$SU(3)$-Symmetry Breaking and Decays of the $\frac{3}{2}^-$ Baryons*

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(Received 31 May 1968; revised manuscript received 26 September 1968)

There exist several coupling-constant sum rules describing the interaction of three arbitrary octets in the presence of octet breaking of $SU(3)$. For decays of the $\frac{3}{2}^-$ baryon γ octet to D-wave baryon–pseudoscalar-meson composites, one of the sum rules involves observable modes only. Comparison of the sum rule with experimental data yields a test of the octet-dominance hypothesis, and in addition is potentially a probe of mixing phenomena in excited baryon systems.

I. INTRODUCTION

EXTENSIVE experimental investigation over the past decade has established that the baryon resonance region may be characterized by $SU(3)$ multiplets of particles, and that there exists a remarkable pattern of symmetry breaking which transforms as an octet (octet dominance). The latter feature is sometimes useful as a phenomenological tool in assigning particles to specific $SU(3)$ multiplets or in checking the consistency of such assignments. The best-known examples of this are the Gell-Mann–Okubo mass sum rules[1] for octets and decuplets (see Sec. II). The importance of octet mass and coupling-constant sum rules to baryon spectroscopy is heightened by the possibility that the following two phenomena exist in the baryon system:

(i) Mixing—an effect produced by the presence of at least two particles having the same spin and parity and roughly comparable masses. As a consequence, $SU(3)$-representation mixing may occur, leading to physical effects which can simulate breaking of $SU(3)$. A case in point is the ϕ-ω mixing in the vector-meson system, which was first detected by observing the violation of an octet mass sum rule.[2] There is now mounting evidence that mixing also occurs in the baryon system as well[3–8] (see Secs. III and IV).

(ii) Chew has pointed out[9] that if $SU(3)$, like shell structure in nuclear physics, has a dynamical origin, there may be many particles for which $SU(3)$ is meaningless, i.e., $SU(3)$ may hold only for certain groups of poles. The octet mass and coupling-constant sum rules could well play a role in exposing these particles, if they exist.

We have derived several coupling-constant sum rules describing the interaction of three arbitrary octets in the presence of octet breaking of $SU(3)$. One of the sum rules is particularly simple and can be used to study decay modes of the $\frac{3}{2}^-$ baryon γ octet to D-wave baryon–pseudoscalar-meson composites.[10] The derivations are given in Sec. II. In Sec. III we confront the sum rule with experimental data, first discussing the extraction of phenomenological coupling constants from decay widths. The latter is a nontrivial problem, as emphasized in Refs. 5 and 8. In addition, we study carefully the implication of mixing between the isoscalar particles $Y_0'(1520)$ and $Y_0'(1690)$, and the consequent effect that this has on the empirical coupling constants. We summarize our results and present our conclusions in Sec. IV.

II. MASS AND COUPLING-CONSTANT OCTET SUM RULES

It is true that $SU(3)$ is a rather badly broken symmetry. To make this statement more precise, we use $SU(3)$ tensor analysis to describe the masses of particles which comprise the filled baryon multiplets. To preserve hypercharge and isospin invariance, the breaking of $SU(3)$ can transform only as an $I=Y=0$ member of the **1**-, **8**-, **27**-, and **64**-dimensional multiplets (our discussion is restricted to octets and decuplets in this paper). We ignore effects of electromagnetic and weak interactions, and thus use isospin-invariant units such as the nucleon rather than charge states to describe the baryons. Using the Wigner-Eckart theorem, we can express the mass M of baryon b in multiplet $\mathbf{B}$ ($\mathbf{B}=\mathbf{8}$ or $\mathbf{10}$) as

$$M(\mathbf{B},b)=\sum_{Z,\gamma} Y_Z(\mathbf{B},b)\begin{pmatrix}\mathbf{B} & \mathbf{Z} & \mathbf{B}_\gamma \\ b & 0 & b\end{pmatrix}, \qquad (1)$$

where

$$\begin{pmatrix}\mathbf{B} & \mathbf{Z} & \mathbf{B}_\gamma \\ b & 0 & b\end{pmatrix}$$

is an $SU(3)$ isoscalar factor,[11] the sum goes over multiplets **1**, **8**, **27**, and **64** as explained above, and the index γ differentiates between the f,d couplings of two

* Supported in part by the National Science Foundation.

