Particle Transfer Reactions on Deformed Nuclei*

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In particle transfer reactions, the effects of strong coupling in the initial and final nuclei can be important, but an exact evaluation is exceedingly complicated numerically. The approximate procedure of Iano and Austern is modified in this work so that the only change from the standard distorted-wave Born-approximation (DWBA) expression is a modification of the optical potentials for the bombarding and exiting particles. The procedure is compared with the Iano-Austern theory and equivalent results are obtained for a ^{24}Mg$(d, p)^{25}$Mg test case. Comparison with experiment is made for the ^{24}Mg$(p, d)^{23}$Mg reaction at 27.3 MeV which was measured along with the necessary elastic proton data. A significant improvement in the angular distributions is obtained using the modified DWBA. The method is applied to the ^{182}W$(d, p)^{183}$W work of Siemssen and Erskine and increases the predicted cross sections as required by the data.

I. INTRODUCTION

PARTICLE transfer reactions such as deuteron stripping are usually analyzed by a single-step distorted-wave Born-approximation (DWBA) calculation. Such reactions have been studied in many regions of the periodic table and have yielded valuable nuclear-structure information. However, in nuclei where there are strongly coupled excited states, such as in the region of deformed nuclei, two-step processes via excited states can give important contributions to the stripping reaction amplitude. There are two ways that these effects can affect the result: The selection rules for the one-step process can be violated and the two-step amplitude can interfere with the one-step amplitude. Penney and Satchler[1] have pointed out that these two-step processes can be taken into account by replacing the elastic scattering distorted-wave solutions in the stripping amplitude by the corresponding solutions to a set of coupled equations for the ground and excited states in the incoming and outgoing channels. This approach, while straightforward and correct, requires a vast computational effort.

In order to reduce the amount of work involved in obtaining the second-order corrections to the stripping amplitude, Iano and Austern[2] have employed a number of approximations to express the corrections caused by the coupling of inelastic channels in the initial and final states as derivatives with respect to the optical model radius of the direct stripping amplitude. We have extended the simplifying assumptions of Iano and Austern[2] so that the optical-model potentials used in the initial and final states are modified. The application of these further approximations to cases where the inelastic states are described by the collective model is to change the optical-model potentials by addition of derivatives of the optical-model potentials. These changes are equivalently expressed as changes in the radii of the potentials.

In Sec. II the theory of nuclear transfer reactions for deformed nuclei is presented and the various approximations necessary to obtain a DWBA-like form are presented. The calculations are compared with the more accurate Iano-Austern[2] result and good agreement is found. The major comparison of theory and experiment is given in Sec. III for the ^{24}Mg$(p, d)^{23}$Mg reaction at 27.3 MeV. The angular distributions for pickup reactions in this region are sensitive to the details of the analysis and thus it seemed appropriate to obtain elastic proton data along with the (p, d) measurements and to pick an energy where deuteron elastic data were available.[3] The confrontation of the modified DWBA theory with the data shows a marked improvement over the usual DWBA in the angular distributions although a discrepancy in absolute strengths remains. Section IV compares the theory with the 12-MeV ^{182}W$(d, p)^{183}$W data of Siemssen and Erskine[4,5] using "best-fit" optical parameters. The predicted cross sections are increased as required by the data.

II. THEORY

A. Formal Development

The usual stripping amplitude using the zero-range approximation is given by

$$T_{fi}^{(0)} = D_0 \langle \chi_f^{(-)}(\mathbf{k}_f, \mathbf{r}_p) \phi_f^{I_f}(\mathbf{r}_n, \xi) \mid \delta(\mathbf{r}_p - \mathbf{r}_n) \mid \times \chi_i^{(+)}(\mathbf{k}_i, \mathbf{r}_d) \phi_i^{I_i}(\xi) \rangle. \quad (1)$$

The $\chi_i^{(+)}(\mathbf{k}_i, \mathbf{r}_d)$ and $\chi_f^{(-)}(\mathbf{k}_f, \mathbf{r}_p)$ functions are the distorted waves for the incident deuteron and exiting proton with outgoing and incoming scattered wave boundary conditions, respectively; $\phi_i^{I_i}(\xi)$ and $\phi_f^{I_f}(\mathbf{r}_n, \xi)$ are the initial nucleus and final nucleus wave functions, and D_0 is the zero-range normalization constant. The replacement of the distorted waves with the solution to the set of coupled equations which describe the strongly

* Work supported in part by the U.S. Atomic Energy Commission.
[1] S. K. Penney and G. R. Satchler, Nucl. Phys. **53**, 145 (1964).
[2] P. J. Iano and N. Austern, Phys. Rev. **151**, 853 (1966).
[3] G. Mairle and U. Schmidt-Rohr, Max-Planck-Institut, Heidelberg Report No. 19651 V113, (unpublished).
[4] R. H. Siemssen and J. R. Erskine, Phys. Rev. Letters **19**, 90 (1967).
[5] R. H. Siemssen and J. R. Erskine, Phys. Rev. **146**, 911 (1966).

excited levels in the initial and final nuclei gives us the expression for the scattering amplitude,

$$T_{fi}=D_0\langle\Psi_f^{(-)}(\mathbf{r}_p,\mathbf{r}_n,\xi)\mid\delta(\mathbf{r}_p-\mathbf{r}_n)\mid\Psi_i^{(+)}(\mathbf{r}_d,\xi)\rangle, \quad (2)$$

which takes into account the higher-order processes. The wave functions Ψ in Eq. (2) are solutions to the set of coupled equations with incoming waves in the entrance channel and incoming $(-)$ or outgoing $(+)$ scattered waves in all the channels represented by the designated coordinates at infinity. One writes the initial and final Hamiltonians as

$$H_i=H_N(\xi)+K_d+V_d(r_d)+\Delta V_d(\mathbf{r}_d,\xi),$$
$$H_f=H_N(\mathbf{r}_n,\xi)+K_p+V_p(r_p)+\Delta V_p(\mathbf{r}_p,\mathbf{r}_n,\xi), \quad (3)$$

where $H_N(\xi)$ and $H_N(\mathbf{r}_n,\xi)$ are the Hamiltonians of the target and residual nuclei, K_d and K_p are the kinetic-energy operators, V_d and V_p are the optical-model potentials, and ΔV_d and ΔV_p are the coupling between the free particles and their respective nuclei. The solutions of the coupled equations can be written formally as

