tended to a multiple integral, we can write

$$I=\left[\frac{1}{q_2{}^T}\left(1+\frac{q_{3x}+q_{4x}+\cdots+q_{nx}}{q_{1x}}\right)\right]_{\Omega_0} q_1{}^T \times\int d\Omega\,|f_n|^2 e^{-b(q_2{}^T+q_3{}^T+\cdots+q_n{}^T)}, \quad \text{(A13)}$$

where the square bracket is evaluated at a point Ω_0 in the phase space lying between the limits of integration. It is reasonable to assume a value for this quantity which is the average of this quantity. Now the sign of $q_{3x}+q_{4x}+\cdots+q_{nx}$ has roughly equal probability of its being positive or negative [as can be seen from Eq. (A5)] and hence the value of $(q_{3x}+q_{4x}+\cdots+q_{nx})_{\text{av}}$ can be taken to be zero. We can then put

$$\left[\frac{1}{q_2{}^T}\left(1+\frac{q_{3x}+q_{4x}+\cdots+q_{nx}}{q_{1x}}\right)\right]_{\Omega_0}=\frac{1}{(q_2{}^T)_{\text{av}}}$$

and find

$$\frac{dg_n(q_1{}^T)}{dq_1{}^T}=b\left(1-\frac{q_1{}^T}{(q_2{}^T)_{\text{av}}}\right)g_n(q_1{}^T). \quad \text{(A14)}$$

If further $q_1{}^T\sim(q_2{}^T)_{\text{av}}$, then $dg_n(q_1{}^T)/dq_1{}^T\simeq 0$ and the left-hand side of the inequality in (A9) would be zero. From Eq. (A14) and the magnitude of $2b\sim 6$ BeV/c, we find in any case that inequality in Eq. (A9) would be satisfied for most values of $q_1{}^T$ which are physically relevant. Hence we have shown that $\sigma_n(q_1{}^T)$ is approximately a damped exponential in $q_1{}^T$ as in (A4).

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Superconvergence Relations and Possible New Resonances in Pion-Baryon Scattering*

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Superconvergence relations for the $B^{(+)}$ amplitudes at $t=0$ for pion-baryon scattering are well saturated by low-lying resonances, except for the one for the π-Σ scattering. This remarkable disparity suggests that some new π-Σ resonances may exist. From the unitarity condition on the p-wave π-Σ scattering, it is shown that the resonances, if any, are in $(I,J^P)=(0,\frac{3}{2}^+)$ and $(2,\frac{3}{2}^+)$ states.

1. INTRODUCTION

THERE are five isospin-symmetric amplitudes of the pion-octet-baryon scatterings $\pi-N$, $\pi-\Lambda$, $\pi-\Sigma$, and $\pi-\Xi$. Analyses of the superconvergence relations (SCR) for the five (spin-flip) amplitudes are summarized in Sec. 2. A remarkable disparity is pointed out; the SCR's are very well saturated by low-lying resonances *except for the one for the* π-Σ *scattering*. This may be interpreted as an indication of new resonances in the π-Σ scattering. In Sec. 3 possible resonances in the p-wave π-Σ scattering are examined. It is shown on the basis of unitarity that the resonances, if they exist, are in $(I,J)=(0,\frac{3}{2})$ and $(2,\frac{3}{2})$ states. Results are discussed in Sec. 4.

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2. SUPERCONVERGENCE RELATIONS

If a scattering amplitude $f(\nu)$, which satisfies an unsubtracted dispersion relation, behaves like $\nu f(\nu)\to 0$ as $\nu\to\infty$, then it satisfies a superconvergence relation

$$\int \text{Im} f(\nu)d\nu=0. \quad (1)$$

Here ν is the invariant energy variable at a fixed-momentum transfer. A modified SCR has been proposed which may apply even when the condition $\nu f(\nu)\to 0$

is not satisfied.[1,2] Consider the spin-flip amplitude B of the pion-baryon scattering. According to the Regge-pole hypothesis, the asymptotic form of B is given by $\nu^{\alpha(t)-1}$, where α is the Regge trajectory for the fixed momentum transfer t. The leading trajectories are the P and P' in the $I=0$ and the ρ in the $I=1$ channel. Since all these trajectories have $\alpha>0$ near $t=0$, B does not satisfy the SCR.[1] However, if one subtracts the relevant Regge trajectory contributions, one can get $\nu(B-B_{\text{Regge}})\to 0$ as $\nu\to\infty$ so that $B-B_{\text{Regge}}$ satisfies the SCR.

