PHYSICAL REVIEW VOLUME 177, NUMBER 4 20 JANUARY 1969

# Proton Excitation in $V^{51}$ †

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(Received 8 August 1968)

The $Ti^{50}(He^3,d)V^{51}$ reaction has been studied at 16.4-MeV incident energy with an over-all energy resolution of 20 keV full width at half maximum (fwhm). 57 levels in $V^{51}$ were identified up to 9.5-MeV excitation, and corresponding deuteron angular distributions were recorded in the angular interval 7° to 50°. Spectroscopic information has been extracted for 27 of the strongest or most well-isolated transitions by means of a distorted-wave analysis of the differential cross sections. The results are compared with nuclear-structure model predictions.

## I. INTRODUCTION

THE present investigation of the $Ti^{50}(He^3,d)V^{51}$ reaction forms part of a systematic study of the $Z=23$ nuclei by means of the $(He^3,d)$ reaction. The results obtained for $V^{47}$ and $V^{49}$ have already been published.[1,2]

## II. RESULTS AND ANALYSIS

The experimental techniques, data handling, and distorted-wave analysis used in the present experiment are identical to those described for the $Ti^{48}(He^3,d)$ reaction in Ref. 2. A deuteron spectrum from the $Ti^{50}(He^3,d)$ reaction measured at an incident energy of 16.4 MeV is presented in Fig. 1. The energy resolution is 20 keV full width at half maximum (fwhm). Since the target[3] contained an appreciable amount of $Ti^{48}$, a number of $(He^3,d)$ transitions originating from this isotope were also observed and their identification was aided by the results from Ref. 2.

Deuteron angular distributions were also measured at 16.4 MeV, using a thicker target that yielded an energy resolution of about 45 keV fwhm. The measured

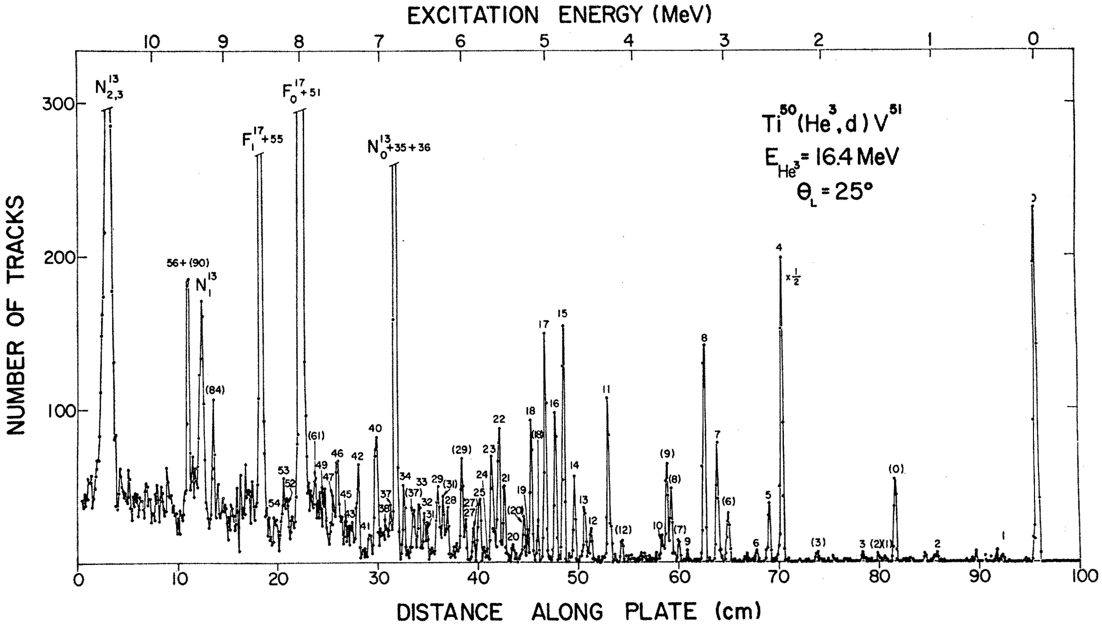

FIG. 1. Deuteron spectrum from the $Ti^{50}(He^3,d)V^{51}$ reaction at 16.4 MeV and $\theta_{lab}=25°$. Groups shown numbered without parentheses correspond to levels in $V^{51}$ and those with parentheses to levels in $V^{49}$ (Ref. 2), the latter being due to approximately 18% of $Ti^{48}$ in the target.