$$\Psi_i^{(+)}(\mathbf{r}_d,\xi)=[1+(E^+-H_i)^{-1}\Delta V_d(\mathbf{r}_d,\xi)]\chi_i^{(+)}(\mathbf{r}_d),\phi_i(\xi),$$
$$\Psi_f^{(-)}(\mathbf{r}_p,\mathbf{r}_n,\xi)=[1+(E^--H_f)^{-1}\Delta V_p(\mathbf{r}_p,\mathbf{r}_n,\xi)]\times\chi_f^{(-)}(\mathbf{r}_p)\phi_f(\mathbf{r}_n,\xi), \quad (4)$$

where the functions $\chi_i^{(+)}(\mathbf{r}_d)\phi_i(\xi)$ and $\chi_f^{(-)}(\mathbf{r}_p)\phi_f(\mathbf{r}_n,\xi)$, satisfy

$$[H_N(\xi)+K_d+V_d(r_d)-E]\chi_i^{(+)}(\mathbf{r}_d)\phi_i(\xi)=0,$$
$$[H_N(\mathbf{r}_n,\xi)+K_p+V_p(r_p)-E]\chi_f^{(-)}(\mathbf{r}_p)\phi_f(\mathbf{r}_n,\xi)=0. \quad (5)$$

The substitution of the coupled-channel solutions $\Psi_i^{(+)}$ and $\Psi_f^{(-)}$ into the expression for the stripping amplitude gives

$$T_{fi}=D_0\langle\chi_f^{(-)}(\mathbf{r}_p)\phi_f(\mathbf{r}_n,\xi)\mid[1+\Delta V_p(E^+-H_f)^{-1}]\times\delta(\mathbf{r}_p-\mathbf{r}_n)[1+(E^+-H_i)^{-1}\Delta V_d]\mid\chi_i^{(+)}(\mathbf{r}_d)\phi_i(\xi)\rangle. \quad (6)$$

This expression is equivalent to that given by Penney and Satchler.[1] The exact evaluation of this expression requires the solutions to the coupled-channel problem plus extensive numerical work to perform the necessary overlap integrals.[6] The computational labor is an order of magnitude more involved than the usual DWBA; in addition, computer programs are not generally available.

In a recent paper Iano and Austern[2] have approximated the expression for the stripping amplitudes to a form which is more easily evaluated but which still requires more computational effort than the usual DWBA, and is not amenable to the existing computer codes. We reproduce their derivation here since our results are an extension of their formulation for the extra contributions resulting from the coupling of excited states into the entrance and exit channels.

Iano and Austern[2] assumed that the coupling interaction $\Delta V(\mathbf{r},\xi)$ can be written in the form

$$\Delta V(\mathbf{r},\xi)=V_\lambda(r)\,Y_\lambda(\mathbf{r}\cdot\xi), \quad (7)$$

so that only a single term in the Legendre polynomial expansion of the interaction which couples in the excited states is important in the inelastic processes. Two other basic approximations are (a) the adiabatic approximation, which considers the ground and excited states in each channel to be degenerate, and (b) the partial-wave expansions of Green's functions $(E^\pm-H)^{-1}$ which are a slowly varying function of the partial waves l, so that the spherical harmonic factor of the interaction $\Delta V(\mathbf{r},\xi)$ commutes with the Green's function. In terms of operator notation, the latter approximation is

$$(E^\pm-H)^{-1}Y_\lambda(\mathbf{r}\cdot\xi)\approx Y_\lambda(\mathbf{r}\cdot\xi)\,(E^\pm-H)^{-1}. \quad (8)$$

The validity of this commutation has been discussed by Austern and Blair[7] for the case of strongly absorbed particles. With these approximations the expressions for the coupled-channel wave functions are

$$[1+(E^+-H_i)^{-1}\Delta V_{\lambda d}(\mathbf{r}_d,\xi)]\chi_i^{(+)}(\mathbf{r}_d)\phi_i(\xi)\approx[\phi_i(\xi)+Y_\lambda(\mathbf{r}_d\cdot\xi)\phi_i(\xi)\,(E^+-H_i)^{-1}V_{\lambda d}(r_d)]\chi_i^{(+)}(\mathbf{r}_d), \quad (9)$$

$$[1+(E^--H_f)^{-1}\Delta V_{\lambda p}(\mathbf{r}_p,\mathbf{r}_n,\xi)]\chi_f^{(-)}(\mathbf{r}_p)\phi_f(\mathbf{r}_n,\xi)\approx[\phi_f(\xi)+Y_\lambda(\mathbf{r}_p\cdot\xi)\phi_f(\mathbf{r}_n,\xi)(E^--H_f)^{-1}V_{\lambda p}(r_p)]\chi_f^{(-)}(\mathbf{r}_p).$$

Substitution of these results into the expression for the stripping amplitude gives

$$T_{fi}\approx T_{fi}^{(1)}=D_0\langle\chi_f^{(-)}(\mathbf{r})\times\mid[F(\mathbf{r})+V_{\lambda p}(r)\,(E^+-H_f)^{-1}F(\mathbf{r})\,\kappa_p+\kappa_d F(\mathbf{r})\,(E^+-H_i)^{-1}V_{\lambda d}(r)+V_{\lambda p}(r)\,(E^+-H_f)^{-1}\times F(\mathbf{r})\,\kappa_{pd}(E^+-H_i)^{-1}V_{\lambda d}(r)]\mid\chi_i^{(+)}(\mathbf{r})\rangle, \quad (10)$$

where

$$F(\mathbf{r})\equiv\int d\xi\phi_f^*(\mathbf{r},\xi)\phi_i(\xi)$$

and

$$\kappa_d=\kappa_p=\int d\xi\phi_f^*(\mathbf{r},\xi)\,Y_\lambda(\mathbf{r}\cdot\xi)\phi_i(\xi)/F(\mathbf{r}),$$

$$\kappa_{pd}=\int d\xi\phi_f^*(\mathbf{r},\xi)[Y_\lambda(\mathbf{r}\cdot\xi)]^2\phi_i(\xi)/F(\mathbf{r}). \quad (11)$$

The correction factors $\kappa=\kappa_d=\kappa_p$ are in general functions

[6] T. Tamura and C. E. Watson, Phys. Letters **25B**, 186 (1967); Y. Dupont and M. Chabre, *ibid.* **26B**, 362 (1968).

