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Universal Theory of Primary Interactions and Nucleon-Nucleon Scattering*

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A dynamical theory of nucleon-nucleon scattering is proposed, employing Sudarshan's universal coupling scheme for all primary interactions. The basic nature of all interactions is assumed to be vector and axial-vector. The contribution to the nuclear force, therefore, comes from vector, axial-vector, and pseudoscalar (longitudinal components of axial-vector fields) meson exchanges. The resulting potential has all the requisite features of a two-nucleon force, notable among them being the natural occurrence of a repulsive core at short distances. The various phase shifts and mixing parameters are then compared with experimental results. This being a parameterless theory, complete agreement is, of course, not expected; however, the over-all agreement of our results with experiments is very good. Another important feature of the present work is the occurrence of a *CP*-violating nuclear force, responsible for induced *CP*-violating effects in weak interactions. The consequences of this force with respect to the polarization and asymmetry measurements in nucleon-nucleon scattering are found to be consistent with the available experimental results, which have large uncertainties.

1. INTRODUCTION

EVER since the fundamental discovery of Yukawa,[1] that the force between two nucleons may be due to the exchange of a meson, no subject has attracted more attention than the nuclear-force problem. The subsequent development of the nuclear-force problem (and, of course, nucleon-nucleon scattering) has been mainly along two parallel lines: purely phenomenological analyses coupled closely with experiments, and meson theory. Over the years, a closer examination shows that there has been surprisingly little interaction between these two lines. The phenomenological analyses and related general theoretical considerations were developed more or less without recourse to meson theory, and the meson theory proceeded along its own path, developing into an abstract discipline, taking very little help from experiments, from pion-nucleon scattering if at all. The relative independence of two approaches has proved to be a boon, since a more or less *complete* phenomenological description of proton-proton scattering (and a relatively less complete description of neutron-proton scattering) up to about 400 MeV is now available.

The main result of these phenomenological analyses has been the construction of a two-nucleon potential, which, in general, contains a large number of parameters (the strengths and ranges of the various types of forces, e.g., tensor force, spin-orbit force, etc.). However, the general form of the two-nucleon potential (if such a nonrelativistic concept is applicable to the description of relativistic scattering) is more or less agreed upon,[2] in that it must contain spin-dependent, tensor, and spin-orbit forces, and must also contain a hard core.[3] Perhaps the nuclear force also does contain a short-range force proportional to $\mathbf{L}\cdot\mathbf{L}$ (where $\mathbf{L}$ is the orbital angular momentum).[4,5]

On the other hand, the progress in meson theory as applied to the two-nucleon problem has been very meager (mainly due to the formulational difficulties of quantized field theories). Perhaps the two most important advancements in the two-nucleon problem, due to meson theory, are (i) the recognition due to Wick[6] and to Taketani *et al.*[7] that the nuclear force must essentially contain three ranges, the longest one due to one-pion exchange, the intermediate one due to two-pion exchange (or, in modern language, the exchange of ρ, ω, and η mesons), and, finally, the shortest range consisting of an extremely repulsive interaction (hard core); and (ii) the derivation of a hard core from meson theory.[8] The essential difficulties in a meson-theoretic approach to the nuclear-force problem are, however, formulational ones (e.g., whether interacting field theories are empty of content and whether they are self-consistent

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[1] H. Yukawa, Proc. Phys. Math. Soc. Japan 17, 48 (1935).

[2] See, for example, J. L. Gammel and R. M. Thaler, Progr. Cosmic Ray Phys. 5, 99 (1960); M. J. Moravcsik, *The Two-Nucleon Interaction* (Clarendon Press, Oxford, 1963). The last-mentioned contains a long list of references on phenomenological two-nucleon potentials.

[3] The change of sign of the $^1S_0$ phase shift confirmed for the first time the existence of a hard core, which had been suspected for some time.

[4] T. Hamada and I. D. Johnston, Nucl. Phys. 34, 382 (1962).

[5] K. E. Lassila, M. H. Hull, Jr., H. M. Ruppel, F. A. McDonald, and G. Breit, Phys. Rev. 126, 881 (1962).

[6] G. C. Wick, Nature 142, 993 (1938).

[7] M. Taketani, S. Nakamura, and M. Sasaki, Progr. Theoret. Phys. (Kyoto) 7, 45 (1952).

[8] M. M. Lévy, Phys. Rev. 88, 725 (1952); A. Klein, *ibid.* 94, 1052 (1954).

and how should one interpret the divergences of quantum field theory). On the other hand, the reliability of the approximations used (e.g., the static approximation or the adiabatic approximation) to derive an essentially nonrelativistic potential from the quantum field theory is also questionable. The situation, therefore, is extremely confused and deserves more attention.

However, the foregoing discussion is not meant to suggest that there have been no attempts to synthesize the phenomenological and meson-theoretic points of view. The Signell-Marshak[9] and Gartenhaus[10] potentials the modified analysis scheme,[11] and the various versions of the boundary-condition models[12] are attempts to synthesize the two parallel approaches, but these are comparatively recent developments and rather limited in their scope.

Thus it is quite clear that, after all is said and done, one always ends up with a potential involving many parameters (often as many as 20 or 30), which gives a good fit to the nucleon-nucleon scattering data up to about 400 MeV. The need for a theory which relies on its internal consistency rather than on a large number of coupling constants is now compelling.

Recently, Sudarshan[13] has proposed such a universal theory, which generalizes the important and well-recognized vector and axial-vector interaction theory[14,15] of weak interactions to all primary interactions. The principle of chiral invariance which led originally to the universal $V-A$ theory of weak interactions was used in an essential manner to investigate the nature of all interactions. Sudarshan suggested that perhaps all interactions are due to the mediation of vector and axial-vector fields. The vector fields satisfied $\partial_\mu V_\mu(x)=0$, while the axial vector fields had a longitudinal component to be interpreted as a pseudoscalar meson. One is thus led to a coupling scheme of baryons with vector and axial-vector fields which in its nonrelativistic form satisfies an $SU(4)$ symmetry.[16] The various coupling constants become related and give rise to many interesting results,[13,17] including the pleasant one $g_A/g_V =(25/18)^{1/2}$.[18] The whole coupling scheme therefore contains only a single over-all coupling constant which may be chosen to be the universal coupling constant of the ρ meson with the conserved isospin current. This theory, therefore, does not contain any free parameters and, as discussed before, seems to be ideally suited for our purpose.

It should be emphasized, however, that such a universal theory can, at best, only hope to be an *effective* replacement of the hadron dynamics. The apparatus of perturbation theory, called in modern language the "tree-diagram" approximation, can therefore be hopefully used. The renormalizability of the interaction scheme is a question which we do not propose to answer, as well as other formulational questions. We simply accept the fact that Sudarshan's theory might be an "effective" way to describe hadron dynamics[19] and proceed to investigate the nuclear-force problem, using this theory.

In recent times, a great deal of effort has gone into the construction of *effective* Lagrangians from chiral symmetry,[20] which, in general, reproduce the results obtained from current algebras and the hypothesis of partially conserved axial-vector current (PCAC). However, these Lagrangians are, in general, nonlinear in character and thus differ *essentially* from Sudarshan's theory, which employs linear Yukawa-type couplings only.

The aim of the present paper is to evaluate the two-nucleon force, employing this universal theory, and to test whether the resulting nucleon-nucleon potential (which is highly energy-dependent and nonlocal) provides a dynamical explanation of the nucleon-nucleon scattering data. The task is enormous, and it is heartening to note that our results are in good agreement (at least as good as achieved by anyone else in the past) with experimental data. The novel feature of the present work is the presence of an extremely momentum-dependent force which violates time-reversal invariance. The consequences of this term with respect to the polarization and asymmetry parameters in high-energy (~600-MeV) nucleon-nucleon scattering are examined and found to be consistent with experimental data, which are very unsatisfactory. At low energies, this term is entirely negligible. A more meaningful comparison of the T-noninvariant term in nucleon-nucleon scattering must await better experimental measurements.

The paper is arranged as follows. Section 2 contains a brief review of Sudarshan's theory which forms the

[9] P. S. Signell and R. E. Marshak, Phys. Rev. **109**, 1229 (1958).
[10] S. Gartenhaus, Phys. Rev. **100**, 900 (1955).
[11] M. H. MacGregor, M. J. Moravcsik, and H. P. Noyes, Phys. Rev. **123**, 1835 (1961); J. K. Perring, Nucl. Phys. **30**, 424 (1962).
[12] G. Breit and W. Bounicius, Phys. Rev. **75**, 1029 (1949); H. Feshbach and E. Lomon, *ibid.* **102**, 891 (1956); A. M. Saperstein and L. Durant, *ibid.* **104**, 1102 (1956).
[13] E. C. G. Sudarshan, Nature **216**, 979 (1967); Proc. Roy. Soc. (London) **305A**, 319 (1968).
[14] E. C. G. Sudarshan and R. E. Marshak, in Proceedings of the Padua Conference on Mesons and Recently Discovered Particles, 1967 (unpublished).
[15] R. P. Feynmnan and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[16] An extension of this theory to include strange particles gives a value of the Cabibbo angle $\tan\theta=m_\pi/m_K$. Also an application to nonleptonic and leptonic K-meson decays seems to work out. See R. H. Graham and S. K. Yun, Syracuse University Report No. SU-1206-146 (unpublished); P. Ramond (to be published).
[17] T. Pradhan, E. C. G. Sudarshan, and R. P. Saxena, Phys. Rev. Letters **20**, 79 (1968).
[18] This result has also been deduced by many authors. See, for example, J. Schwinger, Phys. Rev. Letters **18**, 923 (1967); P. G. O. Freund, *ibid.* **19**, 189 (1967).
[19] The word "effective" should be understood to imply that the heavy-vector and axial-vector mesons produce the same effect as would be produced by two or more pion exchanges.
[20] Working from the hypothesis that the pion transforms nonlinearly under chiral transformations, a number of authors have derived the so-called effective Lagrangians. See, for example, S. Weinberg, Phys. Rev. **166**, 1568 (1968); J. A. Cronin, *ibid.* **161**, 1483 (1967); F. Gürsey and P. Chang, *ibid.* **164**, 1752 (1967); J. Wess and B. Zumino, *ibid.* **163**, 1727 (1967); J. Schwinger, *ibid.* **167**, 1432 (1968); B. W. Lee and H. T. Nieh, *ibid.* **166**, 1507 (1968).

starting point of our work. In Sec. 3, we have derived the two-nucleon potential and compared it with other existing potentials. Our method of numerical solution of the Schrödinger equation with an energy-dependent potential is outlined in Sec. 4. In Sec. 5, we calculate the time-reversal-noninvariant effects and compare them with the existing experimental data. A detailed comparison of the nucleon-nucleon scattering phase shifts with the experiments is made in Sec. 6. Lastly, in Sec. 7 we provide a brief discussion of our results and a possible outlook for a universal theory of hadron dynamics.

