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Energy-Loss Straggling of Protons in Silicon*

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The energy-loss straggling distributions for protons incident on a 200-μ silicon surface-barrier detector were measured in the energy region between 5 and 42 MeV. Experimental results were compared to the predictions of the Vavilov theory and found to be in excellent agreement at the higher energies. However, a systematic difference between theory and experiment was observed for proton energies less than 20 MeV, the discrepancy increasing with decreasing incident proton energy. This effect is due to appreciable variation of proton energy over the thickness of the detector. Tschalär's distribution functions, which specifically consider this variation, yield good fits to the data for energy losses as much as 80% of the incident proton energy.

I. INTRODUCTION

THE energy lost by a charged particle in a thin absorber is subject to fluctuations, because of the statistical nature of the energy-loss process. Thus, the energy Δ deposited in such an absorber by a monoenergetic beam of charged particles will be characterized by a distribution function $\phi(\Delta)$ known as the energy-loss straggling distribution. For moderately thick absorbers, that is, for those in which the mean energy loss $\bar{\Delta}$ is much greater than the maximum energy $\epsilon_{\max}$ which can be transferred to an electron in a single collision, Bohr[1] showed that the distribution function is Gaussian. Landau[2] investigated the opposite extreme in which $\bar{\Delta}$ is much less than $\epsilon_{\max}$, and found that the distribution function in this case is highly asymmetric with a pronounced high-energy tail.

Symon[3] and Vavilov[4] have derived theories depending on $\epsilon_{\max}$ which predict distributions intermediate between the Gaussian and the Landau forms. The parameter describing their distributions is the ratio $\kappa=\xi/\epsilon_{\max}$, where the parameter ξ is directly related to $\bar{\Delta}$ and is defined in Ref. 5. For $\kappa \ll 1$, the predicted distribution is of the Landau type, while for $\kappa \gg 1$ it becomes Gaussian.

Previous work at isolated values of κ in the intermediate region[6,7] and a systematic survey[8,9] of the Landau transition region ($\kappa<0.1$) have shown good agreement between the predictions of Vavilov or Symon and experiment. We have investigated the Gaussian transition region ($\kappa>1$), obtaining data for protons incident on silicon in the range $0.4\leq\kappa\leq 30.4$.

II. EXPERIMENTAL METHOD

A. Beam Line

Figure 1 is a schematic diagram of the elements of the beam line. A proton beam from the Michigan State University isochronous cyclotron was focused by a set of quadrupole doublets on the object slit S1 of an energy analysis system. To facilitate rapid variation of the energy of the incident beam in small steps, a degrader was placed 15 cm in front of this slit. The degrader, an aluminum wedge 7.5 cm long by 2.5 cm wide, had a maximum thickness of 0.9 cm and tapered to a sharp edge. It could be remotely positioned in the path of the beam. In this way, we were able to rapidly change the energy of the beam in steps of 2 MeV over the energy region from 20 to 42 MeV with a primary beam of 42.4±0.1 MeV, and from 4 to 26 MeV with a primary beam of 28.4±0.1 MeV. The lowest energy obtainable in the first case was determined by loss of beam intensity due to multiple scattering and energy straggling effects in the degrader.