[7] N. Austern and J. S. Blair, Ann. Phys. (N.Y.) **33**, 15 (1965).

of $\mathbf{r}$. The numerators of the correction factors contain angular momentum values obtained by coupling $\mathbf{J}_i+\mathbf{J}_f$, the initial plus final nuclear spins, to λ, the angular momentum transferred by the inelastic scattering in the initial and final channels. However, in order to be able to keep within the framework of the standard DWBA we shall restrict the angular momentum in the numerator of κ to be the same as the zeroth-order term $F(\mathbf{r})$. These angular momenta from the correction terms will add coherently with the zeroth-order term when the amplitude is squared and summed over the initial and final magnetic substates. The neglected terms will add to the cross section incoherently and will change the cross section in second order in the coupling. These neglected terms will give rise to the "l-forbidden" transitions and thus are not a correction to a larger zeroth-order term. This restriction then allows one to express the κ's as functions of the radius r only and are roughly constant for the large radii that dominate the stripping amplitude and will be discussed further in the framework of the collective model. If we neglect the coupling terms ΔV in H_i and H_f and make use of the collective model form of $V_\lambda(r)$

$$V_\lambda(r)=\beta_\lambda R(dV(r,R)/dR), \tag{12}$$

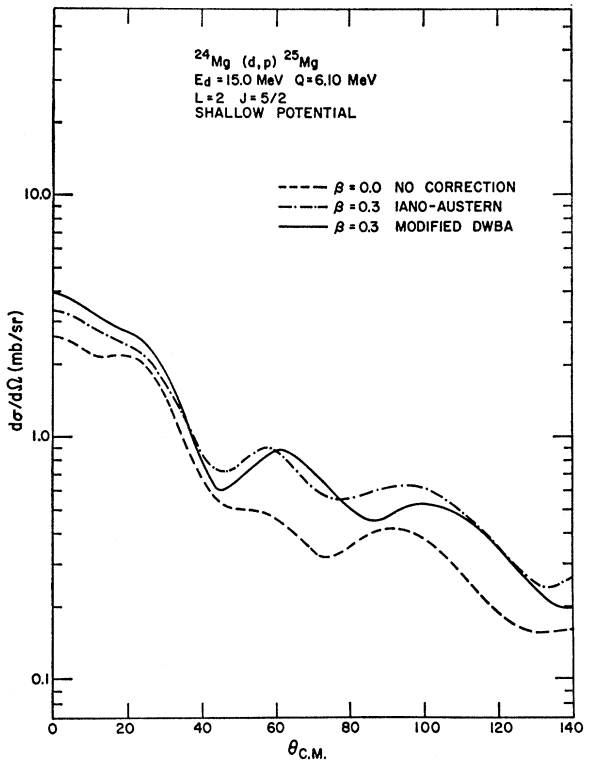

FIG. 1. Single-particle cross section for the transition to the $\frac{5}{2}^+$ ground state of ^{25}Mg for 15-MeV deuterons comparing the usual uncorrected DWBA result and the approximations of Iano and Austern with the modified DWBA calculation. The optical-model parameters for the distorted waves are those used by Iano and Austern for their shallow-well calculations.

where β_λ is the deformation parameter and R is the radius parameter in the optical model, then we can make use of the identity

$$d\psi^\pm/dR=[E^\pm-H_0-V(R)]^{-1}(dV/dR)\psi^\pm, \tag{13}$$

where the function $\psi^\pm$ is a solution with incoming $(-)$ or outgoing $(+)$ waves of the Hamiltonian H_0+V, to express the second and third terms in the brackets of Eq. (10) as

$$\beta_{\lambda p}\kappa_p R_p(d/dR_p)T_{fi}^{(0)}+\beta_{\lambda d}\kappa_d R_d(d/dR_d)T_{fi}^{(0)}. \tag{14}$$

The first term in Eq. (10) is the usual DWBA form, i.e., Eq. (1). Ignoring the higher-order fourth term in Eq. (10), the corrections to the stripping amplitude are given by Eq. (14), which can be expressed in terms of derivatives of the stripping amplitude with respect to the optical-model radius parameter. This result is equivalent to that given by Iano and Austern[2] for l-allowed transitions.

We extend these results by introducing one further approximation in order to express the result in terms of changes in the optical-model potential which generate the distorted waves.

By adding and subtracting a term

$$V_{\lambda p}(r)(E^+-H_f)^{-1}\kappa_p F(\mathbf{r})\kappa_d(E^+-H_i)^{-1}V_{\lambda d}(r)$$

to the expression for the approximate amplitude $T_{fi}^{(1)}$, we can write, with a bit of rearrangement,

$$\begin{aligned}T_{fi}^{(1)}=D_0\langle[1+\kappa_p(E^--H_f)^{-1}V_{\lambda p}(r)\chi_f^{(-)}(\mathbf{r})\mid F(\mathbf{r})\mid\\ \times[1+\kappa_d(E^+-H_i)^{-1}V_{\lambda d}(r)\chi_i^{(+)}(\mathbf{r})\rangle\\ +D_0\langle\chi_f^{(-)}(\mathbf{r})\mid V_{\lambda p}(r)(E^+-H_f)^{-1}\\ \times(\kappa_{pd}-\kappa_p\kappa_d)F(\mathbf{r})(E^--H_i)^{-1}V_{\lambda d}(r)\mid\chi_i^{(+)}(\mathbf{r})\rangle.\end{aligned} \tag{15}$$

If one wants the first-order contributions to T_{fi} due to the inelastic excitations in each channel, the second term of $T_{fi}^{(1)}$ can be ignored.

The expression in Eq. (15) is simplified by combining the coupling terms in H_f and H_i and approximating their effect by

$$\begin{aligned}H=E_0(\xi)+T+V(r)+\Delta V(\mathbf{r},\xi)\approx\bar{H},\\ \bar{H}\equiv E_0(\xi)+T+V(r)+\kappa V_\lambda(\mathbf{r},\xi).\end{aligned} \tag{16}$$

This yields

$$\begin{aligned}T_{fi}^{(1)}\approx D_0\langle[1+\kappa_p(E^--\bar{H}_f)^{-1}V_{\lambda p}(r)\chi_f^{(-)}(\mathbf{r})\mid F(\mathbf{r})\mid\\ \times[1+\kappa_d(E^+-\bar{H}_i)^{-1}V_{\lambda d}(r)\chi_i^{(+)}(\mathbf{r})\rangle+O(V_\lambda^2).\end{aligned} \tag{17}$$

This replacement again introduces corrections to the approximate stripping amplitude of second order in the interaction V_λ. We can rewrite the above expression by noticing that the factors

$$[1+\kappa_p(E^--\bar{H}_f)^{-1}V_{\lambda p}(r)]\chi_f^{(-)}(\mathbf{r}),$$
$$[1+\kappa_d(E^--\bar{H}_i)^{-1}V_{\lambda d}(r)]\chi_i^{(+)}(\mathbf{r})$$

are solutions to single-particle Hamiltonians with potentials $V(r)+\kappa V_\lambda(r)$. Thus the expression in Eq. (17) is of the usual DWBA form [compare with Eq. (1)] and standard DWBA codes can be used.

