State-Dependent Effective Charge in the $2p$-$1f$ Shell*

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Motivated by a desire to understand the electric quadrupole transition rates in ^{58}Ni, calculations of the effective charge for both neutrons and protons were carried out, first with ^{40}Ca as a core and then with ^{56}Ni. The calculation was done in perturbation theory using the Kallio-Kolltveit interaction. Concerning state dependence, it was first observed and then proved that in the limit in which energy differences in the $2p$-$1f$ shell were small compared with $2\hbar\omega$ excitations, the effective charge depended on the initial and final *orbital* angular momentum of a given transition, but when these l's were specified, it was independent of the initial and final j values. The effective-charge correction was much bigger for a neutron than for a proton. This may have the effect of reducing the isovector part of the quadrupole operator and hence causing $\Delta T=1$ transitions to be inhibited. The effective charge is substantially larger with ^{56}Ni as a core than with ^{40}Ca as a core, but is somewhat too small to explain the $E2$ transition from the first excited 2_1^+ stage to ground. The effect of state dependence in the examples considered was to change certain $E2$ rates by a factor of 1.5 to 2. In ^{58}Ni, the $E2$ ratio $2_2^+\to0_1^+/2_2^+\to2_1^+$, if calculated with shell-model wave functions, is *extremely* sensitive to the two-body interaction that is used. For example, it is about *sixty* times smaller (and closer to experiment) if Kuo's matrix elements, which are derived from a realistic interaction, are used rather than matrix elements chosen to give a least-squares fit to the energy levels of the nickel isotopes. If only one of the matrix elements obtained from the energy fit is changed by 0.3 MeV, the ratio becomes forty-two times smaller, also closer to the experimental value. The possibility that a low-lying 2^+ state was basically a $3p$-$1h$ state was examined. The lowest two such states had very weak $E2$ transitions to ground and therefore did not at all resemble the one-phonon state or the electric quadrupole state. By themselves, these states fail as candidates, not only for the 2_1^+ state but also the 2_2^+ state because they radiate more to the ground than to the 2_1^+ state.

I. INTRODUCTION

THE need to keep the description of electromagnetic properties of nuclei restricted to the action of a few valence nucleons leads to the concept of "effective charge." Probably the best known example is ^{17}O, which in the shell model is described as one neutron outside closed shells. The zero charge of the neutron implies that electric transitions involving only a change of state in the neutron should be essentially zero. Still, the observed $E2$ transition between the ground and 0.87 MeV single-particle levels is comparable to the one in ^{17}F, where the transition can be ascribed to the proton outside closed shells. Thus, the valence neutron in ^{17}O behaves as if it had an "effective" electric charge of order unity. Mottelson[1] gives a simple description of the situation in terms of the extra neutron inducing a quadrupole moment in the core. From a more microscopic point of view this means that particle-hole configurations are admixed to the independent-particle states. These admixtures are taken into consideration by Horie and Arima[2] to account for the quadrupole moments in odd-neutron nuclei, and by Blin-Stoyle[3] for the deviations in magnetic moments. De Shalit[4] gives an estimate for the effective charge of neutrons arising from the protons in excited configurations on a shell model basis.

The ^{17}O case mentioned before was treated by several authors,[5] and it was found that core-excited configurations (or core polarization) account both for the ground-state quadrupole moment and the $E2$ transition rate.

Clearly the effective charge can be state dependent. Its origin lies in the particle-hole admixtures to the zero-order wave functions, and these admixtures can be different for different states. This "state dependence" of the effective charge can affect the calculated electric moments and transition rates. In this paper we examine these questions, motivated by the experimental data recently obtained in ^{58}Ni by the Rutgers[6] and Bartol[7] groups. Shell-model calculations,[8,9] using ^{56}Ni as a core and one effective charge for the neutrons, sometimes fail to reproduce electromagnetic transition rates and branching ratios by extremely large factors.[8] We present here a detailed calculation of effective

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[2] H. Horie and A. Arima, Phys. Rev. **99**, 778 (1955). See, also, R. D. Amado, and R. J. Blin-Stoyle, Proc. Phys. Soc. (London) **A70**, 532 (1957).

[3] R. J. Blin-Stoyle, Proc. Phys. Soc. (London) **A66**, 1158 (1953). See, also, A. Arima and H. Horie, Progr. Theoret. Phys. (Kyoto) **11**, 509 (1954).

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[8] S. Cohen, R. D. Lawson, M. H. Macfarlane, S. P. Pandya, and M. Soga, Physica **160**, 903 (1967).

[9] N. Auerbach, Phys. Rev. **163**, 1203 (1967).

charges with ^{40}Ca and ^{56}Ni cores, and study their state dependence. The sensitivity of the ^{58}Ni results to the state-dependence of the effective charge and to various modifications in the wave functions is also investigated in detail.