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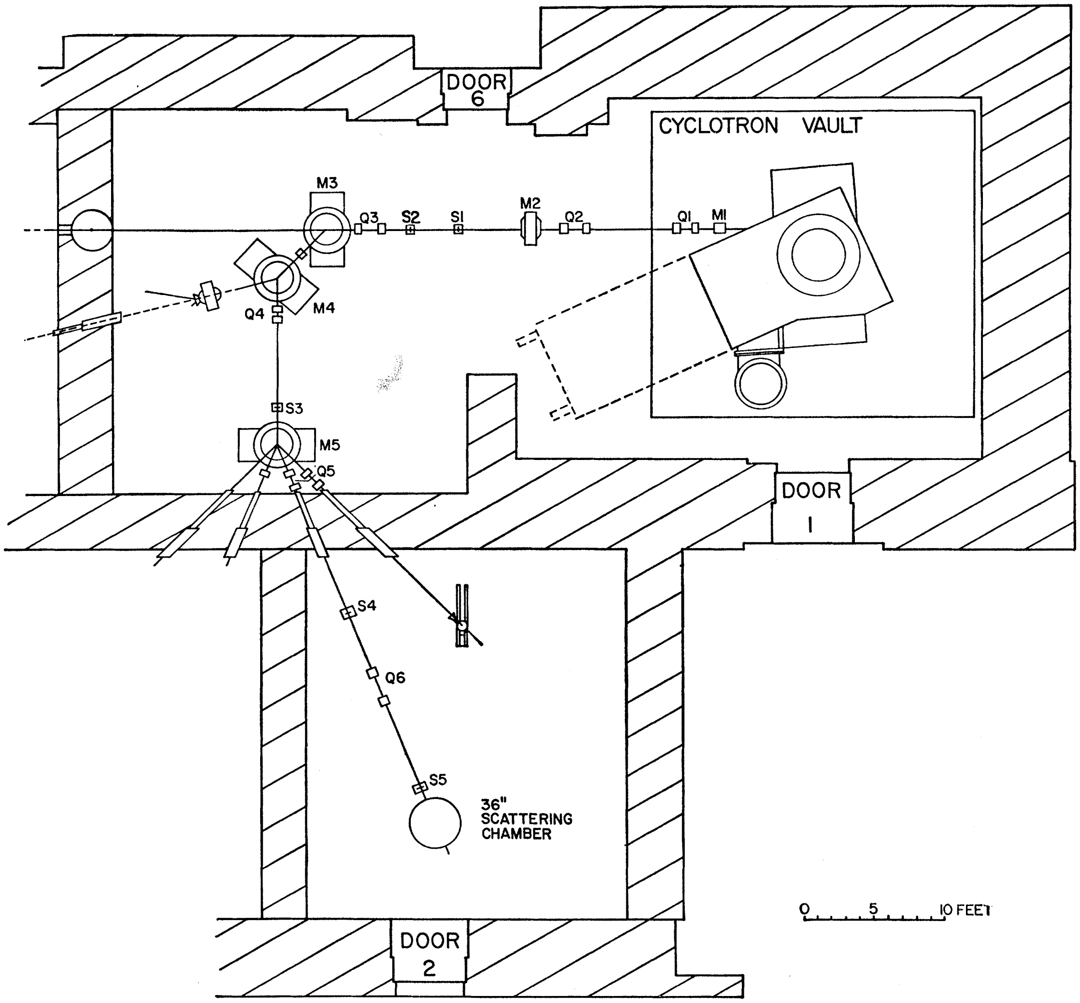

FIG. 1. Scale drawing of the beam transport system. The locations of the defining apertures (S1–S5), quadrupole doublets (Q1–Q6), and bending magnets (M3–M5) are indicated.

The degraded beam was transmitted through the object slit S1 and the divergence limiting slit S2, bent through 90° by the energy analyzing magnets M3 and M4, and focused on the image slit S3. The properties of this beam transport system have been investigated previously. In particular, the energy resolution of the transmitted beam as a function of slit openings,[10] and the energy of the analyzed beam as a function of magnet field strength[11] have been calculated. The slit widths used were 0.25 by 0.25 cm for the object and image slits, and 0.30 by 0.30 cm for the divergence slit. With these settings, the energy resolution of the transmitted beam is 8 parts in 10^4 full width at half-maximum (FWHM). The magnetic field at M3 was monitored throughout the experiment by a NMR field probe, and the measured field strengths were used to calculate the transmitted beam energy to within ±0.1 MeV.

The analyzed beam was deflected into the experimental area by magnet M5, and was focused first on a 0.95-cm-square slit S4, and then on a 0.3-by-0.5-cm defining aperture in the scattering chamber.

B. Detector and Electronics

A 200-μ totally depleted silicon surface-barrier detector with an active region 0.7 cm in diam was placed in the direct beam line 30 cm from the defining aperture at the entrance to the scattering chamber, and oriented to be perpendicular to the beam to within ±0.5°. The thickness of the detector was determined