Balázs and Cornwall[2] examined this modified SCR for the isospin-symmetric amplitude $B^{(+)}$ of the π-N scattering at $t=0$ in detail. Assuming resonance dominance in the low-energy region, the modified SCR,

$$g^2+\frac{1}{2\pi^2}\int_\mu^\infty \text{Im}(B^{(+)}-B_{\text{Regge}}{}^{(+)})d\nu=0\,, \qquad (2)$$

results in a sum rule

$$g^2+\sum_{I,l} C_I R_l{}^I+(\text{Regge contribution})=0 \qquad (3)$$

with

$$R_l{}^I=(\Gamma_{l+}{}^I/2k_{l+}{}^3)\{(l+1)[(M_{l+}{}^I-m)^2-\mu^2] -2l(l+1)mM_{l+}{}^I\}+(\Gamma_{l-}{}^I/2k_{l-}{}^3)\{l[(M_{l-}{}^I-m)^2-\mu^2] +2l(l+1)mM_{l-}{}^I\}\,. \qquad (4)$$

Here m and μ are the baryon and pion mass, respectively, $M_{l\pm}{}^I$ and $\Gamma_{l\pm}{}^I$ are the mass and width of the resonance whose isospin is I, spin is $J=l\pm\frac{1}{2}$, and parity is $(-1)^{l+1}$. The $k_{l\pm}$ is the c.m. momentum corresponding to the total mass $M_{l\pm}{}^I$. The C_I is the square of a Clebsch-Gordan coefficient with respect to a relevant isospin combination: $C_I=\frac{1}{3}$ and $\frac{2}{3}$ for $I=\frac{1}{2}$ and $\frac{3}{2}$, respectively. Dass and Michael[3] have done a similar analysis for all the pion-octet-baryon scatterings.

A remarkable feature of both analyses[2,3] is that, except for the π-Σ case, the sum rule is very well satisfied by the contribution of the $J^P=\frac{3}{2}^+$ baryon decuplet, indicating nearly perfect cancellation among higher resonance terms and the Regge contribution. To summarize the situation, let us quote the results of Dass and Michael.[3]

$$g_{\pi NN}{}^2+R(\Delta)=14.8-14.6\,, \quad \text{for} \quad \pi\text{-}N \qquad (5)$$

$$g_{\pi\Lambda\Sigma}{}^2+R(Y_1^*)=11.2-11.4\,, \quad \text{for} \quad \pi\text{-}\Lambda \qquad (6)$$

$$g_{\pi\Xi\Xi}{}^2+R(\Xi^*)=3.7-3.3\,, \quad \text{for} \quad \pi\text{-}\Xi. \qquad (7)$$

The coupling constants $g_{\pi\Lambda\Sigma}$ and $g_{\pi\Xi\Xi}$ are determined by assuming the $SU(3)$ symmetry with the F-D mixing parameter $\alpha=D/(F+D)=0.75$ and putting $g_{\pi NN}{}^2=15$.[4]

For the π-Σ scattering, there are two independent components of the isospin-symmetric amplitude, say, B_1+B_2 and B_0+2B_2, where the suffix indicates the isospin in the s channel. They are related to those in the t channel, denoted by $\tilde{B}_I$, as follows:

$$B_1+B_2=\tfrac{2}{3}(\tilde{B}_0-\tilde{B}_2)\,, \qquad (8)$$

$$B_0+2B_2=\tilde{B}_0+2\tilde{B}_2\,. \qquad (9)$$

The results of Dass and Michael were given for the t-channel amplitudes $\tilde{B}_0$ and $\tilde{B}_2$, but we rearrange them in the forms of (8) and (9) so that the effect of Y_1^* enters into only one of them. Thus we obtain

$$g_{\pi\Sigma\Sigma}{}^2+R(Y_1^*)=3.7-3.3\,, \quad \text{for} \quad B_1+B_2 \qquad (10)$$

$$g_{\pi\Lambda\Sigma}{}^2 \qquad =11.2\,, \quad \text{for} \quad B_0+2B_2\,. \qquad (11)$$