II. EFFECTIVE-CHARGE CALCULATION

The calculation of an $E(L)$ transition-matrix element up to first order in perturbation theory is indicated schematically in Fig. 1. The first graph represents the zero-order contribution which, of course, vanishes for a neutron. The remaining graphs give the effects of core polarization. The calculation was performed using interaction matrix elements derived from the Kallio-Kolltveit interaction.[10]

Let us write the quadrupole operator in the form $Q^L=Q^{L0}+Q^{L1}$, where 0 and 1 refer to isoscalar and isovector, and where, in particular

$$Q^{20}=[\sum_i (3\cos^2\theta_i-1)r_i^2]\tfrac{1}{2}(\epsilon_n+\epsilon_p),$$

$$Q^{21}=[\sum_i (3\cos^2\theta_i-1)r_i^2]\tfrac{1}{2}(\epsilon_n-\epsilon_p)\tau_0{}^1(i), \quad (1)$$

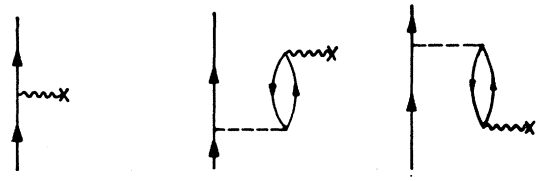
where $\tau_0{}^1=+1$ for a neutron and -1 for a proton and where, strictly speaking, $\epsilon_n=0$ and $\epsilon_p=1$. (Do not confuse the *epsilons* here with the e's we shall use to designate effective charges.) Indeed, in calculating the effective charges we set $\epsilon_n=0$ and $\epsilon_p=1$, but we shall also discuss the possibility of using other values in order to simulate higher-order effects.[11,12]

Up to first order in perturbation theory, the $E2$ matrix element for the transition of one particle beyond a closed shell from a state J_i to a state J_f can be written as

$$EL=E(L0)+E(L1),$$

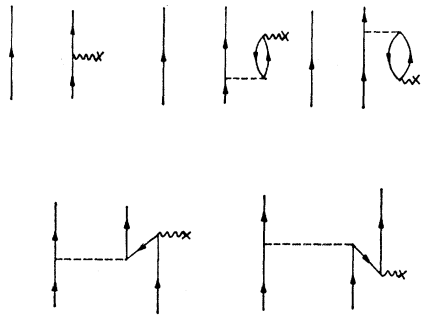
where

$$E(LT)=[T\tfrac{1}{2}\,0\,M_T\mid \tfrac{1}{2}\,M_T]\times\langle\psi^{Jf\,1/2}[(Q^{LT}+\delta Q^{LT})\psi^{Ji\,1/2}]^{Jf\,1/2}\rangle \quad (2)$$



+ exchange graphs

FIG. 1. Contributions to the effective charge up to first order in perturbation theory.



+ exchange graphs

FIG. 2. Zero-order and first-order graphs of electromagnetic transition rates when there are two particles beyond a closed shell. The last two graphs describe effects that are not included by using an effective charge.

and

$$B(EL)=[(2J_f+1)/(2J_i+1)]\mid EL\mid^2.$$

For brevity we designate the first-order matrix element by δQ, and we find

$$\delta Q^{LT}=\sum_{P,H}(1+P_{PH})(-1)^{P+H+L+T+1}\times\{(2H+1)/[(2P+1)(2J_i+1)(2J_f+1)\times 4]\}^{1/2}\times\sum_{I_0T_0}U(LHJ_fI_0,PJ_i)U(T\tfrac{1}{2}\,\tfrac{1}{2}\,T_0,\tfrac{1}{2}\,\tfrac{1}{2})\times\langle\psi^H[Q^{LT}\psi^P]^H\rangle(2I_0+1)(2T_0+1)\times\langle((1-P_{12})[J_fP]^{I_0T_0}\mid V\mid[J_iH]^{I_0T_0}\rangle/\Delta E, \quad (3)$$

where P is above the Fermi sea, H is below, and P_{PH} exchanges the labels P and H.

The above expression, or its equivalent, has been used by many people, starting from Horie and Arima.[2,13] We have nevertheless included it here for completeness.

Since we are interested in Ni^{58} we indicate in Fig. 2 the graphs for *two* particles beyond a closed shell. We note that all but the last two graphs can be taken into account by replacing the matrix element $\langle J_fQJ_i\rangle$ by $\langle J_fQ+\delta QJ_i\rangle$. In the last graph both nucleons are involved in polarizing the core. This graph is an effect beyond the effective charge. However, one should note that in this graph the particle-hole pair excited from the core is a *neutron* pair and therefore will not contribute unless we assign an effective charge due to higher-order processes. We have found that even if we assign an effective charge of one to the particle-hole pair, the size of this graph is negligible in comparison to the effective-charge graphs. The only exception is when one is dealing with very weak transitions.