The effect of the further approximations to the Iano-Austern treatment is illustrated in Fig. 1, where ^{24}Mg$(d, p)^{25}$Mg stripping to the $\frac{5}{2}^+$ band head level of ^{25}Mg is calculated. This case is particularly convenient since only the $d_{5/2}$ orbital appears so that κ in Eq. (11) is constant (the value is -0.255). The comparison of the two corrected curves is good, which lends support to our additional assumption in Eq. (16).

To summarize, we have replaced the effects of the coupling of the excited states in the initial and final channels by modifying the spherical potential $V(r)$ used in the coupled-channel calculation by an additional term $\kappa V_\lambda(r)$. The procedure has made use of the adiabatic approximation, a smoothness approximation, and a limitation to a first-order term in the coupling.

B. Application to Collective Model

For deformed nuclei, the optical potentials are written[8] as a function of a radius $R(\theta)=R_0[1+\beta_\lambda Y_\lambda(\theta)]$, so one identifies the spherical part as $V(x)$ and the first-order coupling term as $V_\lambda(r)=-\beta_\lambda(R_0/a)\,dV/dx$, with $x\equiv(r-R_0)/a$. The modified potential is thus of the form

$$\Delta V=-\kappa\beta_\lambda(R_0/a)\,dV/dx, \tag{18}$$

where $\kappa=\kappa_p=\kappa_d$ is defined in Eq. (11). A further simplification can be derived from the preceding expression by noting that the modified potential is just the first two terms of the expansion of $V\{[r-R_0(1+\kappa\beta_\lambda)]/a\}$, assuming constant κ, in a power series in $\kappa\beta_\lambda$. This result means that to the order of our approximations, the effects of the coupling processes in the stripping reactions can be represented by a change in the radius of the optical-model potentials which describe the distorted waves used in the stripping amplitude.

We wish to evaluate the coefficient κ for the case when the initial and final nuclei are described by the deformed-nucleus model.[9,10] In this model the strong-coupling function for an axially symmetric nucleus $\phi(\xi)$ of spin I and space projection M is given by

$$\phi(\xi)=[(2I+1)/16\pi^2]^{1/2}\phi(r)\times[\psi_\Omega D_{M\Omega}{}^I(\theta_i)+(-1)^{I-j}\psi_{-\Omega}D_{M,-\Omega}{}^I(\theta_i)], \tag{19}$$

where Ω is the projection of I on the body axis. The wave functions ψ_Ω are taken to be antisymmetrized products of single-particle orbitals $\chi_{\Omega_i}(\mathbf{r}_i)$, which are described by the spherical representation

$$\chi_{\Omega_i}(\mathbf{r}_i)=\sum_{lj}c_{lj\Omega_i}u_{lj}(r_i)\mid lj\Omega_i\rangle,$$

where the radial wave functions $u_{lj}(r_i)$ are normalized to unity. The value of κ when the target nucleus is even-even and the coupling term is due to the quadrupole deformation of the nuclei is given by rewriting Eq. (11);

$$\kappa_{lj}=\sqrt{2}[c_{lj\Omega_i}u_{lj}(r)]^{-1}\sum_{l'j'}c_{l'j'\Omega_i}u_{l'j'}(r)\times\langle lj\Omega_i\mid Y_{20}\mid l'j'\Omega_i\rangle. \tag{20}$$

From this expression one can see that κ_{lj} is a function of r since the orbitals for different l, j are not the same. The addition to the optical potential to modify the DWBA for the two-step processes $\kappa_{lj}V_2(r)$ would have a different radial dependence from $V_2(r)$. For the collective model form for $V_2(r)$ one could not make the change in radius approximation. However, by taking an average value of

$$\langle c_{l'j'\Omega}u_{l'j'}(r)/c_{lj\Omega}u_{lj}(r)\rangle\equiv\alpha_{lj l'j',\Omega} \tag{21}$$

one can define a constant κ_{lj} by

$$\kappa_{lj}=\sqrt{2}\sum_{l'j'}\alpha_{lj l'j',\Omega}\langle lj\Omega\mid Y_{20}\mid l'j'\Omega\rangle \tag{22}$$

and keep the form of the modified DWBA as described. The "average" value in Eq. (21) may be chosen at the radius where the maximum DWBA contribution to the stripping occurs and the result is not sensitive to the exact location as long as it is in the external region. A simpler method is to just take the ratio of Nilsson coefficients,

$$\alpha_{lj l'j',\Omega}\approx c_{l'j'\Omega}/c_{lj\Omega}, \tag{23}$$

thus ignoring differences in the normalized orbitals $u_{lj}(r)$. This approximation was compared with several prescriptions for (21) and was found to be adequate. Because of its lack of arbitrariness, it is used in the following results.

III. ^{24}Mg$(p, d)^{23}$Mg REACTION

A convenient testing ground for the modified DWBA procedure is in the *sd* shell since the deformations are large but the number of levels is small. In addition, the angular distributions are rather sensitive to the choice of optical potentials,[11] especially for pickup reactions, where there is a large angular momentum mismatch.[12] In order to test the modified DWBA theory, (p, d) and (p, p) experiments were performed at the University of Colorado cyclotron at 27.3 MeV; for the deuteron optical parameters, the 11.8 MeV (d, d) experiments of the Heidelberg group[3] were used.

[8] D. M. Chase, L. Wilets, and A. R. Edmonds, Phys. Rev. **110**, 1080 (1958).
[9] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **27**, No. 16 (1953).
[10] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **29**, No. 16 (1955).
[11] H. H. Duhm, D. L. Hendrie, and B. G. Harvey, Lawrence Radiation Laboratory Report No. UCRL-17883 (unpublished).
[12] N. Austern, in *Fast Neutron Physics*, edited by J. B. Marion and J. L. Fowler (Interscience Publishers, Inc., New York, 1963) Vol. II.

