Experimental High-Resolution Investigation and Shell-Model Interpretation of the ^{49}Ca Ground-State Analog

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Excitation functions for ^{48}Ca(p,p) at 165° and 105° have been measured from 1.93 to 2.01 MeV. In addition, the reactions ^{48}Ca$(p,n)^{48}$Sc and ^{48}Ca$(p,n\gamma)^{48}$Sc have been observed. Spins, parities, total and partial widths have been assigned. Eleven resonances are observed, eight of which have $J^\pi=\frac{3}{2}^-$ and are associated with the ^{49}Ca ground-state analog. These $\frac{3}{2}^-$ levels are interpreted in terms of a detailed shell-model calculation. The importance of $4p$-$3h$ and higher configurations is noted.

1. INTRODUCTION

THE breakup of the broad $T_>$ isobaric analog state via an enhancement of the "background" $T_<$ states of the appropriate J^π has been demonstrated in high resolution proton elastic-scattering measurements.[1] Recent interest in the ^{49}Ca ground-state analog has led us to investigate the details of the fine structure associated with that state. Because of the simple closed-shell nature of the ^{48}Ca target, shell-model calculations have been performed to interpret the data.

This region in the compound system ^{49}Sc has previously been observed through the (p,p)[2,3], (p,n),[3] $(p,n\gamma)$[2,4] and (p,γ)[2,5] reactions. Using the Duke University 3-MeV Van de Graaff accelerator and electrostatic-analyzer-homogenizer system,[6] we have measured the (p,p), (p,n), and $(p,n\gamma)$ reactions from 1.93 to 2.01 MeV, with a total energy resolution of about 600 eV. The (p,p) data indicate 11 resonances, with even the weakest anomalies visible in the other exit channels observed. The (p,p) and (p,n) excitation functions have been fit with a multilevel, multichannel computer code MULTI,[7] to extract the resonance parameters E_0, J^π, Γ_p, Γ_n, and Γ for each of the fine-structure resonances. In this manner, eight of the 11 resonances observed are assigned $J^\pi=\frac{3}{2}^-$, in agreement with the spin and parity of the expected analog state. The width of the analog state has been extracted from the fine-structure proton widths. This is compared to predictions from microscopic shell-model calculations and an interpretation of the fine structure is made in terms of multi-particle-hole excitations.

2. EXPERIMENTAL PROCEDURE

The general floor plan of the laboratory in this experiment is shown in Fig. 1. The 1-m radius electrostatic analyzer[8] is used to permit precise and continuous knowledge of the beam energy. The homogenizer system[6] derives an energy correction signal from the analyzer image slits which is used to modulate a positive potential applied to the target. In this manner, the slow time-dependent variations in the beam are effectively canceled, resulting in an energy spread in the incident beam of about 200 eV.

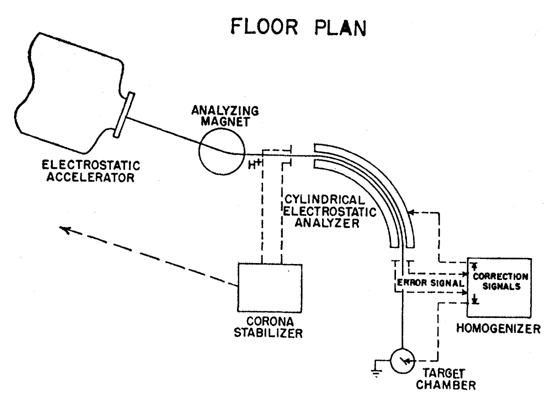

FIG. 1. Floor plan of the laboratory.

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[7] D. L. Sellin, Ph.D. thesis, Duke University, 1968 (unpublished).
[8] A. L. Toller, Ph.D. thesis, Duke University, 1954 (unpublished).

The total effective resolution observed in these measurements is, however, primarily determined by the target. The ^{48}Ca targets were prepared by evaporation of enriched $CaCO_3$ on thin (30–40 μg/cm^2) carbon backings. The $CaCO_3$, purchased from Oak Ridge National Laboratory, was 82% enriched in ^{48}Ca. Because the oxygen contamination of the targets contributed to the energy loss almost as much as did the Ca, the Ca deposit was restricted to about 2 μg/cm^2 in order to maintain a total observed resolution of about 600 eV, as shown in Fig. 2. During the experiment, several changes of targets were required due to carbon deposits developing on the front surface of the target.