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[11] W. J. Gerace and A. M. Green, Nucl. Phys. **A93**, 110 (1967).
[12] G. F. Bertsch, Nucl. Phys. **89**, 673 (1966).
[13] I. Hamamoto and A. Molinari, Phys. Letters **26B**, 649 (1968).

TABLE I. State dependence of the effective charge for ^{40}Ca and ^{56}Ni.

States involved	^{40}Ca e_n	^{56}Ni e_n	^{40}Ca e_p	^{56}Ni e_p
$1f_{7/2}$-$1f_{7/2}$	0.59	1.11	0.21	0.44
$2p_{3/2}$-$1f_{7/2}$	0.41	1.11	0.15	0.36
$1f_{5/2}$-$1f_{7/2}$	0.59	1.14	0.21	0.24
$2p_{3/2}$-$2p_{3/2}$	0.38	0.73	0.14	0.30
$1f_{5/2}$-$2p_{3/2}$	0.41	1.09	0.15	0.48
$2p_{1/2}$-$2p_{3/2}$	0.38	0.73	0.14	0.26
$1f_{5/2}$-$1f_{5/2}$	0.59	1.12	0.21	0.41
$2p_{1/2}$-$1f_{5/2}$	0.41	1.11	0.15	0.47

To obtain the effective charge we set $\epsilon_n=0$ and $\epsilon_p=1$ in the quadrupole operator. The effective charge depends on J_i and J_f and is written

$$e_n(J_iJ_f) \text{ for a neutron } [1+e_p(J_iJ_f)] \text{ for a proton}$$

$$e_{p,n}(J_iJ_f)=\frac{\langle\psi^{J_f}[\delta Q^{L0}-(1/\sqrt{3})\delta Q^{L1}]\psi^{J_i}\rangle}{\langle\psi^{J_f}[Q^0+(1/\sqrt{3})Q^1]\psi^{J_i}\rangle}.$$

Note that the effective charge for a neutron and proton are different as long as δQ^1 is finite and indeed it turns out that the neutron correction is larger than the proton correction. This has been noted by Horie and Arima[2] and others. Note that only the polarization of protons contributes to the EL transition. The valence proton interacts with the polarized protons only via a $T=1$ part of the interaction while the valence neutron can interact also via the $T=0$ part.

III. DISCUSSION OF THE EFFECTIVE CHARGE

In Table I, we present the effective-charge results obtained for a nucleon moving in Ca^{40} and Ni^{56} cores. The first column gives the single particle states involved in the transition. The following four columns present the e_n and e_p obtained for each transition in both cores.

The most striking feature is that the effective charge seems to depend on l_i and l_f, but for a given l_i and l_f it does not depend on j_i and j_f.[14] Thus for example the effective charges for the transitions $f_{7/2}$-$p_{3/2}$, $f_{5/2}$-$p_{3/2}$ and $f_{5/2}$-$p_{1/2}$ are all the same. Another feature of our result is that the effective charge for p orbitals is smaller than for f orbitals.

Although there are very many particle-hole pairs which contribute to the effective charge in ^{40}Ca, it is found that only a few give large contributions. These are $1f_{7/2}1p_{3/2}^{-1}$, $1f_{5/2}1p_{1/2}^{-1}$, $1g_{9/2}1d_{5/2}^{-1}$, and $1g_{7/2}1d_{3/2}^{-1}$. Note that in all cases the angular momentum of the particle is two units greater than that of the hole.

To see why the effective charge is bigger for an $f_{7/2}$-$f_{7/2}$ transition than for a $p_{3/2}$-$p_{3/2}$ transition, we examined in detail the case where the particle-hole pair was $1g_{7/2}$-$1d_{3/2}^{-1}$. This gave the largest single contribution to the effective charge. Note that in Eq. (3) there is a sum over T_0 and I_0. In the $f_{7/2}$-$f_{7/2}$ case the possible values of T_0 and I_0 are (0.2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), and (1, 5); in the $p_{3/2}$-$p_{3/2}$ considerably fewer values are possible—(0, 2), (0, 3), (1, 2), and (1, 3). Actually, the contribution for any given I_0, T_0 which are common to both will be bigger in the $p_{3/2}$-$p_{3/2}$ case. For example, (in arbitrary units) the contribution for $(I_0, T_0)=(0, 3)$ is 2.4 in the $p_{3/2}$-$p_{3/2}$ case but only 1.3 in the $f_{7/2}$-$f_{7/2}$ case. But this is *more* than compensated for by the fact that there are large (0, 4), (0, 5), and (1, 5) contributions (1.3, 3.1, and 3.9, respectively) which are present only in the $f_{7/2}$-$f_{7/2}$ case.

