Comments and Addenda

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Average Resonance Widths in Single-Channel Scattering*

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A proof is given for the relation $T=1-\exp[-2\pi\langle\Gamma\rangle/D]$ which connects the transmission coefficient with the average resonance width-to-spacing ratio.

THE relationship between the average width $\langle\Gamma\rangle$ on the one hand and the average collision matrix $\langle U\rangle$ and the transmission coefficient T on the other hand is given by[1,2]

$$\langle U\rangle=\exp(-2i\phi-\pi\langle\Gamma\rangle/D), \quad (1)$$

$$T=1-\exp(-2\pi\langle\Gamma\rangle/D), \quad (2)$$

where D is the mean spacing of resonances and ϕ is the potential scattering phase shift.

Recently, Ullah proposed a proof of the relation (1).[3] While this proof does not appear to be valid, the approach employed is helpful in that it does suggest a correct derivation of (1) and (2). We first discuss the difficulty in Ullah's derivation and then state a correct proof.

We start from the well-known expression for the average collision matrix at the energy E_0 as given in Ref. 3, Eq. (6):

$$\langle U\rangle=\exp(-2i\phi)\prod_\mu\frac{E_0-\mathcal{E}_\mu+\frac{1}{2}i(I-\Gamma_\mu)}{E_0-\mathcal{E}_\mu+\frac{1}{2}i(I+\Gamma_\mu)}, \quad (3)$$

where the $\mathcal{E}_\mu-\frac{1}{2}i\Gamma_\mu$ are the resonance pole positions in the energy plane and I is the half-width of the averaging interval centered at E_0. In Ref. 3, the factors in the numerator and denominator are multiplied out and arranged in powers of I^{-1}. This leads to a ratio of two series which appears at the bottom of page 1511 in Ref. 3. The derivation of the result (1) depends now on the assertion that in the limit of large I all terms in these series go to zero except those which have the form $(\sum\Gamma_\mu/I)^n/n!$. However, this is clearly not the case. The first term that should vanish is

$$(1/2!)(2i/I)^2\sum_\mu(\mathcal{E}_\mu-E_0+\tfrac{1}{2}i\Gamma_\mu)^2, \quad (4)$$

which contains contributions of the order of unity from, e.g.,

$$(1/2!)(2i/I)^2(\mathcal{E}_\mu-E_0)^2\approx-1, \quad (5)$$

when μ is the index of the "last" resonance with $(\mathcal{E}_\mu-E_0)\approx I$. The approximate value of the expression (4) is found to be one sixth of the total number of resonances.

To obtain a correct proof, we divide the numerator and denominator in Eq. (3) not by factors of I but by $\prod_\mu F_\mu$, where

$$F_\mu=E_0-\mathcal{E}_\mu+\tfrac{1}{2}iI. \quad (6)$$

Then, upon multiplying out the products, we obtain

$$\langle U\rangle=\exp(-2i\phi)\left[1-\sum_\mu\frac{\frac{1}{2}i\Gamma_\mu}{F_\mu}+\sum_{\mu<\nu}\frac{(\frac{1}{2}i\Gamma_\mu)(\frac{1}{2}i\Gamma_\nu)}{(F_\mu F_\nu)}+\cdots\right]\bigg/\left[1+\sum_\mu\frac{\frac{1}{2}i\Gamma_\mu}{F_\mu}+\sum_{\mu<\nu}\frac{(\frac{1}{2}i\Gamma_\mu)(\frac{1}{2}i\Gamma_\nu)}{(F_\mu F_\nu)}+\cdots\right]. \quad (7)$$

If I is sufficiently large compared to $\langle\Gamma\rangle$ and D, the sums in Eq. (6) can now be evaluated by standard means to yield[4]

$$\sum_\mu\tfrac{1}{2}i\Gamma_\mu/F_\mu=\tfrac{1}{2}\pi\langle\Gamma\rangle D, \quad (8)$$

$$\sum_{\mu<\nu}[(\tfrac{1}{2}i\Gamma_\mu)(\tfrac{1}{2}i\Gamma_\nu)/F_\mu F_\nu]=\tfrac{1}{8}\pi^2\langle\Gamma\rangle^2/D^2, \quad (9)$$

etc., which, upon summation, gives the result (1) from which follows, by definition, also (2). One can also write the series in Eq. (7) in terms of single sums and their products as was done by Ullah.[3] In this case, it is easy to see that the "extra" terms analogous to the expression (4) now do indeed go to zero as $(\langle\Gamma\rangle/I)^2$.

The effective averaging interval used in Ref. 3

contains *all* the resonance poles of the collision function U. For such an average, the results (1) and (2) are not generally correct. These results *are* valid for an averaging interval that contains only a small fraction of the resonance poles of U. Such an interval is implicit in the derivations of Eqs. (8) and (9),[4] and is the kind of interval that is of physical interest.

The result (2) in no way contradicts the different formula obtained elsewhere for the relation between the average pole *residue* and the transmission coefficient.[5] The reason for the difference between the average residue and the average widths has been discussed in Sec. E of Ref. 1 and Sec. IV of Ref. 2. A third and still different formula relates the transmission coefficient to the R-matrix strength function.[6]

A simple physical argument for Eq. (2) recently also has been given by Gibbs.[7] This is based on a calculation of the decay rate of the "average" compound nucleus, taking into account its depletion by previous decays. One virtue of this argument is that it can easily be extended to show that Eq. (2) is also expected to hold in the many-channel case, if T is taken to be the channel transmission coefficient for any one of the open channels and $\langle\Gamma\rangle$ is the average partial width for the same channel. The same conclusion had also been deduced in Refs. 1 and 2.

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