These data were taken using a 25-cm scattering chamber containing two movable solid state detectors. The target holder was mounted on insulating supports to allow application of the homogenizer's high-voltage correction signal. The neutrons were detected in BF_3 counters subtending about $\frac{2}{3}\pi$ sr and with an efficiency of about 10%. For γ-ray detection, a 2×2 in. NaI crystal was placed outside the chamber. Absolute energy calibration was checked several times during acquisition of these data by measuring the ^{7}Li$(p,n)^7$Be threshold at 1.8802 MeV. Repeated measurements of a single resonance indicated energy shifts of less than 1.5 keV. We are thus confident that the energy scale used in Fig. 2 is precise within ± 2 keV.

The neutron excitation function in Fig. 2 was obtained after subtraction of background. No such subtraction was made in the γ-ray plot. To eliminate possible effects arising from ^{40}Ca, the (p,p) excitation function for natural Ca(97%^{40}Ca) was measured in steps of 400 eV and with a total energy resolution of about 600 eV. A smooth function corresponding to the 18% isotopic impurity was then subtracted from the elastic-scattering data. In addition, our recent measurements of ^{42,44}Ca(p,p) show that none of these impurities could contribute any effect outside the 1.5% statistics.[9]

3. EXPERIMENTAL RESULTS

The ^{48}Ca(p,p) excitation functions at 105° and 165° from 1.93–2.01 MeV are shown in Fig. 2. Also included in the figure are the ^{48}Ca$(p,n)^{48}$Sc and $(p,n\gamma)$ excitation functions from 1.93 to 2.00 MeV.

Table I gives the results of analysis of all resonances which were observed in the elastic-scattering data. The proton widths $\Gamma_{p\lambda}$, where λ labels the individual fine-structure states, are derived as discussed below and range in precision from about 10% for the widest resonances to about 50% for the narrowest ones; the neutron widths $\Gamma_{n\lambda}$ are, however, perhaps less precise due to a lack of measured absolute cross section. The proton reduced widths are also presented. The partial widths for proton capture have been neglected, an approximation which is justified by the measurements of Ref. 2. Similarly, kinematics allow the (p,α) and (p,p') reaction channels to be neglected.

The partial widths given in Table I were obtained using the multilevel, multichannel R-matrix fitting program MULTI.[7] The resultant fits to the (p,p) and (p,n) data are indicated by the appropriate solid lines in Fig. 2. Although there was some difficulty in distinguishing between very narrow resonances with $J^\pi = \frac{3}{2}^-$ and $\frac{1}{2}^-$, the (p,n) data offered additional helpful criteria. A normalized Gaussian is expected to be a reasonable approximation to the resolution function and was included in the fits as a resolution smear. The fitting procedure does not include R_∞, the effect of distant levels, in the calculation. The penetration factors required in computing the reduced partial widths are calculated in the fitting procedure from Coulomb wave functions generated by the subroutine EXTFN.[10] Throughout the calculations, an interaction radius of 5.79 fm, corresponding to $R_0 = 1.25$ fm, was used.

A sum rule may be applied to the proton widths $\Gamma_{p\lambda}$ of the individual fine-structure resonances yielding the intermediate structure analog state width $\Gamma_p = \sum_\lambda \Gamma_{p\lambda}$ if it is assumed that the widths of the fine-structure states were negligible before enhancement. In this manner, there results a width $\Gamma_p = 1.85$ keV. From optical-model calculations, Ref. 3 has estimated the width of a single-particle resonance, Γ(s.p.), as 3.1 keV for $J^\pi = \frac{3}{2}^-$. Thus these measurements imply a spectroscopic factor $[\Gamma_p/\Gamma(\text{s.p.})]$ of 0.6 for the ground-state analog. The spectroscopic factor for the parent state, the ground state of ^{49}Ca, has been determined from (d,p) stripping measurements to be approximately unity.

