Universal Coupling of the ω Meson and Regge Residues*

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In this paper we consider some of the experimental data bearing on the hypothesis of universality of ω coupling. The low-energy data on the coupling of the ω meson and the high-energy data on the coupling of the ω-type Regge trajectory have been considered. The data seem to support the universality hypothesis. The universal ω coupling constant $g_\omega^2/4\pi$ is estimated to be 4.96 ± 1.24, which is higher than the universal ρ coupling constant $g_\rho^2/4\pi=2.60\pm0.68$, where g_ρ and g_ω are so normalized that $g_\omega=g_\rho$ in the $SU(6)$ limit. To relate the Regge residues of vector trajectories to the universal coupling constant, we factor out the angular momentum dependence of the former and introduce a "reduced" Regge residue and a scale parameter for each trajectory. The scale parameters for the ω and the ρ trajectories turn out to be of the order of 290 and 186 MeV, respectively.

I. INTRODUCTION

THE universality of the coupling of vector mesons was first postulated in 1960 by Sakurai.[1] Gell-Mann and Zachariasen[2] extended these ideas and related the electromagnetic form factors to the vector-meson form factors and established the strength of the coupling of vector mesons to the photon. These ideas led to direct experimental checks. Sakurai[3] has recently compared the relevant experiments which give the ρ-meson coupling constants and has shown that the universality theory is strongly supported regarding the ρ meson. In the case of the ω meson, it was first supposed[1] that it coupled universally to the hypercharge current. However, later experiments indicate that the ω transforms like $2N+Y$, a member of the "Okubo nonet,"[4] where Y is the hypercharge and N is the baryon number. Consequently, the universality theory would say that the ω is coupled to the above linear combination of the hypercharge and baryon number current. Also, the φ meson[4] should be coupled to the strangeness current. There is at present enough experimental data to check on these ideas.

The purpose of this paper is to summarize the experimental evidence in favor of the universality of ω coupling and to discuss the role of Regge residues in analyzing this problem. It should be emphasized that a universal theory deals not with the usual coupling constants but with those evaluated at zero mass of the vector mesons. Fortunately, this is the kind of information one obtains in a study of Regge residues and indeed the universality ratio for the ωKK and ωNN residues has already been reported.[5] A Regge residue is essentially the coupling constant continued to zero mass and to a value of angular momentum, $\alpha(0)$ which is generally around $\frac{1}{2}$ for the vector trajectory instead of the physical value of unity. Consequently, the usual evaluation of coupling constants through strong decay rates, dispersion relations, force parameters, and extrapolations to unphysical regions give the mass-shell (not universal) coupling constants at the correct physical angular momentum while the Regge method gives the coupling constants at the correct zero mass but at the incorrect angular momentum. It appears however, that the angular momentum dependence can be factored out and the resulting "reduced" Regge residue can be studied. It should be pointed out that electromagnetic decays[2] do yield the "zero-mass" coupling constants directly.

The development of this paper is as follows: Section II reviews the vector-meson universality hypothesis from the point of view of the Lagrangian formulation of Kroll, Lee, and Zumino.[6] The form of universality taken in this paper and the definitions of the various coupling constants is given. Section III discusses empirical evidence for the values of the ρ and ω coupling constants. Section IV presents a Regge-pole analysis and gives additional evidence in favor of ω meson universal coupling. Section V compares the Regge analysis made here for $t\geq0$ to those made in reaction studies for $t\leq0$ and compares the two results by extrapolation.