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† Work supported by the National Science Foundation under a contract with the University of Pennsylvania.

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[1] Baruch Rosner and D. J. Pullen, Phys. Rev. **162**, 1048 (1967).

[2] D. J. Pullen, Baruch Rosner, and Ole Hansen, Phys. Rev. **166**, 1142 (1968).

[3] The enriched $Ti^{50}$ metal was obtained from Oak Ridge National Laboratory, Isotopes Sales Division. The isotopic composition of the material was $Ti^{46}$-2%, $Ti^{47}$-1.8%, $Ti^{48}$-17.8%, $Ti^{49}$-2% and $Ti^{50}$-76.4%.

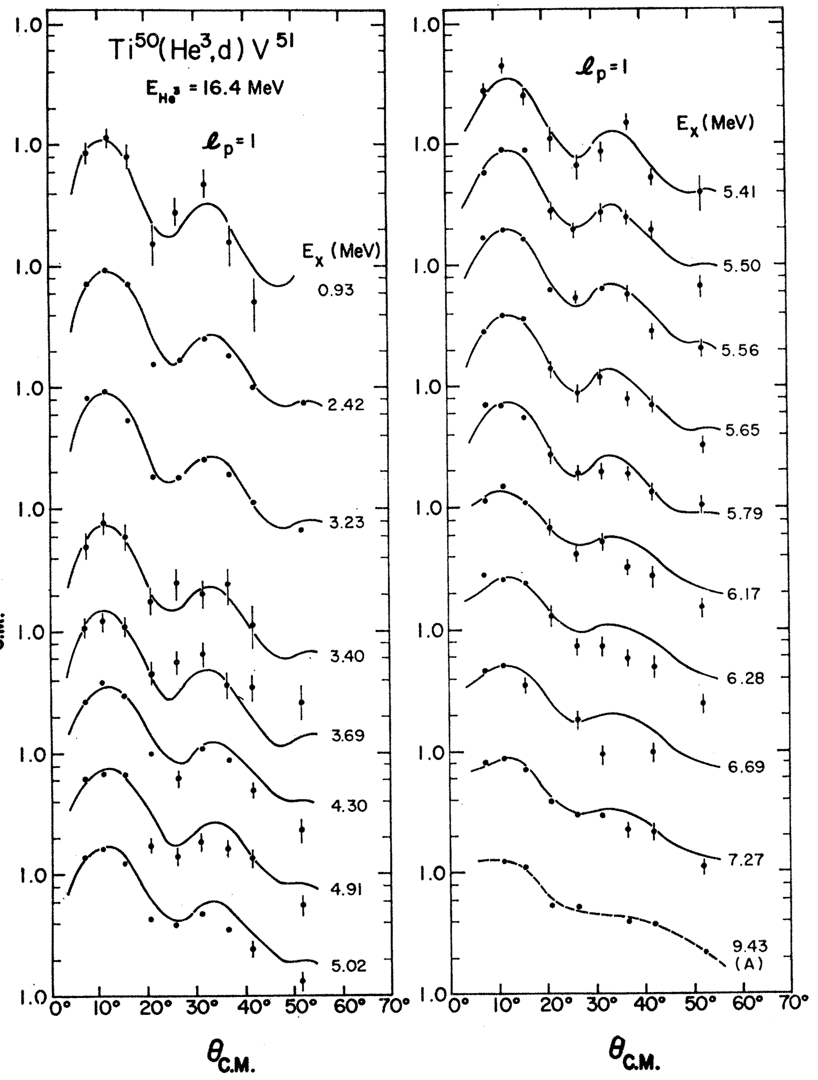

FIG. 2. Deuteron angular distributions corresponding to orbital angular-momentum transfer $l_p=1$ observed in the Ti$^{50}$(He$^3$,$d$)V$^{51}$ reaction. The different distributions are plotted offscale, the true cross-section scale being given in Table I. The full curves are from distorted-waves calculations (see text and Ref. 2). The dashed curve was drawn through the experimental points and has no theoretical significance.