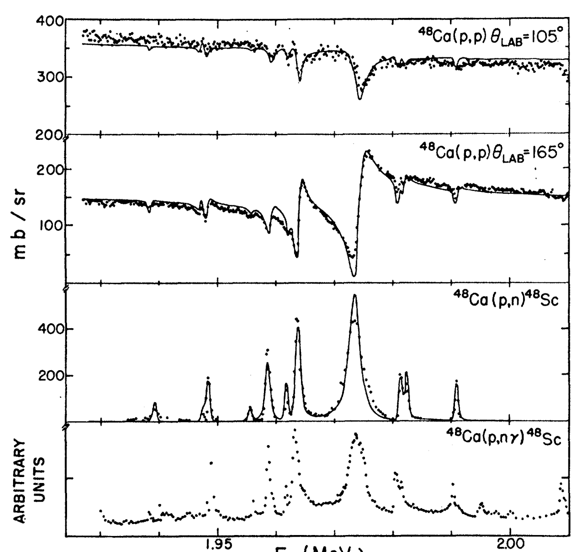

FIG. 2. Excitation functions for ^{48}Ca(p,p), ^{48}Ca$(p,n)^{48}$Sc, and ^{48}Ca$(p,n\gamma)^{48}$Sc. The solid lines through the (p,p) and (p,n) data represent fits derived from R-matrix theory.

[9] J. C. Browne *et al.*, Phys. Letters (to be published).

[10] Coded by M. L. Gursky, Los Alamos Scientific Laboratory.

TABLE I. Resonance parameters.

	Present work						Ref. 2				
λ	$E_{p\lambda}$ (keV)	J^π	$\Gamma_{p\lambda}$ (eV)	$\Gamma_{n\lambda}$ (eV)	$\gamma_{p\lambda}^2$ (keV)	E_p (keV)	J^π	Γ_p (eV)	Γ_n (eV)	Γ_γ (meV)
1	1938±2	$\frac{1}{2}^-$	10	20	0.297	...	...	...	...	...
2	1947±2	$\frac{1}{2}^+$	25	5	0.297	...	...	...	...	...
3	1948±2	$(\frac{3}{2}^-)$	30	70	0.862	1950±2	$(\frac{3}{2}^-)$	≈80	12	...
4	1956±2	$\frac{1}{2}^-$	15	120	0.421	...	...	...	...	...
5	1959±2	$\frac{3}{2}^-$	100±25	500±100	2.773	1959±2	$\frac{3}{2}^-$	≈100	24	10
6	1962±2	$(\frac{3}{2}^-)$	5	10	0.137	...	...	...	...	...
7	1964±2	$\frac{3}{2}^-$	300±50	200±50	8.193	1964±2	$\frac{3}{2}^-$	400±50	40	40
8	1974±2	$\frac{3}{2}^-$	1300±100	600±100	34.360	1975±2	$\frac{3}{2}^-$	1500±200	75	170
9	1981±2	$(\frac{3}{2}^-)$	40	80	1.035	1982±2	$(\frac{3}{2}^-)$	≈70	20	10
10	1982±2	$(\frac{3}{2}^-)$	50	50	1.290	...	...	...	...	...
11	1991±2	$(\frac{3}{2}^-)$	20	50	0.502	1991±2	$(\frac{3}{2}^-)$	≈70	9	7
...	...	...	...	...	...	1.996±2	$(\frac{3}{2}^-)$	≈30	5	...

This difference is most probably due to an uncertainty in the value of Γ(s.p.). It is believed that the error in $\sum_\lambda \Gamma_{p\lambda}$ cannot account for this discrepancy.

4. MICROSCOPIC SHELL-MODEL ANALYSIS

The preceding sections have presented (cf. Fig. 2) the experimental fine-structure observed with high resolution in the vicinity of the expected analog of the ^{49}Ca $\frac{3}{2}^-$ ground state. A total of 11 distinct resonances are found within a range of about 80 keV. Of these levels eight are assigned $\frac{3}{2}^-$, two are given $\frac{1}{2}^-$ and one is determined to be $\frac{1}{2}^+$ (Table I).

If the experiment were to be performed with poorer resolution, one would expect to find a broader intermediate structure resonance over the energy range of the fine-structure levels. This resonance would then be interpreted as the doorway analog of the ^{49}Ca $\frac{3}{2}^-$ state. Equivalently one may simulate the results of such an experiment by energy averaging the fine structure with a resolution function to obtain the desired intermediate resonance. A theoretical analysis of the intermediate structure resonance yields information which may be compared to our detailed shell-model description.

