Nucleation of Superconductivity above H_{c3}†

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(Received 8 July 1968)

The recent calculations of van Gelder concerning an upper critical field $H_{c4}>H_{c3}$ for a wedge-shaped specimen are discussed in the light of the surface-nucleation field of a slab of finite thickness. It is shown that H_{c4} is not a new critical field and that it is related to the size-dependent surface-nucleation field.

IN a recent paper van Gelder[1] has suggested that two intersecting vacuum interfaces might have a nucleation field above H_{c3}. He calculates a critical field H_{c4} which, for small values of the angle 2α between the vacuum interfaces, is

$$H_{c4}\geq(\sqrt{3}/2\alpha)H_{c2}\,, \tag{1}$$

where H_{c2} is the bulk-nucleation field $\sqrt{2}\kappa H_c$. He claims that as $\alpha\rightarrow 0$ the value of $H_{c4}\rightarrow\infty$, from which he concludes nucleation of superconductivity for any field above H_{c3}.

We want to point out that H_{c4} is a misinterpretation of the surface-nucleation field H_{c3}, and that critical fields larger than $1.695H_{c2}$ are a well-known fact, theoretically as well as experimentally.

Let us first consider the case of an infinite slab of thickness d with the applied magnetic field parallel to the surface planes. Ginzburg and Landau[2] have shown that, when $d\ll\xi$, a type-I superconductor nucleates at a second-order phase transition point and that the nucleation field H_n is

$$\frac{H_{c2}}{H_n}=\frac{1}{\sqrt{12}}\frac{d}{\xi}. \tag{2}$$

Saint-James and de Gennes[3] have calculated the nucleation field for a slab of thickness d, and if one converts their figure, which is plotted in reduced units, one gets the same results as Eq. (2) for $d/\xi\ll 1$. A similar plot was obtained by Schultens[4] and also by the author[5] in a general investigation of the nucleation of superconductivity at a second-order phase transition point. In Fig. 1 we show our results for H_{c2}/H_n as a function of d/ξ. The surface-nucleation field H_n is size-dependent, and $H_n=1.695H_{c2}$ when $d\gg\xi(T)$. When $d\lesssim\xi$, Eq. (2) applies and the size-dependent critical field and the surface-nucleation field $H_n(d)$ are the same and indistinguishable. This result is essentially contained in Refs. 2–5 for a slab, and in Refs. 6 and 7 for a cylinder, though the results are plotted in reduced units which do not make the size dependence transparent on first sight. When $d/\xi<1.84=d_0/\xi$,[4,5] no vortices appear in the slab, and when $d>d_0$, a vortex structure appears at H_n which has a similarity to the vortex structure of the mixed state in a bulk specimen. At H_n, superconductivity nucleates near (not at) the surface for $d>d_0$, and when $d<d_0$, nucleation occurs in the center of the slab.[5] When $d/\xi\lesssim 1.6$, Eq. (2) is well satisfied.

If we now consider a slab whose surfaces are almost parallel ($2\alpha\ll\frac{1}{2}\pi$), then the thickness of the slab is a function of y, the distance from the vertex, as shown in Fig. 1. The direction of the magnetic field is perpendicular to the paper. Because $2\alpha\ll\frac{1}{2}\pi$, the above considerations for the slab will also be approximately applicable here. As surface nucleation is tied to the surface for $d>d_0$ and to the center of the slab for $d<d_0$, we readily see that superconductivity will nucleate at some part of the wedge-shaped sample for all fields $H_0\geq 1.695\ H_{c2}$, owing to the size dependence of H_n and not to some new mechanism of nucleation. When 2α is increased to π, the specimen becomes a semi-infinite half-space and surface nucleation occurs at $1.695H_{c2}$. Therefore, it is not difficult to see that for a wedge-shaped sample the value of $H_n(d(y))$ must be smaller than the corresponding value of $H_n(d)$ for a slab with parallel surfaces.

Hence we may conclude that the value of H_{c4} of Ref. 1 must be smaller than the corresponding size-

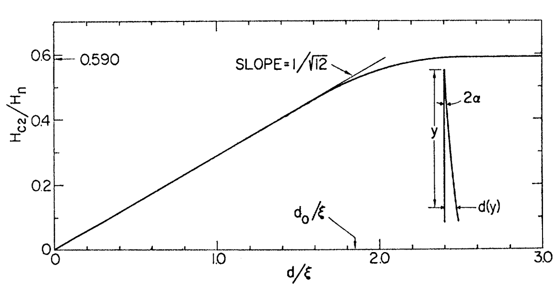

FIG. 1. The surface-nucleation field H_n of an infinite slab as a function of thickness d. $\xi=\xi(T)$ is the temperature-dependent coherence length and $H_{c2}=\sqrt{2}\kappa H_c$. The applied magnetic field is parallel to the surfaces of the slab. For details regarding the wedge-shaped specimen, see the text.

† Based on work sponsored by the Metallurgy Branch, Division of Research, U. S. Atomic Energy Commission, under Contract No. AT(04-3)-701.

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[7] C. Dalmasso and E. Pagiola, Nuovo Cimento **35**, 812 (1965).

dependent surface-nucleation field for a slab with parallel surfaces when related to a certain thickness $d \approx 2\alpha y$ on the wedge-shaped specimen. The author does not believe that one can obtain a critical field larger than the size-dependent surface-nucleation field (corresponding to a certain geometry) which is different in nature from the surface-nucleation field[3] within the framework of the linearized Ginzburg-Landau equations.

Erratum

Moment-Method Calculation of Magnetization and Interspin-Energy Diffusion, ALFRED G. REDFIELD AND W. N. YU [Phys. Rev. **169**, 443 (1968)]. The calculated exchange energy diffusion coefficient D_E was in error; it should be half as large as given because the denominator in the exchange-diffusion version of (13) is $\hbar^2 \mathrm{Tr} E_i \sum_j E_j$, not $\hbar^2 \mathrm{Tr} E_i^2$. Therefore, the following changes should be made: In the abstract, and in Eq. (34), change 0.67 to 0.34; in the next to last paragraph of Sec. IV A, delete both occurrences of "twice"; in the last paragraph of Sec. IV A, change "good" to "poor." We thank Professor D. L. Huber for pointing out this error.