[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); S. Okubo, Progr. Theoret. Phys. (Koyoto) **27**, 949 (1962).
[2] J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962).
[3] R. D. Tripp *et al.*, Nucl. Phys. **B3**, 10 (1967).
[4] N. Masuda and S. Mikamo, Phys. Rev. **162**, 1517 (1967).
[5] M. Goldberg *et al.*, Nuovo Cimento **45**, 169 (1966).
[6] O. W. Greenberg and M. Resnikoff, Phys. Rev. **163**, 1844 (1967).
[7] A. N. Mitra and Marc Ross, Phys. Rev. **158**, 1630 (1967).
[8] R. H. Capps, Phys. Rev. **158**, 1433 (1967).
[9] G. F. Chew, *The Analytic S-Matrix* (W. A. Benjamin, Inc., New York, 1966), p. 97.
[10] Only after this paper had been submitted for publication did the author become aware that the sum rule (8) had previously been derived, but not applied, by V. Gupta and V. Singh, Phys. Rev. **136**, B782 (1964). The sum rules (9) are evidently linear combinations of those appearing in the Gupta-Singh paper.
[11] J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963).

TABLE I. $SU(3)$ tensor analysis of baryon masses. (a) and (b) describe the filled octet multiplets, whereas (c) and (d) describe the decuplet. The input masses are given in (a) and (c), and the reduced tensors of Eq. (1) are given in (b) and (d). All units are in MeV. The data are taken from Refs. 12 and 18.

		(a)		
	N	Λ	Σ	Ξ
$\frac{1}{2}^+$	938.9	1115.6	1193.2	1317.5
$\frac{3}{2}^-$	1525	1690	1660	1815
$\frac{5}{2}^-$	1680	1827	1767	1930
		(b)		
	Y_1	Y_{8d}	Y_{8f}	Y_{27}
$\frac{1}{2}^+$	1151	92.8	−378.6	13.7
$\frac{3}{2}^-$	1669	−22	−290	25
$\frac{5}{2}^-$	1794	−61	−250	−14
		(c)		
	N^*	Y_1^*	Ξ^*	Ω^-
$\frac{3}{2}^+$	1236	1385	1531	1672
		(d)		
	Y_1	Y_8	Y_{27}	Y_{64}
$\frac{3}{2}^+$	1383	−414	−9.4	2.5

octets to a third octet. For instance, the nucleon and Σ baryons have the expansions

$$N=Y_1-(\tfrac{1}{10}\sqrt{5})Y_{8d}+\tfrac{1}{2}Y_{8f}+(1/3\sqrt{5})Y_{27}\,,\qquad \Sigma=Y_1+(1/\sqrt{5})Y_{8f}-(1/9\sqrt{5})Y_{27}\,, \tag{2}$$

where we let the particle symbol represent the mass of the particle. Equation (1) may be inverted to determine the $SU(3)$ tensors from the measured particle masses. At present there are three filled baryon octets[12]: $\frac{1}{2}^+$, $\frac{3}{2}^-$, and $\frac{5}{2}^-$; and one filled decuplet: $\frac{3}{2}^+$. The mass tensors implied by the data are shown in Table I. It is evident that singlet and octet tensors dominate the others, and in the limit that only these tensors are nonzero, one has the Gell-Mann–Okubo mass formulas for the $\frac{1}{2}^+$, $\frac{3}{2}^-$, and $\frac{5}{2}^-$ octets:

$$N+\Xi=\tfrac{1}{2}(3\Lambda+\Sigma)\,; \tag{3}$$

and for the $\frac{3}{2}^+$ decuplet:

$$\Omega-\Xi^*=\Xi^*-Y_1^*=Y_1^*-N^*\,. \tag{4}$$

Each of these holds empirically to better than 1%, although the mass splittings involved can be of order 20%. Notice in Table I that the **27** mass tensor is twice as large for the $\frac{3}{2}^-$ octet as for the $\frac{1}{2}^+$ and $\frac{5}{2}^-$ octets. This has been taken by some to be a manifestation of $SU(3)$ mixing in the $\frac{3}{2}^-$ octet.[3,4]

Although *equally valid*, octet sum rules for coupling constants[13–15] have proved less useful because of the paucity of experimental information. The lightest baryons occur as bound states, and hence do not decay strongly, whereas partial widths for highly excited, typically inelastic baryon resonances are hard to measure. In fact, the only previous successful application of a coupling-constant sum rule is the description by Gupta and Singh[14] and by Becchi *et al.*[15] of the transition $(\frac{3}{2}^+,\mathbf{10})\to(\frac{1}{2}^+,\mathbf{8})\otimes(0^-,\mathbf{8})$, which includes, for instance, the P-wave decay $N^*(1236)\to N\pi$. Expressed in terms of isospin-invariant coupling constants,[16] the sum rule reads

$$\sqrt{2}g(N^*N\pi)+2g(\Xi^*\Xi\pi)=3g(Y_1^*\Lambda\pi)+\tfrac{1}{2}(\sqrt{6})g(Y_1^*\Sigma\pi)\,. \tag{5}$$

