Resnikoff[6] have shown that for the $SU(6)$ limit, a multiplet structure of $\mathbf{70}\otimes\mathbf{1}$ is likely to occur for the negative-parity baryons. This has also been demonstrated by Capps,[8] whose arguments are consistent with bootstrap theory. The content of $\mathbf{70}\otimes\mathbf{1}$ includes two octets and a decuplet with spin and parity $\frac{3}{2}^-$. Since a precise estimate of the masses of these particles is not possible at this time, further word on the mixing conjecture must wait for further experimental study of Ξ excited states. If the current $\Gamma(\Xi'\Sigma\bar{K})$ upper bound is upheld, then the concept of $\Xi'(1815)$ mixing will be enhanced. However, should the value of $\Gamma(\Xi'\Sigma\bar{K})$ turn out to be larger, octet dominance will probably be sufficient to explain the $\frac{3}{2}^-$ decay widths.

Note added in proof: A related work on the negative parity baryon spectrum has been done by S. Pakvasa and S. F. Tuan, Nucl. Phys. (to be published).

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Baryon Excitation Form Factors and Asymptotic Chiral Symmetry

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An explicit multipole decomposition of the vector and pseudovector N-$N^*(1236)$ transition form factors is made, and the predictions of asymptotic chiral symmetry are discussed. A general consistency between the chiral predictions and the simplest assumptions on multipole behavior is found for one choice of decuplet chiral representation.

1. INTRODUCTION

It has been found[1] that assigning the octet baryons to a definite chiral representation leads to several very well satisfied relations among the various vector and pseudovector N-N transition form factors. In order to obtain these relations, it was necessary to interpret the predictions of chiral symmetry as holding only for very large spacelike momentum transfers.[2-4] The reasoning behind this point of view is that chiral symmetry is an expression of the tendency of nucleons to retain their helicities when they engage in interactions with other particles. At low energies, they are not particularly successful and emit pions as an attempt to compensate. However, at high energies (which we shall interpret as large momentum transfers for the case of form factors), the nucleon mass may be considered negligible and nucleons tend to behave as massless spin-$\frac{1}{2}$ objects which do conserve their helicities.

In order to test the relations at large spacelike momentum transfer (q^2), it is necessary to have some idea of the functional dependence of the form factors. For the case of the vector N-N transition form factors, the e-p scattering experiments give us a lot of information. These experiments[5] indicate that it is the Sachs form factors[6] which are simple functions of q^2 and that these behave as $(1+q^2/M_V{}^2)^{-2}$, where $M_V{}^2=0.71$ BeV^2. About the axial-vector N-N form factors and about all the N-N^* form factors, much less is known and we will have to rely mainly on analogy, although there is, of course, some available information from electroexcitation, photoexcitation, and neutrino excitation experiments. If we take the analogy with the vector N-N case seriously, the most natural assumption is that the objects which behave simply as functions of q^2 are the analogs of the Sachs form factors, these analogs being the relativistic multipoles introduced by Durand, DeCelles, and Marr.[7] Thus the first part of this paper is

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[1] J. Schechter and G. Venturi, Phys. Rev. Letters **19**, 276 (1967).

[2] Y. Nambu, in *Group Theoretical Concepts and Methods in Elementary Particle Physics*, edited by F. Gürsey (Gordon and Breach, Science Publishers, Inc., New York, 1964).

[3] A. Logunov, V. Mescheryakov, and A. Tavkhelidze, in *Proceedings of the International Conference on High-Energy Physics, CERN, 1962* (CERN Scientific Information, Geneva, Switzerland, 1962), p. 151.

[4] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **18**, 761 (1967).

[5] See, for example, L. Chan, K. Chen, J. Dunning, Jr., N. Ramsey, J. Walker, and R. Wilson, Phys. Rev. **141**, 1298 (1966).

[6] See, for example, L. N. Hand, P. G. Miller, and R. Wilson, Rev. Mod. Phys. **35**, 335 (1963).

[7] L. Durand, P. DeCelles, and R. Marr, Phys. Rev. **126**, 1882 (1962). We shall refer to this paper by DDM for brevity.

devoted to an explicit multipole decomposition of the vector and pseudovector N-N and N-N^* form factors. These do not all appear to have been given before, so this section may be of use to experimentalists. In particular, we suggest that neutrino production experiments be analyzed in terms of the axial-vector multipoles. It is reasonable to expect that the formulas involved will thereby become much simpler, in the same way[6] that the introduction of multipoles diagonalizes the Rosenbluth formula for electron-proton scattering.

The second part of this paper is a review of the asymptotic chiral $SU(3)\times SU(3)$ predictions for the N-N transition form factors. One interesting feature is that the d/f ratio of the axial vector current at $q^2=0$ is correctly given by the asymptotic chiral symmetry but would be predicted completely incorrectly if the chiral symmetry were applied at $q^2=0$. The reason for this situation is that at $q^2=0$ chiral symmetry relates the d/f ratio to the static charge multipole, while for large q^2 it relates the d/f ratio to the magnetic multipole.

In the third part of this paper we discuss the asymptotic chiral $SU(3)\times SU(3)$ predictions for the N-N^* transition form factors. A feature similar to that in the N-N case is observed; namely, if chiral symmetry were to be used at $q^2=0$, the sign of one axial-vector coupling constant would contradict experiment. This contradiction is avoided by using the asymptotic symmetry. We find that in this case there are three possible sets of chiral predictions. Only one, however, is consistent with a behavior for the form factors which is the exact analog of the N-N case. This enables us to express a preference for the asymptotic chiral representation of the spin-$\frac{3}{2}$ baryons.

2. MULTIPOLE DECOMPOSITION OF FORM FACTORS

Let us denote the octet of vector currents by $(V_b{}^a)_\mu$ and the octet of pseudovector currents by $(P_b{}^a)_\mu$, where μ is the Lorentz index and a and b are $SU(3)$ indices. The definitions may be made precise by first defining their "charges":

$$A_b{}^a=+i\int d^3x\,(V_b{}^a)_4, \tag{1a}$$

$$B_b{}^a=+i\int d^3x\,(P_b{}^a)_4; \tag{1b}$$

next, identifying the electric charge,

$$Q=-A_1{}^1; \tag{2}$$

and finally assuming (to establish the relative scale of V and P) Gell-Mann's $SU(3)\times SU(3)$ commutation relations,[8]

$$[A_b{}^a,A_d{}^c]=\delta_d{}^aA_b{}^c-\delta_b{}^cA_d{}^a, \tag{3a}$$

$$[B_b{}^a,B_d{}^c]=\delta_d{}^aA_b{}^c-\delta_b{}^cB_d{}^a, \tag{3b}$$

$$[A_b{}^a,B_d{}^c]=\delta_d{}^aB_b{}^c-\delta_b{}^cB_d{}^a. \tag{3c}$$

The usual vector and pseudovector form factors of the octet baryons are defined by the expressions