Now it is clear that the sum rules are very well satisfied, except for the last one, (11). This remarkable disparity seems to suggest the existence of a resonance, say, Y_0^* with $J^P=\frac{3}{2}^+$. In the above formulas, $Y_0^*(1405)$ with $J^P=\frac{1}{2}^-$ has not been included because its effect is negligible. In general, the effects of an s-wave resonance is extremely small unless its width is enormous; hence the simplest candidate is a p-wave resonance.

It may be added that, as is seen from (4), contributions from two baryons, with $J=l+\frac{1}{2}$ and $J=l-\frac{1}{2}$, have opposite signs. It is thus possible that effects of resonances of similar structure cancel each other.[2,3] For example, for the π-N scattering, $N(1688,\frac{5}{2}^+)$ and $\Delta(1920\frac{7}{2}^+,)$ give about 2.5 and -3.5, respectively. One may wonder that the contributions of higher Y_0^* resonances may accumulate and cancel $g_{\pi\Lambda\Sigma}{}^2$ in (11). This is very unlikely, however. Observed Y_0^* resonances are $(1520,\frac{3}{2}^-)$, $(1690,\frac{3}{2}^-)$, $(1815,\frac{5}{2}^+)$, $(1830,\frac{5}{2}^-)$, and $(2100,\frac{7}{2}^-)$.[5] Contributions from these resonances are all small, and they are positive except for the one from $(1830,\frac{5}{2}^-)$. Therefore, the effects of the higher Y_0^* resonances, if appreciable, can only worsen the situation.

3. UNITARITY AND POSSIBLE RESONANCES

Motivated by the drastic disparity pointed out in Sec. 2, we have examined the possibility of resonances in the π-Σ scattering. Amati and Fubini[6] showed that, for the π-N scattering, the quantum numbers (I,J) of the resonance $\Delta(1236)$ as well as the effective $\pi N\Delta$ coupling constant can be determined by the unitarity condition of the scattering amplitude. Their method

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can be applied to other pion-baryon scatterings. For example, for the π-Λ scattering one can easily show why the $Y_1^*(1385)$ has $J=\frac{3}{2}$ rather than $J=\frac{1}{2}$. The π-Σ case is less straightforward, so let us describe it in some detail.

For simplicity, we ignore the baryon recoil, i.e., neglect terms of order ω/m, where ω is the pion c.m. energy, and take units such that the pion mass $\mu=1$. The dispersion relation for the p-wave scattering amplitude for $f_\alpha(\omega)$ is[7]

$$\mathrm{Re}f_\alpha(\omega)=q^{-1}\sin\delta_\alpha\cos\delta_\alpha=b_\alpha(\omega)+\frac{q^2}{12\pi^2}\int_1^\infty\frac{dq'}{\omega'}\times\left(\frac{\sigma_\alpha(\omega')}{\omega'-\omega}+\frac{\sum_\beta A_{\alpha\beta}\sigma_\beta(\omega')}{\omega'+\omega}\right)+\cdots. \quad (12)$$

