Rev. Mod. Phys. 38, 298 (1966).

[9]By way of formal analogy, we notice that a similar degeneracy arises in describing the properties of an isotropic Heisenberg ferromagnetic spin system. There, the Hamiltonian operator is invariant to rotations of the spin space so that while the zero-temperature ground state finds all N spins aligned parallel, the orientation of this alignment is not specified. Consequently, the ensemble average (expectation value) of the spin-variable net magnetization must be identically zero. However, by introducing a uniform external magnetic field which causes the individual spins to point along a particular axis, one fixes a preferred direction, thereby destroying the rotational symmetry of the Hamiltonian. The lowest eigenstate of the system is then such as to result in a non zero net magnetic moment. In the limit as this external field vanishes, a finite spontaneous magnetization persists up to the critical temperature (Curie point) T_C. Above this temperature, the spontaneous ordering disappears and a net moment can be achieved only with a finite external field.

[10]T. Berlin and M. Kac, Phys. Rev. 86, 821 (1952). We wish to thank Professor C. Domb for first pointing out this correspondence to us. The work of Gunton and Buckingham (Ref. 2) and Langer (Ref. 3) discuss the similarities in point.

[11]See F. London, Superfluids (Dover Publications, Inc., New York, 1961), Vol. 2. The referee has pointed out that the Bose function $F_S(z)$ is discussed in detail by A. Erdelyi, Higher Transcendental Functions, (McGraw-Hill Book Co., Inc., New York,) Vol. I, Sec. 11.

[12]R. Griffiths, J. Chem. Phys. 158, 176 (1967), has investigated the thermodynamic properties associated with such functional forms for the equation of state. For a general review of the subject see L. Kadanoff *et al.*, Rev. Mod. Phys. 39, 395 (1967). The critical indices are the notation standardized by Fisher. See "Nature of Critical Points," in Lectures in Theoretical Physics, (University of Colorado Press, Boulder, Colorado, Vol. VIIC.

[13]Handbook of Mathematical Functions NBS-AMS 55 (U. S. Government Printing Office, Washington, D. C.).

[14]Similar results for the correlations on the two-dimensional Ising model have been demonstrated by L. Kadanoff and R. Hecht, Phys. Rev. 158, 1557 (1967).

[15]M. S. Green, to be published.

PHYSICAL REVIEW VOLUME 176, NUMBER 1 5 DECEMBER 1968

Conductivity of a Plasma in a Steady Magnetic Field. II

T. Pradhan and B. Dasgupta
Saha Institute of Nuclear Physics, Calcutta, India
(Received 9 July 1968)

With the aid of techniques used in our previous paper I, we prove, without recourse to any approximation, that $k_\mu k_\nu \sigma_{\mu\nu}(\vec{k}, \omega)/k^2$ is independent of the magnetic field strength.

I. INTRODUCTION

The magnetic field dependence of the complex-conductivity tensor of a magnetoplasma was studied by us in a recent paper[1] (herein referred to as I) by employing the Kubo formalism. We considered the "frequency-dependent" conductivity tensor $\sigma_{\mu\nu}(\omega)$ in that paper. The problem of the magnetic field dependence of the "wave number and frequency-dependent" complex-conductivity tensor $\sigma_{\mu\nu}(\vec{k}, \omega)$ is of much greater importance, particularly in the study of the propagation of electromagnetic waves in a magnetoplasma. The problem here is far more difficult than was the $k=0$ case dealt with in I. The extraction of the explicit magnetic field dependence of $\sigma_{\mu\nu}(\vec{k}, \omega)$ has not so far proved to be a success. However, answers have been obtained to simpler questions such as how can the invariant (rotationally) of $\sigma_{\mu\nu}(\vec{k}, \omega)$ that is independent of magnetic field strength be found. From a study of the problem in the Vlassov approximation, it is known that $k_\mu k_\nu \sigma_{\mu\nu}(\vec{k}, \omega)/k^2$, i.e.,[2] σ_l is independent of the magnetic field strength if $\vec{k}$ is parallel to the direction of the magnetic field. It is also possible to obtain this result, without recourse to any approximation, from the fact that the flow of electric current is not affected by imposing a steady magnetic field in the direction of the flow of current.[1] In the present paper, we wish to show by using Kubo formalism that even when $\vec{k}$ is not parallel to the magnetic field[3] the invariant $k_\mu k_\nu \sigma_{\mu\nu}(\vec{k}, \omega)/k^2$ is independent of the magnetic field.

II. DERIVATION OF THE RESULT

We consider a fully ionized homogeneous plasma with electrons moving against a fixed, neutralizing, smeared-out, positive-ion background. The conductivity tensor $\sigma_{\mu\nu}(\vec{k},\omega)$ is the Fourier transform of the response function $K_{\mu\nu}(\vec{x},\omega)$, defined by the nonlocal relation

$$J_\mu(\vec{x},\omega)=\int d\vec{x}' K_{\mu\nu}(\vec{x}-\vec{x}',\omega)E_\nu(\vec{x}',\omega), \quad \mu,\nu=1,2,3 \tag{1}$$

between the electric current $J_\mu(\vec{x},\omega)$ and the applied field $E_\mu(\vec{x},\omega)$.