2. REVIEW OF SUDARSHAN'S THEORY

This theory is based on the hypothesis that the three primary interactions—weak, electromagnetic, and strong—are all characterized by a single coupling constant. The primary weak interactions couple leptons among themselves and to vector and axial-vector fields, but not to baryons. The basic electromagnetic interaction couples the Maxwell field to charged leptons and neutral vector fields, and once again, not the baryons. The only strong interactions in this theory are due to the coupling of vector and axial-vector fields with baryons and among themselves. All the interactions are assumed to be universal. The electromagnetic and weak interactions of baryons thus become derived properties through the mediation of vector and axial-vector fields. In what follows, we shall borrow heavily from Sudarshan's paper.[13]

As mentioned above, the Maxwell field $\mathcal{A}_\lambda$ couples to leptons ψ and neutral vector fields V_λ and W_λ, corresponding to ρ and ω mesons. The couplings are

$$e\bar{\psi}\gamma_\lambda\psi\mathcal{A}_\lambda$$

and

$$e'(m_\rho^2 V_\lambda + m_\omega^2 W_\lambda)\mathcal{A}_\lambda. \tag{2.1}$$

The coupling of baryons to vector fields is chosen to be

$$\bar{N}(\tfrac{1}{2}g\boldsymbol{\tau}\cdot\mathbf{V}_\lambda\gamma_\lambda + \tfrac{1}{2}g'\boldsymbol{\tau}\cdot\mathbf{V}_{\lambda\nu}\tfrac{1}{2}\sigma_{\lambda\nu} + \tfrac{1}{2}g_0 W_\lambda\gamma_\lambda + \tfrac{1}{2}g_0' W_{\lambda\nu}\tfrac{1}{2}\sigma_{\lambda\nu} + \tfrac{1}{2}g_{00}\phi_\lambda\gamma_\lambda + \tfrac{1}{2}g_{00}'\phi_{\lambda\nu}\tfrac{1}{2}\sigma_{\lambda\nu})N. \tag{2.2}$$

e, e', g, g', g_0, g_0', g_{00}, and g_{00}' are dimensionless coupling constants. The vector fields are described by 10-component Duffin-Kemmer objects.[21] The four components V_λ and the six antisymmetric tensor components $V_{\lambda\nu} = (1/m_\rho)(\partial_\lambda V_\nu - \partial_\nu V_\lambda)$ describe the ρ field. For the ω and ϕ fields the corresponding quantities are W_λ, $W_{\lambda\nu}$, ϕ_λ, and $\phi_{\lambda\nu}$. Since there are no scalar mesons in nature, one assumes that

$$\partial_\mu \mathbf{V}_\mu = 0, \quad \partial_\mu W_\mu = 0, \quad \partial_\mu \phi_\mu = 0. \tag{2.3}$$

The current that couples with photons is thus strictly conserved. The effective nucleon-photon coupling can now be worked out and gives (equality of electron and proton charges)

$$e' = -e/g, \quad g_0 = g. \tag{2.4}$$

To derive further relations between coupling constants, Sudarshan assumes an $SU(4)$ symmetry. The nucleons and the $I=J=\frac{3}{2}$ N^* form a 20-dimensional representation. The vector field components are assumed to constitute a 15-dimensional representation. Its contents are ($\boldsymbol{\varrho}_i, (1/m_\phi)\epsilon_{ijk}\partial_j\phi_k$, and $(1/m_\rho)\epsilon_{ijk}\partial_j\boldsymbol{\varrho}_k$).[22] This gives for the coupling constants

$$g'/g = 5/3, \quad g_{00}' = g, \quad g_{00} = g_0' = 0. \tag{2.5}$$

The resulting anomalous moments are in good agreement with experiments.

The weak interactions couple leptons among themselves and to vector and axial-vector fields. The basic couplings are

$$\tfrac{1}{2}\sqrt{2}G(\bar{u}\gamma_\lambda(1+\gamma_5)\nu_\mu)^\dagger(\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e) \tag{2.6}$$

and

$$G'(m_\rho^2 V_\lambda + m_A^2 A_\lambda)(\bar{e}\gamma_\lambda(1+\gamma_5)\nu_l), \tag{2.7}$$

where G is the universal Fermi coupling constant and is related to the μ-decay lifetime. It turns out that $G = (2.43\times10^{-7})m_\pi^{-2}$. V_λ and A_λ are charged vector and axial-vector fields. Since there are no scalar quanta and one would like the vector current to be conserved, we choose $\partial_\lambda V_\lambda \equiv 0$. The axial-vector field, however, has a longitudinal component, which is nothing but the pseudoscalar pion field. We choose for A_λ the Stückelberg form

$$A_\lambda = B_\lambda + (\xi/m_\pi)\partial_\lambda\phi_\pi, \tag{2.8}$$

where the transverse part B_λ describes a real axial-vector meson (say, the A_1 meson) satisfying $\partial_\lambda B_\lambda \equiv 0$. For pion momenta on the mass shell, we have the familiar PCAC condition[23] built in, in (2.8):

$$\partial_\lambda A_\lambda = (\xi/m_\pi)\Box\phi_\pi = \xi m_\pi \phi_\pi. \tag{2.9}$$

One can now write the effective vector (weak) interaction of baryons using (2.2) and (2.7):

$$G' = -G/g. \tag{2.10}$$

For computing the axial-vector contribution to weak interactions, we choose the following strong interaction (the arguments are the same as before):

$$f\bar{N}[\gamma_\lambda\gamma_5\tfrac{1}{2}\boldsymbol{\tau}\cdot\mathbf{A}_\lambda + (f'/f)\tfrac{1}{2}\sigma_{\lambda\nu}\gamma_5\tfrac{1}{2}\boldsymbol{\tau}\cdot\mathbf{A}_{\lambda\nu}]N, \tag{2.11}$$

with similar couplings with other axial-vector fields. The axial-vector β-decay coupling constant is now given by

$$-(g_A/g_V) = f/g. \tag{2.12}$$

Note that the idea of a nonrelativistic symmetry [$SU(4)$] which led us to the nontrivial coupling-constant relations (2.5) forces us to construct another meson

[21] R. Delbourgo, A. Salam, and J. Strathdee, Proc. Roy. Soc. (London) **A284**, 146 (1965).

[22] R. H. Capps, Phys. Rev. Letters **14**, 31 (1965); J. G. Belinfante and R. E. Cutkosky, *ibid.* **14**, 33 (1965).

[23] M. Gell-Mann and M. M. Lévy, Nuovo Cimento **16**, 705 (1960); Y. Nambu, Phys. Rev. Letters **4**, 380 (1960).

matrix (containing axial-vector mesons also). However, the nonrelativistic form of (2.11),

$$\tfrac{1}{2}fN^\dagger\boldsymbol{\tau}\cdot\sigma_j[\mathbf{A}_j+(f'/f)\mathbf{A}_{0j}]N, \tag{2.13}$$

contains a CP-noninvariant term (the term involving $\mathbf{A}_{0j}$). We should not put $f'/f=0$ because of the experimental confirmation of CP violation[24] in neutral K-meson decays. Any further choice would be totally arbitrary. $f'/f=\pm1$ would correspond to maximal violation. We make this choice, and the normalized meson matrix (15-dimensional) thus has the components $(\boldsymbol{\varrho}_0,Z_j,(\frac{1}{2}\sqrt{2})(\mathbf{A}_j+\mathbf{A}_{0j}))$ where Z_λ is an isosinglet axial-vector field whose pseudoscalar component is the η meson: $Z_\lambda\to(\xi/m_\eta)\partial_\lambda\phi_\eta$. With the above choice, the coupling constants satisfy the relations

$$f=f'=(25/18)^{1/2}g. \tag{2.14}$$

The axial-vector-to-vector ratio (2.12) now becomes

$$-(g_A/g_V)=(25/18)^{1/2}, \tag{2.15}$$

in excellent agreement with experiments.[25]

The effect of the CP-violating term in (2.13) in normal β decay is negligible (because of its large momentum dependence) and cannot be tested at the moment. The effects in hyperon decays and μ-capture reactions should be much larger. However, the fact that this term provides a natural vehicle for CP violation in weak interaction but not in electromagnetic phenomena is of great interest in view of the experiments on $K_L{}^0$ and $K_S{}^0$ decays. We shall return to the question of CP (or T) violation in strong interactions (particularly, nuclear force) in a later section.

We are now in a position to write the *complete* strong-interaction Lagrangian of Sudarshan's theory which does not contain any parameters apart from the overall coupling constant. It looks like

$$\begin{aligned}\mathcal{L}_{\text{strong}}=g\bar{N}[&\gamma_\lambda\tfrac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{\varrho}_\lambda+(g'/g)\tfrac{1}{2}\sigma_{\lambda\nu}\tfrac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{\varrho}_{\lambda\nu}+\tfrac{1}{2}(g_0/g)\gamma_\lambda\omega_\lambda\\&+(g_{00}'/g)\tfrac{1}{2}\sigma_{\lambda\nu}\phi_{\lambda\nu}+(f/g)\gamma_\lambda\gamma_5\tfrac{1}{2}\boldsymbol{\tau}\cdot\mathbf{A}_\lambda+\tfrac{1}{2}(f_0/g)\gamma_\lambda\gamma_5Z_\lambda\\&+(f'/g)\tfrac{1}{2}\sigma_{\lambda\nu}\gamma_5\tfrac{1}{2}\boldsymbol{\tau}\cdot\mathbf{A}_{\lambda\nu}+\tfrac{1}{2}(f_0'/g)\sigma_{\lambda\nu}\gamma_5\tfrac{1}{2}\phi_{\lambda\nu}]N.\end{aligned} \tag{2.16}$$

Equations (2.5) and (2.14) determine the coupling-constant relations. The couplings of pseudoscalar mesons are via the Stückelberg decomposition (2.8).

We shall choose this Lagrangian as our basis and compute nuclear force employing the exchange of vector, axial-vector, and pseudoscalar mesons in the next section.