$$\begin{aligned}(p_0p_0'/M^2)^{1/2}\langle N'(p')|(V_b{}^a)_\mu|N(p)\rangle &=i\bar{u}(p')\{\gamma_\mu[d_1{}^V(q^2)D_b{}^a+f_1{}^V(q^2)F_b{}^a]\\&\quad+(\sigma_{\mu\nu}q_\nu/2M)[d_2{}^V(q^2)D_b{}^a+f_2{}^V(q^2)F_b{}^a]\}u(p),\end{aligned} \tag{4a}$$

$$\begin{aligned}(p_0p_0'/M^2)^{1/2}\langle N'(p')|(P_b{}^a)_\mu|N(p)\rangle &=i\bar{u}(p')\{\gamma_\mu\gamma_5[d_1(q^2)D_b{}^a+f_1(q^2)F_b{}^a]\\&\quad+(iq_\mu/2M)\gamma_5[d_2(q^2)D_b{}^a+f_2(q^2)F_b{}^a]\}u(p),\end{aligned} \tag{4b}$$

where M is the octet baryon mass, $D_b{}^a$ and $F_b{}^a$ are the symmetric and antisymmetric $SU(3)$ matrices, and $q_\mu=(p-p')_\mu$. $SU(3)$ symmetry was assumed in writing Eqs. (4); this has the consequence that all the octet vector form factors are known if the usual neutron and proton form factors are known. In order to make Eqs. (4) less unwieldly, we shall introduce the abbreviations

$$(H_{1,2}{}^V)_a{}^b=d_{1,2}{}^V(q^2)D_a{}^b+f_{1,2}{}^V(q^2)F_a{}^b, \tag{4c}$$

$$(H_{1,2}{}^A)_a{}^b=d_{1,2}(q^2)D_a{}^b+f_{1,2}(q^2)F_a{}^b, \tag{4d}$$

and furthermore we shall not write the $SU(3)$ indices explicitly.

The conservation law for the vector current,

$$(p_0p_0'/M^2)^{1/2}\langle N'|q_\mu(V_b{}^a)_\mu|N\rangle=0,$$

is satisfied automatically by Eq. (4a). If we assume that the axial-vector current is conserved, we would, on the other hand, get a relation between $H_1{}^A$ and $H_2{}^A$:

$$H_1{}^A-(q^2/4M^2)H_2{}^A=0. \tag{5a}$$

Since the axial-vector current is only *partially* conserved, we must, following Nambu,[9] replace this equation by

$$H_1{}^A-[(q^2+\mu^2)/4M^2]H_2{}^A=0, \tag{5b}$$

where μ is the mass of the pseudoscalar meson.

The vector and pseudovector form factors[10] for the $N_+{}^*$-n transition can be defined by

$$\begin{aligned}(p_0p_0'/MM^*)^{1/2}\langle n(p')|(V_1{}^2)_\mu|N_+{}^*(p)\rangle &=\bar{u}(p')\gamma_5[F_1\delta_{\mu\nu}+iF_2\gamma_\mu p_\nu'\\&\quad+F_3p_\nu'(p'+p)_\mu+F_4p_\nu'(p'-p)_\mu]u_\nu(p),\end{aligned} \tag{6a}$$

[8] M. Gell-Mann, Physics 1, 63 (1964).

[9] Y. Nambu, Phys. Rev. Letters 4, 380 (1960). Actually, we should probably modify Eq. (5b) somewhat for $q^2\gg\mu^2$. For example, if it is assumed that the axial-vector current is dominated by a pseudoscalar meson pole and an axial-vector meson pole, Eq. (5b) is modified to

$$H_1{}^A-\left(\frac{M_a{}^2}{M_a{}^2-\mu^2}\right)\left(\frac{q^2+\mu^2}{4M^2}\right)H_2{}^A=0,$$

where M_a is the axial-vector meson mass. (See Ref. 1.)

[10] The F_i and G_i are related to the f_i and g_i of Y. T. Chiu, J. Schechter, and Y. Ueda [Phys. Rev. 150, 1201 (1966)] by $f_i=-F_i$; $g_i=+G_i$. There the full $SU(3)$ matrix element is given with final and initial states reversed and the connection is made with the notation of C. H. Albright and L. S. Liu, Phys. Rev. 140, B748 (1965).

$$(p_0p_0'/MM^*)^{1/2}\langle n(p')|(P_1{}^2)_\mu|N_+{}^*(p)\rangle = \bar{u}(p')[G_1\delta_{\mu\nu}+iG_2\gamma_\mu p_\nu' + G_3p_\nu'(p'+p)_\mu+G_4p_\nu'(p'-p)_\mu]u_\nu(p). \quad (6b)$$

For purposes of identification it may help to note that the combination $(V_1{}^2)_\mu+(P_1{}^2)_\mu$ enters into the leptonic weak-interaction Hamiltonian in the form

$$H_w=(G/\sqrt{2})\cos\theta\, l_\mu(V_1{}^2+P_1{}^2)_\mu+\text{H.c.}, \quad (7)$$

where G is the Fermi constant, θ is the Cabibbo angle, and l_μ is the lepton current. All the other baryon-decuplet form factors are proportional to those of Eqs. (6) when $SU(3)$ symmetry is assumed. Thus there is no loss of generality in just considering Eqs. (6). Furthermore, we may show that if the currents $(V_b{}^a)_\mu$ and $(P_b{}^a)_\mu$ go over into $(-1)^{\delta_{\mu4}}(V_a{}^b)_\mu$ and $(-1)^{\delta_{\mu4}}(P_a{}^b)_\mu$, respectively, under both the CP and Hermitian conjugation operations (as would quark currents, for example) the form factors in Eqs. (6) are real.

Conservation of vector current leads to the following relations among the four vector form factors:

$$F_1+(M+M^*)F_2+(M^{*2}-M^2)F_3+q^2F_4=0, \quad (8)$$

where M is the octet mass and M^* is the decuplet mass.