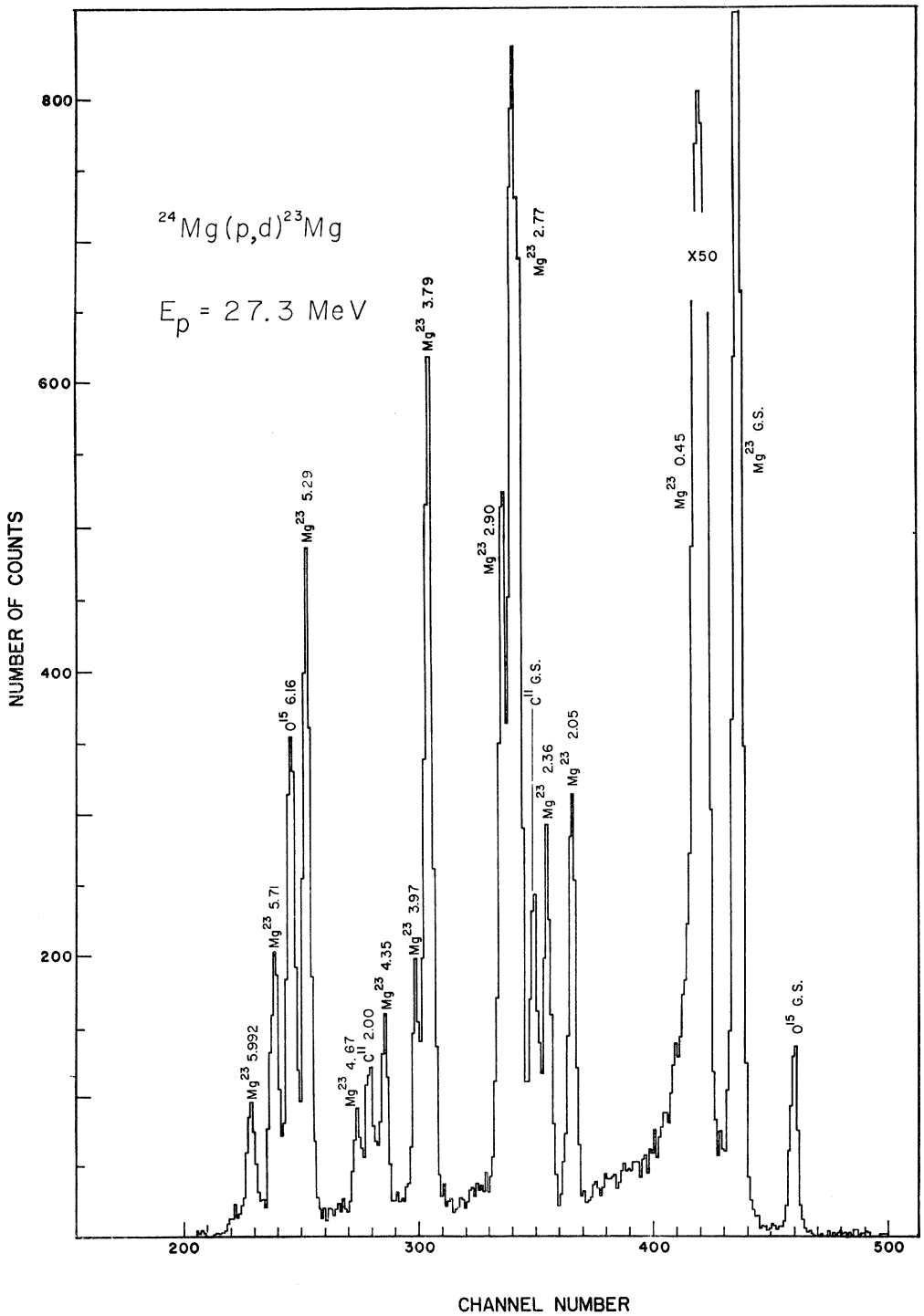

FIG. 2. The spectrum of deuterons observed at 30° from the $^{24}Mg(p, d)^{23}Mg$ reaction.

A. Experimental Method

A (27.3±0.1)-MeV proton beam from the University of Colorado cyclotron[13] was focused through a beam-transport system to a 0.2-cm spot on a 0.7 mg/cm² isotopically pure ^{24}Mg target. The integrated beam current from a Faraday cup was stored in the first channel of a ND160 pulse-height analyzer which also stored the deuteron spectra, thus eliminating analyzer dead-time corrections.[14]

[13] D. A. Lind, J. J. Kraushaar, R. Smythe, and M. E. Rickey, Nucl. Instr. **18**, 62 (1962).

[14] P. W. Allison, Rev. Sci. Instr. **35**, 1728 (1964).

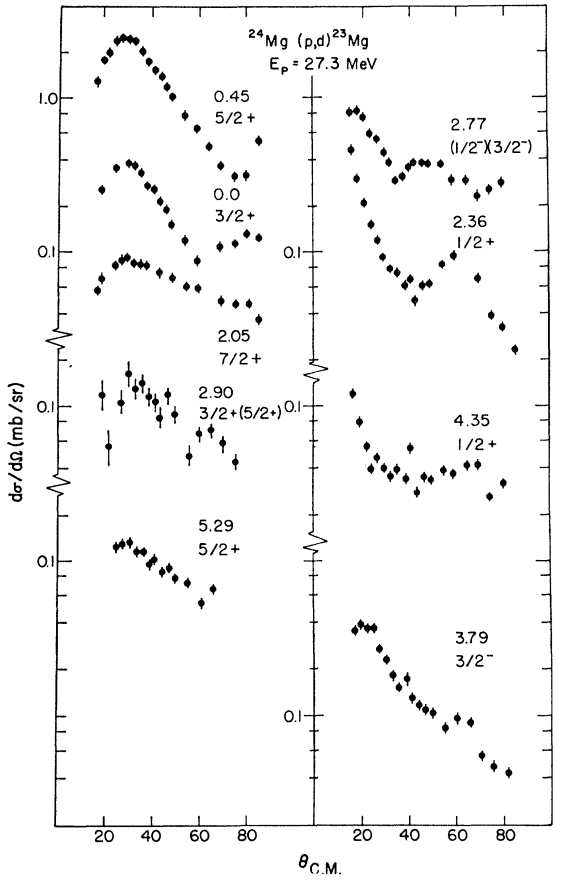

FIG. 3. Differential cross sections for the ^{24}Mg(p, d)^{23}Mg reaction at 27.3 MeV. Only resolved deuteron groups are shown (see text). The spin assignments are taken from Ref. 17.