II. UNIVERSALITY HYPOTHESIS

A formal description of the neutral vector mesons in a universal Lagrangian formulation has been given by Kroll, Lee, and Zumino.[6] They introduce field strength tensors for the vector fields V_μ:

$$G_{\mu\nu}{}^V\equiv\frac{\partial}{\partial x^\mu}V_\nu-\frac{\partial}{\partial x^\nu}V_\mu. \tag{1}$$

The field equations for ρ_μ, ϕ_μ, and ω_μ are

$$\partial G_{\mu\nu}{}^\rho/\partial x_\mu-m_\rho^2\rho_\nu=g_\rho J_\nu{}^\rho=\mathcal{T}_\nu{}^\rho\,, \tag{2a}$$

$$\partial G_{\mu\nu}{}^\omega/\partial x_\mu-m_\omega^2\omega_\nu=\mathcal{T}_\nu{}^\omega\,, \tag{2b}$$

and

$$\partial G_{\mu\nu}{}^\phi/\partial x_\mu-m_\phi^2\phi_\nu=\mathcal{T}_\nu{}^\phi\,, \tag{2c}$$

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[5] C. Levinson, H. Lipkin, and N. Wall, Phys. Rev. Letters **17**, 1122 (1966).
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where the currents $\mathcal{T}_\nu^\rho$, $\mathcal{T}_\nu^\omega$, and $\mathcal{T}_\nu^\phi$ are conserved and

$$\int d^3x\, J_0^\rho = I_Z\,, \tag{3}$$

the Z component of the isospin. The Lagrangian is chosen so that the electromagnetic-current operator is given by

$$J_\mu^\gamma = -(m_\rho^2/g_\rho)\rho_\mu - \tfrac{1}{2}g_Y^{-1}(\cos\theta_Y\, m_\phi^2\phi_\mu - \sin\theta_Y\, m_\omega^2\omega_\mu)\,, \tag{4}$$

where parameters θ_Y, θ_N, g_Y, and g_N are introduced such that

$$\begin{aligned} g_Y Y_\mu &= \cos\theta_Y\, \mathcal{T}_\mu^\phi - \sin\theta_Y\, \mathcal{T}_\mu^\omega\,, \\ g_N N_\mu &= \sin\theta_N\, \mathcal{T}_\mu^\phi + \cos\theta_N\, \mathcal{T}_\mu^\omega\,, \end{aligned} \tag{5}$$

and Y_μ and N_μ are hypercharge and baryon currents.

These are conserved currents and the spatial integrals of their time components are the hypercharge (Y) and the baryon number (N) operators. All fields above are renormalized quantities. The particular form of universality we wish to discuss in this paper corresponds to the assumption that

$$\begin{aligned} \mathcal{T}_\mu^\omega &= (2N_\mu + Y_\mu)g_\omega/2\,, \\ \mathcal{T}_\mu^\phi &= (Y_\mu - N_\mu)g_\phi/\sqrt{2}\,. \end{aligned} \tag{6}$$

These equations define the g_ω and g_ϕ used in this paper. This assumption follows if the ϕ is a pure singlet of $SU(4)$ and the ω is in the **15** representation. The g_ρ, g_ϕ, and g_ω defined here become equal in the nonet symmetry limit. The smallness of the decay $\phi \to \rho + \pi$ and the observed ϕ and ω masses have been used to argue in favor of this assumption.[7] We would like to consider Eq. (6), however, simply as an assumption independent of any symmetry considerations and to show what physical consequences it leads to when incorporated in a universal scheme.

The assumed forms of $\mathcal{T}_\mu^\omega$ and $\mathcal{T}_\mu^\phi$ lead to two constraints on θ_Y, θ_N, g_Y, and g_N, namely,

$$\begin{aligned} g_Y \cos\theta_N &= -g_N \sin\theta_Y\,, \\ g_N \cos\theta_Y &= -2g_Y \sin\theta_N\,. \end{aligned} \tag{7}$$

Eliminating g_Y and g_N, we find

$$\tan\theta_Y \tan\theta_N = \tfrac{1}{2}\,. \tag{8}$$

If we take $\theta_Y = \theta_N$, as in the mass mixing model of Kroll, Lee, and Zumino,[6] we find that $\theta_Y = \theta_N = 35°$, which compares well with their values of 32° and 39° in two variations of this model.