3. NUCLEON-NUCLEON POTENTIAL

The computation of nucleon-nucleon potential from the choice of the Lagrangian [Eq. (2.16)] is a straightforward one.[26] Since the Lagrangian contains the interaction of nucleons with pseudoscalar (π and η), vector (ρ, ω, and ϕ), and axial-vector (A_1, D, and E) mesons,[27] we shall have a nuclear force consisting of three distinct ranges (on account of the different masses of the mesons involved), as visualized by earlier authors.[6,7] The longest-range force will, of course, come from the exchange of a single π meson. The intermediate-range force, which in traditional meson theory comes from 2π exchange (probably from $K\bar{K}$ exchange as well) and more complicated Feynman diagrams, shall emerge in the present work in an *effective* way from the exchange of η, ρ, ω, and ϕ. The inner core of nuclear force (often called the hard core) comes from axial-vector-meson exchanges[28] (a part also comes from vector-meson exchanges). In addition, we have a CP-violating force coming from the pseudotensor interaction term of the axial-vector mesons.

Our procedure of calculation is simply to compute the various Born terms corresponding to the exchange of pseudoscalar, vector, and axial-vector mesons and evaluate their Fourier transforms in the nonrelativistic limit.[29] The nonlocality coming from the difference in relativistic and nonrelativistic phase-space factors (m/E) is confined to regions of the order of a nucleon Compton wavelength and shall be approximated by a polynomial dependence on momentum *à la* Wong.[26] The resulting potentials will thus be seen to be extremely momentum-dependent.

We shall choose to write our potential in the following form[30] (apart from the nonlocal factor m/E):

$$\begin{aligned}U(r)=U_c+U_\sigma(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)+U_TS_{12}+U_{LS}(\mathbf{L}\cdot\mathbf{S})+m^{-2}\\\times[\boldsymbol{\sigma}_1\cdot\mathbf{k}U_{\sigma p}\boldsymbol{\sigma}_2\cdot\mathbf{k}+(\boldsymbol{\sigma}_1\leftrightarrow\boldsymbol{\sigma}_2)]+(i/2m)[\boldsymbol{\sigma}_1\cdot\mathbf{k}U_{TV}\boldsymbol{\sigma}_2\cdot\mathbf{r}\\+\boldsymbol{\sigma}_1\cdot\mathbf{r}U_{TV}\boldsymbol{\sigma}_2\cdot\mathbf{k}+(\boldsymbol{\sigma}_1\leftrightarrow\boldsymbol{\sigma}_2)],\end{aligned} \tag{3.1}$$

where U_c is the pure central potential, U_σ is the spin-dependent central potential responsible for singlet-triplet splitting, U_T is the tensor force, U_{LS} is the spin-orbit force, $U_{\sigma p}$ is the force which couples spin and momenta, and U_{TV} is the time-reversal-violating nuclear force. The various operators are standard and are defined below for completeness:

$$\begin{aligned}S_{12}&=(3/r^2)(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2,\\\mathbf{L}&=\mathbf{r}\times\mathbf{k},\\\mathbf{S}&=\tfrac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2),\\\mathbf{k}&=-i\partial/\partial\mathbf{r}.\end{aligned} \tag{3.2}$$

[24] J. H. Christenson, J. W. Cronin, V. L. Fitch, and F. Turlay, Phys. Rev. Letters **13**, 138 (1964).

[25] Using the algebra of currents, the value of g_A/g_V was obtained by S. Adler, Phys. Rev. Letters **14**, 1051 (1965); W. I. Weisberger, *ibid.* **14**, 1047 (1965).

[26] D. Y. Wong, Nucl. Phys. **55**, 212 (1964).

[27] The E meson in this work has been assumed to be an axial-vector particle, while the present experimental indications are that it may be a pseudoscalar.

[28] The fact that the axial-vector mesons provide the inner core of the nuclear force is vital from the point of view of the universal theory, since the hypothesis of chiral invariance demands the vector and axial-vector nature of all interactions. See Ref. 13 for details.

[29] V. V. Babikov, Nucl. Phys. **76**, 665 (1966).

[30] The potential (3.1) has been written in a symmetrized form for convenience.

All the potential terms are assumed to be functions of $|\mathbf{r}|$ and k^2.

A. Pseudoscalar Potential

As mentioned before, the pseudoscalar mesons arise in our theory as longitudinal components of axial-vector fields. Their coupling constants are determined from the relation

$$A_\lambda = B_\lambda + (\xi/m_P)\partial_\lambda \phi_P\,, \tag{3.3}$$

where ξ is a predetermined universal parameter [see Eq. (2.9)] and m_P is the mass of the pseudoscalar meson. The axial-vector coupling constant f_A is now related to the dimensionless pseudoscalar coupling constant (pseudovector coupling) f_P by the relation

$$f_P = f_A \xi. \tag{3.4}$$

The Born term corresponding to the exchange of a pseudoscalar meson is simply given by

$$T = (f_P/m_P)^2 O_I \bar{u}(p_3)(\gamma\cdot q)\gamma_5 u(p_1)(t-m_P^2)^{-1} \times \bar{u}(p_4)(\gamma\cdot q)\gamma_5 u(p_1)\,. \tag{3.5}$$

(See Fig. 1 for kinematics.) Here O_I is the isotopic spin operator equal to $\frac{1}{4}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$ for the pion ($I=1$) and $\frac{1}{4}$ for the η meson ($I=0$). In what follows we shall drop the isospin dependence, since it is a trivial multiplicative factor. The nonrelativistic form of the above expression is now written in the convenient form

$$T = (f_P/m_P)^2 (\chi_1^\dagger \boldsymbol{\sigma}_1\cdot\mathbf{q}\chi_1)(\chi_2^\dagger\boldsymbol{\sigma}_2\cdot\mathbf{q}\chi_2)(t-m_P^2)^{-1}. \tag{3.6}$$

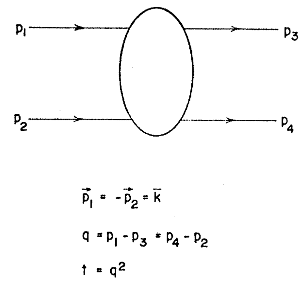


FIG. 1. Feynman diagram for nucleon-nucleon scattering, illustrating the kinematics.

The evaluation of the potential is now straightforward:

$$\begin{aligned} U_c &= 0\,,\\ U_\sigma &= (f_P^2/12\pi)Z_0(m_P r)\,,\\ U_T &= (f_P^2/12\pi)Z_2(m_P r)\,,\\ U_{LS} &= U_{\sigma p} = U_{TV} = 0\,, \end{aligned} \tag{3.7}$$

where

$$Z_0(mr) = e^{-mr}/r$$

and

$$Z_2(mr) = (3/m^2r^2 + 3/mr + 1)e^{-mr}/r\,. \tag{3.8}$$

Note that higher-derivative terms in (2.16) (involving $Z_{\lambda\nu}$ and $A_{\lambda\nu}$) do not contribute to the pseudoscalar force. The above potential is the same as the one obtained before by many authors.[26]

B. Vector Potential

The Born term, corresponding to the exchange of a vector meson, is (apart from isospin factors)

$$T = [g^2\bar{u}(p_3)\gamma_\mu u(p_1)\bar{u}(p_4)\gamma_\nu u(p_2) + (g'^2/m_V^2)\bar{u}(p_3)\sigma_{\mu\alpha}q_\alpha u(p_1)\bar{u}(p_4)\sigma_{\nu\beta}q_\beta u(p_2) - (gg'/m_V) \times (\bar{u}(p_3)\gamma_\mu u(p_1)\bar{u}(p_4)\sigma_{\nu\beta}q_\beta u(p_2) + \bar{u}(p_3)\sigma_{\mu\alpha}q_\alpha u(p_1)\bar{u}(p_4)\gamma_\nu u(p_2))]\left(\frac{-g_{\mu\nu}+q_\mu q_\nu/m_V^2}{t-m_V^2}\right). \tag{3.9}$$

The nonrelativistic form of the above expression can now be arranged in the form (3.1) and the appropriate Fourier transforms yield the well-known potential

$$U_c = \left[\frac{g^2}{4\pi}\left(1+\frac{m_V^2}{2m^2}+\frac{m_V^4}{64m^4}+\frac{2k^2}{m^2}+\frac{k^2m_V^2}{16m^4}-\frac{k^2m_V^4}{128m^6}\right)-\frac{gg'}{4\pi}\left(\frac{m_V}{m}+\frac{m_V^3}{4m^3}+\frac{k^2m_V}{m^3}-\frac{k^2m_V^3}{16m^5}\right) +\frac{g'^2}{4\pi}\left(\frac{m_V^2}{4m^2}+\frac{m_V^4}{64m^4}+\frac{k^2m_V^2}{16m^4}-\frac{k^2m_V^4}{128m^6}\right)\right]Z_0(m_V r)\,, \tag{3.10}$$

$$U_\sigma = \left[\frac{g^2}{4\pi}\left(\frac{m_V^2}{6m^2}+\frac{m_V^4}{64m^4}+\frac{k^2m_V^2}{24m^4}-\frac{k^2m_V^4}{128m^6}\right)-\frac{gg'}{4\pi}\left(\frac{2m_V}{3m}+\frac{m_V^3}{4m^3}+\frac{2k^2m_V}{3m^3}-\frac{k^2m_V^3}{16m^5}\right) +\frac{g'^2}{4\pi}\left(\frac{2}{3}+\frac{m_V^2}{2m^2}+\frac{m_V^4}{64m^4}+\frac{4k^2}{3m^2}+\frac{k^2m_V^2}{24m^4}-\frac{k^2m_V^4}{128m^6}\right)\right]Z_0(m_V r)\,, \tag{3.11}$$

$$U_T = \left[-\frac{g^2}{4\pi}\left(\frac{m_V^2}{12m^2}+\frac{k^2m_V^2}{48m^4}\right)+\frac{gg'}{4\pi}\left(\frac{m_V}{3m}+\frac{k^2m_V}{3m^3}\right)-\frac{g'^2}{4\pi}\left(\frac{1}{3}+\frac{2k^2}{3m^2}+\frac{k^2m_V^2}{48m^4}\right)\right]Z_2(m_V r)\,, \tag{3.12}$$