P-wave coupling constants g may be extracted from observed decay widths Γ with the formula[17] (also see Sec. III)

$$g^2=\frac{\frac{1}{2}\Gamma}{k^3}\frac{6M_R}{E+M}\,, \tag{6}$$

where k is the decay momentum, M_R is the resonance mass, and E and M are the decay-baryon energy and mass. A comparison of Eq. (5) with experiment[18] is given in Table II. Notice that although $SU(3)$ breaking may shift the value of a given coupling constant by almost 40% (the $\Xi^*\Xi\pi$ transition), the octet sum rule holds to better than 1%. It is clear that apparently large deviations from $SU(3)$ symmetry need not vitiate the usefulness of the symmetry.

With the aim of ultimately studying decays of the $\frac{3}{2}^-$ baryon octet, we consider here transitions involving three *arbitrary* octets, $B'\to BP$, in the presence of $SU(3)$-symmetry breaking. There are 17 independent coupling constants, e.g., $N'N\pi$, $Y_1'N\bar{K}$, etc., each given for the limit of pure $SU(3)$ in terms of the f,d parameters. If the $SU(3)$ symmetry is broken by some effect transforming as an octet, then a coupling $g(\alpha+\beta\to\gamma)$ between members of the three distinct octets B', B, and P must in general be described by additional parameters

TABLE II. Coupling-constant sum rule for $\frac{3}{2}^+$ baryon decuplet. All couplings are normalized with respect to the $N^*N\pi$ transition. The empirical values are determined from experimental decay parameters with Eq. (3) of the text. The sum rule is Eq. (2) of the text.

	Isospin-invariant coupling constants		Contribution to sum rule	
	$SU(3)$	Empirical	$SU(3)$	Empirical
$N^*N\pi$	1.000	1.000	1.414	1.41
$\Xi^*\Xi\pi$	0.707	0.47	1.414	0.94
$Y_1^*\Lambda\pi$	0.707	0.61	2.121	1.83
$Y_1^*\Sigma\pi$	0.577	0.44	0.707	0.54

[12] The $\frac{5}{2}^-$ octet has apparently been completed with the substantiation of a particle, $\Xi^*(1930)$, with the correct properties. See J. Alitti *et al.*, Phys. Rev. Letters **21**, 1119 (1968).

[13] M. Muraskin and S. Glashow, Phys. Rev. **132**, 482 (1963).

[14] V. Gupta and V. Singh, Phys. Rev. **135**, B1442 (1964).

[15] C. Becchi, E. Eberle, and G. Morpurgo, Phys. Rev. **136**, B808 (1964).

[16] In this paper we use isospin-invariant coupling constants only. These may be related to couplings of particular charge states with appropriate $SU(2)$ Clebsch-Gordan coefficients.

[17] J. G. Rushbrooke, Phys. Rev. **143**, 1345 (1966).

[18] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).

$X_N(i,j)$:

$$g(\alpha+\beta\to\gamma)=g_{SU(3)}(\alpha+\beta\to\gamma)+\sum_{N,i,j}X_N(i,j)\begin{pmatrix}8 & 8 & \mathrm{N}_i\\ \gamma & 0 & \gamma\end{pmatrix}\begin{pmatrix}8 & 8 & \mathrm{N}_j\\ \alpha & \beta & \gamma\end{pmatrix}, \quad (7)$$

the sum on N going over **1**, **10**, $\mathbf{\overline{10}}$, **27**, and **8** and the indices i,j representing f,d couplings for **N**=**8**. The quantities in parentheses are $SU(3)$ isoscalar factors. In brief, each of the 17 couplings is expressible in terms of 10 parameters, two from $SU(3)$ and eight from the breaking, so that seven sum rules relating the couplings may be constructed. However, most of these are not suitable for confrontation with experiment, containing, for instance, decay modes of the η, etc. However, we have derived the following useful sum rule[10,19]:

$$4[g(N'N\pi)+g(\Xi'\Sigma\bar{K})]=(\sqrt{6})[g(Y_1'N\bar{K})+g(Y_1'\Sigma\pi)]+6\left(\frac{g(Y_0'N\bar{K})}{\sqrt{2}}+\frac{g(Y_0'\Sigma\pi)}{\sqrt{3}}\right). \quad (8)$$