Coupling constants γ_V are sometimes defined so that the expression for the electromagnetic current is given by $J_\mu^\gamma = -\sum_V (m_V^2/\gamma_V)V_\mu(x)$, where V means any vector meson. In terms of the constants defined in Eq. (6), we have

$$\gamma_\rho = g_\rho\,, \quad \gamma_\phi = (3/\sqrt{2})g_\phi = 2g_Y/\cos\theta_Y\,,$$
and
$$\gamma_\omega = 3g_\omega = -2g_Y/\sin\theta_Y\,. \tag{9}$$

The model used by Van Royen and Weisskopf[8] and by Dar and Weisskopf[9] implies

$$\gamma_\rho = -m_\rho/m_\pi\,, \quad \gamma_\omega = -3m_\omega/m_\pi\,, \quad \gamma_\phi = 3m_\phi/\sqrt{2}m_\pi\,. \tag{10}$$

Equation (10) was arrived at from their values[8] of g_V:

$$g_\rho = -m_\rho/m_\pi\,, \quad g_\phi = m_\phi/m_\pi\,, \quad \text{and} \quad g_\omega = -m_\omega/m_\pi\,, \tag{11}$$

or

$$g_\rho^2/4\pi = 2.54\,, \quad g_\omega^2/4\pi = 2.72\,, \quad \text{and} \quad g_\phi^2/4\pi = 4.58\,. \tag{12}$$

The form factors for the vector currents are normalized by using the fact that the space integrals of the time components of the vector currents can be expressed in terms of isospin, hypercharge, and baryon number. For the matrix elements with respect to a pseudoscalar meson (M) we define form factors.

$$\langle P'|\mathcal{T}_\mu^V(0)|P\rangle = g_{VMM}F_{VMM}(t) \times [(P_\mu' + P_\mu)/(4P_0P_0')^{1/2}] \tag{13}$$

with $t = (P - P')^2$, where

$$g_{\rho MM}F_{\rho MM}(0) = g_\rho I_z(M)\,, \tag{14a}$$

$$g_{\omega MM}F_{\omega MM}(0) = \tfrac{1}{2}g_\omega[2N(M) + Y(M)]\,, \tag{14b}$$

$$g_{\phi MM}F_{\phi MM}(0) = g_\phi[Y(M) - N(M)]/\sqrt{2}\,, \tag{14c}$$

where $I_z(M)$ is the value of I_z for the meson M, and similarly for $N(M)$ (baryon number) and $Y(M)$ (hypercharge). If the vector mesons were stable, then we would normalize $F_{VMM}(m_V^2) = 1$. However, following Gell-Mann and Zachariasen[2] we must normalize [see their equation (3.161)]:

$$F_{VMM}(t) = [\Delta_{F1}(t)/\Delta_F(t)]V_{VMM}(t)\,,$$

where $V_{VMM}(m_V^2) = 1$ and $\Delta_{F1}(t)$ is the renormalized propagator and $\Delta_F(t)$ is the free propagator $(t - m_V^2)^{-1}$. $V_{VMM}(t)$ is the proper vertex function for VMM.

The baryon form factors are defined (for baryon x) by

$$\langle P'|\mathcal{T}_\mu^V(0)|P\rangle = (\bar{u}_x(P')|[g_{Vxx}F_{Vxx}^{(1)}(t)\gamma_\mu + i(\mu_{Vxx}/2M_x)F_{Vxx}^{(2)}(t)\sigma_{\mu\nu}(P' - P)^\nu]|u_x(P)) \times (m_x^2/P_0P_0')^{1/2} \tag{15}$$

and

$$g_{\rho xx}F_{\rho xx}^{(1)}(0) = g_\rho I_z(x)\,, \tag{16a}$$

$$g_{\omega xx}F_{\omega xx}^{(1)}(0) = g_\omega[2N(x) + Y(x)]/2\,, \tag{16b}$$

$$g_{\phi xx}F_{\phi xx}^{(1)}(0) = g_\phi[Y(x) - N(x)]/\sqrt{2}\,. \tag{16c}$$

Just as in the meson case, $F_{Vxx}^{(1)}(m_\rho^2) = 0$ and is

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[8] R. Van Royen and V. F. Weisskopf, Nuovo Cimento **50**, 617 (1967).