Similarly, the partial conservation of the axial-vector current leads to the relation

$$G_1+(M^*-M)G_2+(M^{*2}-M^2)G_3+(q^2+\mu^2)G_4=0. \quad (9)$$

Now, having written down the usual form-factor decomposition for the N-N and N-N^* transition matrix elements, let us give the multipole decompositions. These are defined in terms of the Breit-frame matrix elements of the currents. We shall follow explicitly the notation of DDM[7] and define the Breit-frame matrix elements between an initial state[11] of momentum p along the $+z$ axis, spin s, and helicity λ, and a final state of spin s' and helicity λ' as

$$\Gamma_{s'\lambda';s\lambda}{}^{(\mu)}=\langle p0s'\lambda'|e^{i\pi J_2}V_\mu|p0s\lambda\rangle \quad (10)$$

for a vector current V_μ. In Eq. (10), the $SU(3)$ indices were suppressed and $e^{i\pi J_2}$ is a rotation of 180° about the y axis which directs the final-state momentum along the negative z axis. For the pseudovector current, we similarly define

$$\Gamma_{s'\lambda';s\lambda}{}^{5(\mu)}=\langle p0s'\lambda'|e^{i\pi J_2}P_\mu|p0s\lambda\rangle. \quad (11)$$

The zeroth component of the current can then be simply expanded in terms of charge multipoles, $Q_J(s',s)$, by

$$\Gamma_{s'\lambda';s\lambda}{}^{(0)}=(-1)^{2s'}\sum_{J=0}\begin{pmatrix}s' & J & s\\ \lambda' & 0 & \lambda\end{pmatrix}Q_J(s',s), \qquad (-1)^J=(-1)^\pi, \quad (12)$$

[11] The states of Eq. (10) are normalized covariantly, while those of Eq. (6a), for example, are not. However, the whole left-hand side of Eq. (6a) is properly covariant and may be identified with Eq. (10).

where the Wigner 3-j symbols have been used and $(-1)^\pi$ is the relative final-initial state parity. The threshold behavior of Q_J is p^J.

The plus and minus components of the current $[V_\pm=\mp(1/\sqrt{2})(V_1\pm iV_2)]$ can be expanded in terms of electric and magnetic multipoles as

$$\Gamma_{s'\lambda';s\lambda}{}^{(\pm)}=(-1)^{2s'}\sum_{J=1}\begin{pmatrix}s' & J & s\\ \lambda' & \pm1 & \lambda\end{pmatrix} \times\{\tfrac{1}{2}[1+(-1)^{J+\pi}]E_J(s',s) \pm\tfrac{1}{2}[1-(-1)^{J+\pi}]M_J(s',s)\}. \quad (13)$$

E_J behaves as p^{J-1} at threshold while M_J behaves as p^J.

Because of the conservation of the vector current the expansion of the third component does not give anything new.[7]

We may make an analogous expansion of the zeroth component of the pseudovector current in terms of "axial charge" multipoles,[12] $Q_J{}^5(s',s)$:

$$\Gamma_{s'\lambda';s\lambda}{}^{5(0)}=(-1)^{2s'}\sum_{J=0}\begin{pmatrix}s' & J & s\\ \lambda' & 0 & \lambda\end{pmatrix}Q_J{}^5(s',s), \qquad (-1)^J=-(-1)^\pi. \quad (14)$$

The threshold behavior of $Q_J{}^5$ is p^J.

The analog of the electric and magnetic multipole expansion for the pseudovector case is

$$\Gamma_{s'\lambda';s\lambda}{}^{5(\pm)}=(-1)^{2s'}\sum_{J=1}\begin{pmatrix}s' & J & s\\ \lambda' & \pm1 & \lambda\end{pmatrix} \times\{\tfrac{1}{2}[1-(-1)^{J+\pi}]E_J{}^5(s',s) \pm\tfrac{1}{2}[1+(-1)^{J+\pi}]M_J{}^5(s',s)\}. \quad (15)$$

At threshold $E_J{}^5$ behaves as p^{J-1} and $M_J{}^5$ behaves as p^J.

Since the pseudovector current is not conserved there is another set of multipoles corresponding to the expansion of the third component:

$$\Gamma_{s'\lambda';s\lambda}{}^{5(3)}=(-1)^{2s'}\sum_{J=0}\begin{pmatrix}s' & J & s\\ \lambda' & 0 & \lambda\end{pmatrix}L_J{}^5, \qquad (-1)^\pi=-(-1)^J. \quad (16)$$

[12] To derive the multipole expansion for the third and fourth components of an axial-vector current $j_\mu{}^5$ which satisfies the partial-conservation law $\partial_\mu j_\mu{}^5=c\phi$, where c is some constant and ϕ is a pseudoscalar field, we may proceed analogously to the treatment of DDM for the case of a conserved current. Following their notation, we would have in this case

$$\Gamma_{s'\lambda';s\lambda}{}^{5(3)}=-R(-1)^{s'-\lambda'}\langle s'\lambda'|e^{-i\nu K_3/2}j_3{}^5e^{-i\nu K_3/2}|s\lambda\rangle -\frac{icR\sinh\frac{1}{2}\sigma}{p_0-p_0'}\langle p0s'\lambda'|e^{i\pi J_2}\phi|p0s\lambda\rangle,$$

where

$$R=\left[\frac{2p}{p_0-p_0'}\sinh\frac{\sigma}{2}-\cosh\frac{\sigma}{2}\right]^{-1}.$$

In this form both the first and second terms can be expanded as relativistic multipoles. A similar formula holds for the fourth component. We note that since the equation $\partial_\mu j_\mu{}^5=c\phi$ is actually nothing more than a definition of ϕ, this derivation is valid for any nonconserved current.

The threshold behavior of $L_J{}^5$ is p^{J-1} for $j\neq 0$ and p for $j=0$. In deriving[12] Eq. (16), we formally assumed partial conservation of the axial vector current. For large-momentum transfer it is expected (and certainly required by asymptotic chiral symmetry) that $\Gamma_{s'\lambda';s\lambda}{}^{5(3)}$ be dependent on the other components. Thus this multipole is not particularly interesting for our present purposes.

To find the connection between the multipoles of Eqs. (12) and (13) and the usual N-N vector form factors of Eqs. (4a) and (4c), it is only necessary to evaluate explicitly Eqs. (12) and (13) with one specific nonvanishing helicity transition on the left-hand side. This gives

$$Q_0=\sqrt{2}[H_1{}^V-(q^2/4M^2)H_2{}^V], \tag{17a}$$

$$M_1=-(\sqrt{6})(p/M)(H_1{}^V+H_2{}^V), \tag{17b}$$

where M is the octet baryon mass. We recognize Eqs. (17) with the threshold dependences factored out as essentially the well-known Sachs form factors. For the axial-vector N-N transition form factors, the connection between the multipoles and the usual decomposition is provided by

$$E_1{}^5=-(\sqrt{6})(q^2+4M^2)^{1/2}(1/2M)H_1{}^A, \tag{18a}$$

$$Q_1{}^5=0, \tag{18b}$$

$$L_1{}^5=-(\sqrt{6})[H_1{}^A-(q^2/4M^2)H_2{}^A]. \tag{18c}$$

Comparison of Eq. (18c) with Eq. (5a) shows that $L_1{}^5$ vanishes in the limit of conserved axial vector current. For a partially conserved axial vector current, Eq. (5b) shows that $L_1{}^5=-(\sqrt{6})(\mu^2/4M^2)H_2{}^A$ and hence should be small at large q^2 (away from the pion pole in $H_2{}^A$).