The discussion in Sec. III is devoted to a specific application of the sum rule (8). As examples of less useful sum rules, we exhibit the following:

$$3[g(Y_1'\Lambda\pi)+g(Y_1'\Sigma\eta)-g(Y_0'\Lambda\eta)]+(\sqrt{6})[g(Y_1'N\bar{K})+g(Y_1'\Xi K)]+\sqrt{2}[g(Y_0'N\bar{K})-g(Y_0'\Xi K)]+g(Y_0'\Sigma\pi)/\sqrt{3}=0, \quad (9)$$

$$2[2g(Y_1'\Lambda\pi)+2g(Y_1'\Sigma\eta)-2g(Y_0'\Lambda\eta)+g(N'N\pi)+g(\Xi'\Sigma\bar{K})-g(\Xi'\Xi\pi)-g(N'\Sigma K)]+(\sqrt{6})[g(Y_1'N\bar{K})+g(Y_1'\Xi K)]-(4/\sqrt{3})g(Y_0'\Sigma\pi)=0.$$

III. COUPLING CONSTANTS AND DECAY WIDTHS

Because there is at present no complete theory of the strong interactions, the extraction of coupling constants from decay widths is beset with ambiguities.[20] The two most popular approaches appear to be use of a nonrelativistic barrier penetration formula based on a square-well potential,[21] and employment of lowest-order perturbation theory to evaluate a relativistic field-theoretic interaction Lagrangian.[17] For the decay $\frac{3}{2}^-\to\frac{1}{2}^+0^-$, one can define the potential-theory coupling constant g_P as

$$g_P^2=\frac{M_R}{M_N}\frac{\Gamma}{k}\frac{1}{\nu_2(kR)}, \quad (10)$$

TABLE III. Comparison of potential- and field-theoretic coupling-constant formulas. Each entry is the ratio of squared potential- to squared field-theoretic coupling constants g_P^2/g_F^2. The ratio is normalized to unity at $M+\mu=1680$ MeV, $\mu/M=1$. The relevant formulas in the text are Eqs. (10) and (13). We take $R=1$ F in Eq. (10).

μ/M \ $M+\mu$ (MeV)	1680	1600	1500	1400	1300	1200	1100
1.0	1	1.9	3.9	6.5	9.9	13.8	18.3
0.8	0.90	1.7	3.5	5.9	9.0	12.7	16.9
0.5	0.74	1.4	2.7	4.6	7.1	10.2	13.9
0.2	0.56	0.87	1.5	2.6	4.2	6.3	9.1
0.08	0.49	0.69	1.0	1.7	2.7	4.3	6.6

where

$$\nu_2(kR)=\frac{(kR)^4}{9+3(kR)^2+(kR)^4}. \quad (11)$$

In the above, we use the same notation as described below Eq. (6) except for the range of the potential R and the nucleon mass M_N. The presence of the latter in (10) is purely artificial, serving to make g_P^2 both dimensionless and of a convenient scale.[3] Aside from the ambiguities arising from the choice of R and the necessity of making g_P^2 dimensionless, the general validity of Eq. (10) is doubtful because it is nonrelativistic. This could be particularly serious for the higher resonances where the momentum of the decay particles is usually appreciable. For example, a decay pion from the transition $Y_0'(1690)\to\Sigma\pi$ carries a momentum of 403 MeV/c, about three times its rest mass.

The field-theoretic coupling constant for the transition $\frac{3}{2}^-\to\frac{1}{2}^+0^-$ is found as follows.[17] We start with the interaction

$$L=(g_F/\mu)\bar{\psi}\gamma_5\psi_\mu'\partial^\mu\phi, \quad (12)$$

where $\bar{\psi}$ and ϕ are free-field creation operators for the $\frac{1}{2}^+$ baryon and 0^- meson, respectively, ψ_μ' is a free-field destruction operator for a $\frac{3}{2}^-$ field, and g_F is the field-theoretic coupling constant. The inverse factor of μ is an arbitrary mass which makes g_F dimensionless as defined in (12). The operator (12), evaluated between appropriate initial and final states, squared and summed over spin, and finally integrated over phase space of the decay particles, implies[17]

$$g_F^2=\frac{3M_R\mu^2}{k^3}\frac{\Gamma}{E-M}, \quad (13)$$

where again the notation is that of Eq. (6). Although this approach is relativistic, it contains the defect of omitting particle *structure*, a property now believed to be common to all hadrons. One conceivable remedy is multiplication of Eq. (12) by a form factor to effectively parametrize the structure. Unfortunately, it is very difficult to evaluate such form factors in a reliable manner, especially when comparing momentum transfers as different as m_π^2 and m_K^2.

[19] The sum rules, Eqs. (8) and (9), have been successfully tested in model calculations. E. Golowich (unpublished).