We thus conclude that the reason the effective charge is larger in the $f_{7/2}$-$f_{7/2}$ case than the $p_{3/2}$-$p_{3/2}$ is that there are more three-particle–one-hole intermediate states in the former case. Our result concerning the "j independence" of the effective charge should be of value to people who are interested in parametrizing the effective charge. In the case of ^{40}Ca only three parameters are required instead of eight (one for each single-particle transition).

Often in the literature the assumption is made that the effective-charge correction is the same for neutron and proton. Our results indicate that the effective charge for the neutron is more than twice as large as the effective charge for the proton. This trend is supported by an empirical analysis of transitions in the $f_{7/2}$ region[15] where it is found that about the same total charge is needed for neutron and proton to give the best fit to several quadrupole moment and $E2$ transition measurements.

If, indeed, it turned out that the neutron and proton had the same effective total charge, then the effective quadrupole operator would be pure isoscalar. It then follows that $\Delta T=1$ $E2$ transitions would vanish. It is not at present clear whether or not there is an inhibition of $\Delta T=1$ transitions in this region. Obviously, an empirical study of this point would be very interesting.

Another point in our results is that the effective charge correction in Ni^{56} is almost twice as large as the one in Ca^{40}. We can write the effective charge in Ni^{56} as

$$e_n=e_n^1+e_n^2,$$

where e_n^1 is due to excitations in which both the particle

[14] The same effect is observed in a recent calculation for the tin isotopes. M. Gmitro, A. Rimini, J. Sawicki, and T. Weber, Phys. Rev. Letters **20**, 1185 (1968).

[15] L. Zamick and J. D. McCullen, Bull. Am. Phys. Soc. **10**, 485 (1965).

and the hole are in the $2p$-$1f$ shell and hence involve relatively small energy denominators ($\sim$5 MeV), and e_n^2 to all other kind of excitations, which will involve $2\hbar\omega$ energy denominators ($\sim$20 MeV). It is then found that e_n^1 and e_n^2 are both significant. For instance for the $f_{5/2}$-$f_{5/2}$ transition, $e_n^1=0.41$ and $e_n^2=0.71$

IV. PROOF OF THE j INDEPENDENCE OF THE EFFECTIVE CHARGE FOR A CENTRAL INTERACTION

We must show that, in the limit where single-particle energy differences in the $2p$-$1f$ shell are small compared with $2\hbar\omega$ excitations, the quantity $e=\delta Q/Q$, which is proportional to

$$\langle\psi^j[V/(E-H)]Q\psi^{j'}\rangle/\langle\psi^jQ\psi^{j'}\rangle$$

does not depend on j and j'. We also assume V is a central interaction.

We take all $2p$-$1h$ intermediate states to have the same energy, $-E_I$. We can then write

$$\langle\psi^jV(E-H)^{-1}Q\psi^{j'}\rangle=(E-E_I)^{-1}\sum_I\langle\psi^jVI\rangle\langle IQj'\rangle,$$

where I represents a $2p$-$1h$ intermediate state. We now use the completeness relation

$$\sum_I I\rangle\langle I+\sum_{j''}j''\rangle\langle j''$$
$$+(\text{states which do not contribute})=1,$$

where $|j''\rangle$ is a $1p$ state. Hence,

$$\left\langle j\left|\frac{V}{E-H}Qj'\right.\right\rangle=\frac{\langle\psi^jVQ\psi^{j'}\rangle}{(E-E_I)\langle\psi^jQ\psi^{j'}\rangle}$$
$$-(E-\bar{E})^{-1}\langle\psi^jV\psi^{j'}\rangle.$$

The numerator of the second term above represents the potential energy of the nucleus in a j state. The difference of this quantity for different values of j is simply the single-particle splitting $\epsilon_j-\epsilon_{j'}$. This is divided by $E-E_I\approx 2\hbar\omega$ and precisely such ratios are being neglected. Alternatively, if one uses an interaction which has no two-body spin-orbit part, e.g. a central interaction, then one will not get any j dependence.

We next consider the numerator of the first term. It is convenient to regard the product VQ as a single entity. It is a rank-two tensor. Consider in particular $\langle\psi_m{}^jVQ_0\psi_m{}^{j'}\rangle$. We expand the single-particle wave functions in terms of m_l and m_s and get this to equal

$$\sum_{m_lm_s}(jm\,|\,l\tfrac{1}{2}m_lm_s)(j'm\,|\,l'\tfrac{1}{2}m_lm_s)$$
$$\times\langle\psi_{m_l}{}^lX_{m_s}{}^{1/2}VQ_0{}^2\psi_{m_l}{}^lX_{m_s}{}^{1/2}\rangle.$$

There will be no cross terms in m_l and m_s because VQ_0 commutes with both L_z and S_z.