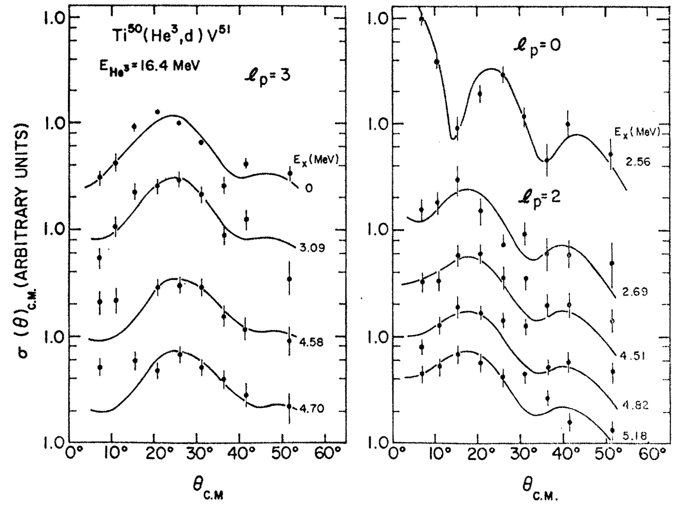

FIG. 3. Deuteron angular pistributions corresponding to orbital ngular-momentum transfer $l_p=3$ (left) and $l_p=0$ and 2 (right) See also caption of Fig. 2.

TABLE I. Results from the $Ti^{50}(He^3,d)V^{51}$ reaction.

| Deuteron group | $E_x$(keV) a | $E_x$(keV) Present[b] | $d\sigma/d\omega$ max (mb/sr)[c] | a | $l_p$ | Spectroscopic strength[d] | Shell-model assignment[e] |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 3.22 | $\frac{7}{2}-$ | 3 | 7.2 | $1f_{7/2}$ |
| 1 | 320 | 321 | | $\frac{5}{2}-$ | | | |
| 2 | 931 | 933 | 0.34 | $\frac{3}{2}-$ | 1 | 0.05 | $2p_{3/2}$ |
| 3 | 1611 | 1601 | | $\frac{11}{2}-$ | | | |
| | 1815 | | | $\frac{9}{2}-$ | | | |
| 4 | 2411 | 2424 | 21.8 | $\frac{3}{2}-$ | 1 | 2.6 | $2p_{3/2}$ |
| 5 | 2548 | 2560 | 1.97 | $\frac{1}{2}+$ | 0 | 0.17 | $2s_{1/2}$ |
| 6 | 2678 | 2685 | 0.19 | $\frac{3}{2}+$ | 2 | 0.21 | $1d_{3/2}$ |
| | 2702 | | | $\frac{15}{2}-$ | | | |
| | 2793 | | | | | | |
| 7 | 3085 | 3092 | 0.69 | | 3 | 1.0 | $1f$ |
| 8 | 3218 | 3227 | 7.31 | | 1 | 0.7 | $2p$ |
| | 3265 | | | | | | |
| | 3282 | | | | | | |
| | 3379 | | | | | | |
| 9 | 3385 | 3398 | 0.25 | | 1 | 0.02 | $2p$ |
| | 3395 | | | | | | |
| | 3454 | | | | | | |
| | 3519 | | | | | | |
| | 3566 | | | | | | |
| | 3579 | | | | | | |
| | 3617 | | | | | | |
| | 3634 | | | | | | |
| | 3668 | | | | | | |
| 10 | 3682 | 3693 | 0.47 | | 1 | 0.05 | $2p$ |
| | 12 levels | | | | | | |
| 11 | | 4303 | 5.41 | | 1 | 0.5 | $2p$ |
| | 4439 | | | | | | |
| 12 | 4487 | 4506 | 0.31 | | 2 | 0.06 | $2d$ |
| | 4512 | | | | | | |
| | 4551 | | | | | | |
| | 4576[f] | | | | | | |
| 13 | | 4584 | 0.25 | | 3 | 0.3 | $1f$ |
| 14 | 45 levels known up to 6648 keV[g] | 4702 | 0.34 | | (3) | 0.4 | $1f$ |
| 15 | | 4817 | 1.28 | | (2) | 0.2 | $2d$ |
| 16 | | 4910 | 2.69 | | 1 | 0.3 | $2p$ |
| 17 | | 5023 | 4.94 | | 1 | 0.5 | $2p$ |
| 18 | | 5184 | 1.06 | | (2) | 0.2 | $2d$ |
| 19 | | 5237 | | | | | |
| 20 | | 5407 | 0.50 | | 1 | 0.04 | $2p$ |
| 21 | | 5503 | 1.97 | | 1 | 0.2 | $2p$ |
| 22 | | 5564 | 2.78 | | 1 | 0.3 | $2p$ |
| 23 | | 5654 | 2.06 | | 1 | 0.2 | $2p$ |
| 24 | | 5703 | | | | | |
| 25 | | 5785 | 1.28 | | 1 | 0.1 | $2p$ |
| 26 | | 5907 | | | | | |
| 27 | | 5961 | | | | | |
| 28 | | 6165 | 1.50 | | 1 | 0.1 | $2p$ |
| 29 | | 6278 | 1.19 | | 1 | 0.1 | $2p$ |
| 30 | | 6355 | | | | | |
| 31 | | 6413 | | | | | |
| 32 | | 6444 | | | | | |
| 33 | | 6506 | | | | | |
| 34 | | 6694 | 0.91 | | (1) | 0.08 | $2p$ |
| 35 | | 6747 | | | | | |
| 36 | | 6806 | | | | | |
| 37 | | 6866 | | | | | |
| 38 | | 6939 | | | | | |
| 39 | | 6989 | | | | | |
| 40 | | 7047 | | | | | |
| 41 | | 7152 | | | | | |
| 42 | | 7272 | 2.12 | | 1 | 0.2 | $2p$ |
| 43 | | 7348 | | | | | |
| 44 | | 7393 | | | | | |
| 45 | | 7442 | | | | | |
| 46 | | 7540 | | | | | |
| 47 | | 7590 | | | | | |
| 48 | | 7633 | | | | | |
| 49 | | 7682 | | | | | |
| 50 | | 7710 | | | | | |
| 51 | | 7940 | | | | | |
| 52 | | 8171 | | | | | |