The resolution function used in this averaging procedure was chosen to be a Gaussian with a width of 8 keV. This width was determined so that the maximum amount of fine structure in the elastic proton data at 165° was smoothed out and the averaged data exhibited only one wide resonance. Assuming that this was a $\frac{3}{2}^-$ level, an attempt was made to fit it with computer program MULTI.[7] A reasonably good fit was obtained with a resonance energy of 1.971 MeV, $\Gamma_p = 2$ keV, and $\Gamma_d = 4.7 \pm 0.4$ keV (where Γ_d is the total width). This single resonance is interpreted as the $\frac{3}{2}^-$ intermediate structure analog level with a $2p$-$1h$ doorway wave function. The method of analysis of this resonance follows the procedure outlined by Beres and Divadeenam.[11,12] The single proton part of the usual analog state wave function gives rise to the proton escape width Γ_p, while the decay of the doorway into $3p$-$2h$ and more complicated states is manifested in the damping (or spreading) width $\Gamma^\downarrow$.

The target nucleus ^{48}Ca is assumed to consist of a closed $1d_{3/2}$ proton shell ($Z=20$) and a closed $1f_{7/2}$ neutron shell ($N=28$). Configuration mixing in the target ground state is neglected. Relative to this core the available single proton particle (hole) states are $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$($1d_{5/2}^{-1}$, $2s_{1/2}^{-1}$, and $1d_{3/2}^{-1}$). For neutrons the single particle (hole)states are $2p_{3/2}$, $2p_{1/2}$, $1f_{5/2}$, and $1g_{9/2}$($1f_{7/2}^{-1}$). The binding energies of these states relative to ^{48}Ca as the zero of energy are shown in Fig. 3. The sources of these experimentally determined energies are given in Ref. 13. The wave functions for neutrons and protons are chosen to have different radial forms. This is accomplished by using a single particle Hamiltonian consisting of a Woods-Saxon well with spin-orbit coupling and (for protons) the Coulomb potential of a uniform sphere of charge. The resultant well parameters are listed in Table II. The $\frac{3}{2}^-$ analog $2p$-$1h$ configuration is $\{[1f_{7/2}(p), 1f_{7/2}^{-1}(n)]^{0^+}, 2p_{3/2}(n)\}$ which has an unperturbed

TABLE II. Woods-Saxon potential parameters (^{49}Sc).

$r_0 = 1.281$ fm $a = 0.5$ fm
$r_c = 1.341$ fm $V_{s0} = 5.4$ MeV
$R = r_0 A^{1/3}$ fm $R_c = r_c A^{1/3}$ fm

Neutron			Proton		
j_n	BE (MeV)	V_0 (MeV)	j_p	BE (MeV)	V_0 (MeV)
$1g_{9/2}$	0.100	48.47	$2p_{1/2}$	3.51	57.12
$1f_{5/2}$	1.184	41.26	$1f_{5/2}$	4.86	55.92
$2p_{1/2}$	3.114	45.43	$2p_{3/2}$	6.01	58.55
$2p_{3/2}$	6.010	46.48	$1f_{7/2}$	9.55	56.19
j_n^{-1}	BE (MeV)	V_0 (MeV)	j_p^{-1}	BE (MeV)	V_0 (MeV)
$1f_{7/2}$	9.93	48.32	$1d_{5/2}$	19.3	55.54
			$2s_{1/2}$	15.7	56.45
			$1d_{3/2}$	15.3	54.01

[11] W. P. Beres and M. Divadeenam, Phys. Rev. Letters **20**, 938 (1968).
[12] W. P. Beres and M. Divadeenam, Nucl. Phys. **117**, 143 (1968).
[13] M. Divadeenam and W. P. Beres, Phys. Rev. Letters **21**, 379 (1968).

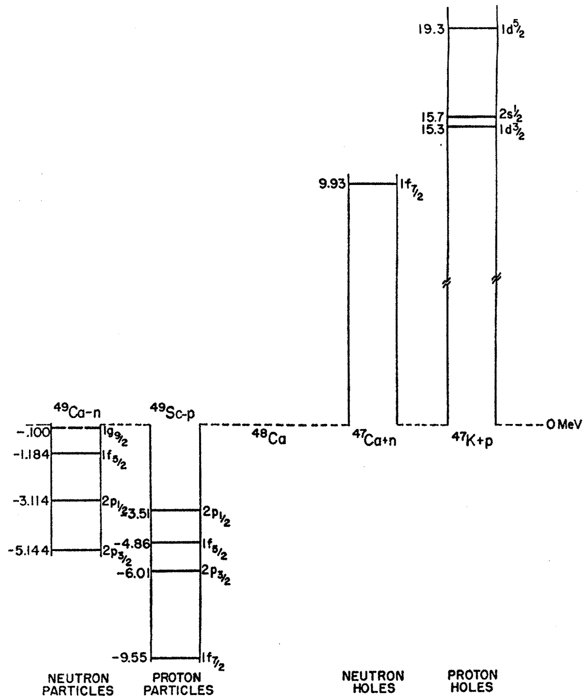

FIG. 3. The particle energies and the negatives of the hole energies are shown relative to ^{48}Ca as the zero of energy. A binding energy of about 0.100 MeV is assumed for the $1g_{9/2}$ neutron particle state. This is indicated by a thick dashed line.