Here the suffix $\alpha=1, 2, \cdots, 6$ stand for $(I,J)=(0,\frac{1}{2})$, $(0,\frac{3}{2})$, $(1,\frac{1}{2})$, $(1,\frac{3}{2})$, $(2,\frac{1}{2})$, $(2,\frac{3}{2})$ in that order, $b_\alpha(\omega)$ is the contribution from the Λ and Σ poles, and $\sigma_\alpha(\omega)$ is the cross section. The dots at the end of (12) indicate effects of the ρ exchange, etc. The Born term $b_\alpha(\omega)$ is, if we ignore the Λ-Σ mass difference which is irrelevant in the following,

$$b_\alpha=(q^2/\omega)(3f_\Lambda^2\lambda_\alpha^{(1)}+2f_\Sigma^2\lambda_\alpha^{(3)}), \quad (13)$$

where f_Λ and f_Σ are the pseudovector coupling constants for the $\pi\Lambda\Sigma$ and $\pi\Sigma\Sigma$ interactions, respectively. $\lambda_\alpha^{(1)}$ and $\lambda_\alpha^{(3)}$ together with the crossing matrix $A_{\alpha\beta}$ will be given later.

Now suppose that there is a narrow resonance in the state γ which dominates the cross section. Then we may assume that

$$\sigma_\alpha=12\pi^2\delta_{\alpha\gamma}f^{*2}q^*\delta(\omega^*-\omega), \quad (14)$$

where q^* and ω^* are the pion-momentum and energy at the resonance, and f^* is an effective $\pi\Sigma Y^*$ coupling constant. Equation (12) then becomes

$$\mathrm{Re}f_\alpha(\omega)=b_\alpha(\omega)+f^{*2}q^2\left(\frac{\delta_{\alpha\gamma}}{\omega^*-\omega}+\frac{A_{\alpha\gamma}}{\omega^*+\omega}\right)+\cdots. \quad (15)$$

If there are more than one resonance, one simply takes a sum with respect to the index γ.

At high energies $\omega\gg\omega^*$, the right-hand side of (15) behaves like

$$(q^2/\omega)[3f_\Lambda^2\lambda_\alpha^{(1)}+2f_\Sigma^2\lambda_\alpha^{(3)}+\sum_\gamma f_\gamma^{*2}(A_{\alpha\gamma}-\delta_{\alpha\gamma})]. \quad (16)$$

It can be shown that the ρ-exchange effect does not increase with q as fast as (16). Also, the smooth part of the cross section cannot give rise to a term which asymptotically behaves like q. On the other hand, unitarity demands that

$$\mathrm{Re}f<\tfrac{1}{2}q^{-1}, \quad (17)$$

which is in contradiction with (16) unless

$$3f_\Lambda^2\lambda_\alpha^{(1)}+2f_\Sigma^2\lambda_\alpha^{(3)}+\sum_\gamma f_\gamma^{*2}(A_{\alpha\gamma}-\delta_{\alpha\gamma})=0. \quad (18)$$

Note that (17) should hold even if the phase shift becomes complex. A resonance or resonances, if any, have to meet this requirement (18), at least approximately.

The amplitude $f_\alpha(\omega)$ satisfies the crossing relation

$$f_\alpha(\omega)=\sum_\beta A_{\alpha\beta}f_\beta(-\omega), \quad (19)$$

which implies

$$\mathbf{A}\boldsymbol{\lambda}=-\boldsymbol{\lambda}, \quad (20)$$

$$\mathbf{A}^2=1, \quad (21)$$

where the vector $\boldsymbol{\lambda}$ stands for $\boldsymbol{\lambda}^{(1)}$ and $\boldsymbol{\lambda}^{(3)}$. If we define a vector $\boldsymbol{\lambda}^{(\gamma)}$ with the components

$$\lambda_\alpha^{(\gamma)}=A_{\alpha\gamma}-\delta_{\alpha\gamma}, \quad (22)$$

then it also satisfies (20). In fact, $\boldsymbol{\lambda}^{(1)}$ and $\boldsymbol{\lambda}^{(3)}$ are nothing but $\boldsymbol{\lambda}^{(\gamma)}$ with $\gamma=1$ and 3, respectively. The explicit form of $\mathbf{A}$ is given as a direct product of the crossing matrices for the spin and isospin,[8]

$$\mathbf{A}=\frac{1}{6}\begin{pmatrix}2&-6&10\\-2&3&5\\2&3&1\end{pmatrix}\times\frac{1}{3}\begin{pmatrix}-1&4\\2&1\end{pmatrix}. \quad (23)$$