We can now apply the Wigner-Eckart theorem to the *orbital* angular momentum and get

$$\langle\psi_m{}^jVQ_0\psi_m{}^{j'}\rangle=[\sum_{m_lm_s}(jm\,|\,l\tfrac{1}{2}m_lm_s)(j'm\,|\,l'\tfrac{1}{2}m_lm_s)$$
$$\times(2l'0m_l\,|\,lm_l)]\langle\psi^l[VQ^2\psi^{l'}]^l\rangle.$$

We have thus factored the expression and we note that the second factor has no j dependence. We can factor the denominator in the same way and we thus obtain

$$\langle\psi_m{}^jVQ_0\psi_m{}^{j'}\rangle/\langle\psi_m{}^jQ_0\psi_m{}^{j'}\rangle=\langle\psi^l(VQ^2\psi^{l'})^l\rangle/\langle\psi^l(Q^2\psi^{l'})^l\rangle.$$

The right-hand side is clearly independent of j and j'.

V. STATES OF ^{58}Ni IN THE TWO-PARTICLE MODEL

Shell-model calculations of the nickel isotopes, including ^{58}Ni, were carried out by Auerbach,[9] McGrory,[16] Lawson, MacFarlane, and Kuo,[17] and by S. Cohen *et al.*[8] (see Ref. 18). For Ni^{58} the calculation is very easy—the configuration consists of a closed ^{56}Ni core and two particles in the $p_{3/2}$, $f_{5/2}$ and $p_{1/2}$ orbitals. The calculation of Auerbach and of the Argonne group is similar in that both made a least-squares fit of interaction-matrix elements, the criterion being that the energy levels are to be fitted as well as possible; indeed, very good fits to energy levels were obtained. This leads the authors to believe that the low-lying states—in particular the 0_1^+, 2_1^+, and 2_2^+ states—can basically be described as "effective" two particle shell model states. In simple cases, the hope is that an effective charge will take care of more complicated configurations. But in general transition rates depend so drastically on the detailed structure of the wave functions that the use of simply an effective charge is an oversimplification. This is dramatically exemplified in the case of ^{58}Ni. The $E2$ transition rates involving the three states mentioned above come out rather poorly.[6] They were calculated in the same way by both Auerbach and Cohen *et al.*: an effective charge was chosen so that the $2_1^+\rightarrow 0_1^+$ transition is fitted to the experimental value. The *same effective charge* was then used for all the other transitions. An effective charge of 1.9 was needed by Cohen *et al.* and about the same by Auerbach. (In his report, Auerbach quotes a value 1.1, but we repeated the calculation and found that 1.9 is the correct value.)

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[16] J. B. McGrory (private communication).

[17] R. D. Lawson, M. H. Macfarlane, and T. T. S. Kuo, Phys. Letters **22**, 168 (1968).

[18] It should be mentioned that the shell-model calculations of Refs. 8 and 9 were performed by fitting the 2.78 and 2.94 MeV levels as having, respectively, spins 0^+ and 2^+. Later assignments (Ref. 7) show that the correct spins are 2^+ and 0^+, respectively. A fitting to the corrected spins could therefore yield slightly modified two-particle matrix elements.

TABLE II. Transition rates in ^{58}Ni.

Transition	Experiment (e^2f^4)	Cohen *et al.* $e_n=1.9$	Auerbach $e_n=1.9$	Kuo's matrix elements $e_n=1.9$	Kuo's matrix elements $e_n=1.1$	Kuo's matrix elements; our effective charge	Auerbach modified $e_n=1.9$
$2_2^+\rightarrow 0_1^+$	0.36 ± 0.11		17.6	38.5	13.0	9.5	9.2
$2_2^+\rightarrow 2_1^+$	190 ± 60		0.61	76.8	25.6	16	13.4
$2_2^+\rightarrow 0_1^+/2_2^+\rightarrow 2_1^+$	1.9×10^{-3}	33.1	29.00	0.51	0.51	0.59	0.69

This should be compared with the values of 0.73 and 1.1 that we obtain for p-p and for f-p or f-f transitions, respectively. It is perhaps worth mentioning that the inclusion of higher-order corrections would increase the values we obtained.[11,12] The nucleons involved in the particle-hole excitations describing the polarization of the core would also acquire an effective charge arising from higher-order excitations. This could be simulated by changing the values of ϵ_n and ϵ_p in the definition of the quadrupole operator from the bare values 0 and 1 to larger ones.[12]