[20] For instance, see J. D. Jackson, Nuovo Cimento 34, 1644 (1964). Also see Ref. 5.

[21] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), especially Chaps. VII–IX.

TABLE IV. Coupling-constant sum rule for $\frac{3}{2}^-$ baryon octet. All couplings are normalized relative to the $N'N\pi$. The sum rule is Eq. (8). A mixing angle $\theta=-12°$ is used in determining the $Y_0'N\bar{K}$, $Y_0'\Sigma\pi$ empirical coupling constants. Equation (13) has been used to extract the couplings from decay widths.

	Isospin-invariant coupling constants			Contribution to sum rule	
	$SU(3)$[a]	Empirical (no mixing)	Empirical (mixing)	$SU(3)$	Empirical (mixing)
$N'N\pi$	1.0	1.0	1.0	4.00	4.00
$\Xi'\Sigma\bar{K}$ [b]	1.0	<0.26	<0.26	4.00	<1.0
$\Xi'\Sigma\bar{K}$ [c]	1.0	1.76	1.76	4.00	7.0
$Y_1'N\bar{K}$	−0.16	−0.41	−0.41	−0.182	−1.0
$Y_1'\Sigma\pi$	0.98	1.22	1.22	2.182	2.99
$Y_0'N\bar{K}$	1.04	0.46	0.81	4.182	3.44
$Y_0'\Sigma\pi$	0.46	1.02	0.68	1.818	2.36

[a] We take $f=0.6$, $f+d=1$. See Ref. 3.
[b] Based on data taken from Ref. 18.
[c] Based on data taken from Ref. 26.

To gain some quantitative feeling for the seriousness of the difference between the formulas (10) and (13), we have performed a model calculation involving the decay of a fictitious particle of mass 1700 and width 100 MeV into a D-wave baryon–pseudoscalar-meson composite of mass $M+\mu$ and meson-to-baryon mass ratio μ/M. The potential- and field-theoretic coupling constants are normalized to be equal in the nonrelativistic limit, $M+\mu=1680$ MeV, $\mu/M=1$. The results are given in Table III, where we exhibit g_P^2/g_F^2 for a variety of decay masses. Clearly the transition from threshold or nonrelativistic kinematics to the relativistic domain can lead to important differences between the two approaches. For the D-wave decays studied in this paper, e.g., $Y_0'(1690)\rightarrow\Sigma\pi$ ($M+\mu=1331$ MeV, $\mu/M=0.12$) and $Y_0'(1690)\rightarrow N\bar{K}$ ($M+\mu=1435$ MeV, $\mu/M=0.53$), the kinematics are such that differences between formulas (10) and (13) of order 40% could appear. Because we have no *a priori* justification for either Eqs. (10) or (13), the sum rule (8) will be tested with both types of coupling constants. However, we are somewhat more inclined to believe the results of (13), the field-theoretic approach, because of its adherence to Lorentz invariance.

Before Eq. (8) can be confronted with the data, we must study the possibility that $SU(3)$ mixing is present. With the input $N'=1525$ MeV, $\Xi'=1815$ MeV, and $Y_1'=1660$ MeV, the Gell-Mann–Okubo mass formula

$$Y_0'=\tfrac{1}{3}(2\Xi'+2N'-Y_1') \tag{14}$$

implies a mass of 1673 MeV for the isoscalar member of the $\frac{3}{2}^-$ γ octet. If one associates the $Y_0'(1690)$ with this octet and attributes the 17-MeV discrepancy entirely to mixing between the $Y_0'(1690)$ and $Y_0'(1520)$, a mixing angle $|\theta|=16°$ is found,[3] where θ is defined by

$$\begin{aligned} Y_0^{(8)}&=\cos\theta\, Y_0'(1690)+\sin\theta\, Y_0'(1520)\,,\\ Y_0^{(1)}&=-\sin\theta\, Y_0'(1690)+\cos\theta\, Y_0'(1520)\,, \end{aligned} \tag{15}$$

the states $Y_0^{(8)}$ and $Y_0^{(1)}$ transforming according to octet and singlet representations, respectively. [This parametrization is standard,[3,4,22] although not entirely compelling as applied here. For instance, the assumption that effects of mixing and $SU(3)$ breaking separate cleanly is not obvious.] We believe that it is somewhat extreme to blame the entire 17-MeV discrepancy on $SU(3)$ mixing. Instead, if we adopt the criterion that the **27** mass tensor of the $\frac{3}{2}^-$ octet should roughly equal those of the $\frac{1}{2}^+$ and $\frac{5}{2}^-$ octets, we find a mixing angle $|\theta|=12°$. We use this value in the following. Although the sign of the mixing angle is not determined by the mass sum rule (14), we can obtain it from coupling-constant values. The octet $Y_0^{(8)}$ and singlet $Y_0^{(1)}$ coupling to the $\Sigma\pi$ and $N\bar{K}$ channels can be found from the observed $Y_0'(1690)$ and $Y_0'(1520)$ decays by using (15):