[9] A. Dar and V. F. Weisskopf (unpublished).

normalized to $F_{VMM}^{(1)}(t)=[\Delta_{F1}(t)/\Delta_F(t)]V_{Vxx}^{(1)}(t)$ and $V_{Vxx}^{(t)}(m_\rho^2)=1$. Thus, $F_{VMM}(t)$ and $F_{Vxx}^{(1)}(t)$ have poles at the point $t=m_V^2-im_V\Gamma_V$, where Γ_V is the decay rate of the vector meson. The residue at this pole is $-m_V\Gamma_V$. Hence, the lifetime of the vector particle determines $F(t)$ in the neighborhood of $t=M_\rho^2$ and universality determines $F(t)$ in the neighborhood of $t=0$.

III. COUPLING CONSTANTS

There is strong empirical evidence that $g_\rho^2/4\pi \simeq 2.5\pm0.3$. The decay $\phi^0\to K^++K^-$ indicates (cf. Ref. 6 for the various assumptions made) that $g_\phi^2/4\pi=2.8\pm0.6$.

With our assumption Eq. (6), the ratio of $\Gamma(\omega\to\pi^0+\gamma)$, the partial decay rate for the decay of the ω into $\pi^0+\gamma$ to $\Gamma(\pi^0\to\gamma+\gamma)$, the decay rate for π^0-meson decay becomes

$$\frac{\Gamma(\omega\to\pi^0+\gamma)}{\Gamma(\pi^0\to\gamma+\gamma)}=\frac{1}{\alpha}\left(\frac{m_\omega^2-m_\pi^2}{m_\omega m_\pi}\right)^3\left(\frac{g_\omega^2}{4\pi}\right)\frac{3}{2}. \qquad (17)$$

Similarly the partial decay rate for the decay of an ω into a lepton pair becomes

$$\Gamma(\omega\to l^++l^-)=(\alpha^2/27)(g_\omega^2/4\pi)^{-1}(1+2m_l^2/m_\omega^2)\times(1-4m_l^2/m_\omega^2)^{1/2}m_\omega\,, \qquad (18)$$

where m_ω, m_π, and m_l are the masses of the ω, π^0, and lepton.

In the recent compilation of data on particles and resonances, Rosenfeld *et al.*[10] quote 12.2±1.3 MeV for the width of ω and a branching ratio of $(9.7\pm0.8)\%$ for the mode $\Gamma(\omega\to\pi^0+\gamma)$. The mean lifetime of π^0 has been quoted[10] to be $(0.89\pm0.18)\times10^{-16}$ sec. Using these values in (17), we get $g_\omega^2/4\pi=4.26\pm0.11$. Using this value for the coupling constant in (18), we predict the value $(3\pm0.8)\times10^{-5}$ for the branching ratio $\Gamma(\omega\to e^++e^-)/\Gamma$ (all). The most recent experimental value, due to Hertzbach *et al.*,[11] for this branching ratio is $(5.3_{-2.0}^{+1.5})\times10^{-5}$.