We focus our attention on the ratio $2_2^+\rightarrow 0_1^+/2_2^+\rightarrow 2_1^+$. Experimentally, this ratio is very small, i.e., there is hardly any cross-over transition; theoretically, the ratio is very large. The disagreement between theory and experiment is of the order 10^4. The experimental ratio is easily explained on the basis of a vibrational model. The 0_1^+ state would be a zero-phonon state, the 2_1^+ a one-phonon state and the 2_2^+ state a two-phonon state. There cannot be any transition between a pure two-phonon state and a zero-phonon state. In the rest of the even Ni isotopes such a difference in the predictions of the shell model and the vibrational model does not arise. The cross-over inhibition can be explained in terms of shell-model wave functions for the 2_2^+ states containing mainly seniority 2, which cannot therefore be identified as two-phonon states.[8] It is therefore of interest to examine more closely the case of ^{58}Ni. Its spectrum can be nicely fitted using the same interaction as for the rest of the Nickel isotopes, by two-particle wave functions which in no way resemble a one- or two-phonon state.

Our initial idea was to investigate to what extent the concept of one effective charge is an oversimplification, and see if a possible state dependence of it could be of importance in the theoretical estimates of the transition rates.

But before discussing the state dependence of the effective charge, we consider an even more basic question: are the results for the $B(E2)$'s sensitive to the two body interaction (or matrix elements) that is used? To answer this question we redid the Ni^{58} calculation, using the matrix elements of Kuo, which were derived from the realistic Hamada Johnston two-nucleon interaction. These matrix elements had already been used, of course, by Lawson, Macfarlane and Kuo.[16]

The results are listed in Table II. We find that there is a *tremendous difference in the results* of using *Kuo's* matrix elements compared to those of Auerbach. In particular, the ratio $2_2^+\rightarrow 0_1^+/2_2^+\rightarrow 2_1^+$, which was about 30 in the results of Auerbach and Cohen *et al.*, is now 0.51 with Kuo's matrix elements. This is much closer to experiment, although there is still a long way to go. If one examines the individual contributions of each shell-model configuration to the $B(E2)$'s it becomes apparent that the main reason for the difference between the results obtained using Kuo's or Auerbach's matrix elements is the vanishing amplitude of the $p_{3/2}p_{1/2}$ configuration in the 2_2^+ state in Auerbach's

TABLE III. Wave functions of 2_1^+ and 2_2^+ states in Ni^{58}.

Matrix elements	State	$p_{3/2}^2$	$p_{3/2}f_{5/2}$	$p_{3/2}p_{1/2}$	$f_{5/2}^2$	$f_{5/2}p_{1/2}$
Auerbach	2_1	0.696	0.180	−0.633	0.191	0.215
Auerbach modified	2_1	0.684	0.225	−0.602	0.233	0.253
Kuo	2_1	0.875	0.163	−0.356	0.241	0.154
Auerbach	2_2	0.422	−0.674	0.001	−0.478	−0.373
Auerbach modified	2_2	0.691	−0.448	0.378	−0.327	−0.268
Kuo	2_2	0.482	−0.252	0.669	−0.373	−0.343

case. Moreover, comparing both sets of matrix elements one can see that $\langle p_{3/2}p_{1/2}J=2 \mid V \mid p_{3/2}2J=2\rangle$ is the one that differs most. Auerbach quotes a value of 0.83 MeV while Kuo gives 0.27 MeV. We decided then to check the sensitivity of Auerbach's results to a small change in the $\langle p_{3/2}p_{1/2}J=2 \mid V \mid p_{3/2}{}^2J=2\rangle$ matrix element only. We modified it from 0.83 to 0.5 MeV. In Table III we list the wave functions for the $2_1{}^+$ and $2_2{}^+$ states for the three sets of matrix elements: Kuo's, Auerbach's without and with the single modification mentioned above. The change induced by the small change in one of Auerbach's matrix elements is very appreciable only in the $2_2{}^+$ state,[19] as can be seen in Table III. Next, we go on to calculate the transition rates with the new wave functions obtained for the 2^+ state. The results are included in Table II. It should be noted that the effect of modifying one matrix element by 0.3 MeV is indeed striking. The ratio $2_2{}^+\to 0_1{}^+/2_2{}^+\to 2_1{}^+$ goes from the previously mentioned value of about 30 to a new value 0.69. Also the individual transitions, though still far from being satisfactorily explained, are in better agreement. In particular the $2_2{}^+\to 2_1{}^+$ rate goes from 0.61 e^2 to 13.35 e^2f^4, to be compared with 190 ± 60 e^2f^4 obtained experimentally.