TABLE I. (*continued*)

| Deuteron group | $E_x$(keV) Present[b] | $d\sigma/d\omega$ max (mb/sr)[c] | $l_p$ | Spectroscopic strength[d] | Shell-model assignment[e] |
|---|---|---|---|---|---|
| 53 | 8218 | | | | |
| 54 | 8305 | | | | |
| 55 | 8501 | | | | |
| 56 | 9433 | 4.19 | (1) | 0.6[h] | $2p_{3/2}$, $T=\frac{7}{2}$ |

[a] Energy levels are from Ref. 5. The spin-parity assignment for the 2424-keV level (group 4) is based on the present work, and the remaining assignment are from Ref. 6.

[b] From 0 to 3 MeV the uncertainty in the excitation energy ($E_x$) is ±15 keV. Between 3 and 6 MeV the uncertainty is ±20 keV and above 6 MeV it is ±30 keV.

[c] The uncertainty in the cross-section scale was estimated to be ±30%.

[d] The strength is defined as $(2j+1)S(j)$, where $j$ is the transferred angular momentum and $S(j)$ is the spectroscopic factor in a formalism without isospin. If the isospin is introduced, then $S(j)=\langle 3\ 3\ \frac{1}{2}\ -\frac{1}{2}\,|\,T\frac{5}{2}\rangle^2 S(j,T)$, where $T$ is $\frac{5}{2}$ or $\frac{7}{2}$. Unless otherwise stated, it was assumed that $j=l+\frac{1}{2}$. (See, also, the text and Table III of Ref. 2).

[e] When no isospin is stated, $T=\frac{5}{2}$. The shell model assignments were used in the distorted wave analysis, and were based on the observed $l_p$ values.

[f] This state must be different from the 4584-keV state observed in the ($He^3$,$d$) study. The spin of the 4576-keV level is from $\frac{9}{2}^-$ to $\frac{15}{2}^-$ (Ref. 5), whereas that for the 4584-keV state must be $\frac{5}{2}^-$ or $\frac{7}{2}^-$.