In the case of M_1, experiment indicates that we must factor out the threshold dependence to find a "simple" function of q^2. For $E_1{}^5$ the threshold factor is just unity but, as we shall see, asymptotic chiral symmetry suggests that we take out the factor $(q^2+4M^2)^{1/2}$ to obtain a "simple" function of q^2.

The connection of the N-N^* multipoles with the usual form factors is made more complicated by the fact that the initial and final states have different masses. This has the consequence that the momentum transfer $q_\mu=(p-p')_\mu$ is no longer always a spacelike vector. A convenient generalization of q_μ is a spacelike vector which we denote by Q_μ (see Appendix) and whose square is related to q^2 by

$$Q^2=q^2+(M^{*2}-M^2)^2/[q^2+2(M^{*2}+M^2)]. \tag{19}$$

Evidently Q_μ goes over into q_μ when the final and initial masses become equal. It might be more meaningful to consider form factors as a function of Q^2 rather than q^2. In the Breit frame the magnitude of each particle's space momentum is simply given by

$$|\mathbf{p}|\equiv p=\tfrac{1}{2}\sqrt{(Q^2)}. \tag{20}$$

Details of the calculation of the N-N^* transition multipoles in terms of the form factors of Eqs. (6) are given in the Appendix. With this warning, and with the hope that no confusion results, we shall use the same symbols for the N-N^* multipoles as we did for the N-N multipoles. The vector N-N^* transition multipoles are

$$M_1=p(F_1Y-F_2X), \tag{21a}$$

$$E_2=-(\sqrt{5})p(F_1Y+F_2X), \tag{21b}$$

$$Q_2=2\left(\frac{5}{3}\right)^{1/2}\frac{1}{M^*}|\mathbf{p}|^2[-F_1+(M^*-M)F_2-K^2F_3+(M^2-M^{*2})F_4]Y, \tag{21c}$$

where for compactness we have used the symbols

$$X=\frac{1}{2}\left(\frac{-K^2}{MM^{*3}}\right)^{1/2}[(\tfrac{1}{4}Q^2+M^2)^{1/2}+M]^{1/2}\times[(\tfrac{1}{4}Q^2+M^{*2})^{1/2}+M^*]^{-1/2}\times[(-K^2)^{1/2}+M^*-M], \tag{22a}$$

$$Y=\frac{1}{2}\left(\frac{1}{MM^*}\right)^{1/2}\left[\left(\frac{Q^2}{4}+M^2\right)^{1/2}+M\right]^{-1/2}\times\left[\left(\frac{Q^2}{4}+M^{*2}\right)^{1/2}+M^*\right]^{-1/2}\times[(-K^2)^{1/2}+M^*+M], \tag{22b}$$

$$(-K^2)^{1/2}=\tfrac{1}{2}[(Q^2+4M^2)^{1/2}+(Q^2+4M^{*2})^{1/2}]. \tag{22c}$$

In Eqs. (21), the momentum dependence at threshold in the Breit frame has been explicitly factored. Proceeding by analogy to the case of the N-N vector form factors would lead us to expect that it is the remaining factor which should have a "simple" dependence on q^2 (or Q^2).

The pseudovector N-N^* transition multipoles are given by

$$E_1{}^5=\left(\frac{1}{-K^2}\right)^{1/2}\left(2M^*XG_1-\frac{K^2Q^2Y}{4M^*}G_2\right), \tag{23a}$$

$$M_2{}^5=-|\mathbf{p}|^2\left(\frac{5}{-K^2}\right)^{1/2}\frac{K^2Y}{M^*}G_2, \tag{23b}$$

$$Q_1{}^5=p\frac{2X}{(-K^2)^{1/2}}[G_1-(M+M^*)G_2+K^2G_3+(M^{*2}-M^2)G_4], \tag{23c}$$

$$L_1{}^5=X\left(\frac{Q^2+4M^{*2}}{-K^2}\right)^{1/2}G_1+\frac{(M^*-M)}{2M^*}Q^2YG_2+Q^2XG_4, \tag{24}$$

where the same abbreviations as before have been used and again the threshold dependences have been factored.

In accordance with the discussion given after Eq. (18a) we would expect the functions with "simple" Q^2 dependences to be the multipoles with both the threshold factors and the $(-K^2)^{1/2}$ factor removed.

3. ASYMPTOTIC CHIRAL SYMMETRY FOR N-N TRANSITIONS

In this section we shall review the N-N case[1] in order to extract the features which will be useful in the treatment of the N-N^* case.

The predictions of asymptotic chiral symmetry for the form factors of Eqs. (4a) and (4b) are

$$d_1(q^2)=d_1^V(q^2)\,, \tag{25a}$$

$$f_1(q^2)=f_1^V(q^2)\,, \tag{25b}$$

$$d_2(q^2)=f_2(q^2)=d_2^V(q^2)=f_2^V(q^2)=0\,. \tag{26}$$

The derivation of these equations is similar to the one which will be given in the next section for the N-N^* case. We interpret these equations as holding only at large spacelike q^2. Equation (26) will be interpreted to mean that the subscript-2 (induced) form factors fall off faster in q^2 than the subscript-1 form factors.

The faster falloff of $d_2(q^2)$ and $f_2(q^2)$ is guaranteed by Eqs. (5) which we must certainly accept in the asymptotic chiral limit. We can verify that $d_2^V(q^2)$ and $f_2^V(q^2)$ fall off faster than $d_1^V(q^2)$ and $f_1^V(q^2)$ if we assume that the usual phenomenological fit to nucleon form factors can be extrapolated to very large q^2. The usual fit is in terms of G_E and G_M which are, respectively, proportional to Q_0 of Eq. (17a) and M_1 of Eq. (17b) with threshold factor, p taken out. Particular linear combinations of d^V and f^V correspond to neutron and proton form factors and are labelled by superscripts n and p. Knowing these, it is possible to solve for $d_{1,2}{}^V$ and $f_{1,2}{}^V$ and obtain

$$d_1^V(q^2)=-\tfrac{3}{2}(1+q^2/4M^2)^{-1}\left(G_E{}^n+\frac{q^2}{4M^2}G_M{}^n\right), \tag{27a}$$

$$d_2^V(q^2)=-\tfrac{3}{2}(1+q^2/4M^2)^{-1}(G_M{}^n-G_E{}^n)\,, \tag{27b}$$

$$f_1^V(q^2)=(1+q^2/4M^2)^{-1}\times\left[G_E{}^p+\tfrac{1}{2}G_E{}^n+\frac{q^2}{4M^2}(G_M{}^p+\tfrac{1}{2}G_M{}^n)\right], \tag{27c}$$

$$f_2^V(q^2)=(1+q^2/4M^2)^{-1}\times(G_M{}^p+\tfrac{1}{2}G_M{}^n-G_E{}^p-\tfrac{1}{2}G_E{}^n)\,. \tag{27d}$$