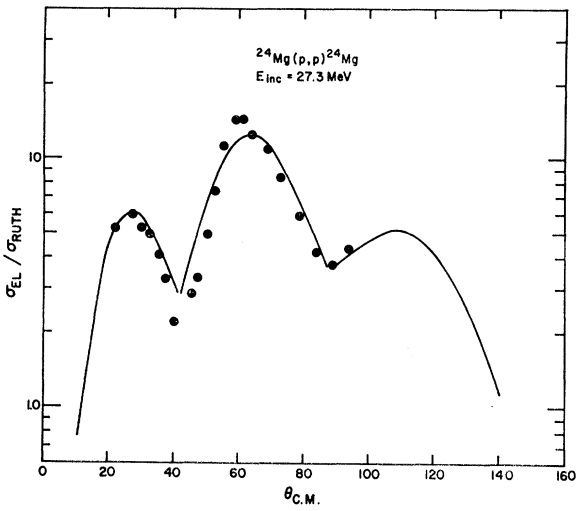

FIG. 4. Differential cross sections for the elastic scattering of 27.3-MeV protons from ^{24}Mg. The solid curve is the optical-model fit to the data using the parameters given in Table I.

A semiconductor detector telescope cooled to dry-ice temperature was mounted in a remote-controlled arm of the scattering chamber. Particle identification was accomplished using the detector telescope ΔE signal from a 0.07-mm ORTEC surface-barrier counter, the $E-\Delta E$ signal from a locally manufactured 2.4-mm Si(Li) counter and a pulse multiplier circuit.[15] The ΔE and $E-\Delta E$ signals were fed to the differential inputs of a TC200 amplifier which produced a summed energy pulse suitably shaped for the pulse-height analyzer.

The detector telescope solid angle of 0.264 msr and beam currents of 0.02 and 0.5 μa on target required data runs of from 30 to 50 min. The over-all resolution of the system was 100 keV full width at half-maximum. A typical deuteron spectrum is shown in Fig. 2.

B. Experimental Results

Data were taken at $2\frac{1}{2}°$ intervals from 15° to 50° and in 5° intervals from 50° to 80°. The angular distributions are shown in Fig. 3. Levels were also observed at 3.97, 4.67, 5.75, and 5.99 MeV with insufficient counting statistics to merit extraction of an angular distribution. Carbon contaminant contributions to the spectra were subtracted by normalizing the carbon peak yield (when separated) to relative angular distributions of the ^{12}C(p, d) reaction at 30 MeV.[16] This resulted in an additional ±2% error in the 2.78-MeV level at 40° to 47.5° due to subtraction uncertainties. The 2.00-MeV ^{11}C level obscured only the 4.67 MeV ^{23}Mg level whose angular distribution, as previously mentioned, was not analyzed. The 6.16-MeV ^{15}O level obscured the 5.75-MeV level whose angular distribution was also not analyzed. Apart from the above contaminant considerations, the errors shown in Fig. 3 are statistical. Errors due to beam integration are less than 1%. The errors due to target nonuniformity and solid angle measurements (~5%) result in an uncertainty of 6% in the absolute cross sections. The spins of the levels as shown in Fig. 3 are those determined by Dubois and Earwaker[17] in a particle-γ correlation experiment. Additional levels in ^{23}Mg from a (p, d) experiment[18] at 34 MeV were seen

TABLE I. Optical potential parameters for the ^{24}Mg(p, d)^{23}Mg reaction at 27.3 MeV.

	V (MeV)	r_0 (F)	a (F)	W_0 (MeV)	r_I (F)	a_I (F)
Deuteron	87.48	1.055	0.85	29.2	1.369	0.738
Proton	43.53	1.16	0.75	3.70	1.370	0.48

[15] G. L. Miller and V. Radeka, Brookhaven National Laboratory Report No. BNL-6952 (unpublished).
[16] N. S. Chant, P. S. Fisher, and D. K. Scott, 1964 PLA Progress Report, Rutherford High Energy Laboratory, p. 55 (unpublished).
[17] J. Dubois and L. G. Earwaker, Phys. Rev. **160**, 925 (1967).
[18] R. L. Kozub and E. Kashy, Michigan State University, Cyclotron Laboratory Annual Report, June 1968 (unpublished).

at 3.86 and 5.32 MeV but were not resolved in our data.

The measurement of the elastic proton scattering and the optical-model analysis of the data is straightforward and is adequately described elsewhere.[19] The data are plotted in Fig. 4 along with the six-parameter optical-model fit, the parameters being given in Table I. The table also gives our best-fit parameters to the data of Ref. 3 for 11.8-MeV deuteron elastic scattering. It is seen that the proton and deuteron geometries are nearly equal and that $V_d \approx 2V_p$, as desired.

C. Modified DWBA Analysis

In the analysis of the 27.3-MeV ^{24}Mg$(p, d)^{23}$Mg data, optical potential parameters were taken to be those that fit the elastic scattering. In principle, the parameters should be obtained via a coupled-channels analysis which differs from the optical-model result, especially in the imaginary part.[20] However, for a qualitative demonstration of the effects of the strong coupling on nucleon transfer reactions, the simple optical potential should suffice as a zero-order starting point.

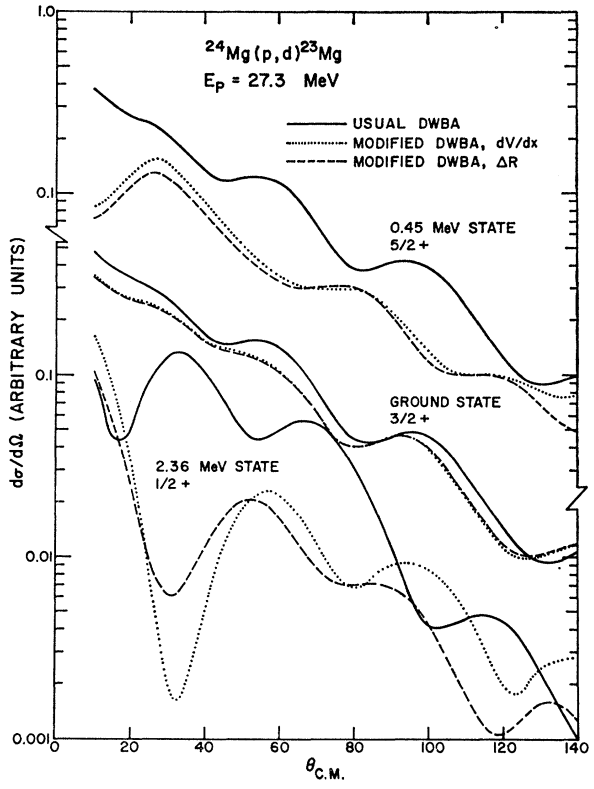

FIG. 5. Calculated DWBA angular distributions for ^{24}Mg(p, d)-^{23}Mg at E_p=27.3 MeV. The solid curve uses the conventional DWBA; the dotted curve gives the results of the modified theory with a dV/dx addition to the optical potentials; the dashed curve is obtained by an equivalent change in potential radii. The calculations use the DWUCK computer program with parameters given in Table I and the text.

