b{}^c\phi_a{}^d-(m_\Xi-m_\Sigma)[\bar{N}_b{}^3N_3{}^b+2if\bar{N}_c{}^3\gamma_5N_3{}^b\phi_b{}^c-if(\bar{N}_b{}^3\gamma_5N_d{}^b\phi_3{}^d+\bar{N}_b{}^d\gamma_5N_3{}^b\phi_d{}^3)\\
&-2f^2\bar{N}_c{}^3N_3{}^b\phi_a{}^c\phi_b{}^a+2f^2(\bar{N}_c{}^3N_d{}^b\phi_b{}^c\phi_3{}^d+\bar{N}_b{}^dN_3{}^c\phi_c{}^b\phi_d{}^3)-f^2(\bar{N}_b{}^3N_d{}^b\phi_a{}^d\phi_3{}^a+\bar{N}_b{}^dN_3{}^b\phi_a{}^3\phi_d{}^a)]\\
&-(m_p-m_\Sigma)[\bar{N}_3{}^aN_a{}^3-2if\bar{N}_3{}^a\gamma_5N_d{}^3\phi_b{}^d+if(\bar{N}_3{}^a\gamma_5N_a{}^b\phi_b{}^3+\bar{N}_b{}^a\gamma_5N_a{}^3\phi_3{}^b)-2f^2\bar{N}_3{}^aN_d{}^3\phi_b{}^d\phi_a{}^b\\
&+2f^2(\bar{N}_3{}^aN_d{}^b\phi_b{}^3\phi_a{}^d+\bar{N}_b{}^dN_a{}^3\phi_3{}^b\phi_d{}^a)-f^2(\bar{N}_3{}^aN_a{}^b\phi_c{}^3\phi_b{}^c+\bar{N}_b{}^aN_a{}^3\phi_c{}^b\phi_3{}^c)]+\cdots.
\end{aligned} \tag{22}$$

The net result of using Eq. (22) is to replace the nonderivative coupling constants given after Eq. (9) by

$$g_{p\Lambda K}=-(3f/\sqrt{6})(m_\Lambda+m),\quad g_{p\Sigma K}=(-f/\sqrt{2})(m_\Sigma+m),\quad g_{n\Sigma K}=-f(m_\Sigma+m).$$

Substituting these into Eq. (13) gives the results $a_s(K^+p)\simeq-0.56\mu^{-1}$ and $a_s(K^+n)\simeq-0.28\mu^{-1}$, which are rather poor.

Another possibility is to assume the symmetry breaking to transform as $T_{3'}{}^3+T_3{}^{3'}$. Then we may replace $(\mathcal{L}_1+\mathcal{L}_{\Delta M})$ by

$$\begin{aligned}
\mathcal{L}_1+\mathcal{L}_{\Delta M}\rightarrow\ &-m_\Sigma[\bar{L}_c{}^aM_{b'}{}^cR_{d'}{}^{b'}M_a{}^{d'}+\bar{R}_{c'}{}^{a'}M_b{}^{c'}L_d{}^bM_{a'}{}^d]\\
&+(m_\Sigma-m_\Xi)[\bar{L}_c{}^3M_{a'}{}^cR_{3'}{}^{a'}+\bar{R}_{c'}{}^{3'}M_a{}^{c'}L_3{}^a]+(m_\Sigma-m_p)[\bar{L}_3{}^aM_a{}^{c'}R_{c'}{}^{3'}+\bar{R}_{3'}{}^{a'}M_{a'}{}^cL_c{}^3]\\
\simeq&-m_\Sigma\bar{N}_b{}^aN_a{}^b+2im_\Sigma f(\bar{N}_c{}^a\gamma_5N_b{}^c-\bar{N}_b{}^c\gamma_5N_c{}^a)\phi_a{}^b+2m_\Sigma f^2(\bar{N}_c{}^aN_b{}^c+\bar{N}_b{}^cN_c{}^a)\phi_d{}^b\phi_a{}^d-4m_\Sigma f^2\bar{N}_c{}^aN_d{}^b\phi_b{}^c\phi_a{}^d\\
&+(m_\Sigma-m_\Xi)[\bar{N}_c{}^3N_3{}^c+2if\bar{N}_c{}^3\gamma_5N_3{}^a\phi_a{}^c-2f^2\bar{N}_c{}^3N_3{}^a\phi_d{}^c\phi_a{}^d]\\
&+(m_\Sigma-m_p)[\bar{N}_3{}^cN_c{}^3-2if\bar{N}_3{}^a\gamma_5N_c{}^3\phi_a{}^c-2f^2\bar{N}_3{}^aN_c{}^3\phi_b{}^c\phi_a{}^b]+\cdots.
\end{aligned} \tag{23}$$