Let us look at the transitions in more detail. The two-particle wave functions are of the form

$$\psi^I=\sum C^I(J_1J_2)\,(\psi^{j_1}\psi^{j_2})^I,$$

and the transition matrix element has the structure

$$\sum_{j_1j_2j_3j_4} C^{I_i}(J_1J_2)C^{I_f}(J_3J_4)\,\langle[J_3J_4]^{I_f}[Q[J_1J_2]^{I_i}]^{I_f}\rangle.$$

We can thus speak of a component of the $E2$ matrix element, where each component is specified by the numbers j_1, j_2, j_3, and j_4. The individual contributions are listed in Tables IV and V for an effective charge $e_n=1.1$. Table IV gives the $2_2\to 0_1$ and Table V the $2_2\to 2_1$ contributions. The first columns give j_1, j_2, j_3, and j_4. The following three columns correspond to the case of Kuo, Auerbach, and Auerbach modified wave functions, respectively. It turns out that in the $2_1\to 0_1$ transition, all components have the same phase. This explains why this transition is so strong, and also why the transition is not very sensitive to small changes in the wave function.

TABLE IV. Individual contributions to the transition rate $2_2\to 0_1$.

j_1	j_2	j_3	j_4	Kuo	Auerbach	Auerbach modified
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	2.25	+0.00	+1.26
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	4.00	+0.01	+2.25
$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	−4.07	−3.52	−5.75
$\frac{5}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1.17	1.28	0.89
$\frac{5}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	1.15	1.28	0.93
$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	0.82	2.17	1.44
$\frac{3}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	0.45	1.24	0.83
$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	2.28	2.99	2.07

TABLE V. Individual contributions to the transition rate $2_2\to 2_1$.

j_1	j_2	j_3	j_4	Kuo	Auerbach	Auerbach modified
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	1.07	+0.00	+1.08
$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	−1.09	−1.70	−2.77
$\frac{5}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	0.16	0.31	0.22
$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	0.13	1.04	0.69
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	3.71	+0.00	+1.69
$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	−0.54	−1.16	−0.75
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	0.13	+0.00	+0.15
$\frac{5}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	0.27	0.41	0.30
$\frac{3}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	0.09	0.35	0.25
$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	0.32	0.58	0.40
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	0.27	+0.00	+0.20
$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	0.19	0.19	0.31
$\frac{5}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	0.13	0.16	0.12
$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	−0.06	−0.02	−0.01
$\frac{5}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	0.07	0.09	0.06
$\frac{5}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	0.47	0.40	0.30
$\frac{3}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	0.07	0.14	0.10
$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	−0.33	−0.34	−0.23

In the $2_2\to 0_1$ transition it was observed that all components except *one*, had the same sign. The exception was the case when $j_1=j_2=p_{3/2}$ and $j_3=j_4=p_{3/2}$. We remember that the $2_2\to 0_1$ transition is very weak, essentially zero. Theoretically, the only way to get cancellation is for the one component $p_{3/2}{}^2\to p_{3/2}{}^2$ to cancel all the rest. But this simply does not happen, because, as can be seen from Table III, there are many large components other than $p_{3/2}{}^2$. The use of a state dependent effective charge could have helped provided that somehow, the effective charge for $p_{3/2}\to p_{3/2}$ were much larger than in other states. On the contrary, our calculation indicates that it is smaller. It thus seems impossible on the $2p$ model to get a cancellation of this matrix element because of the fact that the burden of the cancellation is on only one of many terms.

In the $2_2{}^+\to 2_1{}^+$ there are first of all many more terms than in the $2_2{}^+\to 0_1{}^+$ transition, and there are several positive and several negative terms. In such a situation

[19] G. T. Garvey and I. Kelson, Phys. Rev. Letters **16**, 197 (1966).

the result is extremely sensitive to the details of the wave functions.

It should be emphasized that the large changes in the $E2$ rates for different interactions are due not to small changes in the wave functions but rather to very large ones (see Table III). The statement is that small changes in the two-body matrix elements can induce large changes in the wave functions and hence large changes in $E2$ transition rates.

In the above we refer to changes in the two-particle wave functions. One should not forget that the use of an effective charge masks the effect of many configurations other than $2p$, e.g. $3p$-$1h$, that are contributing to the transition (without them the effective charge would be zero). These other configurations may have a very small probability, but still they seriously affect the transition rates.

VI. POSSIBILITY OF LOW-LYING THREE-PARTICLE-ONE-HOLE STATES

In calculating the effective charge, one treats the $3p$-$1h$ states as weak perturbations of the basic $2p$ states. But what if one of the low-lying states was a $3p$-$1h$ state with a possible weak admixture of a $2p$ state? Then the above approach would break down.