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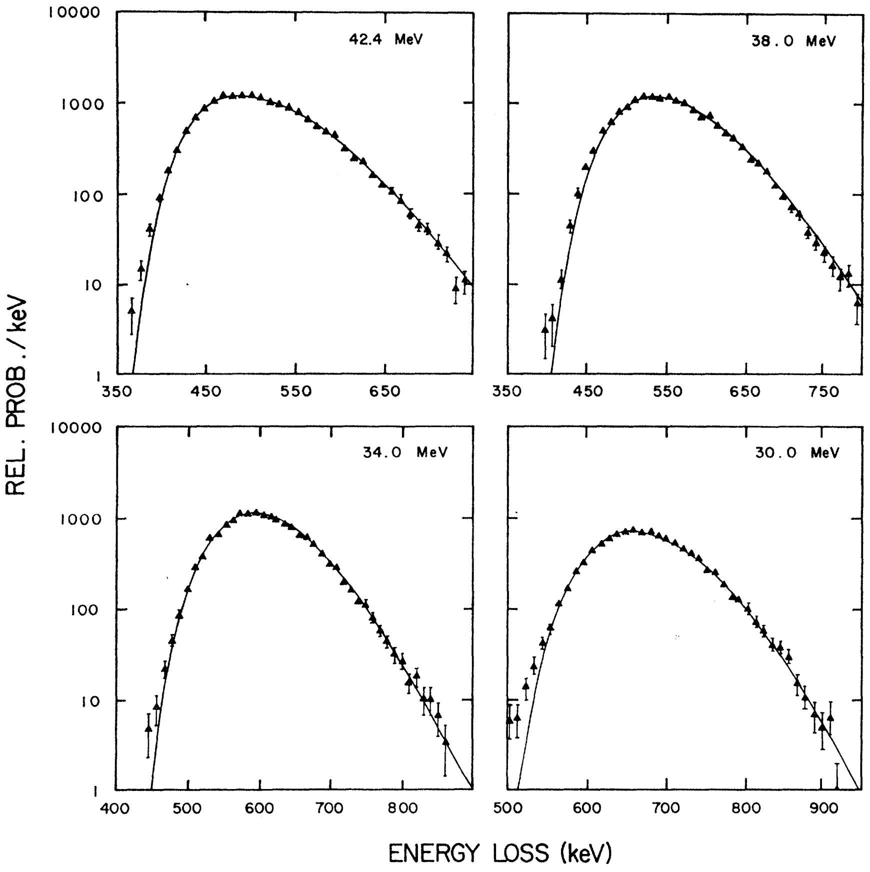

FIG. 2. Straggling distributions for 30.0- to 42.4-MeV protons. Error bars indicate statistical uncertainties only. The solid curves are the predictions of the Vavilov theory.

to be 45.6±0.4 mg/cm^2 by fitting the mean proton energy loss from 5 to 12 MeV to the range-energy table for silicon.[12] In a preliminary analysis assuming this thickness, the measured mean energy loss agreed with the table to within ±0.5% from 12 to 42 MeV.

The signals from this detector were amplified by a charge sensitive FET preamplifier followed by a shaping amplifier with 2-μsec time constants. The resulting pulse-height spectra were analyzed, using a conventional 1024-channel analyzer. The best energy resolution obtained was 16.4-keV FWHM, measured with a 4-MeV proton beam which stopped in the detector.

The gain and zero point of the detector-amplifier-analyzer system were monitored by inserting test pulses from a precision pulser into the preamplifier. The data were partially analyzed during the course of the experiment using the SDS Sigma 7 computer at the cyclotron laboratory, as an additional check against drifts.

The beam current extracted from the cyclotron was adjusted to give a count rate in the detector of less than 1000 counts/sec, in order to avoid pulse pile-up problems. The current required to maintain this rate varied from less than 10 nA for minimum degradation, to more than 1 μA for maximum degradation, in which case multiple scattering and energy-loss straggling effects in the degrader cause large beam losses.

III. ANALYSIS AND RESULTS

Energy-loss spectra for protons in the detector described are shown in Figs. 2–5. The error bars shown indicate statistical uncertainties only. The solid curves are the predictions of the Vavilov theory, as calculated by a FORTRAN program originally written by Seltzer and Berger[5] and modified by one of us (J.K.) for the Sigma 7 computer. The theoretical distributions obtained have been normalized to the experimental data, using a least-squares fitting criterion. In addition, they have been corrected for the finite resolution of the detection system and the energy spread in the incident beam. The detector resolution (16.4 keV) and the beam width (8 parts in 10^4) were added in quadrature, and the resultant Gaussian resolution function was folded into the theoretical distributions. This correction