The net result of using Eq. (23) is to replace the nonderivative coupling constants by

$$g_{p\Lambda K}=-(2f/\sqrt{6})(2m+m_\Sigma),\quad g_{n\Sigma K}=-2m_\Sigma f,$$
$$g_{p\Sigma K}=-\sqrt{2}m_\Sigma f,$$

to replace the $4mf^2$ term in Eq. (13a) by $2(m+m_\Sigma)f^2$, and to replace the $2mf^2$ term in Eq. (13b) by $2m_\Sigma f^2$ Then we have the results

$$a_s(K^+p)\simeq-0.66\mu^{-1}\quad\text{and}\quad a_s(K^+n)\simeq-0.28\mu^{-1},$$

which are also poor.

[11] See J. A. Cronin, Ref. 1.

A more unusual way of introducing symmetry breaking of the $T_3{}^3+T_{3'}{}^{3'}$ type is to replace the baryon part of the "kinematic" Lagrangian, Eq. (4b), by

$$-\frac{m}{m_\Sigma}(\bar{L}_b{}^a\sigma_\mu\partial_\mu L_a{}^b+\bar{R}_{b'}{}^{a'}\tilde{\sigma}_\mu R_{a'}{}^{b'})$$

$$-\frac{(m_\Sigma-m)}{m_\Sigma}(\bar{L}_3{}^a\sigma_\mu\partial_\mu L_a{}^3+\bar{R}_{3'}{}^{a'}\tilde{\sigma}_\mu\partial_\mu R_{a'}{}^{3'})$$

$$-\frac{m(m_\Sigma-m_\Xi)}{m_\Sigma m_\Xi}(\bar{L}_b{}^3\sigma_\mu\partial_\mu L_3{}^b+\bar{R}_{b'}{}^{3'}\tilde{\sigma}_\mu\partial_\mu R_{3'}{}^{b'})$$

$$=-\bar{p}\gamma_\mu\partial_\mu p-\frac{m}{m_\Lambda}\bar{\Lambda}\gamma_\mu\partial_\mu\Lambda-\frac{m}{m_\Sigma}\bar{\Sigma}\gamma_\mu\partial_\mu\Sigma$$

$$-\frac{m}{m_\Xi}\bar{\Xi}\gamma_\mu\partial_\mu\Xi\,, \quad (24)$$

and to delete the $\mathfrak{L}_{\Delta M}$ term. The resulting Lagrangian gives rise to an inverse Gell-Mann–Okubo formula:

$$3/m_\Lambda+1/m_\Sigma=2/m+2/m_\Xi\,,$$

which is satisfied to 3.4% as compared to 0.9% for the usual one. (Such a formula is extremely poor for the pseudoscalar mesons, however.) At any rate, the result of using Eq. (24) is to multiply the Λ propagator, for example, by a factor of m_Λ/m. This changes the scattering lengths to $a_s(K^+p)\simeq-0.50\mu^{-1}$ and $a_s(K^+n)\simeq-0.21\mu^{-1}$, which are again about twice as large as they should be.

Thus none of the schemes proposed in this section seems as suitable as the original one where the baryon $SU(3)$ symmetry-breaking term transformed as (8,8) under the chiral group.

[(3,3*),(3*,3)] REPRESENTATION FOR BARYONS

We may choose to group a unitary singlet [presumably the $\frac{1}{2}^- Y_0^*(1405)$] together with the octet baryons to form the [(3,3*),(3*,3)] representation. This choice was originally suggested by Gell-Mann[12] on the grounds that the kinematical term in the Lagrangian gives a D-type axial-vector current instead of the F-type corresponding to the [(8,1),(1,8)] representation. However, since we have seen [Eq. (6)] that the D and F parts of the axial current can be arbitrarily adjusted by adding the term $\mathfrak{L}_2$ of Eq. (4e) this objection to [(8,1)(1,8)] is not serious.[13] The analog of Eq. (1) is, for this representation,

$$N_a{}^b=\begin{pmatrix}L_a{}^{b'}\\ R_{a'}{}^b\end{pmatrix},\quad \bar{N}_a{}^b\equiv(\bar{R}_a{}^{b'},\bar{L}_{b'}{}^a)\,. \quad (25)$$