$$U_{LS}=\left[-\frac{g^2}{4\pi}\left(\frac{3m_V^2}{2m^2}+\frac{m_V^4}{16m^4}+\frac{k^2m_V^2}{8m^4}-\frac{k^2m_V^4}{32m^6}\right)-\frac{gg'}{4\pi}\left(\frac{4m_V}{m}+\frac{m_V^3}{m^3}-\frac{2k^2m_V}{m^3}-\frac{k^2m_V^3}{4m^5}\right)\right.$$
$$\left.-\frac{g'^2}{4\pi}\left(\frac{3m_V^2}{2m^2}+\frac{m_V^4}{16m^4}+\frac{k^2m_V^2}{8m^4}-\frac{k^2m_V^4}{32m^6}\right)\right]Z_1(m_Vr)\,, \quad (3.13)$$

$$U_{\sigma p}=\left[-\frac{g^2}{4\pi}\left(\frac{m_V^2}{16m^2}-\frac{k^2m_V^2}{32m^4}\right)+\frac{gg'}{4\pi}\left(\frac{m_V}{m}-\frac{k^2m_V}{4m^3}\right)-\frac{g'^2}{4\pi}\left(2+\frac{m_V^2}{16m^2}-\frac{k^2m_V^2}{32m^4}\right)\right]Z_0(m_Vr)\,, \quad (3.14)$$

$$U_{TV}=0\,, \quad (3.15)$$

where

$$Z_1(mr)=\left(\frac{1}{mr}+\frac{1}{m^2r^2}\right)e^{-mr}/r\,. \quad (3.16)$$

C. Axial-Vector Potential

Using the axial-vector couplings given before [Eq. (2.16)], the axial-vector-meson exchange amplitude may be written

$$T=[f^2\bar{u}(p_3)\gamma_\mu\gamma_5u(p_1)\bar{u}(p_4)\gamma_\nu\gamma_5u(p_2)+(f'^2/m_A^2)\bar{u}(p_3)\sigma_{\mu\alpha}q_\alpha\gamma_5u(p_1)\bar{u}(p_4)\sigma_{\nu\beta}q_\beta\gamma_5u(p_2)-(ff'/m_A)$$
$$\times(\bar{u}(p_3)\gamma_\mu\gamma_5u(p_1)\bar{u}(p_4)\sigma_{\nu\beta}q_\beta\gamma_5u(p_2)+\bar{u}(p_3)\sigma_{\mu\alpha}q_\alpha\gamma_5u(p_1)\bar{u}(p_4)\gamma_\nu\gamma_5u(p_2))]\left(\frac{-g_{\mu\nu}+q_\mu q_\nu/m_A^2}{t-m_A^2}\right). \quad (3.17)$$

The resulting potentials are

$$U_c=\left[-\frac{f^2}{4\pi}\left(\frac{m_A^4}{64m^4}+\frac{k^2m_A^2}{16m^4}-\frac{k^2m_A^4}{128m^6}\right)\right]Z_0(m_Ar)\,, \quad (3.18)$$

$$U_\sigma=\left[-\frac{f^2}{4\pi}\left(\frac{2}{3}-\frac{m_A^2}{3m^2}+\frac{m_A^4}{64m^4}-\frac{k^2m_A^2}{24m^4}-\frac{k^2m_A^4}{128m^6}\right)\right.$$
$$\left.-\frac{f'^2}{4\pi}\left(\frac{1}{3}+\frac{m_A^2}{12m^2}+\frac{2k^2}{3m^2}\right)\right]Z_0(m_Ar)\,, \quad (3.19)$$

$$U_T=\left[\frac{f^2}{4\pi}\left(\frac{1}{3}+\frac{m_A^2}{12m^2}+\frac{k^2m_A^2}{48m^4}\right)\right.$$
$$\left.-\frac{f'^2}{4\pi}\left(\frac{1}{3}+\frac{m_A^2}{12m^2}+\frac{2k^2}{3m^2}\right)\right]Z_2(m_Ar)\,, \quad (3.20)$$

$$U_{LS}=\left[-\frac{f^2}{4\pi}\left(\frac{m_A^2}{2m^2}-\frac{m_A^4}{16m^4}-\frac{k^2m_A^2}{8m^4}+\frac{k^2m_A^4}{32m^6}\right)\right]$$
$$\times Z_1(m_Ar)\,, \quad (3.21)$$

$$U_{\sigma p}=\left[-\frac{f^2}{4\pi}\left(2-\frac{m_A^2}{16m^2}+\frac{k^2m_A^2}{32m^4}\right)\right]Z_0(m_Ar)\,, \quad (3.22)$$

$$U_{TV}=\left[-i\frac{ff'}{4\pi}\left(2m_A+\frac{k^2m_A}{m^2}\right)\right]Z_1(m_Ar)\,. \quad (3.23)$$

The last term [Eq. (3.23)] violates time-reversal invariance and its consequences will be discussed in detail in Sec. 5.[31]

Note that in writing the above potentials all energy-dependent factors except the phase-space factor m/E have been expanded in powers of k^2 and terms up to k^2 are retained. The relativistic phase-space factor is multiplicative to all terms and will be approximated by $m/E\sim1-k^2/2m^2$.

In order to introduce these potentials in a Schrödinger equation we first estimate their energy dependence. Should it turn out that our potentials are sensitively dependent upon energy (in the low-energy region, say, up to ~300 MeV), we shall be forced to treat k^2 as an operator and the resulting differential equations will be of a very complicated form. To be allowed to treat the momentum-dependent potentials written above as c-number functions of momenta (operators otherwise), we should be able to show that the momentum dependence is small. We shall therefore write the total potential due to all the eight mesons (including the phase-space factor m/E) in the form

$$U(r,k^2)=U_c+(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)U_\sigma+S_{12}U_T+\mathbf{L}\cdot\mathbf{S}U_{LS}$$
$$+\boldsymbol{\sigma}_1\cdot\hat{k}U_{\sigma p}\boldsymbol{\sigma}_2\cdot\hat{k}\,, \quad (3.24)$$

[31] The existence of this term is vital because in weak interactions this term provides a natural vehicle for CP violation as an induced strong-interaction effect, while in pure strong interactions it is not so disastrous, as we shall discuss later.

TABLE I. The relative strengths of various kinds of potentials arising from the exchange of pseudoscalar, vector, and axial-vector mesons.

	Energy (MeV)	Meson exchanged π	η	ρ	ω	ϕ	A_1	E	D
A	0	0.0	0.0	0.0092	0.0531	0.0124	−0.0057	−0.0178	0.0
	140	0.0	0.0	0.0112	0.0569	0.0121	0.0066	−0.0188	0.0
	300	0.0	0.0	0.0133	0.0606	0.0118	0.0075	−0.0197	0.0
B	0	0.00051	0.00051	0.0697	0.0048	0.0500	0.1527	0.0032	−0.0767
	140	0.00049	0.00049	0.0754	0.0047	0.0520	−0.1580	0.0022	−0.0813
	300	0.00047	0.00047	0.0811	0.0046	0.0539	−0.1631	0.0012	−0.0859
C	0	0.00051	0.00051	−0.0207	−0.0023	−0.0131	0.0	0.1141	−0.0767
	140	0.00049	0.00049	−0.0240	−0.0022	−0.0145	−0.0700	0.1106	−0.0813
	300	0.00047	0.00047	−0.0274	−0.0022	−0.0160	−0.0205	0.1065	−0.0859
D	0	0.0	0.0	0.0508	−0.0421	−0.0857	−0.1185	−0.1778	0.0
	140	0.0	0.0	0.0530	−0.0407	−0.0828	−0.1123	−0.1693	0.0
	300	0.0	0.0	0.0551	−0.0392	−0.0795	−0.1055	−0.1597	0.0
E	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	140	0.0	0.0	−0.0123	−0.0001	−0.0058	−0.0300	−0.0291	0.0
	300	0.0	0.0	−0.0253	−0.0002	−0.0119	−0.0616	−0.0598	0.0

$\hat{k}$ being the unit vector along $\mathbf{k}$. Here

$$
\begin{aligned}
U_c &= \sum_{i=1}^{8} A_i Z_0(\mu_i r)\,,\\
U_\sigma &= \sum_{i=1}^{8} B_i Z_0(\mu_i r)\,,\\
U_T &= \sum_{i=1}^{8} C_i Z_2(\mu_i r)\,,\\
U_{LS} &= \sum_{i=1}^{8} D_i Z_1(\mu_i r)\,,\\
U_{\sigma p} &= \sum_{i=1}^{8} E_i Z_0(\mu_i r)\,,
\end{aligned}
\tag{3.25}
$$

where μ_i is the mass of the meson exchanged and A_i, B_i, C_i, D_i, and E_i are polynomials in k^2. Table I shows the dependence of A_i, $B_i,\cdots$, upon energy. In most cases it is seen that the momentum dependence is very small. In some cases (e.g., A for ρ and ω), it is about 30%. Since we are interested in doing a phase-shift analysis at low energies only, we are probably justified in treating this small momentum dependence as a c-number.

We now perform a partial-wave projection and plot the effective partial-wave potentials[32] in Fig. 2 for various energies. A comparison of our potentials with other recent ones obtained by Sugawara and von Hippel[33] and by Reid[34] leads us to make the following remarks:

(i) The ${}^1S_0$ potential shows the well-known hard core and changes sign at about 0.6 or 0.7 F. In the present work, this is a natural consequence of Sudarshan's universal theory. Our potential has the same shape, passes through zero at about the same point, and is slightly less attractive as compared to the potential given in Ref. 34. However, in Reid's work, this hard-core behavior was contrived at by a judicious choice of his parameters.

(ii) In general, our potentials (see Fig. 2) are in good agreement with the phenomenological ones used by Reid; however, in some partial waves, notably, ${}^3P_2$, our potential is even opposite in sign as compared to Reid. This difference will be reflected in the phase shifts and will be commented upon later.

(iii) In the so-called zero-parameter fit by Sugawara and von Hippel,[33] the extreme short-range potential is obtained by considering fourth-order diagrams including the effects of N^* production in intermediate states. These effects presumably give rise to the hard core. However, in the present work the exchange of heavy axial-vector mesons seems to simulate the effect of inelastic channels in the intermediate state. It is, therefore heartening to note that in our simple-minded lowest-order calculations it is possible to simulate the effects of high-mass inelastic channels, provided that due cognizance is given to axial-vector mesons and their universal couplings.

4. SCHRÖDINGER EQUATION AND METHOD OF SOLUTION

In Sec. 3, we have discussed the energy dependence of our potential. We shall therefore treat the k^2 dependence of the potential as a c-number[35] and attempt to solve the Schrödinger equation for various partial waves. Our method of solution is quite standard,[32,36] and we shall only sketch it briefly for the sake of completeness.