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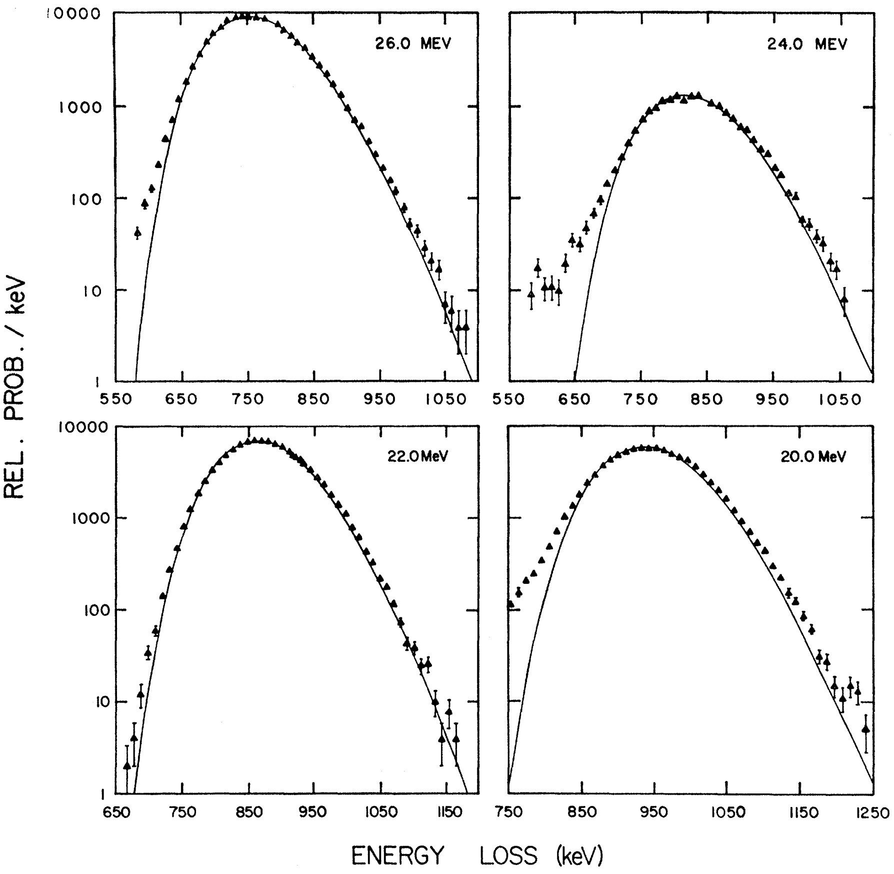

FIG. 3. Straggling distributions for 20.0- to 26.0-MeV incident proton energy.

was small compared to the total width of the distribution. For example, the full width at half-maximum of the resolution function was 34.5 keV for the data taken at 38 MeV. The corresponding correction to the width of the theoretical distribution was 2.9%, increasing it from 137- to 141- keV FWHM.

The agreement between theory and experiment is excellent for incident proton energies greater than 30 MeV; the Vavilov theory correctly predicts the shape of the distribution to 1% of the peak value. At 30 MeV and below, the probability of small energy losses seems to be enhanced above the theoretical prediction in some cases. This is not a systematic discrepancy, as can be seen by comparing the data taken at 22 MeV with the 24- and 26-MeV data, and again from the 14-, 16-, and 18-MeV data. A possible explanation of this effect is that it is due to protons which pass through the edge of the detector and thus deposit a smaller fraction of their energy in the sensitive volume of the detector. The probability that this will occur depends on the size of the beam spot at the detector, that is, on the focusing properties of the beam transport system. During the initial setup, the 42.4-MeV proton beam was focused on a scintillator at the location of the detector, prior to its insertion into the beam line. Settings for the focusing magnets were then computed as a function of proton energy to be used for the lower-energy runs. It is possible that small cumulative errors in these computed settings could have caused an increase in the beam size below 30 MeV, thus enabling some protons to escape from the edges of the detector.

An additional effect is observable in the data taken at 20 MeV and below, where there seems to be a systematic difference between theory and experiment. The predicted widths are smaller than those observed, and the magnitude of the discrepancy increases with decreasing proton energy. This is a result of the approximation, made in the derivation of the Vavilov theory, that the energy of the incident particle does not change significantly over the thickness of the target. Recently, Tschalär[13,14] has developed a theory of straggling for large energy losses, which does not make this approximation. We have fitted Tschalär's distributions to the