Here, $N_a{}^b$ is no longer traceless; the trace is related to the unitary $\frac{1}{2}^-$ singlet, Z by $\mathrm{Tr}(N_b{}^a)=\sqrt{3}\gamma_5 Z$. Then the chiral invariant Lagrangian density which replaces $\mathfrak{L}_1$ may be written as

$$\mathfrak{L}_1'=(m+c)[\bar{L}_{b'}{}^a M_{e'}{}^d R_{f'}{}^g\epsilon_{adg}\epsilon^{b'e'f'}$$

$$+\bar{R}_g{}^{f'}M_d{}^{e'}L_a{}^{b'}\epsilon^{gda}\epsilon_{f'e'b'}]+\left(\frac{m_{Y_0}}{3}-\frac{2m}{3}-c\right)$$

$$\times[\bar{L}_{b'}{}^a M_a{}^{b'}R_{c'}{}^d M_d{}^{c'}+\bar{R}_b{}^{a'}M_{a'}{}^b L_c{}^{d'}M_{d'}{}^c]$$

$$+c[\bar{L}_{a'}{}^b M_b{}^{c'}R_{c'}{}^d M_d{}^{a'}+\bar{R}_a{}^{b'}M_{b'}{}^c L_c{}^{d'}M_{d'}{}^a]\,, \quad (26)$$

where m is the degenerate octet mass, m_{Y_0} is the Y_0^* mass, and c is an arbitrary constant which eventually drops out in our calculation.

The coefficients in Eq. (26) were chosen so that the first terms in the expansion of the meson matrices reproduce the octet and singlet mass terms. The derivative-coupling part of the Lagrangian which replaces $\mathfrak{L}_2$ is given by

$$\mathfrak{L}_2'=\alpha'[\bar{R}_e{}^{b'}\tilde{\sigma}_\mu R_{a'}{}^e(M_c{}^{a'}\overleftrightarrow{\partial}_\mu M_{b'}{}^c)+\bar{L}_{e'}{}^b\sigma_\mu L_a{}^{e'}$$

$$\times(M_{c'}{}^a\overleftrightarrow{\partial}_\mu M_b{}^{c'})]+\beta'[\bar{R}_a{}^{e'}\tilde{\sigma}_\mu R_{e'}{}^b(M_{c'}{}^a\overleftrightarrow{\partial}_\mu M_b{}^{c'})$$

$$+\bar{L}_{a'}{}^e\sigma_\mu L_e{}^{b'}(M_c{}^{a'}\overleftrightarrow{\partial}_\mu M_{b'}{}^c)]+h[\bar{L}_{a'}{}^b\sigma_\mu L_c{}^{d'}$$

$$\times(M_b{}^{a'}\overleftrightarrow{\partial}_\mu M_{d'}{}^c)+\bar{R}_a{}^{b'}\tilde{\sigma}_\mu R_{c'}{}^d(M_{b'}{}^a\partial_\mu M_d{}^{c'})]\,, \quad (27)$$

where

$$\alpha'=\tfrac{1}{4}(1-g_A)\,,$$

$$\beta'=\tfrac{1}{4}(-1-F+D)\,,$$

and h is a parameter which we will use to fit the Y_0^* width.

To complete the phenomenological Lagrangian for this case we must add also the kinematic terms which we do not give explicitly and the symmetry-breaking term Eq. (8). We note that the Y_0^* has the correct mass even without symmetry breaking.

With the above interaction we find (after somewhat lengthy calculations) that $a_s(K^+n)$ is still given by Eq. (13b), but with β replaced by β'. Numerically, $a_s(K^+n)=-0.09\mu^{-1}$ with $m=m_p$. For K^+-proton scattering we must include also the graph with Y_0^* exchange. The result is

$$a_s(K^+p)=\frac{m}{(4\pi)(m+\mu_K)}\left\{-8\mu_K(\alpha'-\beta')f^2-\tfrac{4}{3}f^2(m_{Y_0}-2m)\right.$$

$$\left.+8hf^2\mu_K-\sum_Y\frac{(f_{pYK}\mu_K-g_{pYK})^2}{P_Y m_Y+m-\mu_K}\right\}\,, \quad (28)$$

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[12] M. Gell-Mann, Physics **1**, 63 (1964).