Now the following empirical relations seem to hold very well as far as the form factors have been measured:

$$G_M{}^n/\mu_n=G_M{}^p/\mu_p=G_E{}^p\equiv\Phi_V(q^2)\,, \tag{28a}$$

$$G_E{}^n=0\,, \tag{28b}$$

where $\mu_p\simeq 2.79$ and $\mu_n\simeq -1.91$. Substituting Eqs. (28) into Eqs. (27) gives the following asymptotic behaviors:

$$d_1^V(q^2)\sim-\tfrac{3}{2}\mu_n\Phi_V(q^2)\,, \tag{29a}$$

$$d_2^V(q^2)\sim-\tfrac{3}{2}\mu_n\frac{4M^2}{q^2}\Phi_V(q^2)\,, \tag{29b}$$

$$f_1^V(q^2)\sim(\mu_p+\tfrac{1}{2}\mu_n)\Phi_V(q^2)\,, \tag{29c}$$

$$f_2^V(q^2)\sim(\mu_p+\tfrac{1}{2}\mu_n-1)\frac{4M^2}{q^2}\Phi_V(q^2)\,. \tag{29d}$$

It is clear from Eqs. (29) that the induced vector form factors do fall away faster as a function of q^2 when we accept the empirical Eqs. (28). Thus, Eqs. (25) and (26) appears to be mutually consistent.

Now let us consider the parametrization of the axial-vector form factors. From Eqs. (5) and (18) we see that $H_1{}^A$ [defined in terms of $d_1(q^2)$ and $f_1(q^2)$ by Eq. (4d)] is the only independent form factor and is essentially the same as our multipole $E_1{}^5$. However, if we require both that Eqs. (25) be satisfied and that the "simple" function we introduce to describe the $E_1{}^5$ multipole fall off with q^2 in the same way as $\Phi_V(q^2)$, we are led to the requirement that $E_1{}^5$ with the quantity $(q^2+4M^2)^{1/2}$ factored out should have a "simple" behavior. Namely, we should write

$$d_1(q^2)=d_1(0)\Phi_A(q^2)\,, \tag{30a}$$

$$f_1(q^2)=f_1(0)\Phi_A(q^2)\,, \tag{30b}$$

where $\Phi_A(q^2)$ falls off in the same way as $\Phi_V(q^2)$.

Taking the ratio of Eqs. (30a) and (30b) and equating this to the ratio of Eqs. (29a) and (29c) by Eqs. (25) gives the remarkable result

$$d_1(0)/f_1(0)=-3\mu_n/(2\mu_p+\mu_n)\simeq 1.56\,. \tag{31}$$

This is very close to the usually accepted value[13] of 1.7 for this well-known ratio. It is in marked contrast to the result we would obtain by using Eqs. (25) at $q^2=0$ rather than at *large* q^2. In that case, reference to Eqs. (27a) and (27c) shows that we would predict $d_1(0)/f_1(0)=0$. It is also evident from Eqs. (27a) and (27c) that the reason for this difference is that at low q^2 we are picking out the d/f ratio of the static charge multipole (G_E), while at large q^2 we are picking out the d/f ratio of the M_1 multipole (G_M).

Next we note that the function $\Phi_V(q^2)$ seems to be experimentally[5] given by

$$\Phi_V(q^2)=(1+q^2/M_V{}^2)^{-2}\,, \tag{32}$$

where $M_V{}^2=0.71$ BeV^2. This form is not well understood theoretically, although it can be predicted in certain models.[14] The quantity M_V is presumably something like a vector meson mass. The most straight-

[13] W. Willis *et al.*, Phys. Rev. Letters **13**, 291 (1964).
[14] See, for example, A. O. Barut, D. Corrigan, and H. Kleinert, Phys. Rev. Letters **20**, 167 (1968).

forward analogy for the axial-vector case would lead us to write

$$\Phi_A(q^2)=(1+q^2/M_A{}^2)^{-2}. \qquad (33)$$

Taking the sum of Eqs. (25a) and (25b) at large q^2 then gives the formula

$$g_A\equiv d_1(0)+f_1(0)=(M_V/M_A)^4(\mu_p-\mu_n). \qquad (34)$$

This has the numerical consequence that

$$(M_A/M_V)^4\simeq 3.99,$$

which seems extremely reminiscent of Weinberg's relation[15] for vector and axial-vector mesons:

$$(M_A/M_V)^4=4.$$

Thus, the fact that asymptotic chiral symmetry for the N-N transitions leads to such a consistent picture when combined with the most natural extrapolations of our present data encourages us to go on and investigate the N-N^* case.

Finally, we remark on a different way of deriving Eqs. (25) and (26), which were first derived by assuming the octet baryons to belong to a definite chiral representation. Let us simply require that the corresponding (same J) vector and pseudovector multipoles given by Eqs. (17) and (18) become equal at large q^2. Writing this out explicitly, we then have

$$H_1{}^V(q^2)+H_2{}^V(q^2)\sim H_1{}^A(q^2), \qquad (35a)$$

$$H_1{}^V(q^2)-(q^2/4M^2)H_2{}^V(q^2)\sim 0, \qquad (35b)$$

$$H_1{}^A(q^2)-\frac{q^2}{4M^2}H_2{}^A(q^2)\sim 0. \qquad (35c)$$

These equations come, respectively, from equating M_1 to $E_1{}^5$, Q_0 to zero, and $L_1{}^5$ to zero. Upon noting Eqs. (4c) and (4d) we see that Eqs. (35b) and (35c) are equivalent to Eq. (26) while Eq. (35a) then becomes equivalent to Eqs. (25).

4. ASYMPTOTIC CHIRAL SYMMETRY FOR N-N^* TRANSITIONS

A. Introduction

First we shall give in sections B, C, and D the predictions of chiral symmetry and then in Secs. E and F discuss the consistency of these predictions with "reasonable" behavior of the multipoles and whatever experimental data are available.

Once we have assigned the octet baryons to a definite chiral representation there are two choices for the decuplet baryons. These lead to two different sets of chiral predictions (Secs. B and C). One set appears at this stage to be favored over the other. Still a different set of predictions (Sec. D) can be gotten by equating the corresponding vector and pseudovector multipoles, as discussed previously for the N-N case. This is a phenomenological prescription which works in the N-N case but does not appear to be favored here.