[g] It is not possible to match the present ($He^3$,$d$) results with the ($d$,$p$) data of de Lopez *et al.*, (Ref. 5) for $E_x>4.6$ MeV.

[h] State unbound. The strength was estimated by keeping the single-particle wave function bound near 0 keV, and using the correct kinematics.

distribution for 27 of the most intense or well-isolated deuteron groups are shown in Figs. 2 and 3. The excitation energies, maximum differential cross sections,[4] $l_p$ values, and spectroscopic strengths are listed together in Table I. The potentials $BA1$, $PE1$, and $PR0$ of Table II (Ref. 2) were used for the extraction of the spectroscopic strengths. Also shown in Table I are the excitation energies and spin-parity assignments taken from the literature.[5,6]

## III. SUM-RULE ANALYSIS

The measured spectroscopic strength sums for $1f$ and $2p$ transitions to $V^{49}$ and $V^{51}$ are shown in Table II, and are compared with the values expected from the corresponding $1f_{7/2}$ Ti ground states. The $V^{49}$ data are from the Appendix of this report. In view of the $j=l+\frac{1}{2}$ assumption for the final-state spin values that underlies the distorted-wave analysis,[2] the strength sums of Table II probably err on the low side, with the $1f$ strength being more strongly affected[2] by the spin-orbit coupling than the $2p$ strength. It appears, nevertheless, that most of the $T$-lower $2p$ and $1f$ strengths have indeed been observed in these experiments.

The observation of $1d_{3/2}$ and $2s_{1/2}$ strengths in the present work indicates that the $Ti^{50}$ ground state contains core-excited components.

## IV. COMPARISON WITH OTHER EXPERIMENTS

$V^{51}$ has been extensively studied by the MIT group using the $V^{50}$($d$,$p$), $V^{51}$($p$,$p'$), and $Ti^{50}$($He^3$,$d$) reactions.[5,7,8] In Table I are shown the excitation energies from Ref. 5, and it appears that the present values are systematically higher than those by about 10 keV (19 keV in the case of group 12). A comparison with the data of Ref. 8 shows somewhat larger discrepancies in the excitation energies. A reasonable identification of the stronger transitions reported here with the stronger transitions given in Ref. 8 is proposed in Table III. The energy discrepancy is seen to increase toward higher excitation energy, where it amounts to about 70 keV. [In the $Ti^{48}$($He^3$,$d$) experiment of Ref. 2, an over-all agreement to better than 15 keV is found with the magnetic-spectrograph data from Heidelberg[9].] With two exceptions all the present $l_p$-value assignments agree with those of the MIT group. For the two exceptions we have tentative $l_p=2$ assignments, whereas $l_p=3$ is assigned in Ref. 8. The present strengths are larger than the MIT strengths by about 30% for $E_x<3$ MeV, the discrepancy increasing to almost a factor of 2 at $E_x\sim5$ MeV. The ($He^3$,$d$) study of Ref. 8 was performed at an incident energy of 10 MeV, whereas the present study employed 16.4-MeV $He^3$ ions. In the case of the $Ti^{48}$($He^3$,$d$) study, the spectroscopic strengths of the Appendix agree well with the 18-MeV Heidelberg values of Ref. 9.

## V. DISCUSSION

The level spectrum of $V^{51}$ has recently been calculated by Auerbach[10] with a configuration space comprising

TABLE II. $T_{\text{lower}}$ strength sums.[a]

| Target | | Configuration $1f$ | $2p$ |
|---|---|---|---|
| $Ti^{48}$ | Expt.[b] | 8.2 | 4.5 |
| | Theory | 10.4 | 4.8 |
| $Ti^{50}$ | Expt. | 8.9 | 6.0 |
| | Theory | 11.1 | 5.1 |

[a] Pure $1f_{7/2}$ ground states and good isospin were assumed in the evaluation of the theoretical strengths.

[b] See the Appendix.

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[4] A reexamination of the $Ti^{48}$($He^3$,$d$) $V^{49}$ cross sections reported in Ref. 2 was undertaken and an error of a factor of 1.91 was discovered. The corrected $Ti^{48}$($He^3$,$d$) cross sections and spectroscopic strengths are given in the Appendix and the sum-rule data in Table II.

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[9] D. Bachner, R. Santo, H. H. Duhm, and R. Bock, Nucl. Phys. **A106**, 577 (1968).