[13] This is also found to be the case for asymptotic chiral $SU(3)\times SU(3)$. See J. Schechter and G. Venturi, Phys. Rev. Letters **19**, 276 (1967).

where P_Y is the parity (± 1) of hyperon Y and

$$g_{p\Lambda K}=-\frac{2}{\sqrt{6}}mf,\qquad f_{p\Lambda K}=-\frac{4f}{\sqrt{6}}(2\alpha'+\beta')$$

$$g_{p\Sigma K}=+\sqrt{2}mf,\qquad f_{p\Sigma K}=-\frac{4f\beta'}{\sqrt{2}},$$

$$g_{pY_0K}=-\frac{2f}{\sqrt{3}}(m_{Y_0}-m),\quad f_{pY_0K}=-\frac{4f}{\sqrt{3}}(\beta'-\alpha')-2\sqrt{3}hf.$$

It is not hard to check that Eq. (28) also reduces to Eq. (14a) in the CA limit where $\mu_K\ll m$ and the *octet* baryon masses are taken to be degenerate. In Eq. (28) only the parameter h is unknown. We may determine it from the width for the $Y_0^*\to\Sigma\pi$ decay which is also predicted from our Lagrangian. The formula for the width Γ is

$$\Gamma=\frac{3g^2}{4\pi}|\mathbf{p}_\Sigma|\frac{E_\Sigma+m_\Sigma}{m_{Y_0}},\tag{29a}$$

with

$$g=-\frac{2f}{\sqrt{3}}(m_{Y_0}-m)-\frac{2f}{\sqrt{3}}(m_{Y_0}-m_\Sigma)(D-1)-2\sqrt{3}hf(m_{Y_0}-m_\Sigma).\tag{29b}$$

Taking[14] $\Gamma=50$ MeV gives $|g|=0.83$ and $h=-0.50$ or -0.79. For $h=-0.50$, we then calculate the scattering length $a_s(K^+p)$ as $-0.27\mu^{-1}$, which is quite a reasonable result [$h=-0.79$ leads to the unphysical value $a_s(K^+p)\simeq-0.01\mu^{-1}$].

Thus, the [(3,3*),(3*,3)] baryon representation also gives good predictions for the K^+p scattering lengths. In addition the Y_0^* width is given correctly. These results correspond to the type of symmetry breaking given in Eq. (8) rather than the type where the breaking term has a very simple chiral transformation property.

It may be worthwhile to note that the assumption of chiral symmetry does not relate the Y_0^* mass and couplings to the octet mass and couplings. We were free to treat these as parameters to be fit from the data. No additional parameters were introduced for the scattering, however.

[14] See, for example, A. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).

DISCUSSION

(i) In the same way that the (soon to be measured) pion-pion s-wave scattering lengths have been recently[15] regarded as criteria for distinguishing different phenomenological pion-pion interaction Lagrangians, we may regard the (already measured) K^+-nucleon s-wave scattering lengths as points of reference for investigating the validity of $SU(3)$ in the interaction of mesons and baryons.

(ii) We have found that the simplest $SU(3)$ invariant nonlinear chiral Lagrangian with symmetry breaking only in the mass terms gives good answers for the K^+-nucleon s-wave scattering lengths. The expressions we get reduce in the limit of kaon zero-momentum transfer and the usual kaon PCAC to the current-algebra expressions, as they must. The extrapolation in this case turns out to be non-negligible.

(iii) Our results are good for both the [(8,1),(1,8)] and [(3,3*),(3*,3)] baryon representations. Furthermore, the answers depend very little on the D/F ratio of the axial-vector current which our Lagrangian is set up to fit.

(iv) The fact that this model works so well leads us to believe that it may be worthwhile to consider extensions to other processes and more ambitious theories.

ACKNOWLEDGMENTS

It is a pleasure to thank Dr. J. Cronin for helpful discussions and suggestions. After preparing this manuscript, we received a preprint by Kumar and Ramachandran, in which K-nucleon scattering lengths are computed with a somewhat similar Lagrangian. However, there are many differences between the two works. A Lagrangian similar to ours has also been proposed by B. W. Lee [Phys. Rev. **170**, 1359 (1968)].

[15] S. Weinberg, Phys. Rev. **166**, 1568 (1968).