$$\begin{aligned} Y_0^{(8)}N\bar{K}&=\cos\theta\, Y_0'(1690)N\bar{K}+\sin\theta\, Y_0'(1520)N\bar{K}\,,\\ Y_0^{(1)}N\bar{K}&=-\sin\theta\, Y_0'(1690)N\bar{K}+\cos\theta\, Y_0'(1520)N\bar{K}\,,\\ Y_0^{(8)}\Sigma\pi&=\cos\theta\, Y_0'(1690)\Sigma\pi+\sin\theta\, Y_0'(1520)\Sigma\pi\,,\\ Y_0^{(1)}\Sigma\pi&=-\sin\theta\, Y_0'(1690)\Sigma\pi+\cos\theta\, Y_0'(1520)\Sigma\pi\,. \end{aligned} \tag{16}$$

Equation (13) and the current values for decay widths[18] imply

$$\begin{aligned} &g(Y_0'(1520)\Sigma\pi)=-1.27\,,\\ &g(Y_0'(1520)N\bar{K})=1.13\\ \text{and}\quad &\\ &g(Y_0'(1690)\Sigma\pi)=0.75\,,\\ &g(Y_0'(1690)N\bar{K})=0.34\,. \end{aligned} \tag{17}$$

Although the magnitude of the mixing angle is small, $|\theta|=12°$, the size of the $Y_0'(1520)$ couplings relative to those of the $Y_0'(1690)$ implies that the mixing phenomenon has an appreciable effect on the $Y_0^{(8)}$ couplings. Results are given in columns 1–3 of Table IV. The value for the octet parameter $f=0.6$ ($f+d=1$) is taken from Ref. 3 and is consistent with previous dynamical models of the $\frac{3}{2}^-$ octet.[23] We find that for a negative mixing angle, $\theta=-12°$, the agreement of $Y_0^{(8)}N\bar{K}$ and $Y_0^{(8)}\Sigma\pi$ with $SU(3)$ is markedly improved, as is that of the singlet $Y_0^{(1)}$.[24] The combined impact of these results is to support the concept of baryon mixing.

We are now ready to compare the sum rule (8) with experiment. Except for the $Y_1'(1660)$ and $\Xi'(1815)$ decays, the data used are taken from Ref. 18. The $Y_1'(1660)\rightarrow\Sigma\pi$ mode has recently been analyzed by Primer *et al.* and by Button-Shafer, who obtain consistent branching ratios.[25] We take an average of their results. For the $Y_1'(1660)\rightarrow N\bar{K}$ decay, we take the

[22] R. F. Dashen and D. H. Sharp, Phys. Rev. **133**, B1585 (1964).

[23] For instance, we cite Y. Hara, Phys. Rev. **133**, 1565 (1964); J. J. Brehm, *ibid.* **136**, B216 (1964).

[24] $SU(3)$ implies a ratio of coupling constants $g(Y_0^{(1)}\Sigma\pi)/g(Y_0^{(1)}N\bar{K})=-1.23$. The unmixed empirical ratio is -0.89, whereas a mixing angle $\theta=-12°$ implies a ratio -1.17.

[25] M. Primer *et al.*, Phys. Rev. Letters **20**, 610 (1968); J. Button-Shafer, *ibid.* **21**, 1123 (1968).

branching ratio used by Tripp *et al.*[3] At present, there exist two mutually inconsistent values of the $\Xi'(1815)\to\Sigma\bar{K}$ decay width. The Rosenfeld compilation[18] lists an upper bound $\Gamma(\Xi'\Sigma\bar{K})\lesssim 0.4$ MeV, whereas a very recent analysis by Alitti *et al.*[26] implies $\Gamma(\Xi'\Sigma\bar{K})=18$ MeV, although with estimated errors, 12 MeV $\lesssim\Gamma(\Xi'\Sigma\bar{K})\lesssim 28$ MeV. Numerical results based on each of these $\Gamma(\Xi'\Sigma\bar{K})$ values are given in Table IV. Equations (13) and (16) are used in determining the numbers shown there. The right-hand side of Eq. (8) gives

$$(\sqrt{6})[g(Y_1'N\bar{K})+g(Y_1'\Sigma\pi)]+6\left(\frac{g(Y_0'N\bar{K})}{\sqrt{2}}+\frac{g(Y_0'\Sigma\pi)}{\sqrt{3}}\right)=7.8\,,$$

whereas we find for the left-hand side

$$4[g(N'N\pi)+g(\Xi'\Sigma\bar{K})]<5\,,^{18}$$
$$4[g(N'N\pi)+g(\Xi'\Sigma\bar{K})]=11\,.^{26}$$