The matrix $\mathbf{A}$ has six eigenvalues, which are ±1 because of (21). Since $\mathrm{Tr}\mathbf{A}=0$, as is seen from (23), three of the six eigenvalues are $+1$ and others are -1. Thus there are three independent vectors which satisfy the eigenvalue equation (20), two of them being $\boldsymbol{\lambda}^{(1)}$ and $\boldsymbol{\lambda}^{(3)}$.[9] For the third one it is convenient to take $\boldsymbol{\lambda}^{(6)}$, which corresponds to $(I,J)=(2,\frac{3}{2})$. Other three-vectors defined by (22) can be expressed as linear combinations of the three basis vectors as follows:

$$\begin{aligned}\boldsymbol{\lambda}^{(2)}&=-5\boldsymbol{\lambda}^{(1)}-2\boldsymbol{\lambda}^{(3)}-2\boldsymbol{\lambda}^{(6)}, &&\text{for } (I,J)=(0,\tfrac{1}{2})\\ \boldsymbol{\lambda}^{(4)}&=-4\boldsymbol{\lambda}^{(1)}-(8/3)\boldsymbol{\lambda}^{(3)}-2\boldsymbol{\lambda}^{(6)}, &&\text{for } (I,J)=(0,\tfrac{3}{2}) \quad (24)\\ \boldsymbol{\lambda}^{(5)}&=10\boldsymbol{\lambda}^{(1)}+5\boldsymbol{\lambda}^{(3)}+4\boldsymbol{\lambda}^{(6)}, &&\text{for } (I,J)=(2,\tfrac{1}{2}).\end{aligned}$$

When the third term on the left-hand side of (18) is expressed in terms of $\boldsymbol{\lambda}^{(1)}$, $\boldsymbol{\lambda}^{(3)}$, and $\boldsymbol{\lambda}^{(6)}$, the coefficient of $\boldsymbol{\lambda}^{(6)}$ must vanish. Hence it is clear that we have to assume at least two resonances so that their $\boldsymbol{\lambda}^{(6)}$ terms cancel each other. Two resonances can be those with $\gamma=2$ and 6, or $\gamma=4$ and 6. There is no possibility for $\gamma=5$, because the coefficients of $\boldsymbol{\lambda}^{(1)}$ and $\boldsymbol{\lambda}^{(3)}$ in the resonance terms must be altogether negative. The two solutions are

$$\tfrac{3}{5}f_\Lambda^2=f_\Sigma^2=f_2^{*2}=2f_6^{*2}, \quad \text{for} \quad \gamma=2 \text{ and } 6 \quad (25)$$

[7] In the integrals in (12), contributions from below the threshold are ignored.

[8] A. Komatsuzawa, R. Sugano, and Y. Nogami, Progr. Theoret. Phys. (Kyoto) **21**, 51 (1959); R. H. Capps and M. Nauenberg, Phys. Rev. **118**, 593 (1960).

[9] In the π-Λ case, (20) has only one solution. In the π-N case there are two solutions, which are identical because of the symmetry between spin and isospin.

and

$$\tfrac{3}{4}f_\Lambda{}^2=\tfrac{3}{4}f_\Sigma{}^2=f_4{}^{*2}=2f_6{}^{*2} \quad \text{for} \quad \gamma=4 \text{ and } 6. \tag{26}$$

Since we wanted a resonance with $(I,J)=(0,\frac{3}{2})$, we prefer the solution (25) rather than (26). We do not have to obtain the $Y_1^*(1385)$, because it is obtained in the π-Λ channel and also Y_1^* is very weakly coupled with the π-Σ channel. It is gratifying to note that the ratio $(f_\Lambda/f_\Sigma)^2=5/3$ leads to the F-D mixing parameter $\alpha=0.69$. If we include the mass factor in the conversion from the pseudoscalar to pseudovector coupling, we get $\alpha=0.70$ instead. This is in good agreement with the value determined in other context.[4]