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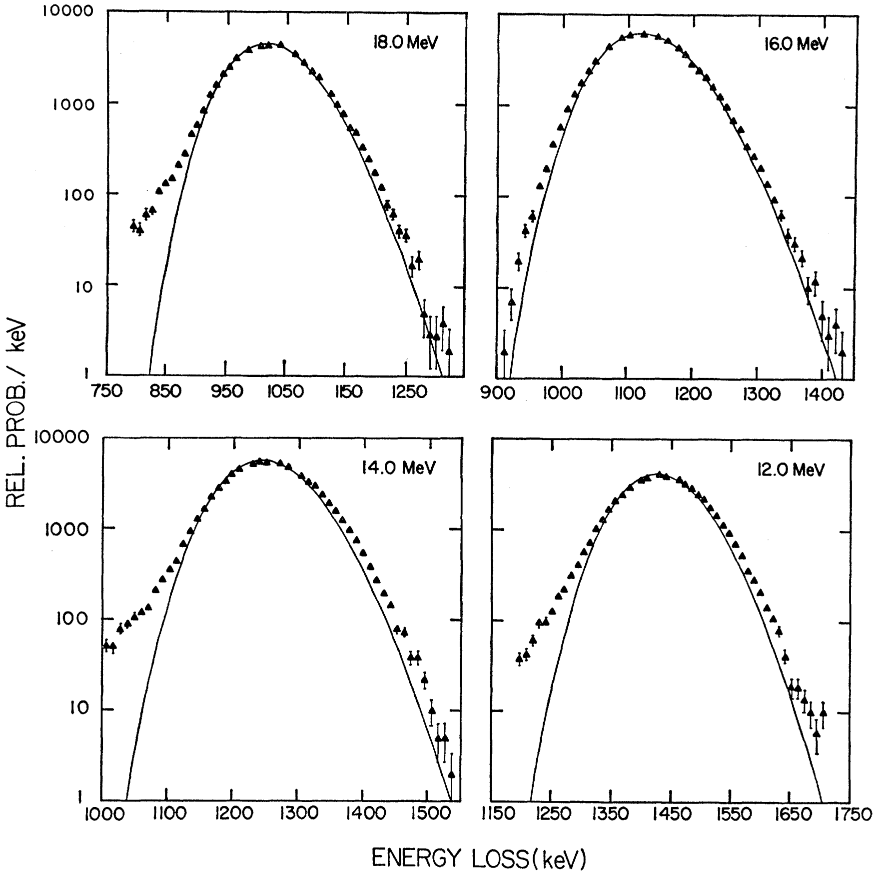

FIG. 4. Straggling distributions for 12.0- to 18.0-MeV incident proton energy.

data taken at 5.0 and 7.0 MeV, the energy lost in the detector being 76 and 34% of the initial energy, respectively. The results are shown in Fig. 5. We conclude that the method of Tschalär can predict straggling distributions accurately for very large energy losses, of the order of 80% of the initial energy. In addition, our data indicate that the variation of the proton energy over the target thickness can produce noticeable deviations from the Vavilov distribution for energy losses as small as 5% of the initial energy.

It should be noted that in all the comparisons between theory and experiment we have neglected several processes which could possibly affect the distributions. The first is the increase in effective target thickness due to multiple scattering in the detector.[14,15] This effect will be most important at the lower energies. For example, at 5.06 MeV, with a residual energy of about 1.33 MeV, the rms deviation is approximately 4°, while at 38.0 MeV it is less that 1°. The corresponding corrections to the path length are 0.2% and less than 0.01%, respectively. We have also neglected the effect of the loss of high-energy electrons, or δ rays, produced near the surface of the detector, and the possibility of energy-loss fluctuations due to nonstatistical effects such as channeling. Finally, the distributions of Vavilov and Tschalär are calculated on the assumption that the atomic collision cross section follows a $1/\epsilon^2$ law. Indications of the changes to be expected from a more realistic collision spectrum can be found in Ref. 16. For the relatively large energy losses of the present experiment, the changes are expected to be small.

IV. CONCLUSION

The Vavilov theory for energy-loss straggling of charged particles in matter has been tested for protons incident on a 200-μ silicon detector in the energy region from 5.0 to 42.4 MeV. This corresponds to a range in the parameter κ of 30.4 to 0.44, which covers the transition region to a Gaussian distribution. There is excellent agreement between theory and experiment for proton energies greater than 20 MeV. Below this energy, a systematic energy-dependent discrepancy appears which has been shown to be due to the variation of proton energy over the thickness of the target. The method of Tschalär, which takes this effect into consideration, yields good fits to the observed distributions for energy losses as much as 80% of the incident proton energy.