B. Favored Decuplet Representation

The "left-handed" $SU(3)$ of chiral $SU(3)\times SU(3)$ is generated by $\frac{1}{2}(A_b{}^a+B_b{}^a)$ of Eq. (1) and will be designated by unprimed tensor indices. The right-handed $SU(3)$ is generated by $\frac{1}{2}(A_b{}^a-B_b{}^a)$ and will be designated by primed tensor indices. In a representation of of the Dirac matrices where γ_5 is diagonal we may then write the octet baryon as

$$N_b{}^a=\begin{pmatrix} L_b{}^a \\ R_{b'}{}^{a'}\end{pmatrix},\quad \bar{N}_b{}^a\equiv(\bar{R}_{b'}{}^{a'}\bar{L}_b{}^a). \qquad (36)$$

We shall denote this representation as [(8,1),(1,8)] corresponding to the representations of the entries in the upper and lower parts of the spinor. Other representations which contain the octet also bring in additional particles and do not appear to be favored in this scheme.[16]

Let us first assign the decuplet baryons to the [(1,10),(10,1)] representation. Then we have

$$(D_{abc})_\mu=\begin{pmatrix}(L_{a'b'c'})_\mu \\ (R_{abc})_\mu\end{pmatrix},\quad \bar{D}_\mu{}^{abc}\equiv(\bar{R}_\mu{}^{abc}\bar{L}_\mu{}^{a'b'c'}). \qquad (37)$$

To find the chiral predictions, it is only necessary to construct an effective operator out of Eqs. (36) and (37) which transforms like the weak current, namely, (8,1). This operator will correspond to the combination $V_\mu+P_\mu$. Thus we have for the effective weak current operator

$$\begin{aligned}V_{b\mu}{}^a+P_{b\mu}{}^a&=\epsilon^{fga}[A\bar{L}_g{}^e(R_{efb})_\mu+B\partial_\nu\bar{L}_g{}^e\partial_\mu(R_{efb})_\nu \\ &\qquad+C\partial_\mu\partial_\nu\bar{L}_g{}^e(R_{efb})_\nu] \\ &=\tfrac{1}{2}\epsilon^{fga}[A\bar{N}_g{}^e(1-\gamma_5)(D_{efb})_\mu \\ &\qquad+B\partial_\nu\bar{N}_g{}^e(1-\gamma_5)\partial_\mu(D_{efb})_\nu \\ &\qquad+C\partial_\mu\partial_\nu\bar{N}_g{}^e(1-\gamma_5)(D_{efb})_\nu], \qquad (38)\end{aligned}$$

where A, B, and C are some arbitrary real constants. Comparison of Eq. (38) with Eq. (6) then leads to the predictions

$$F_1(q^2)=-G_1(q^2), \qquad (39a)$$

$$F_2(q^2)=G_2(q^2)=0, \qquad (39b)$$

$$F_3(q^2)=-G_3(q^2), \qquad (39c)$$

$$F_4(q^2)=-G_4(q^2), \qquad (39d)$$

where q^2 is to be considered very large.

[15] S. Weinberg, Phys. Rev. Letters 18, 507 (1967). See also Ref. 4.

[16] For discussions of notations and representations see M. Gell-Mann, Physics 1, 63 (1964); P. Freund and Y. Nambu, Ann. Phys. (N. Y.) 32, 201 (1965); R. Marshak, N. Mukunda, and S. Okubo, Phys. Rev. 137, B698 (1965); Y. Hara, *ibid.* 139, B134 (1965); J. Schechter and Y. Ueda, *ibid.* 144, 1338 (1966).

C. Alternative Decuplet Representation

The [(10,1),(1,10)] decuplet representation corresponds to the spinor decomposition

$$(D_{abc})_\mu=\begin{pmatrix}(L_{abc})_\mu\\(R_{a'b'c'})_\mu\end{pmatrix}. \tag{40}$$

The effective current operator which takes the place of Eq. (38) in this case is

$$\begin{aligned}V_{b\mu}{}^a+P_{b\mu}{}^a&=A'\partial_\nu\bar{L}_g{}^e\sigma_\mu(L_{efb})_\nu\epsilon^{fga}\\&=\tfrac{1}{2}A'\epsilon^{fga}\partial_\nu\bar{N}_g{}^e(1-\gamma_5)\gamma_\mu(D_{efb})_\nu,\end{aligned} \tag{41}$$

where A' is an arbitrary real constant and $\sigma_\mu=(-\boldsymbol{\sigma},i)$. Comparison of Eq. (41) with Eq. (6) gives the asymptotic chiral predictions for this case:

$$F_2(q^2)=-G_2(q^2), \tag{42a}$$

$$\begin{aligned}F_1(q^2)=G_1(q^2)=F_3(q^2)=G_3(q^2)\\=F_4(q^2)=G_4(q^2)=0\end{aligned} \quad (\text{large } q^2). \tag{42b}$$

D. Equating Multipoles

Before discussing our favored set of predictions, we shall list the predictions which result from the intuitive prescription of equating multipoles.

Taking the asymptotic limits of the multipoles in Eqs. (21) and (23) and equating corresponding (same J) vector and pseudovector multipoles gives rise to the following large q^2 predictions:

$$F_1(q^2)\sim+G_1(q^2), \tag{43a}$$

$$F_1(q^2)\sim-\tfrac{1}{2}(q^2/M^*)[F_2(q^2)+G_2(q^2)], \tag{43b}$$

$$F_1+(M-M^*)F_2-q^2F_3+(M^{*2}-M^2)F_4\sim0, \tag{43c}$$

$$G_1-(M+M^*)G_2-q^2G_3+(M^{*2}-M^2)G_4\sim0, \tag{43d}$$

$$G_1+(M^*-M)G_2+q^2G_4\sim0. \tag{43e}$$

E. Comparison with Experiment

Here we shall show that the predictions of Eqs. (39), when combined with a model which is the simplest analog of the N-N case, lead to results which are consistent with experiment and also to one prediction which should be testable in the future.

First let us briefly summarize the experimental[17] situation as follows:

(a) $F_1(0)=-5.6,\quad G_1(0)\simeq-1.$[18]

(b) The M_1 multipole dominates the vector transition; i.e., the E_2 multipole is small. Not much is known about Q_2.

(c) The fall off in q^2 (for the M_1 transition) seems to be about the same, or somewhat faster than in the N-N case.

For simplicity, we shall set $M^*=M$ in our analysis. In view of the lack of present experimental knowledge this seems acceptable. In this limit the vector transition multipoles of Eqs. (21) can be written as

$$\tilde{M}_1=\left[F_1-\frac{1}{2M}(q^2+4M^2)F_2\right]\Big/M, \tag{44a}$$

$$\tilde{E}_2=-5^{1/2}\left[F_1+\frac{1}{2M}(q^2+4M^2)F_2\right]\Big/M, \tag{44b}$$

$$\tilde{Q}_2=-2(5/3)^{1/2}\left[F_1-(q^2+4M^2)F_3\right]\Big/M^2. \tag{44c}$$

In Eqs. (44) the tilde signifies that the threshold dependence has been taken out.