[32] See, for example, M. L. Goldberger and K. M. Watson, *Collision Theory* (John Wiley & Sons, Inc., New York, 1964), Chap. 7, p. 384.
[33] H. Sugawara and F. von Hippel, Phys. Rev. **172**, 1764 (1968).
[34] R. Reid, Ph.D. thesis, Cornell University (unpublished).
[35] If one regards the momentum dependence of the potential as an operator dependence, the resulting Schrödinger equations are extremely complicated. See, for example, M. Razavy, G. Field, and J. S. Levinger, Phys. Rev. **125**, 269 (1962).
[36] See B. L. Scott, thesis (unpublished); R. A. Bryan and B. L. Scott, Phys. Rev. **135**, B434 (1964).

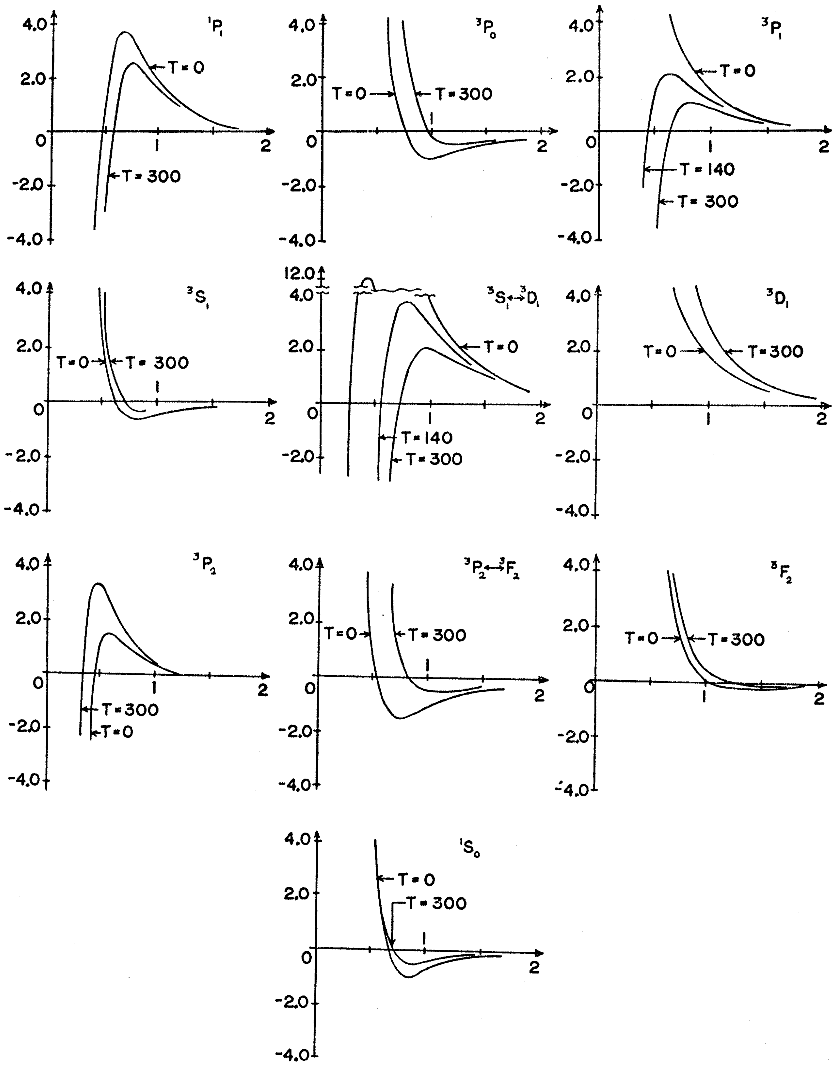


FIG. 2. Plot of momentum-dependent partial-wave potentials for laboratory kinetic energies of 0 and 300 MeV.

For the singlet state ($l=J$, $S=0$) and the unmixed triplet state ($l=J$, $S=1$), the radial Schrödinger equation is (in what follows, our discussion follows very closely that of Refs. 32 and 36)

$$\left(\frac{d^2}{dr^2}+k^2-\frac{J(J+1)}{r^2}\right)\omega_{JS;JS}{}^J(r) = v_{JS;JS}{}^J(r,k^2)w_{JS;JS}{}^J(r). \quad (4.1)$$

The wave function w (and the potential v) are written for the transition $w_{lS;l',S}{}^J$. To solve for the scattering phase shift, one looks at the asymptotic form of the varial wave function

$$w_{JS,JS}{}^J(r) \underset{r\to\infty}{\sim} \text{const}\,\sin(kr-\tfrac{1}{2}J\pi+\delta_{JS;JS}{}^J). \quad (4.2)$$

The phase shift can now be extracted from (4.1) and (4.2) by matching the logarithmic derivatives of the solution of (4.1) and the asymptotic wave function at a point $r=r_m$ such that $r_m \gg 1/\mu$,[37] μ being the mass of the lightest meson exchanged (in our case, the π meson).

For the mixed partial waves we follow the convention of Blatt and Biedenharn[32] and define the radial wave functions by

$$\left(\frac{d^2}{dr^2}+k^2-\frac{J(J-1)}{r^2}\right)u_{\alpha,\beta}{}^J(r) = v_{J-1,1;J-1,1}{}^J(r,k^2) \times u_{\alpha,\beta}{}^J(r)+v_{J-1,1;J+1,1}{}^J(r,k^2)w_{\alpha,\beta}{}^J(r), \quad (4.3)$$

$$\left(\frac{d^2}{dr^2}+k^2-\frac{(J+1)(J+2)}{r^2}\right)w_{\alpha,\beta}{}^J(r) = v_{J+1,1;J+1,1}{}^J(r,k^2)w_{\alpha,\beta}{}^J(r) + v_{J+1,1;J-1;1}{}^J(r,k^2)u_{\alpha,\beta}{}^J(r), \quad (4.4)$$

[37] In actual practice we found that at about $r_m=1/8m_\pi$ the asymptotic conditions are well satisfied.

where α and β are indices denoting the two eigenchannels. Equations (4.3) and (4.4) will have two linearly independent solutions (labeled 1 and 2) which, in general, will not be the eigensolutions. To find the eigenwaves, one must find the linear combination of solutions 1 and 2, wherein the $l=J-1$ part has the same phase shift as the $l=J+1$ part. The two linearly independent solutions are written in the form (dropping the superscript J)

$$\psi_{1,2}=\frac{u_{1,2}(r)}{r}\mathcal{Y}_{J-1,1,m}{}^{J}+\frac{w_{1,2}(r)}{r}\mathcal{Y}_{J+1,1,m}{}^{J}, \quad (4.5)$$

where $u_{1,2}$ and $w_{1,2}$ are any two solutions of (4.3) and (4.4), and $\mathcal{Y}_{lSm}{}^{J}$ are the spherical harmonics. The eigenwaves, of course, must have the form

$$\psi_\alpha(r)\underset{r\to\infty}{\sim}[j_{J-1}(kr)-\tan\delta_\alpha\, n_{J-1}(kr)]\mathcal{Y}_{J-1,1,m}{}^{J} +\eta_\alpha[j_{J+1}(kr)-\tan\delta_\beta\, n_{J+1}(kr)]\mathcal{Y}_{J+1,1,m}{}^{J} \quad (4.6)$$

and

$$\psi_\beta(r)\underset{r\to\infty}{\sim}\eta_\beta[j_{J-1}(kr)-\tan\delta_\beta\, n_{J-1}(kr)]\mathcal{Y}_{J-1,1,m}{}^{J} +[j_{J+1}(kr)-\tan\delta_\beta\, n_{J+1}(kr)]\mathcal{Y}_{J+1,1,m}{}^{J}. \quad (4.7)$$

Here j and n are the spherical Bessel functions and Neumann functions, $\delta_{\alpha,\beta}$ are the eigenphase shifts, and $\eta_{\alpha,\beta}$ are the mixing parameters which are related by the unitarity condition $\eta_\alpha/\eta_\beta=-1$.

Assuming the eigenwaves to be linear combinations of the solutions (4.5), we again match logarithmic derivatives at $r=r_m$ to determine the eigenphases $\delta_{\alpha,\beta}$ and the mixing parameters. The resulting algebra is quite complicated but standard.[36] We merely quote the results

$$\tan\delta_{\alpha,\beta}=-\tfrac{1}{2}B\pm[(\tfrac{1}{2}B)^2-C]^{1/2}, \quad (4.8)$$

where

$$B=(\alpha_1\beta_4-\alpha_4\beta_1+\alpha_3\beta_2-\alpha_2\beta_3)/(\alpha_3\beta_4-\alpha_4\beta_3), \quad (4.9)$$

$$C=(\alpha_1\beta_2-\alpha_2\beta_1)/(\alpha_3\beta_4-\alpha_4\beta_3), \quad (4.10)$$

$$\alpha_{1,2}=j_{J-1}(kr)\frac{d}{dr}\left(\frac{u_{1,2}}{r}\right)-kj_{J-1}'(kr)u_{1,2}, \quad (4.11)$$

$$\alpha_{3,4}=-n_{J-1}(kr)\frac{d}{dr}\left(\frac{u_{1,2}}{r}\right)+kn_{J-1}'(kr)u_{1,2}. \quad (4.12)$$

The quantities $\beta_{1,2,3,\ldots}$ are obtained from $\alpha_{1,2,3,\ldots}$ by replacing u's with w's and $J-1$ with $J+1$. The mixing parameters η_α and η_β are given by

$$\eta_\alpha=-\frac{\alpha_1+\alpha_3\tan\delta_\alpha}{\beta_1+\beta_3\tan\delta_\alpha}=-\frac{\alpha_2+\alpha_4\tan\delta_\alpha}{\beta_2+\beta_4\tan\delta_\alpha},$$
$$\eta_\beta=-\frac{\beta_1+\beta_3\tan\delta_\beta}{\alpha_1+\alpha_3\tan\delta_\beta}=-\frac{\beta_2+\beta_4\tan\delta_\beta}{\alpha_2+\alpha_4\tan\delta_4}. \quad (4.13)$$

The unitarity condition $\eta_\alpha/\eta_\beta=-1$ will only be used as a check on the calculations. The sign ambiguity in Eq. (4.8) can be settled by noting that in the absence of noncentral forces the eigenwaves α and β get decoupled.[36] If in the absence of noncentral forces $\delta_\beta \gtrless \delta_\alpha$, one should use the upper (lower) sign in Eq. (4.8) for $\tan\delta_\alpha$ and the lower (upper) sign for $\tan\delta_\beta$.