According to Kubo[4]

$$K_{\mu\nu}(\vec{x}-\vec{x}',\omega)=\beta\int\int d\Gamma d\Gamma' j_\mu(\vec{x}) j_\nu(\vec{x}) R(x,v|x',v';\omega), \tag{2}$$

where $\vec{j}(\vec{x})=e\sum_{r=1}^{N}\vec{v}_r\delta(\vec{x}-\vec{x}_r)$,

$d\Gamma$ is the element of phase-space volume, and (x,v) is the totality of position and velocity coordinates, and $R(x,v|x',v';\omega)$ is the product of the Green's function of the Liouville operator for the system and the N-particle distribution. Therefore the conductivity tensor can be written as

$$\sigma_{\mu\nu}(\vec{k},\omega)=e^2\beta\sum_{r,s}\int\int d\Gamma d\Gamma' v_{r\mu}v_{s\nu}'\exp[i\vec{k}\cdot(\vec{x}_r-\vec{x}_s')]R(x,v|x',v';\omega). \tag{3}$$

We shall now use techniques employed in I to prove that $k_\mu k_\nu\sigma_{\mu\nu}(\vec{k},\omega)$ is independent of the magnetic field. We multiply both sides of the Liouville equation for the unprimed coordinates,

$$\left\{i\omega+\sum_{j=1}^{N}\left[\vec{v}_j\cdot\frac{\partial}{\partial\vec{x}_j}+\frac{e}{m}\left(\sum_{i\neq j}^{N}\frac{\partial}{\partial\vec{x}_j}\frac{e}{|\vec{x}_j-\vec{x}_i|}+\frac{1}{c}\vec{v}_j\times B\right)\cdot\frac{\partial}{\partial\vec{v}_j}\right]\right\}R(x,v|x',v';\omega)$$
$$=\delta(x-x')\delta(v-v')f_N(x,v) \quad , \tag{4}$$

by $\exp(i\vec{k}\cdot\vec{x}_r)$ and both sides of the Liouville equation for primed coordinates,

$$\left\{-i\omega+\sum\left[\vec{v}_j{}'\cdot\frac{\partial}{\partial\vec{x}_j{}'}+\frac{e}{m}\left(\sum_{i\neq j}^{N}\frac{\partial}{\partial\vec{x}_j{}'}\frac{e}{|\vec{x}_j{}'-\vec{x}_i{}'|}+\frac{1}{c}\vec{v}_j{}'\times\vec{B}\right)\cdot\frac{\partial}{\partial\vec{v}_j{}'}\right]\right\}R(x,v|x',v',\omega)$$
$$=-\delta(x-x')\delta(v-v')f_N(x',v'), \tag{5}$$

by $\exp(-i\vec{k}\cdot\vec{x}_s{}')$ and then integrate the resulting equations over the phase space and obtain the following identities

$$i\omega\int d\Gamma\exp(i\vec{k}\cdot\vec{x}_r)R(x,v|x',v';\omega)-ik\mu\int d\Gamma\exp(i\vec{k}\cdot\vec{x}_r)v_{r\mu}R(x,v|x',v';\omega)=\exp(i\vec{k}\cdot\vec{x}_r)f_N(x',v'), \tag{6}$$

$$i\omega\int d\Gamma'\exp(-i\vec{k}\cdot\vec{x}_s{}')R(x,v|x',v';\omega)-ik_\nu\int d\Gamma'\exp(-ik\cdot\vec{x}_s{}')v_{s\nu}{}'R(x,v|x',v';\omega)=\exp(-i\vec{k}\cdot\vec{x}_s)f_N(x,v). \tag{7}$$

Integrating Eq. (6) over the rest of the phase-space variables and using Eq. (7), we finally get

$$k_\mu k_\nu\sigma_{\mu\nu}(\vec{k},\omega)/k^2=\frac{e^2\beta\omega}{k^2}\sum_{r,s}\int\int d\Gamma d\Gamma'\exp[i\vec{k}\cdot(\vec{x}_r-\vec{x}_s{}')]R(x,v|x',v';\omega)$$
$$+\frac{e^2\beta\omega}{k^2}\sum_{r,s}\int d\Gamma\exp[i\vec{k}\cdot(\vec{x}_r-\vec{x}_s{}')]f_N(x,v). \tag{8}$$

In I we proved that $\int\int dv dv' R(x,v|x',v';\omega)$ is independent of magnetic field strength. It therefore follows from Eq. (8) that $k_\mu k_\nu\sigma_{\mu\nu}(\vec{k},\omega)/k^2$ is independent of magnetic field strength.

---

[1] T. Pradhan and B. Dasgupta, Phys. Rev. 160, 184 (1967).

[2] $\sigma_{\mu\nu}(\vec{k},\omega)=(\delta_{\mu\nu}-k_\mu k_\nu/k^2)\sigma_{tr}+(k_\mu k_\nu/k^2)\sigma_l$ when $\vec{k}\parallel\vec{B}$ where $\vec{B}$ is the steady magnetic field.

[3] When $\vec{k}$ is not parallel to $\vec{B}$, $\sigma_{\mu\nu}$ cannot be decomposed into transverse and longitudinal parts.

[4] R. Kubo, J. Phys. Soc. (Japan) 12, 570 (1957). However, we use a form of Kubo theory given by S. F. Edwards and J. J. Sanderson, Phil. Mag. 6, 71 (1961) and by R. Balescu, Physica 27, 693 (1961).