For the axial-vector multipoles we shall not only factor out the threshold dependences from Eqs. (23) but also multiply what remains[19] by $(q^2+4M^2)^{1/2}$. This is similar to the procedure we followed in the N-N case. The results are

$$\tilde{E}_1{}^5=(q^2+4M^2)\frac{1}{M}\left(G_1+\frac{q^2}{4M}G_2\right), \tag{45a}$$

$$\tilde{M}_2{}^5=\frac{\sqrt{5}}{M^2}(q^2+4M^2)G_2, \tag{45b}$$

$$\tilde{Q}_1{}^5=\frac{1}{M^2}(q^2+4M^2)[G_1-2MG_2-(q^2+4M^2)G_3]. \tag{45c}$$

The conservation and partial conservation laws, Eqs. (8) and (9), here simplify to

$$F_1+2MF_2+q^2F_4=0, \tag{46a}$$

$$G_1+(q^2+\mu^2)G_4=0. \tag{46b}$$

Equation (46b) is essentially the multipole $L_1{}^5$, which we therefore have not written here explicitly. Comparison of Eqs. (46a) and (44b) shows that, provided F_4 doesn't have a pole at $q^2=0$,

$$\tilde{E}_2(0)=0. \tag{47}$$

This vanishing of the electric quadrupole transition seems to be roughly in accord with experiment and is due to the fact that we neglected the M^*-M mass difference. Insofar as the M^*-M mass difference does not affect the physics of the situation this might be interpreted as a theoretical prediction.

As in the previous N-N case the most useful way to compare the chiral predictions with the multipoles is

[17] For discussions and further references on the vector transitions see A. J. Dufner and Y. S. Tsai [Phys. Rev. **168**, 1801 (1968)]. These authors analyze the data in terms of only the M_1 multipole. For the axial-vector transition see C. H. Albright and L. S. Liu, *ibid.* **140**, B1611 (1965).

[18] This value is rather rough but the relative sign seems to be definite. See Ref. 17; the $SU(6)$ prediction given there is -0.83.

[19] This is necessary to insure that F_1 and G_1 fall off with the same power of q^2 in accordance with the predictions of either Eq. (39a) or (43a).

to solve Eqs. (44)–(46) in reverse. Then we have

$$F_1=\tfrac{1}{2}M\left(\tilde{M}_1-\frac{1}{\sqrt{5}}\bar{E}_2\right), \tag{48a}$$

$$F_2=\frac{-M^2}{q^2+4M^2}\left(\tilde{M}_1+\frac{1}{\sqrt{5}}\bar{E}_2\right), \tag{48b}$$

$$F_3=\frac{\frac{1}{2}M}{q^2+4M^2}\left[\tilde{M}_1-\frac{1}{\sqrt{5}}\bar{E}_2+(\sqrt{\tfrac{3}{5}})M\tilde{Q}_2\right], \tag{48c}$$

$$q^2F_4=\frac{\frac{1}{2}M}{q^2+4M^2}\left(-q^2\tilde{M}_1+\frac{1}{\sqrt{5}}(q^2+8M^2)\bar{E}_2\right), \tag{48d}$$

$$G_1=\frac{M}{q^2+4M^2}\left(\bar{E}_1{}^5-\frac{q^2}{4\sqrt{5}}\tilde{M}_2{}^5\right), \tag{49a}$$

$$G_2=\frac{M^2/\sqrt{5}}{q^2+4M^2}\tilde{M}_2{}^5, \tag{49b}$$

$$G_3=\frac{-M}{(q^2+4M^2)^2}\times\left(M\tilde{Q}_1{}^5-\bar{E}_1{}^5+\frac{1}{4\sqrt{5}}(q^2+8M^2)\tilde{M}_2{}^5\right), \tag{49c}$$

$$G_4=\frac{-1}{q^2+\mu^2}\,\frac{M}{q^2+4M^2}\left(\bar{E}_1{}^5-\frac{q^2}{4\sqrt{5}}\tilde{M}_2{}^5\right). \tag{49d}$$

In the case of the N-N vector transitions it was found empirically [Eqs. (28)] that the two multipoles were proportional to each other. It would therefore be rash to assume anything else for the N-N^* transitions. Let us then write

$$\tilde{M}_1(q^2)/\tilde{M}_1(0)=\bar{E}_2(q^2)/\bar{E}_2(0)=\tilde{Q}_2(q^2)/\tilde{Q}_2(0)=\Phi_V(q^2), \tag{50}$$

$$\bar{E}_1{}^5(q^2)/\bar{E}_1{}^5(0)=\tilde{M}_2{}^5(q^2)/\tilde{M}_2{}^5(0)=\tilde{Q}_1{}^5(q^2)/\tilde{Q}_1{}^5(0)=\Phi_A(q^2), \tag{51}$$

where Φ_V and ϕ_A are assumed to fall off in the same way at large q^2 and as a first guess may be taken to coincide with Eqs. (32) and (33). This will probably have to be slightly modified in the future as more experimental information becomes available.

Now let us consider the chiral predictions of Eqs. (39). First we note that Eq. (39b), which we interpret as meaning that F_2 and G_2 fall off as $1/q^2$ times F_1 and G_1 respectively, holds. For F_2 and F_1 this is seen by substituting Eq. (50) into Eqs. (48a) and (48b). For G_2 and G_1 we must substitute Eqs. (51) into Eqs. (49a) and (49b) and pass to the large q^2 limit.

Next consider the prediction $F_1(q^2)=-G_1(q^2)$. If we were to (incorrectly) use this formula at $q^2=0$, then $F_1(0)$ and $G_1(0)$ would have opposite signs, in contradiction with experiment. However, if we use this formula for large q^2 we see, noting Eq. (49b), that it is $G_2(0)$ not $G_1(0)$ which becomes related to $F_1(0)$. This is exactly analogous to the situation concerning the prediction of the axial vector d/f ratio in the N-N case. The prediction of Eq. (39a) is

$$\tilde{M}_1(q^2)\sim(1/2\sqrt{5})\tilde{M}_2{}^5(q^2), \tag{52}$$

which may be restated through the use of Eqs. (50) and (51) as

$$G_2(0)=(M_V/M_A)^4(1/M)F_1(0). \tag{53}$$

A different assumed form for Φ_V and Φ_A would, of course, lead to some modification of the factor $(M_V/M_A)^4$. In the future, Eq. (53) may be tested experimentally. It would be desirable for neutrino production experiments to be analyzed in terms of multipoles; then we could test the prediction in the form $\tilde{M}_1(0)=(1/2\sqrt{5})\tilde{M}_2{}^5(0)(M_A/M_V)^4$.