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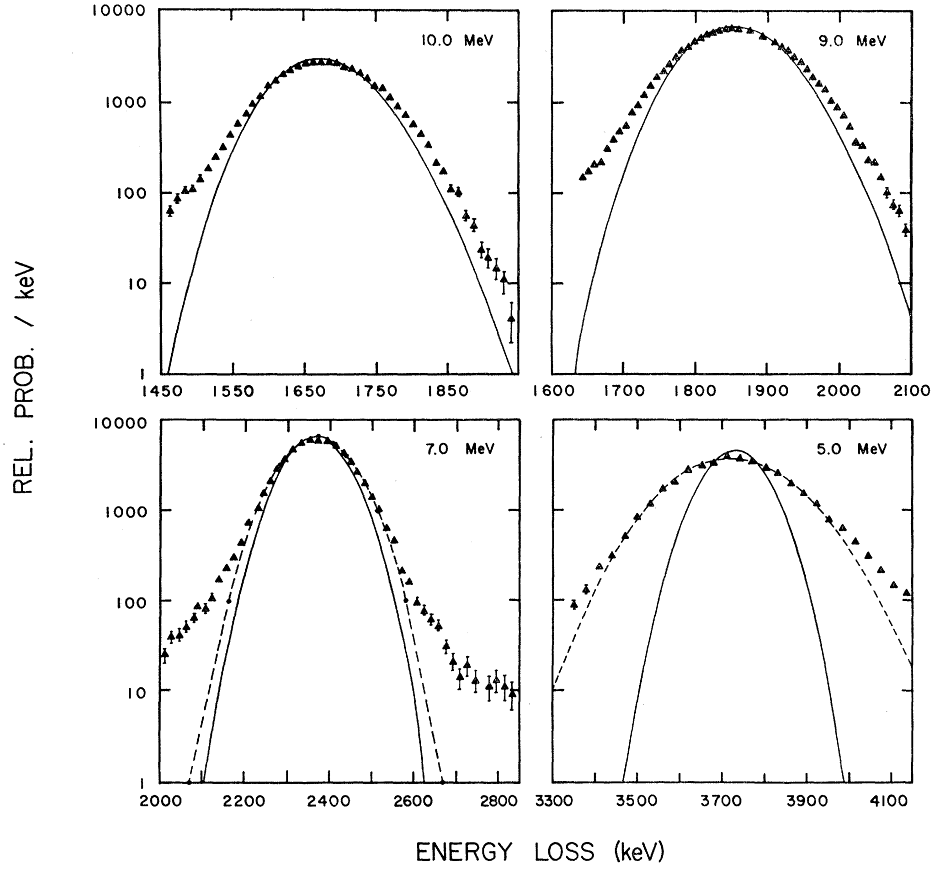

FIG. 3. Straggling distributions for 5.0- to 10.0-MeV incident proton energy. The solid curves are the predictions of the Vavilov theory. The dashed curves are the predictions of Tschalär, which take into account the variation of the proton energy over the detector thickness.

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Theoretical Study on the Anisotropy of the Spin-Lattice Relaxation for Paramagnetic Ions in MgO Crystal

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The anisotropy of the spin-lattice relaxation time with respect to the orientation of the Zeeman field about the crystal axes has been studied theoretically for Co^{2+}, Ni^{2+}, and Cr^{3+} ions in an MgO crystal for both single- and two-phonon processes, using a Debye model of the phonon distribution. For the single-phonon process, the anisotropy in the relaxation time is large for Co^{2+} but small for Ni^{2+} and Cr^{3+}. For the double-phonon process, where the calculations are done without making the long-wavelength approximation, the relaxation time is totally isotropic for Co^{2+}, but for Ni^{2+} and Cr^{3+} it is slightly anisotropic. The deviation from the expected T^{-7} law is found to be significant for the Cr^{3+} ion. The effect of the dipolar interaction on the anisotropic factor for both single- and two-phonon processes is discussed.

I. INTRODUCTION

IN this paper, we present the results of our theoretical investigations on the anisotropy of the spin-lattice relaxation in dilute paramagnetic crystals of cubic (XY_6) symmetry with respect to the Zeeman field orientation about the crystal axes. The objectives of this study are primarily as follows:

(1) The anisotropy associated with the single- and the two-phonon processes have been studied separately,

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