The one-phonon state is mostly a $3p$-$1h$ state, and since it has sometimes been offered as a candidate for the 2_1^+ state, it would be of interest to see if the lowest $3p$-$1h$ state obtained by diagonalization resembles a one-phonon state, or alternately resembles the 2_1^+ state.

Closely related to the above is the electric quadrupole state which is simply the product of the ground-state wave function and the electric quadrupole operator. Such a state would exhaust the $E2$ sum rule. It was mentioned as a possibility by Cohen *et al.*[8] (they did not necessarily believe it) that the 2_1^+ state could largely be such a state. Clearly the quadrupole state is also mostly a $3p$-$1h$ state and again it is of interest to compare with the $2p$-$1h$ state obtained by diagonalization.

Also, if it turns out that the 2_1^+ state is not a $3p$-$1h$ state, then maybe the 2_2^+ state is.

The first problem is to estimate the energies of the $3p$-$1h$ states. We consider two cases, depending on whether the isobaric spin of the three particle is $T_p=\frac{1}{2}$ or $T_p=\frac{3}{2}$.

In estimating the energy, we at first neglect the particle-hole interaction. In the $T_p=\frac{1}{2}$ case the excitation energy is about $E(^{59}\text{Cu})-E(^{56}\text{Ni})+E(^{55}\text{Co})-E(^{58}\text{Ni})=3.9$ MeV. In the $T_p=\frac{3}{2}$ case it is $E(^{59}\text{Ni})-E(^{56}\text{Ni})+E(^{55}\text{Ni})-E(^{58}\text{Ni})=5.9$ MeV. [The nucleus ^{55}Ni is not listed in the mass tables. It was obtained by using the Garvey-Kelson mass formula[19] $E(^{55}\text{Fe})-E(^{55}\text{Ni})=E(^{54}\text{Fe})-E(^{54}\text{Co})+E(^{56}\text{Co})-E(^{56}\text{Ni})$.]

Both energies above are higher than the energies of the 2_1^+ and 2_2^+ states (1.45 and 2.77 MeV), but not so high that they could be ruled out with certainty.

The diagonalization of the $J=2$ $3p$-$1h$ states was carried out in two stages. First the $3p$ states were diagonalized, i.e. ^{59}Ni and ^{59}Cu, and then an $f_{7/2}$ hole was coupled to the low-lying $3p$ wave functions.

The first two low-lying $3p$-$1h$ states had a very simple structure. They were mixtures of two states, one where the $3p$ coupled to $J=\frac{3}{2}$ $T=\frac{1}{2}$ and one where they coupled to $J=\frac{3}{2}$ $T=\frac{3}{2}$. The first component dominated in the lowest state despite the fact that the particle-hole interaction brought the $T_p=\frac{3}{2}$ state down towards the $T_p=\frac{1}{2}$ state.

Since the precise mixing of the $T_p=\frac{1}{2}$ and $T_p=\frac{3}{2}$ states is very sensitive to the parameters that are used, it makes more sense to calculate the transitions for each of these states separately.

Here we calculated the following quantities:

(a) The transition of the $3p$-$1h$ state $J_p=\frac{3}{2}$ $T_p=\frac{1}{2}$ to the 0_1^+ $2p$ state with the result $B(E2)=4.7\ e^2f^4$.

(b) To the 2_1^+ $2p$ state with the result $B(E2)=0.75\ e^2f^4$.

(c) The transition of the $3p$-$1h$ $J_p=\frac{3}{2}$ $T_p=\frac{3}{2}$ to the 0_1^+ $2p$ state, with the result $B(E2)=9.3\ e^2f^4$.

(d) To the 2_1^+ $2p$ state with the result $B(E2)=2.8\ e^2f^4$.

An effective charge of 1.1 and 2.1 were used for the neutron and proton, respectively.

We see that both the states above decay very weakly to ground and hence are not likely candidates for the 2_1^+ state. Secondly, the transition to ground, though weak, is stronger than the transition to the (two particle) 2_1^+ state. This again is contrary to experiment. We found further that the sign of the mixing amplitude between $T_p=\frac{1}{2}$ and $T_p=\frac{3}{2}$ in the lowest $3p$-$1h$ state was such that the $3p$-$1h$ transition to ground would not be cancelled, but rather enhanced by a mixture of these two basic states.

It is possible to concoct a mixture of $3p$-$1h$ and $2p$ states whose decay to ground would be negligible, but since it is so easy to control the mixing amplitudes by adjusting certain parameters, especially the diagonal energies, it is hard to know whether such a calculation should be believed or not.

What *can* be said with reasonable confidence is that the 2_1^+ state is not basically a $3p$-$1h$ state and the 2_2^+ state is not a pure $3p$-$1h$ state.

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