The above analysis does not tell us the masses of the resonances. If we assume that the two resonances, denoted by Y_0^* and Y_2^*, have the same mass, it then follows from (25) that the ratio of their contributions to the amplitude B_0 and B_2 is 2:1. Therefore, if we denote the additional term in $B_1+B_2(10)$ due to Y_0^* and Y_2^* together by R, (10) and (11) are modified as

$$g_{\pi\Sigma\Sigma}{}^2+R(Y_1^*)+R \quad \text{for} \quad B_1+B_2\,, \tag{27}$$

$$g_{\pi\Lambda\Sigma}{}^2+4R \quad \text{for} \quad B_0+2B_2\,. \tag{28}$$

Now, if we assume $R\simeq-2.5$ so that (28) vanishes approximately, the sum rule for B_1+B_2 is somewhat spoiled. However, the over-all consistency can be restored by slightly readjusting α. For example, if we take $\alpha=0.72$ instead of 0.75, Eqs. (6), (7), (27), and (28) become

$$g_{\pi\Lambda\Sigma}{}^2+R(Y_1^*) \quad =10.4-11.4\,, \tag{29}$$

$$g_{\pi\Sigma\Sigma}{}^2+R(\bar{2}^*) \quad =2.9-3.3\,, \tag{30}$$

$$g_{\pi\Sigma\Sigma}{}^2+R(Y_1^*)+R=4.7-3.3+R\,, \tag{31}$$

$$g_{\pi\Lambda\epsilon}{}^2+4R \quad =10.4+4R. \tag{32}$$

Thus, if we assume $R=-(2.3 \text{ to } 2.4)$ all the sum rules are quite well satisfied. If Y_0^* is heavier than Y_2^*, Y_0^*'s effect increases; hence we can get even better results. For the width, if we assume that Y_0^* and Y_2^* both have the same mass 1400 MeV, $\Gamma(Y_0^*)=2\Gamma(Y_2^*)\simeq15$ MeV.

4. DISCUSSION

So far no evidence has been found for the Y_0^* and Y_2^* with $J^P=\frac{3}{2}^+$.[10] For the Y_0^* it may overlap Y_0^* $(1405,\frac{1}{2}^-)$; then the decay angular distribution will deviate from isotropy. Available data do not seem to be accurate enough to rule out such a possibility.[11] If the Y_2^* exists, it would belong to a **27** multiplet in the SU(3) classification, but no other likely candidate for the member of this **27** multiplet has been established. Analyses of peripheral peaks in high-energy scatterings also do not seem to require any **27** multiplet.[12]

On the other hand, the Y_2^* has been *theoretically* found many times in a variety of ways; to name a few, the Chew-Low-type method,[8,13] the strong- or intermediate-coupling theory,[14] the bootstrap and/or N/D method.[15–17] It should be noted, however, that a calculation in the octet model[18] did not indicate any resonance in the 27-dimensional representation of $SU(3)$. The resonance $Y_0^*(\frac{3}{2}^+)$ seems to be more ambiguous and has received little attention. We would like to emphasize, however, that what is needed most in the SCR is the Y_0^* rather than Y_2^*. Theoretically, it is rather difficult not to have the Y_2^* in a dynamical calculation. If the resonances under discussion do not really exist, one can seriously question the validity of many of dynamical methods which have apparently been successful in many respects.

If the Y_2^* is found, but not other members of the **27** multiplet, it might imply that there are two types of particles or resonances. One is made from three quarks, or from a pair of quark and antiquark, by means of some primary interactions, whereas the other is caused by secondary interactions between the particles of the first type. The $SU(3)$ symmetry may be very badly violated for the second type while it holds well for the first type. Of course, this is a pure speculation. Finally we might mention that Bisiacchi and Fronsdal[19] have shown that $S\tilde{U}(3)$, a noncompact version of $SU(3)$, resembles $SU(3)$ in many respects, but $S\tilde{U}(3)$ can accommodate the Y_2^* in the same multiplet as the Δ, Y_1^*, and Ξ^*.

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