For actual numerical solution of the Schrödinger equations, we had to use a cutoff at small distances, since the potential is extremely singular at $r=0$. The terms in our potential (Sec. 3) which have a $1/r^3$-type singularity have a nice cancellation mechanism[18,17] for $I=1$ mesons, but not so for $I=0$ mesons. Such a cancellation of $1/r^3$ terms was visualized in very early days by Schwinger.[38] However, for our complete potential it does not work and we are forced to use a cutoff. We have used two forms of cutoffs: a hard-core cutoff, which means an infinite repulsion for $r\leq r_c$, and a soft-core cutoff, implying $V(r)\equiv V(r_c)$ for all $r\leq r_c$. Our results for various phase-shifts and mixing parameters ($\epsilon_\alpha=\tan^{-1}\eta_\alpha$) along with experimental values[39] are plotted in Fig. 3 for various cutoffs.

5. VIOLATION OF TIME-REVERSAL INVARIANCE

As mentioned before, a part of the two-nucleon interaction via the exchange of axial-vector mesons [Eqs. (2.11) and (3.23)] violates time-reversal invariance. We have chosen the coupling constants in such a way that the violation of T (or CP) invariance is maximal. However, this term is extremely momentum-dependent (as argued before) and its effects will be seen in only high-energy nucleon-nucleon collisions.

Perhaps the simplest way to detect T (or CP) noninvariance in nucleon-nucleon scattering is to measure the polarization P in unpolarized particle collisions and the asymmetry $\mathcal{A}$ in the scattering cross section of totally polarized beams. If T invariance is good, P and $\mathcal{A}$ should be equal. In the present work, however, they will not be equal. To compute the quantity $P-\mathcal{A}$, we rewrite our Feynman amplitude M in the standard form given by Wolfenstein and Ashkin[40] and by Dalitz[41]:

$$M=(u+v)+(u-v)\boldsymbol{\sigma}_1\cdot\mathbf{n}\boldsymbol{\sigma}_2\cdot\mathbf{n}+c(\boldsymbol{\sigma}_1\cdot\mathbf{n}+\boldsymbol{\sigma}_2\cdot\mathbf{n}) +(g-h)\boldsymbol{\sigma}_1\cdot\mathbf{m}\boldsymbol{\sigma}_2\cdot\mathbf{m}+(g+h)\boldsymbol{\sigma}_1\cdot\mathbf{l}\boldsymbol{\sigma}_2\cdot\mathbf{l}. \quad (5.1)$$

The violation of T invariance allows one to add the following term[42,43]:

$$M^{(1)}=T(\boldsymbol{\sigma}_1\cdot\mathbf{l}\boldsymbol{\sigma}_2\cdot\mathbf{m}+\boldsymbol{\sigma}_1\cdot\mathbf{m}\boldsymbol{\sigma}_2\cdot\mathbf{l}), \quad (5.2)$$

where $\mathbf{l}=(\mathbf{k}+\mathbf{k}')/|\mathbf{k}+\mathbf{k}'|$, $\mathbf{m}=(\mathbf{k}'-\mathbf{k})/|\mathbf{k}'-\mathbf{k}|$, and

[38] J. Schwinger, Phys. Rev. **61**, 387 (1942).
[39] M. H. MacGregor, R. A. Arndt, and R. M. Wright, Phys. Rev. **169**, 1128 (1968); R. A. Arndt and M. H. MacGregor, *ibid.* **141**, 873 (1966).
[40] L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952).
[41] R. H. Dalitz, Proc. Phys. Soc. (London) **A65**, 175 (1952).
[42] R. J. N. Phillips, Nuovo Cimento 8, 265 (1958).
[43] L. I. Lapidus, Rev. Mod. Phys. **39**, 689 (1967).

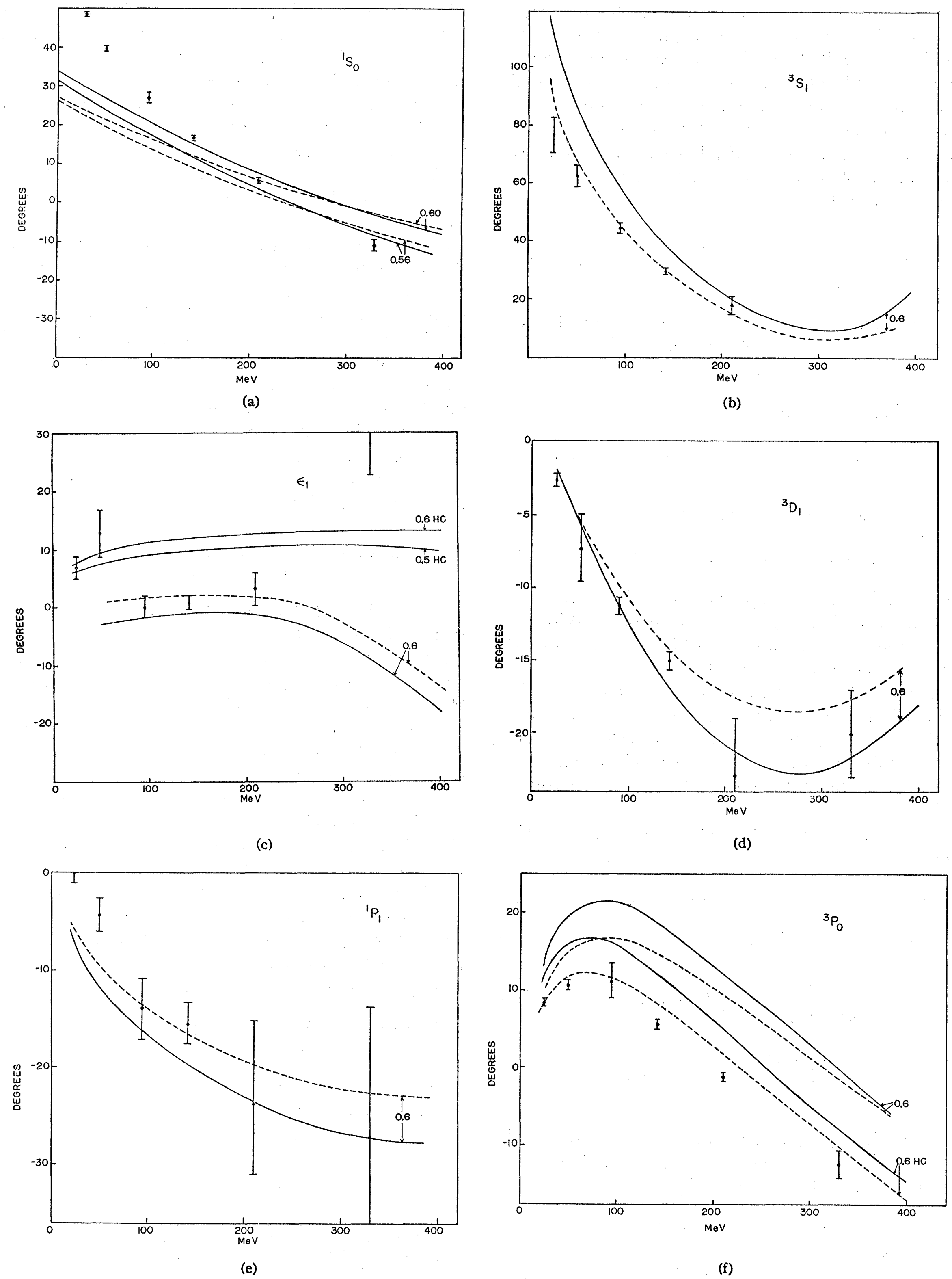
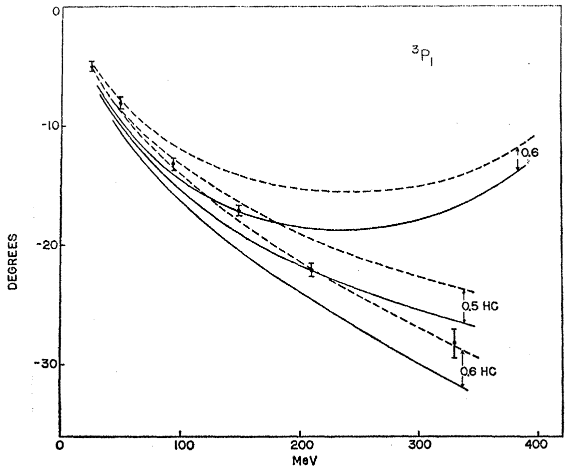
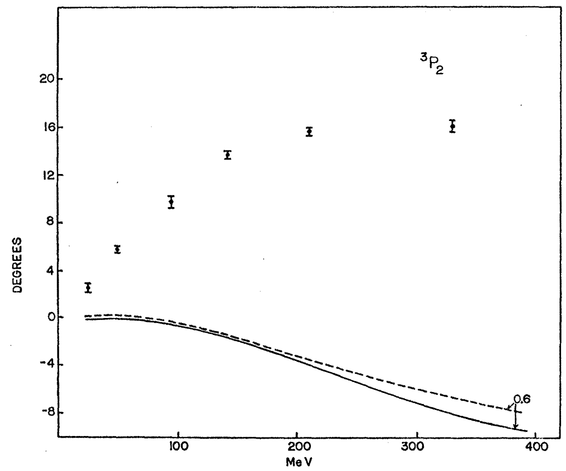


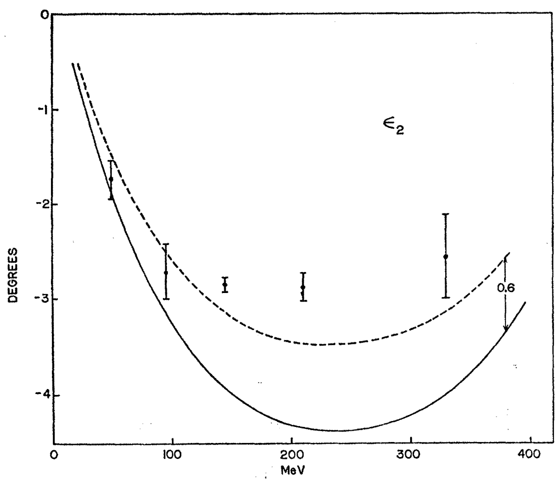
FIG. 3. Our predicted phase shifts and mixing parameters plotted as a function of energy for a soft-core cutoff equal to 0.6 F with two values of coupling constant $g^2/4\pi=6.5$ (dashed lines) and 8.125 (solid lines). At places where a hard-core (HC) cutoff also gives a reasonable fit, our results are plotted side by side for $r_c=0.5$ F. The experimental points are taken from Arndt, MacGregor, and Wright and from Arndt and MacGregor (see Ref. 39). The phases plotted here are the conventional bar phases.