energy of -4.76 MeV (relative to ^{48}Ca$+p$). This level is raised to the energy of the intermediate structure analog resonance (referred to in the preceding paragraph) by assuming an effective two-body residual nuclear interaction. For simplicity, this interaction was chosen to be a radial delta function with the Soper mixture. The strength needed was 755 MeV fm^3. The analog doorway is found to have negligible coupling to $T_<2p$-$1h$ states containing higher energy proton contributions, i.e., $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$.[13] Thus, within this shell model picture, the high-resolution splitting of the $\frac{3}{2}^-$ intermediate structure resonance is due to $3p$-$2h$ and more complicated states.

According to Feshbach *et al.*,[14] the intermediate structure width of the doorway state Γ_d may be written as

$$\Gamma_d=\Gamma^{\uparrow}+\Gamma^{\downarrow}, \quad (1a)$$

where $\Gamma^{\uparrow}$ is the total escape width to the continuum. One may write

$$\Gamma^{\uparrow}=\sum_i \Gamma_i, \quad (1b)$$

where the index i refers to the ith exit channel. In the present experiment the possible open channels are p, n, and γ, so that

$$\Gamma_d=(\Gamma_p+\Gamma_n+\Gamma_\gamma)+\Gamma^{\downarrow}. \quad (2)$$

[14] H. Feshbach, A. K. Kerman, and R. Lemmer, Ann. Phys. (N. Y.) **41**, 230 (1967).

All of the intermediate structure escape widths in the above equation may be obtained experimentally using the relation

$$\Gamma_i=\sum_\lambda \Gamma_{i\lambda}, \quad (3)$$

where the index λ, as mentioned in Sec. 3, indicates the λth fine-structure resonance. In Eqs. (1)–(3) all levels correspond to the same spin and parity. Adding the eight fine-structure widths for $\frac{3}{2}^-$ from table I results in $\Gamma_p=1.85$ keV and $\Gamma_n=1.56$ keV. Following Refs. 11 and 12 a theoretical calculation of Γ_p using shell-model wave functions yields $\Gamma_p=2.0$ keV, in very good agreement with the corresponding experimental value. The γ widths are not measured in the present experiment. The value $\Gamma_\gamma\approx 0.25$ eV has been reported in Ref. 2 and hence is negligible. This yields a total escape width of $\Gamma^{\uparrow}\approx 3.4$ keV. Using the value of $\Gamma_d=4.7\pm 0.4$ keV obtained in our energy average of the fine-structure data we obtain for the experimental damping width $\Gamma_{\text{expt}}{}^{\downarrow}=1.3\pm 0.4$ keV.

The damping width may be calculated theoretically in the manner of Refs. 11 and 12. There are 833 available $3p$-$2h$ states that can couple to the $2p$-$1h$ analog doorway and this results in a value of $\Gamma_{\text{theor}}{}^{\downarrow}\approx 0.6$ keV. We note that both the experimental and theoretical values of $\Gamma^{\downarrow}$ are quite small and of the same order of magnitude. A summary of the experimental and theoretical widths appears in Table III. Columns 1–3 contain the sums of experimental fine-structure widths while column 4 gives the total experimental escape width $\Gamma^{\uparrow}$. Columns 5 and 6 contain, respectively, the intermediate resonance widths Γ_p (for the elastic proton channel) and Γ_d, and column 7 gives the inferred experimental damping width $\Gamma^{\downarrow}=\Gamma_d-\Gamma^{\uparrow}$. The theoretical values of Γ_p and $\Gamma^{\downarrow}$ are shown in columns 8 and 9. The experimental elastic proton width $\Gamma_p=2$ keV was obtained from the intermediate structure without much detailed analysis and differs slightly from the value of 1.85 keV obtained from the fine structure. This is possibly due to errors introduced in the energy averaging procedure and the fact that the average is made over all 11 fine-structure resonances.