If we use the potential-theory coupling constant of Eq. (10) with an interaction range $R=10^{-13}$ cm, we find 6.0 for the right-hand side of Eq. (8) and <4.6, 8.0 for the left-hand side, using the $\Gamma(\Xi'\Sigma\bar{K})$ data of Refs. 18 and 26, respectively. Clearly, the octet coupling-constant sum rule (8) applied to decays of the $\frac{3}{2}^-$ baryons is badly violated by the existing data. However, note that for the $\Gamma(\Xi'\Sigma\bar{K})$ of Rosenfeld, the left-hand side of (8) is too *small*, while the $\Gamma(\Xi'\Sigma\bar{K})$ of Alitti *et al.* implies a value too *large*. For purposes of comparison, we have also shown in Table IV contributions of each coupling constant to the sum rule in the $SU(3)$ limit.

IV. SUMMARY AND CONCLUSIONS

Octet dominance is apparently a basic property of the baryon system. In those situations where the data have been determined with quantitative accuracy, octet mass and coupling-constant sum rules are typically obeyed to within several percent. However, the results of Sec. III show that the $\frac{3}{2}^-$ baryons violate the relation (8) by a magnitude of order 30–60%. We now consider possible explanations for this.

In a phenomenological analysis like that described in this paper, one can only assume that the data are correct. The recent analysis by Alitti *et al.* of the $\Xi'\Sigma\bar{K}$ decay width[26] suggests that the current upper bound for this mode[18] is too small. Indeed, we find that the $N'(1525)$, $Y_1'(1660)$, and $Y_0'(1690)$ couplings along with octet dominance imply $\Gamma(\Xi'\Sigma\bar{K})\cong 5$ MeV. We feel it likely that the violation of (8) is related to uncertainty in $\Gamma(\Xi'\Sigma\bar{K})$ and *we urge that further experimental study of this mode be carried out.*

It is possible that the violation of the sum rule (8) arises from the method used to calculate coupling constants. Our discussion in Sec. III of the extraction procedure from decay widths leaves little doubt that present approximations to the unknown correct procedure can lead to significant numerical errors. In particular, the definition of coupling constants with dimensions and the subsequent artificial elimination of these dimensions via suitable redefinitions is a common malady in the theory of high-spin baryons. Only when coupling constants with a degenerate kinematical property, such as occurrence of a common decay meson, are compared is it possible to circumvent this difficulty with confidence. The remarkable agreement of the $(10,\frac{3}{2}^+)\to(8,\frac{1}{2}^+)\otimes(8,0^-)$ coupling constants of Table II is due partially to the occurrence of decay pions in all transitions considered there. This fortunate experimental situation has made it possible to uncover for these coupling constants the underlying octet-dominance phenomenon. When decays involving pions *and* kaons are compared, the validity of Eqs. (10) and (13) becomes correspondingly more suspect. Our only guide in the calculation presented here is that both formulas (10) and (13) lead to violations in the sum rule (8) of the same sign and magnitude for a given set of input data. This implies that the violation of (8) is real, and is produced either by incorrect data or by some physical effect not accounted for. We conclude by considering the possibility of such an effect, $\Xi'(1815)$ mixing.