TABLE II. Expansion coefficients for a neutron in a deformed ^{23}Mg well.

Band	Level	c_{lj}	κ
$[211]\frac{3}{2}^+$	$d_{3/2}$	+0.247	+0.420
	$d_{5/2}$	−0.967	+0.091
$[200]\frac{1}{2}^+$	$s_{1/2}$	−0.482	+0.675
	$d_{3/2}$	+0.246	+0.884
	$d_{5/2}$	−0.839	+0.399

The states in ^{23}Mg to be considered for analysis are the ground state $(\frac{3}{2}+)$, 0.45-MeV state $(\frac{5}{2}+)$, and 2.36-MeV state $(\frac{1}{2}+)$. The former two are believed[21] to be the first two members of the $[211]\frac{3}{2}^+$ band while the $\frac{1}{2}^+$ state is the band head of the $[200]\frac{1}{2}^+$ band. Assuming a deformation $\beta=0.4$ consistent with inelastic scattering measurements,[22] it is quite straightforward to calculate the wave function of those levels in the collective model. They were obtained using a Woods-Saxon well with parameters $r_0=1.25$ F, $a=0.65$ F, $\lambda=25$ (spin-orbit factor) and the depth V adjusted to fit the known separation energy (yielding $V\approx 70$ MeV). The expansion (or Nilsson) coefficients c_{lj} are presented in Table II for the three cases as is the correction factor κ computed from Eq. (11). In Fig. 5, the results of the two recipes for effective optical potentials in the DWBA are compared with the usual DWBA. For the most sensitive case, the $l=0$ transition to the $\frac{1}{2}^+$ state, a noticeable difference is seen between the cases; fortunately, this difference is small compared with the spherical DWBA and with the uncertainties inherent in the calculations. In the following situations the radius-change prescription will be employed since it seems to be more reasonable for large κ values.

In Fig. 6, the results of the modified DWBA are compared with the experimental data. It is seen that the bad discrepancy for the $l=0$ angular distribution is removed by including the coupling and the $d_{3/2}$ fit is also improved. The correction for the $d_{5/2}$ level is in the right direction, but is too small. The absolute magnitudes, however, are not well explained as the scale factors given in the figure show. The direction of the effect is encouraging, viz., small components are underpredicted so that band mixing and other corrections to the rotational model should selectively enhance these levels. The use of an artificially small deformation (Čujec[23] uses $\beta=0.2$ for ^{25}Mg) to fit the data conceals

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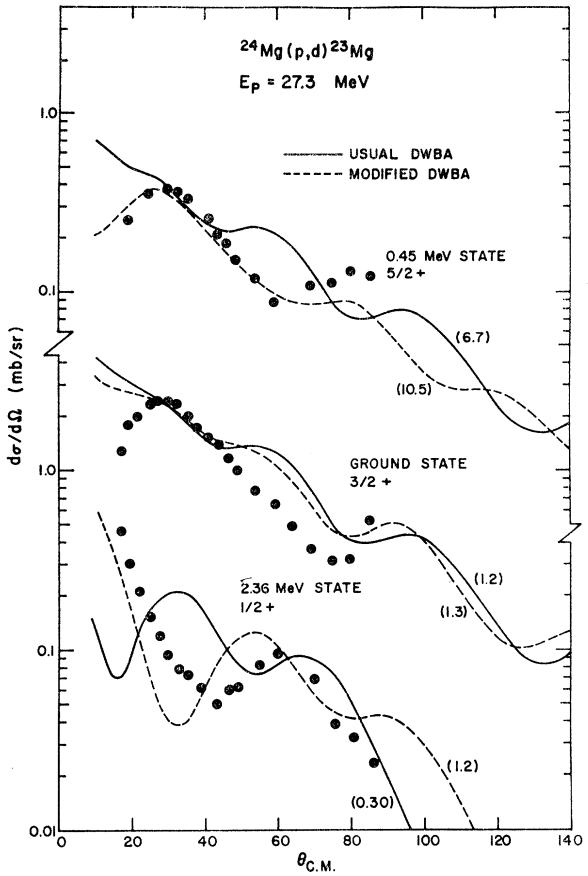

FIG. 6. Comparison of DWBA with the experimental data. The modified DWBA uses the radius-change prescription. The numbers in parentheses give the factor by which the curve was multiplied to "fit" the data, the fit being made by eye.

TABLE III. Calculated results for the ^{182}W$(d, p)^{183}$W reaction.

Band	Level	c_{lj}	κ	$\sigma_{theo}/\sigma_{DWBA}$	$\sigma_{expt}/\sigma_{DWBA}$[a]
$[510]\frac{1}{2}^-$	$p_{1/2}$	...	...	...	...
	$p_{3/2}$	−0.628	−0.888	1.16	1.42
	$f_{5/2}$	−0.534	−0.065	1.21	1.53
	$f_{7/2}$	+0.413	−0.321	...	...
	$h_{9/2}$	+0.354	−0.298	...	...
	$h_{11/2}$	−0.081	−1.41	...	...
$[512]\frac{3}{2}^-$	$p_{3/2}$	+0.403	−0.107	1.22	1.12
	$f_{5/2}$	−0.731	−0.059	1.11	1.50
	$f_{7/2}$	−0.343	−0.335	2.2[b]	1.70
	$h_{9/2}$	+0.405	+0.615	...	...
	$h_{11/2}$	+0.074	−1.41	...	...

[a] Taken from Ref. 4.