Now, inspection of Eqs. (48c), (48d), and (49c) and (49d) shows that assuming the validity of Eqs. (50) and (51), the form factors F_3, F_4, G_3, and G_4 all fall off as $1/q^2$ times F_1 and thus should be considered zero asymptotically in our way of thinking. Nevertheless, setting $F_4(q^2)\sim-G_4(q^2)$ as required by Eq. (39d) does not lead to an absurdity but to Eq. (52) over again. Furthermore, setting $F_3(q^2)\sim-G_3(q^2)$ leads to the plausible result $\tilde{Q}_2(q^2)\sim0$.

Thus the simultaneous consistency of the predictions for the [(1,10),(10,1)] representation and the "natural" assumed forms of Eqs. (50) and (51) gives results in accord with experiment. One of the three axial multipoles is related to the M_1 multipole and the other two are unspecified. It goes without saying that more experimental information is really needed to check this scheme. It would seem quite reasonable to analyze the data in terms of the relativistic multipoles.

F. Other Possibilities

(i) Suppose that instead of assigning the decuplet baryons to the [(1,10),(10,1)] representation we had used the [(10,1),(1,10)] assignment. Then the predictions would be those of Eq. (42). This leads to an immediate contradiction if the "natural" behavior of the multipoles given in Eqs. (50) and (51) is accepted; namely, Eqs. (48a) and (48b) show that F_2 falls off faster at large q^2 than F_1.

(ii) A phenomenological way of obtaining asymptotic chiral predictions is just to equate corresponding vector and axial vector multipoles at large q^2. This procedure leads to Eqs. (43) which also contradict Eqs. (50) and (51). Specifically, if (50) and (51) are substituted into (48a), (48b), (49a), and (49b) and the results are substituted into (43a) and (43b), we derive the fact that $\tilde{M}_1=0$.

(iii) So far we have imposed the requirement that the chiral predictions we adopt be consistent when all the vector and axial vector multipoles are respectively proportional to each other. This is the exact analog of the N-N case. If this restriction is relaxed there is endless room for speculation. We shall mainly refrain from this temptation and only observe that if we assume double-pole forms for $F_1(q^2)$ and $G_1(q^2)$ and substitute in Eq. (43a) we would find $F_1(0)/G_1(0)=(M_A/M_V)^4\simeq+4$, which is about right and as in the N-N case seems to give a good idea of the relative scale between the vector and axial-vector couplings at low momentum transfer.

APPENDIX

Here we present some details which a reader interested in checking our formulas for the multipoles might find helpful.

The helicity matrix elements defined in Eqs. (10) and (11) were calculated by substituting explicit spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ helicity eigenstates into Eqs. (6a) and (6b). For the case of spin $\frac{1}{2}$, the plus and minus helicity eigenstates for a particle moving along the positive z axis with momentum p are given by

$$u^{(+)}(p)=\left(\frac{M+p_0}{2M}\right)^{1/2}\begin{pmatrix}\chi_+\\ [p/(p_0+M)]\chi_+\end{pmatrix},\tag{A1}$$

$$u^{(-)}(p)=\left(\frac{M+p_0}{2M}\right)^{1/2}\begin{pmatrix}\chi_-\\ -[p/(p_0+M)]\chi_-\end{pmatrix},\tag{A2}$$

where

$$\chi_+=\begin{pmatrix}1\\0\end{pmatrix}\quad\text{and}\quad\chi_-=\begin{pmatrix}0\\1\end{pmatrix}.$$

To get the helicity eigenstates for a particle moving along the negative z axis with momentum p, we must apply the rotation operator $e^{-i\pi J_2}$ to (A1) and (A2). In the Dirac-Pauli representation of the γ matrices which we are using $e^{-i\pi J_2}=\gamma_1\gamma_3$. This gives, for a negatively directed particle,

$$u^{(+)}(p)=\left(\frac{M+p_0}{2M}\right)^{1/2}\begin{pmatrix}\chi_-\\ [p/(p_0+M)]\chi_-\end{pmatrix},\tag{A3}$$

$$u^{(-)}(p)=\left(\frac{M+p_0}{2M}\right)^{1/2}\begin{pmatrix}-\chi_+\\ [p/(p_0+M)]\chi_+\end{pmatrix}.\tag{A4}$$

The spin-$\frac{3}{2}$ eigenstates are given, for example, in the paper of Frishman and Gotsman.[20]

Two typical helicity matrix elements are

$$\Gamma_{\frac{1}{2}-\frac{1}{2};\frac{3}{2}\frac{3}{2}}{}^{(-)}=NN'p\left(\frac{1}{p_0'+M}+\frac{1}{p_0+M^*}\right)F_1,\tag{A5}$$

$$\Gamma_{\frac{1}{2}\frac{1}{2};\frac{3}{2}\frac{1}{2}}{}^{(-)}=NN'\frac{p}{\sqrt{3}}\left[\left(\frac{1}{p_0'+M}+\frac{1}{p_0+M^*}\right)F_1+\frac{2}{M^*}(p_0+p_0')\left(1+\frac{p^2}{(p_0'+M)(p_0+M^*)}\right)F_2\right],\tag{A6}$$

where

$$NN'=\left(\frac{(M+p_0')(M^*+p_0)}{4MM^*}\right)^{1/2}.$$

These may be generalized to covariant form with the help of the vectors

$$q_\mu=p_\mu-p_\mu',\tag{A7}$$

$$K_\mu=p_\mu+p_\mu',\tag{A8}$$

$$Q_\mu=q_\mu+K_\mu[(M^{*2}-M^2)/K^2].\tag{A9}$$

K_μ is timelike while Q_μ is spacelike and $Q_\mu K_\mu=0$. It is sufficient to note Eqs. (19) and (20) to achieve the covariant form.

The helicity matrix elements of (A5) and (A6) are connected to the multipoles M_1 and E_2 by Eq. (13). Explicitly, this reads

$$\Gamma_{\frac{1}{2}-\frac{1}{2};\frac{3}{2}\frac{3}{2}}{}^{(-)}=\tfrac{1}{2}M_1-\frac{1}{2\sqrt{5}}E_2,\tag{A10}$$

$$\Gamma_{\frac{1}{2}\frac{1}{2};\frac{3}{2}\frac{1}{2}}{}^{(-)}=\frac{-1}{2\sqrt{3}}M_1-\tfrac{1}{2}(\sqrt{\tfrac{3}{5}})E_2.\tag{A11}$$

Solving Eqs. (A10) and (A11) gives the results stated in Eqs. (21a) and (21b).

[20] Y. Frishman and S. Gotsman, Phys. Rev. **140**, B1151 (1965).