[10] N. Auerbach, Phys. Letters **24B**, 260 (1967).

TABLE III. Comparison of present $Ti^{50}(He^3,d)$ results with those from Ref. 8.

| $E_x$(keV) | | $l_p$ | | $(2j+1)S$[a] | |
|---|---|---|---|---|---|
| Present | b | Present | b | Present | b |
| 0 | 0 | 3 | 3 | 7.2 | 5.6 |
| 933 | 930 | 1 | 1 | 0.05 | 0.05 |
| 2424 | 2406 | 1 | 1 | 2.6 | 1.7 |
| 2560 | 2541 | 0 | 0 | 0.2 | 0.06 |
| 2685 | 2667 | 2 | 2 | 0.2 | 0.06 |
| 3092 | 3075 | 3 | 3 | 1.0 | 0.75 |
| 3227 | 3208 | 1 | 1 | 0.7 | 0.52 |
| 4303 | 4252 | 1 | 1 | 0.5 | 0.25 |
| 4506 | 4445 | (2) | 3 | 0.06 | 0.14 |
| 4584 | 4521 | (3) | 3 | 0.3 | 0.22 |
| 4702 | 4633 | (3) | 3 | 0.4 | 0.30 |
| 4817 | 4755 | (2) | 3 | 0.2 | 0.89 |
| 4910 | 4849 | 1 | 1 | 0.3 | 0.18 |
| 5023 | 4964 | 1 | 1 | 0.5 | 0.24 |
| ... | 5104 | ... | 0 | | 0.01 |
| 5184 | 5127 | (2) | 2 | 0.2 | 0.06 |
| ... | 5170 | ... | 2 | | 0.04 |
| 5503 | 5440 | 1 | 1 | 0.2 | 0.08 |
| 5564 | 5497 | 1 | 1 | 0.3 | 0.17 |
| 5654 | 5585 | 1 | 1 | 0.2 | 0.12 |
| 5785 | 5720 | 1 | 1 | 0.1 | 0.09 |

[a] The strengths of Ref. 8 were evaluated without the use of a spin-orbit coupling term. Under our assumption of $j=l+\frac{1}{2}$, inclusion of a spin-orbit term in the bound-state potential would lead to a reduction in the strengths of Ref. 8.

[b] Reference 8.

TABLE IV. Corrected $Ti^{48}(He^3,d)\ V^{49}$ strengths.

| Deuteron group | $E_x$ (keV) | $d\sigma/d\omega$ max (mb/sr) | $l_p$ | Spectroscopic strength | Shell-model assignment |
|---|---|---|---|---|---|
| 0 | 0 | 2.5 | 3 | 4.8 | $1f_{7/2}$ |
| 1 | 83 | 0.11 | ... | | |
| 2 | 152 | 1.2 | 1 | 0.1 | $2p$ |
| 3 | 751 | 0.76 | 2 | 0.8 | $1d_{3/2}$ |
| 6 | 1663 | 7.3 | 1 | 0.7 | $2p$ |
| 7 | 2185 | 0.80 | 3 | 1.0 | $1f$ |
| 8 | 2265 | 6.9 | 1 | 0.7 | $2p$ |
| 9 | 2307 | 13.0 | 1 | 1.3 | $2p$ |
| 11 | 2820 | 0.74 | 3 | 0.8 | $1f$ |
| 13 | 3401 | 0.36 | ... | | |
| 16 | 3763[a] | 0.92 | (1)<br>(3) | 0.1<br>0.2 | $2p$<br>$1f$ |
| 17 | 3932 | 3.6 | 1 | 0.3 | $2p$ |
| 18 | 4018 | 0.69 | 1 | 0.06 | $2p$ |
| 20 | 4235 | 1.5 | 1 | 0.1 | $2p$ |
| 21 | 4265 | | | | |
| 22 | 4385 | 0.59 | 1 | 0.04 | $2p$ |
| 24 | 4511 | 2.7 | 1 | 0.2 | $2p$ |
| 26 | 4657 | 1.9 | 3 | 1.4 | $1f$ |
| 27 | 4862 | 2.9 | 1 | 0.2 | $2p$ |
| 29 | 4954 | 0.42 | 1 | 0.04 | $2p$ |
| 31 | 5064 | 0.69 | 1 | 0.06 | $2p$ |
| 32 | 5218 | 2.1 | 1 | 0.2 | $2p$ |
| 43 | 6000 | 1.4 | 1 | 0.1 | $2p$ |
| 48 | 6212 | 1.4 | 1 | 0.1 | $2p$ |
| 49 | 6252 | | | | |
| 50 | 6327 | 1.9 | 1 | 0.2 | $2p$ |
| 51 | 6363 | | | | |
| 67 | 7137 | 1.9 | ... | | |
| 71 | 7430 | 1.7 | (1) | | $2p$ |
| 72 | 7478 | | | | |
| 76 | 7783 | 7.4 | (1) | 0.8 | $2p_{3/2}\ T=\frac{5}{2}$ |
| 82 | 8111 | 3.1 | (1) | 0.3 | $2p_{1/2},\ T=\frac{5}{2}$ |