[b] The correction is so large as to change the angular distribution, e.g., the maximum is at 65° rather than at 50° in the uncorrected case. Thus the number is unreliable.

the difficulty. The inclusion of strong-coupling effects into the DWBA does not improve the normalization. Since all level components contribute to the modified DWBA in the same way (i.e., they increase the radius), it is likely that including collective-model corrections will not change this general feature responsible for the improved angular distributions. The desirability of having a larger optical-model radius has been found empirically in some recent ^{24}Mg(^{3}He, α) work.[11]

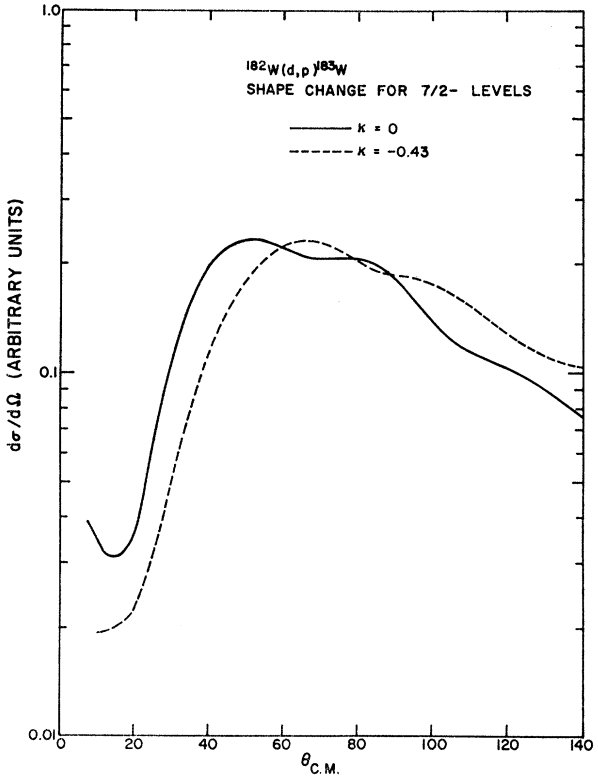

FIG. 7. Comparison of angular distributions for $f_{7/2}$ stripping ^{182}W$(d, p)^{183}$W at 12 MeV. The solid curve is computed assuming no coupling to excited states; the dashed curve is computed with $\kappa = -0.43$, a value similar to that obtained from the $[512]\frac{3}{2}^-$ band. The curves are normalized at their maximum.

IV. ^{182}W$(d, p)^{183}$W REACTION

A discrepancy in the usual DWBA procedure was reported by Siemssen and Erskine[4,5] in their analysis of the ^{182}W$(d, p)^{183}$W reaction using 12-MeV deuterons. Briefly, they found that the theory predicted too small cross sections (or, alternatively, gave spectroscopic factors too large) when best-fit optical potential parameters were employed. Agreement could be achieved using "average" parameters, i.e., those obtained from an analysis of several (usually nondeformed) nuclei.

A possible explanation of the above effect is in the strong coupling due to the permanent deformation of ^{182}W and thus is amenable to the theory described in this

work. Thus we use the best-fit potential (D2P1 in Ref. 5) and assume a deformation $\beta=0.3$. Neutron levels were computed with a Woods-Saxon well with $r_0=1.30$ F, $a=0.65$ F, and $\lambda=39.5$. The corrections due to the coupling are given in Table III along with the fractional increase in predicted cross sections. A change in the desired direction is seen although the $\frac{7}{2}^-$ change is far too large to be reliably calculated in a first-order theory.

Unlike the Mg cases, no large changes in angular distributions occur in the 12-MeV ^{182}W$(d, p)^{183}$W reaction. However, there are small changes which may be detectable in favorable circumstances. As an example, Fig. 7 shows two curves for $f_{7/2}$ stripping, one with $\kappa=0$ (spherical) and another with the $\kappa=-0.43$ appropriate to the $[512]\frac{3}{2}^-$ level at 412 keV. For comparison the curves are normalized at the maximum and they show a difference similar to Fig. 8 of Ref. 5 if the $\kappa=0$ level is taken to be the $[503]\frac{7}{2}^-$ band-head level at 453 keV. Unfortunately, our calculations yield a large negative value of κ for both bands, so that the explanation of the "band dependence" of the angular distribution is premature. Once again, no account has been taken of band mixing in this preliminary work.

V. CONCLUSION

In particle transfer reactions when strong coupling is important, it is possible to approximate the effects of the coupling for allowed transitions while retaining the usual DWBA form. For the case of deformed nuclei the only change is to change the potential radii R_0 to $R_0(1+\kappa\beta)$, where β is the deformation parameter and

$$\kappa=\sqrt{2}c_{lj\Omega}^{-1}\sum_{l'j'}c_{l'j'\Omega}\langle lj\Omega \mid Y_{20} \mid l'j'\Omega\rangle$$

$$=\sqrt{2}c_{lj\Omega}^{-1}\sum_{l'j'}c_{l'j'\Omega}[5(2j'+1)/4\pi(2j+1)]^{1/2}$$

$$\times(2j'0\Omega \mid j\Omega)(2j'0-\tfrac{1}{2} \mid j-\tfrac{1}{2}).$$

Preliminary comparisons with the data in the Mg and W regions are encouraging.

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Photoneutron Cross Sections for As^{75}, Ag^{107}, and Cs^{133}†

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Photoneutron cross sections, including $\sigma[(\gamma, n)+(\gamma, pn)]$ and $\sigma(\gamma, 2n)$ for As^{75}, Ag^{107}, and Cs^{133}, and $\sigma(\gamma, 3n)$ for Cs^{133}, were measured as a function of photon energy from threshold to 30 MeV. The photon energy resolution varied from less than 300 keV at the lowest to 400 keV at the highest energies. The source of radiation was the monoenergetic photon beam obtained from the annihilation in flight of fast positrons. The data were taken at 300-keV intervals. The partial cross sections were determined by neutron-multiplicity counting, and the average neutron energies for both single- and double-photoneutron events were determined simultaneously with the cross-section data by the ring-ratio technique. Nuclear information extracted from the data includes giant-resonance parameters, integrated cross sections and their moments, symmetry energies, quadrupole moments, and level densities. The mass of Ag^{105} was determined from the Ag^{107} $(\gamma, 2n)$ threshold energy. No marked structure was observed in the total photoneutron cross sections, although small deviations from a smooth curve are present, both in and above the giant resonance.

I. INTRODUCTION

THE systematics of the giant electric dipole resonance, which characterizes the absorption of electromagnetic radiation by nuclei in the energy range from about 10 to 30 MeV, have been of interest since the discovery of the giant resonance itself. Only recently, however, with the advent of monoenergetic photon beams and the development of efficient neutron detectors and associated multiplicity-counting techniques, has there been the capability of performing a systematic survey of the giant-resonance phenomenon throughout the periodic table with an accuracy sufficient to choose between competing nuclear models and with the precision necessary to delineate single-nucleon, pairing, and shell effects. In a continuing series of experiments, the photonuclear group at the Livermore electron linear accelerator has undertaken to conduct such a survey. The measurements reported here on the three even-odd nuclei As^{75}, Ag^{107}, and Cs^{133} are part of that series.

Previous photoneutron measurements in the giant-

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