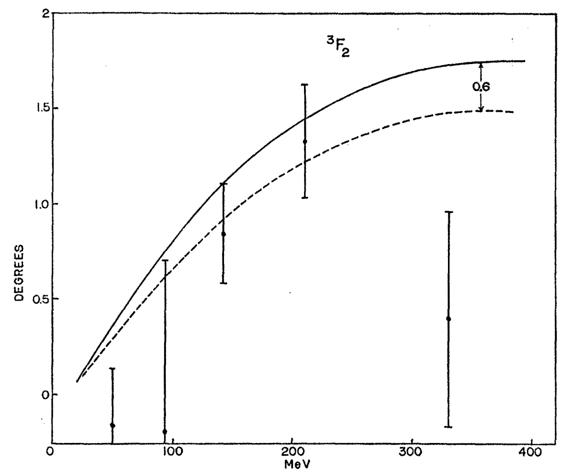
(g)



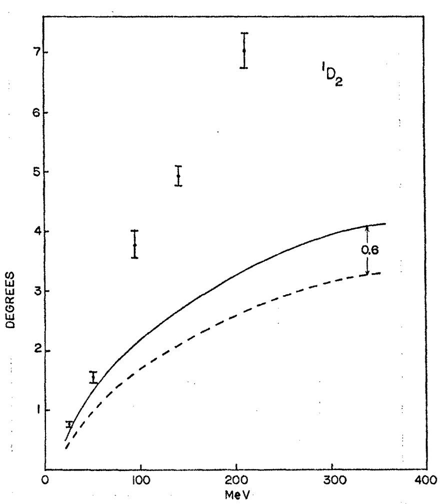
(h)



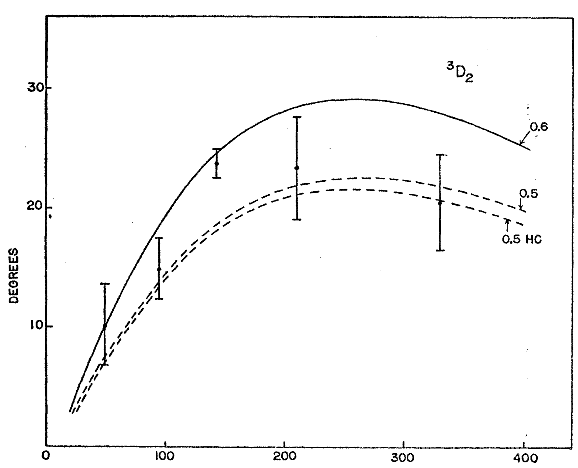
(i)



(j)



(k)



(l)

FIG. 3 (*continued*).

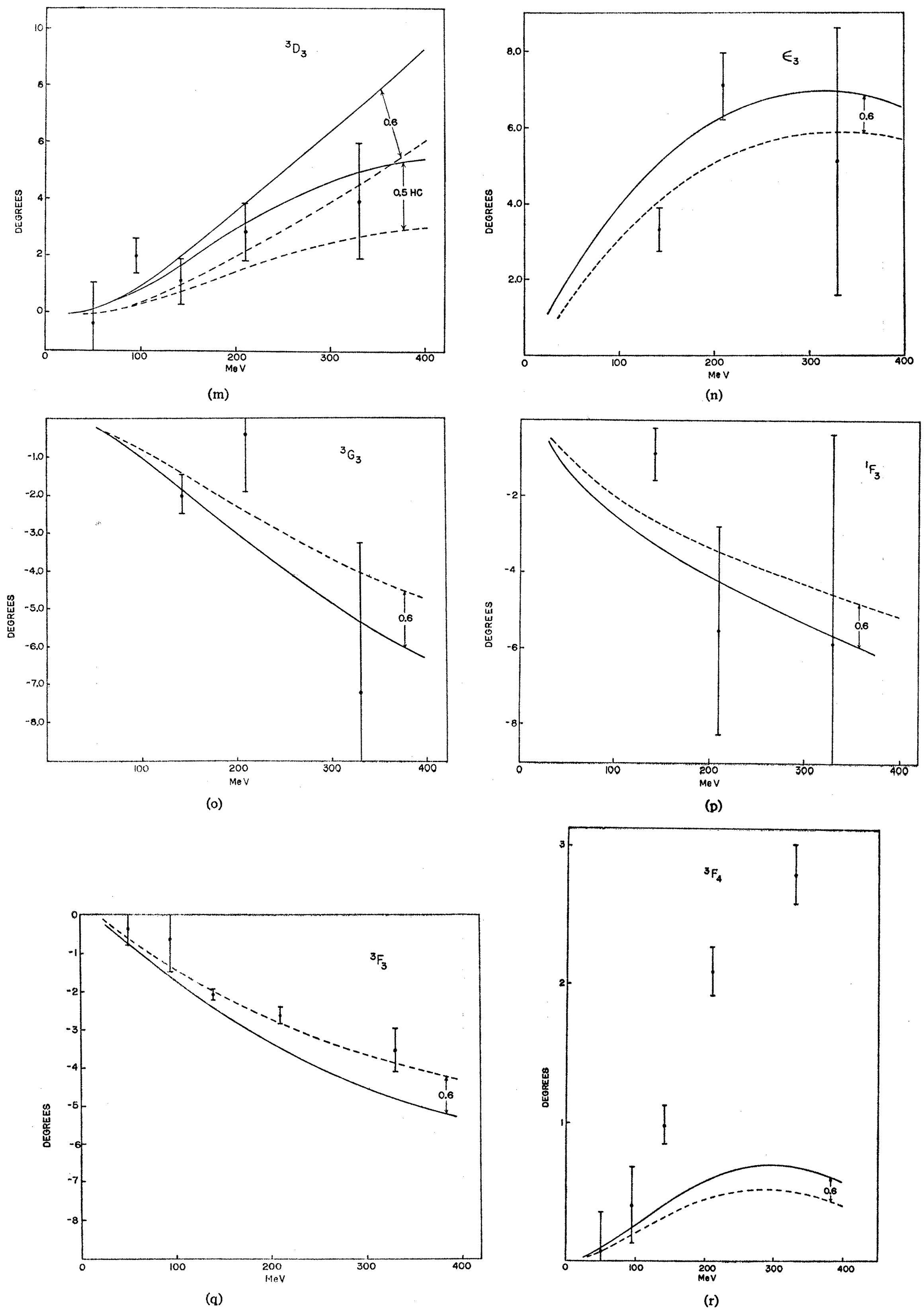


FIG. 3 (*continued*).

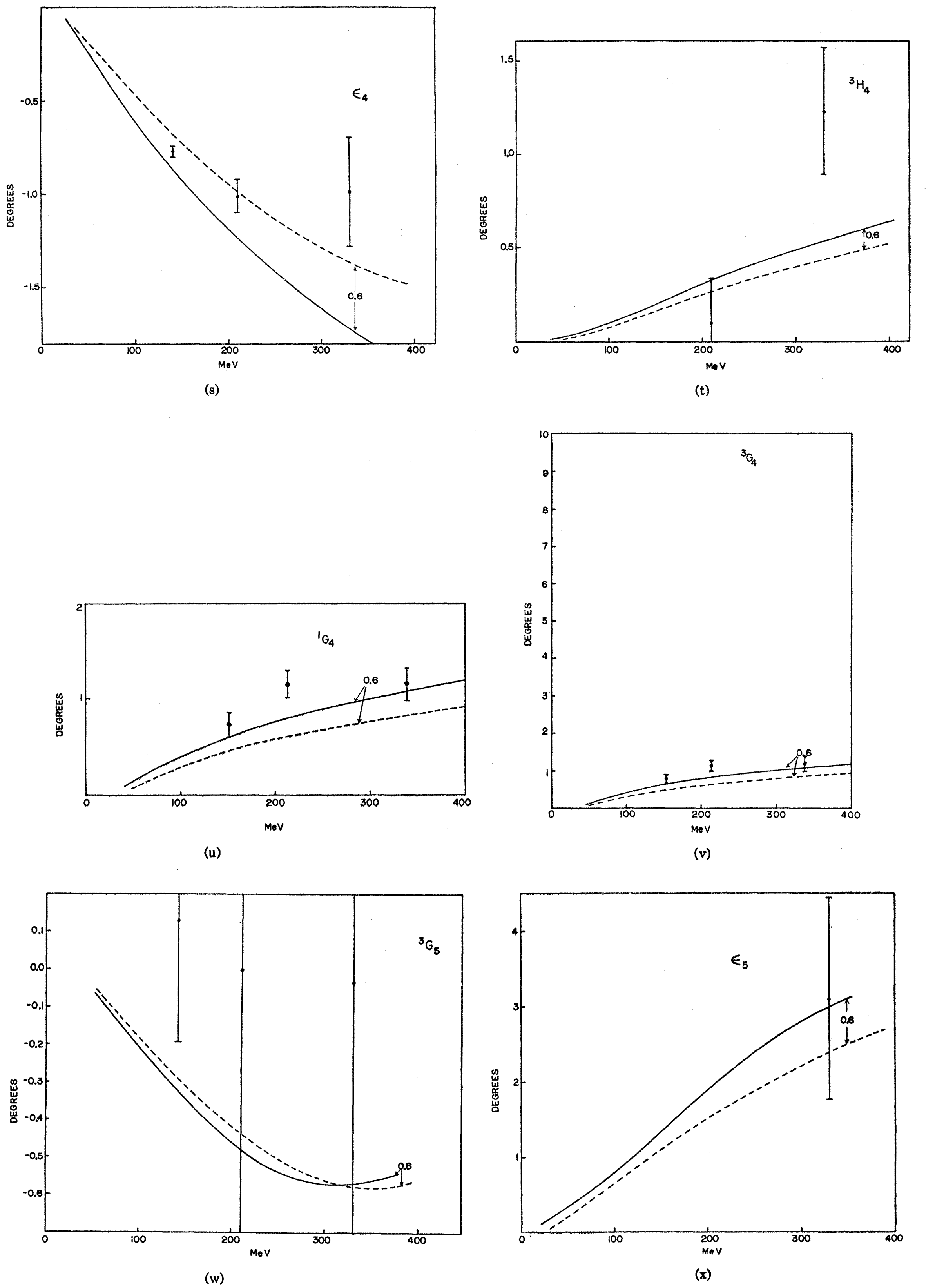


FIG. 3 (*continued*).