As pointed out earlier, there are 833 $\frac{3}{2}^-$ $3p$-$2h$ states populated within an energy span of about 10 MeV measured from proton threshold (i.e., 9.6–19.6 MeV excitation in Sc^{49}). A reasonable approximation to the level density would be to distribute the levels according to an exponential density function. Gilbert and Cameron[15] have extensively analyzed neutron resonances using an exponential function which takes into account both shell and pairing effects. This density for a given

[15] A. Gilbert and A. G. W. Cameron, Can. J. Phys. **43**, 1446 (1965).

TABLE III. Comparison of experimental and theoretical widths. All widths are given in keV.

Experimental							Theoretical	
Fine structure				Intermediate structure				
$\Gamma_p=\sum\Gamma_{p\lambda}$	$\Gamma_n=\sum_\lambda\Gamma_{n\lambda}$	$\Gamma_\gamma=\sum_\lambda\Gamma_{\gamma\lambda}$	$\Gamma\uparrow=\Gamma_p+\Gamma_n+\Gamma_\gamma$	Γ_p	Γ_d	$\Gamma\downarrow=\Gamma_d-\Gamma\uparrow$	Γ_p	$\Gamma\downarrow$
1.85	1.56	≈ 0	3.4	2.0	4.7 ± 0.4	1.3 ± 0.4	2.0	≈ 0.6

J and for both parities is given by

$$\rho(U,J)=\frac{\sqrt{\pi}}{12}\frac{\exp(2\sqrt{aU})}{a^{1/4}U^{5/4}}\times\frac{(2J+1)\exp[-(J+\frac{1}{2})^2/2\sigma^2]}{2\sigma^3\sqrt{2\pi}},\quad(4)$$

where U is the corrected excitation energy, i.e., the incident proton energy plus the separation energy of the last proton plus the pairing energy. The parameter σ is given by

$$\sigma^2=0.0888a(U/a)^{1/2}A^{2/3},\quad(5)$$

where A is the mass number. Finally, a is the level-density parameter.

For a particular parity,

$$\rho(U,J^\pi)=\tfrac{1}{2}\rho(U,J).\quad(6)$$

Equation (4) for $J^\pi=\frac{3}{2}^-$ was integrated over the range of the $3p$-$2h$ spectrum and it was found that $a=3.5$ yielded approximately 833 such states. Using this value of a, Eqs. (4) and (6) predict two levels within an interval of 80 keV about the immediate-structure resonance energy. This number is smaller than the number of $\frac{3}{2}^-$ fine-structure levels observed in the experiment. The $\frac{3}{2}^-$ intermediate-structure analog resonance is only at an incident proton energy of 1.97 MeV. This relatively low energy accounts for the small density of $3p$-$2h$ states predicted by Eq. (4) in the vicinity of the resonance.

Examining the experimental fine structure (cf. Fig. 2 and Table I) one notes that there are two very strong resonances. These two resonances probably correspond to the doublet observed by Jones *et al.*[3] From the discussion of the exponential distribution in the preceding paragraph, it seems plausible to associate these two strong levels primarily with $3p$-$2h$ structure. This is consistent with the very small value of the damping width $\Gamma\downarrow$. The theoretical calculation of $\Gamma\downarrow$ indicates that there are very few $3p$-$2h$ states that couple strongly to the $2p$-$1h$ analog doorway. One would then have to describe the remaining six weak $\frac{3}{2}^-$ high resolution resonances mainly in terms of $4p$-$3h$ and more complicated states. The density of such states in the vicinity of the analog resonance is expected to be much larger than that of the $3p$-$2h$ states. The importance of such complex states in the interpretation of the present data emphasizes the need for a more detailed microscopic description.

Lastly, using the expressions obtained by Mekjian and MacDonald[16] in a K-matrix approach to reaction theory, one predicts a damping width of about 5 keV for the present data. This is to be compared to the value of $\Gamma\downarrow=\Gamma_d-\Gamma\uparrow=1.3\pm0.4$ keV inferred from experiment and the microscopic shell-model prediction of ≈ 0.6 keV. The value of 5 keV is too large and probably arises because the authors[16] implicitly assume that fine structure is no more complicated than $3p$-$2h$ (or hallway) states.

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[16] A. Mekjian and W. MacDonald, Phys. Rev. Letters **18**, 706 (1967).