[a] A doublet.

$(1f_{7/2})^3$ and $(1f_{7/2})^2\ (2p_{3/2})$. The results of this calculation are in good agreement with the $(He^3,d)$ data for the ground state and three lowest $l_p=1$ transitions (see also Ref. 8). It is conspicuous that the first excited $l_p=3$ transition in $V^{51}$ occurs more than 1 MeV higher than in $V^{49}$. (See Table IV and Ref. 2.) This feature suggests that all the $1f_{7/2}$ strength is concentrated in the ground state of $V^{51}$, whereas several low-lying $l_p=3$ transitions in $V^{49}$ may have $1f_{7/2}$ character.

If the strong transition to the 9433-keV state is indeed of $l_p=1$ character, as suggested in Table I, then this probably corresponds to the isobaric analog of the $Ti^{50}(d,p)Ti^{51}$ ground-state transition. The corresponding Coulomb displacement energy, $\Delta E_c$, is then 7751±30 keV, which is, within the experimental uncertainties, the same as that found for the $2p_{3/2}$ state in the $V^{49}$-$Ti^{49}$ pair. In evaluating $\Delta E_c$, the $Q$ value of the $Ti^{50}(d,p)$-$Ti^{51}(0)$ transition was taken from Ref. 11 and that for the $Ti^{50}(He^3,d)V^{51}(0)$ transition from the 1964 mass table.[12] The spectroscopic strength for the $(He^3,d)$ transition to the 9433-keV level is 0.6, whereas the corresponding $(d,p)$ strength, when divided by $2T+1=7$, is 0.4.[13]

## ACKNOWLEDGMENTS

We wish to thank Professor R. Middleton and Professor W. E. Stephens for their continued interest in this work, and Dr. R. H. Bassel for performing part of the distorted-wave analysis. One of us (OH) is indebted to the Rask-Ørsted foundation in Copenhagen, Denmark, for a travel grant. The scanning of the nuclear emulsions was carefully performed by Mrs. M. Scrinivasan, Mrs. C. Coliukus, and Mrs. M. Barnett.

## APPENDIX: ERRATA TO THE $Ti^{48}(He^3,d)V^{49}$ STUDY OF REF. 2

An error of a factor of 1.91 has been found in the evaluation of the thickness of the $Ti^{48}$ target used in Ref. 2. All cross sections and spectroscopic strengths in Ref. 2 should be multiplied by 1.91. The corrected values are given in Table IV, which includes only those transitions for which a cross section was measured. From the data of Tables I and IV the ratio of spectroscopic strengths for the $V^{51}$ and $V^{49}$ ground-state transitions is $S(V^{51})/S(V^{49})=1.5$. The $Ti^{50}$ target employed in the present study also contained $Ti^{48}$ as a contaminant. If the intensities of the deuteron groups leading to the $V^{51}$ and $V^{49}$ ground states are used (see Fig. 1) in conjunction with the isotopic composition of the $Ti^{50}$ material,[3] then a ratio $S(V^{51})/S(V^{49})=1.5$ is again obtained, thus showing the consistency of the target thickness evaluation.

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[13] P. Wilhjelm, O. Hansen, J. Comfort, C. K. Bockelman, P. D. Barnes, and A. Sperduto, Phys. Rev. **166**, 1121 (1968).