$\mathbf{n}=\mathbf{l}\times\mathbf{m}$, $\mathbf{k}$ and $\mathbf{k}'$ being the initial and final momenta in the c.m. system.

Using (5.1) and (5.2), one can compute the polarization P and asymmetry $\mathcal{A}$ parameters. Their difference is given by

$$P-\mathcal{A}=-8\,\mathrm{Im}(T^*h)/\sigma_0, \qquad (5.3)$$

where σ_0 is the differential cross section at any angle. Since this difference is expected to be small, and at any rate the present experimental data are extremely uncertain[44,45] with respect to the parameters P and $\mathcal{A}$, we make only a crude estimate using the Born approximation for the right-hand side of (5.3). In the Born approximation, h is purely real and T is purely imaginary, so that

$$P-\mathcal{A}=-8\,\mathrm{Im}T\,\mathrm{Re}h/\sigma_0. \qquad (5.4)$$

Our results for $P-\mathcal{A}$ are as follows: At 400 (lab energy) and 600 MeV, for an angle of scattering around 40°, the values of $P-\mathcal{A}$ are 0.028 and 0.049, respectively. These are to be compared with 0.0019 ± 0.010 [45] and 0.021 ± 0.027,[44] respectively. In view of the large uncertainties in the experimental measurements, our results can certainly be regarded as being consistent with experiments. We also computed the ratio $|T|^2/\sigma_0$, which can be called a measure of T violation in the theory. The value of this ratio decreases very fast at low energies It varies from 1.5×10^{-4} to 0.8×10^{-3} for lab energies varying from 400 to about 600 MeV. These estimates agree with those of Ref. 44 but not with those of Ref. 45. At the present time, one cannot say with any degree of certainty whether the experiments show any T violation at all. For testing the present theory properly, the experimental accuracy must go up by an order of magnitude.

6. COMPARISON WITH EXPERIMENTS

Looking at the phase-shift and mixing-parameter curves, a few general remarks can be made. In general, the phase shifts prefer a soft core (though some partial waves are better with hard core) at a distance $r_c=0.6$ F. The best value of the over-all coupling constant seems to be $g^2/4\pi=6.5$, corresponding to the width of the ρ meson being 90 MeV.[46,47] Since there seems to be some uncertainly about the width of the ρ meson[48] and considerable spread in the value of the universal ρ coupling obtained from other sources,[49] we have even tried varying $g^2/4\pi$ up to about 8.125 (corresponding ρ width $\sim$115 MeV). As can be seen from the curves, for some partial waves even this value of the coupling constant seems to be all right. However, the over-all prediction of all partial waves seems to be in good agreement with experiments (with some exceptions, of course) with $g^2/4\pi=6.5$ and a soft core at $r_c=0.6$ F.

As mentioned before, the most heartening feature of the present work is the *natural* occurrence of a repulsive core in $^1S_0$ potential (largely due to the axial-vector mesons); however, the slope of the $^1S_0$ phase-shift curve seems to be too small for low energies ($0<E<100$ MeV). Increasing the coupling constant helps a little but not very much. If one uses a hard core, the results become much worse. A tentative reason for the slope being too small can be found by looking at our potential at intermediate and long ranges. The potential is attractive but momentum-dependent; therefore, for low energies the attraction is reduced and hence the slope of the phase-shift curve.

For the $^3S_1$ state our result is extremely good up to about 300 MeV. For higher energies, the phase shift, in general, rises, due to the soft core with attractive potential near $r=0$. In practice, this may not be the case. The hard-core results are uniformly bad for this partial wave. The mixing parameter ϵ_1 can be fitted quite well in the intermediate energy range ($100<E<300$ MeV). The bad fit at lower energies may be due to our calculational inability to satisfy the unitarity condition $\eta_\alpha/\eta_\beta=-1$, while for higher energies the unitarity relation was very well satisfied. For higher energies (>300 MeV) a reason similar to that given above might be responsible. The partial wave $^3D_1$ gives good results with both soft and hard cores up to 200 MeV, beyond which the hard-core result is bad and the soft-core result continues to be good. The numerical dependence of our results on the value of r_c is very weak.

The $^1P_1$ phase shift is repulsive and soft-core results (less repulsive than hard-core) are preferred. The results are insensitive to the value of coupling constants. The $^3P_0$ and $^3P_1$ partial waves are the notable exceptions which seem to prefer a hard core, although the soft-core results have a similar shape and are not so bad. The best value of parameters for these partial waves seems to be $g^2/4\pi=6.5$, $r_c=0.6$ F (hard core). Our results for $^3P_2$ phase shifts are uniformly bad (lower than experiment). The reason for this is the existence of a large attractive $\mathbf{L}\cdot\mathbf{S}$ force arising due to the exchange of heavy axial-vector mesons (see Table I for D and E mesons). For $^3P_0$, $^3P_1$, $^3F_2,\cdots$ partial waves, this force is repulsive and merges with the repulsive core at short distances. For $^3D_1$, $^3D_3$, $^3G_3,\cdots$ partial waves, it is again repulsive due to isotopic spin factors. The only other place where this large attraction at extremely

[44] Yu. M. Kazarinov, Rev. Mod. Phys. **39**, 509 (1967).

[45] R. Handler, S. C. Wright, L. Pondrom, P. Limon, S. Olsen, and P. Kloeppel, Phys. Rev. Letters **19**, 933 (1967).

[46] V. S. Auslander, G. I. Budker, J. N. Peslov, V. A. Sidorov, A. N. Skriusky, and A. G. Khabakhpashev, Phys. Letters **25B**, 433 (1967).

[47] In Sudarshan's theory the $\rho\pi\pi$ coupling consists of a vector as well as a tensor part. [See T. Pradhan, E. C. G. Sudarshan, and R. P. Saxena, Phys. Rev. Letters **20**, 79 (1968), in particular, Eq. (8)]. Due to the intefrerence between the two parts, one obtains a ρ width of 90 MeV for $g^2/4\pi=6.5$.

[48] P. Söding, C. G. Wohl, A. Roos, and W. J. Willis, Rev. Mod. Phys. **40**, 77 (1968).

[49] P. S. Signell and J. W. Durso, Phys. Rev. Letters **18**, 185 (1967).

short distances can give rise to trouble is the $^3F_4$ partial wave, where our results are uniformly bad. However, this should be not regarded as a serious drawback of the present theory, owing to the rather uncertain existence of D and E mesons. The consequences of their absence will not be disastrous elsewhere, the A_1 meson already provides a suitable amount of repulsion at short distances. The other notable features of the nuclear force at intermediate and long ranges are already present in this theory.

As for the rest of the higher partial waves, our results are in good agreement with experiment. They are largely independent of cutoff and mostly favor $g^2/4\pi=6.5$. For some notable exceptions (e.g., ϵ_3, ϵ_5, $^3F_5$), the experimental data seem to be in a bad shape and thus our results cannot be properly compared. It therefore appears that in the present work we have a reasonably good dynamical explanation of nucleon-nucleon scattering data (for $E<400$ MeV) with essentially a single parameter.

7. DISCUSSION AND OUTLOOK

We have thus seen that a largely successful description of nucleon-nucleon scattering can be given on the basis of a "universal theory of primary interactions." The fact that such a theory can account for a large volume of nucleon-nucleon scattering data is in itself not very surprising. After all, all the ingredients of a successful two-nucleon potential (e.g., three distinct ranges, $\mathbf{L}\cdot\mathbf{S}$ force, and a large repulsion at short distances) are found naturally in the theory. From the authors' point of view, the most heartening feature of the theory is the complete absence of any dynamical (or otherwise) parameters. The cutoff distance r_c is also largely a parameter introduced for calculational convenience and does not significantly affect our results for higher partial waves. The fact that the cancellation mechanism[13,17] for the $1/r^3$ singularity of the potential, which works nicely for $I=1$ meson exchanges, does not do so for $I=0$ mesons is unfortunate and forces us to do calculations with an arbitrary parameter. The theory in its basic concepts is otherwise free from parameters. The other so-called zero-parameter fits (e.g., Ref. 33) have quite a few hidden parameters which are chosen to fit other experimental data. In this sense, our work, which relies basically on the internal consistency of Sudarshan's universal theory, may be claimed to be better.

The few failures of the present work are largely due to experimental uncertainties (about the existence of $I=0$ axial-vector mesons), and could probably be explained better if one had more data on axial-vector mesons. We have repeatedly mentioned before that the axial-vector mesons are the main important new feature of this work. The axial-vector mesons provide the extreme short-range force[50] which traditionally was obtained in meson theory via the exchange of two or more pions and inclusion of inelastic channels in fourth-order diagrams. The fact that the axial-vector mesons effectively simulate these effects is, in our opinion, another major success of the theory. Including too many of them without worrying about their experimental existence has led us to a strongly attractive $\mathbf{L}\cdot\mathbf{S}$ force in the $^3P_2$ and $^3F_4$ partial waves. As mentioned before, this is not disastrous, since the absence of D and E mesons (perhaps even of one of them) would improve the situation in these partial waves without upsetting our good results elsewhere. The experimental situation about the existence of the particles should be much clarified before we can reach an unambiguous conclusion regarding their role in the nuclear-force problem. On the other hand, the necessity of the so-called S-wave $\pi\pi$ pair exchange, or, more simply, the scalar mesons, seems to be vitiated now for two reasons: first, the repeated failure of experimenters to successfully find a scalar meson, and, second, the fact that the axial-vector mesons simulate the same effect as shown by the present work.

Apart from the success of the present work, the fact that a large class of phenomena[13,16,17] can be correlated and understood by employing the single idea of the universality of all primary interactions encourages one to hope that such a theory might form an effective basis for studying hadron dynamics.

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[50] In a recent paper [G. Breit, K. A. Friedman, J. M. Holt, and R. E. Seamon, Phys. Rev. **170**, 1424 (1968)] it has been remarked that the existence of a *short*-range nuclear force in Ref. 17 is not very obvious. We submit that the mechanism of axial-vector-meson exchanges in the present work provides a short-range force of the right type to explain many of the features of the two-nucleon system.