Capps[8] and Mitra and Ross[7] have recently questioned the octet assignment of $\Xi'(1815)$ with the following observation. Consider the following $SU(3)$ coupling constants: $N'N\pi$; $f+d$; $\Xi'\Sigma\bar{K}$; $f+d$; $\Xi'\Lambda\bar{K}$; $\frac{1}{3}d-f$; and $\Xi'\Xi\pi$; $f-d$. Most theoretical models[23] of these states have predicted $f\approx d$, and, in particular, Capps[8] has found $f/d=1.2$. Clearly, if $f\approx d$, it is impossible to have both $\Xi'\Sigma\bar{K}$ *and* $\Xi'\Xi\pi$ couplings small. Further, the $N'N\pi$ and $\Xi'\Sigma\bar{K}$ $SU(3)$ coupling constants are equal, regardless of f/d. Yet from Table IV we see that the data are at odds with *both* these statements, and the trouble evidently lies in the very small value of the empirical $\Xi'\Sigma\bar{K}$ coupling constant (if we use the current Rosenfeld compilation[18]). Capps and Mitra and Ross attempt to resolve the dilemma by conjecturing that $\Xi'(1815)$ is not pure octet, but is mixed with some other baryon belonging to a decuplet. Capps can explain the anomalously small $\Sigma\bar{K}$ branching ratio with a mixing angle of 25°. (We refer the reader to Ref. 8 for details.) The weak point in the above argument is the assumption of pure $SU(3)$ symmetry, for we have seen that, for example, the $\frac{3}{2}^+$ resonance coupling shifts may approach 40%, although the octet sum rule, Eq. (5), is almost exactly obeyed. In this paper we have explored the possibility that $SU(3)$ breaking alone can explain the magnitudes of the $\frac{3}{2}^-$ decay widths, and have concluded that it cannot. In particular, the sum rule predicts a $\Xi'\Sigma\bar{K}$ coupling constant much larger than that which the present $\Gamma(\Xi'\to\Sigma\bar{K})$ upper bound[18] implies. The mixing conjecture is not completely arbitrary. In the quark model, Mitra and Ross[7] and Greenberg and

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[26] J. Alitti *et al.*, Brookhaven National Laboratory Report No. BNL-12808 (unpublished).

Resnikoff[6] have shown that for the $SU(6)$ limit, a multiplet structure of $\mathbf{70}\otimes\mathbf{1}$ is likely to occur for the negative-parity baryons. This has also been demonstrated by Capps,[8] whose arguments are consistent with bootstrap theory. The content of $\mathbf{70}\otimes\mathbf{1}$ includes two octets and a decuplet with spin and parity $\frac{3}{2}^-$. Since a precise estimate of the masses of these particles is not possible at this time, further word on the mixing conjecture must wait for further experimental study of Ξ excited states. If the current $\Gamma(\Xi'\Sigma\bar{K})$ upper bound is upheld, then the concept of $\Xi'(1815)$ mixing will be enhanced. However, should the value of $\Gamma(\Xi'\Sigma\bar{K})$ turn out to be larger, octet dominance will probably be sufficient to explain the $\frac{3}{2}^-$ decay widths.

Note added in proof: A related work on the negative parity baryon spectrum has been done by S. Pakvasa and S. F. Tuan, Nucl. Phys. (to be published).

ACKNOWLEDGMENTS

The author is pleased to acknowledge useful conversations with Professor R. H. Capps and Professor L. F. Cook.

PHYSICAL REVIEW VOLUME 177, NUMBER 5 25 JANUARY 1969

Baryon Excitation Form Factors and Asymptotic Chiral Symmetry

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(Received 22 July 1968)

An explicit multipole decomposition of the vector and pseudovector N-$N^*(1236)$ transition form factors is made, and the predictions of asymptotic chiral symmetry are discussed. A general consistency between the chiral predictions and the simplest assumptions on multipole behavior is found for one choice of decuplet chiral representation.

1. INTRODUCTION

IT has been found[1] that assigning the octet baryons to a definite chiral representation leads to several very well satisfied relations among the various vector and pseudovector N-N transition form factors. In order to obtain these relations, it was necessary to interpret the predictions of chiral symmetry as holding only for very large spacelike momentum transfers.[2–4] The reasoning behind this point of view is that chiral symmetry is an expression of the tendency of nucleons to retain their helicities when they engage in interactions with other particles. At low energies, they are not particularly successful and emit pions as an attempt to compensate. However, at high energies (which we shall interpret as large momentum transfers for the case of form factors), the nucleon mass may be considered negligible and nucleons tend to behave as massless spin-$\frac{1}{2}$ objects which do conserve their helicities.

In order to test the relations at large spacelike momentum transfer (q^2), it is necessary to have some idea of the functional dependence of the form factors. For the case of the vector N-N transition form factors, the e-p scattering experiments give us a lot of information. These experiments[5] indicate that it is the Sachs form factors[6] which are simple functions of q^2 and that these behave as $(1+q^2/M_V^2)^{-2}$, where $M_V^2=0.71$ BeV^2. About the axial-vector N-N form factors and about all the N-N^* form factors, much less is known and we will have to rely mainly on analogy, although there is, of course, some available information from electroexcitation, photoexcitation, and neutrino excitation experiments. If we take the analogy with the vector N-N case seriously, the most natural assumption is that the objects which behave simply as functions of q^2 are the analogs of the Sachs form factors, these analogs being the relativistic multipoles introduced by Durand, DeCelles, and Marr.[7] Thus the first part of this paper is

* Supported in part by the U. S. Atomic Energy Commission.

† Supported in part by the National Research Council of Canada.

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