There are no other direct methods presently available for the evaluation of the ω coupling constant from decay processes. Several attempts have been made to evaluate $g_{\omega NN}$, the ω-nucleon coupling constant, from N-N and N-$\bar{N}$ scattering data. According to Eq. (16b) we should expect $g_{\omega NN}=\frac{3}{2}g_\omega/F_{\omega NN}(0)$. Basically these attempts are of three types. The first type employs a single-boson-exchange model for the nucleon-nucleon interaction. The second type uses a dispersion relation for N-N forward scattering amplitude. The discontinuity across the unitarity cut is given by the N-N cross section, while that across the left-hand cut is given by the N-$\bar{N}$ cross section. The dispersive part of the amplitude can be found from the differential cross sections. Using these facts and the dispersion relation, $g_{\omega NN}$ can be found. This calculation was carried out by Hara.[12] The third method uses the Regge hypothesis and was employed by Phillips.[13] This method involves isolating the contribution of the ω trajectory ($I=0$, $C=-1$) to the high-energy scattering amplitude and then extrapolating to $t=m_\omega^2$, where the scattering amplitude should become $2g_{\omega NN}^2s/(t-m_\omega^2)$. The extrapolation from the physical region of the s channel, i.e., $t<0$, to the value $t=m_\omega^2$ requires a fairly reasonable guess for the form of the t dependence of the Regge residue function $\beta(t)$. Some of the results of such investigations are presented in Table I. These values, except for those of Ref. 14, are consistent with the value of $g_\omega^2/4\pi$ found above from meson decays [assuming $F_{\omega NN}(0)=1$], and, thus, lends support to the universality of $g_\omega^2/4\pi$.

TABLE I. Evaluations of $(g_{\omega NN}^2/4\pi)$. The last column gives $(4/9)(g_{\omega NN}^2/4\pi)$ which according to Eq. (14b) should be equal to $(g_\omega^2/4\pi)/F_{\omega NN}^{(1)}(0)$. From meson decays $(g_\omega^2/4\pi)\sim4.96\pm1.24$.

Author	Reference	Method	$(4/9)(g_{\omega NN}^2/4\pi)$
Sakurai	1	L-S term in N-N interaction	4.5
Arndt *et al.*	14	N-N scattering with unitarity	1.8
Hara	12	N-N forward dispersion relation	4.0
Phillips	13	Regge hypothesis	4.5

IV. REGGE-POLE ANALYSIS

We discuss now the information about coupling constants which can be derived from a Regge-pole analysis. According to the Regge hypothesis[15] the elastic scattering amplitude for the (x,y) process at large s can be written as

$$A_{xy}(s,t)=-\pi^{3/2}\sum_i\frac{\Gamma(\alpha_i+\frac{1}{2})}{\Gamma(\alpha_i+1)}(2\alpha_i+1)\times\left[\frac{1\pm e^{-i\pi\alpha_i}}{2\sin(\pi\alpha_i)}\right]\frac{\beta_{ix}(t)}{(q_{x\bar{x}})^{\alpha_i}}\frac{\beta_{iy}(t)}{(q_{y\bar{y}})^{\alpha_i}}s^{\alpha_i}. \qquad (19)$$

The summation index i runs over the different Regge trajectories which we will take to be Pomeranchuk, ρ^0, ω, R^0 ($I=1$), and P' ($I=0$). We will assume that the vertex functions discussed just below Eq. (16) are slowly varying and hence the form functions do not vanish at $t=0$. We thus conclude from Eq. (16c) that $g_{\phi xx}=0$ whenever x is a nonstrange particle. This in turn implies, as shown below, that the Regge residue

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[13] R. J. N. Phillips, Phys. Letters 3, 21 (1962).

[14] R. A. Arndt, R. A. Bryan, and M. H. MacGregor, Phys. Rev. 152, 1490 (1966).

[15] See, for example, E. D. Squires, *Complex Angular Momentum and Particle Physics* (W. A. Benjamin, Inc., New York, 1963).

$\beta_{\phi x}(m_\phi^2)=0$ for nonstrange x. Using the further assumptions about the t dependence of $\beta(t)$ discussed below, this will allow us to neglect the ϕ trajectory completely for the processes considered here where at least one of the particles is not strange. In the factor $(1\pm e^{-i\pi\alpha_i})$ the plus sign has to be taken for trajectories which are even under charge conjugation and the minus sign for the odd ones. $\alpha_i=\alpha_i(t)$ is the trajectory function. The dimensionless Regge-pole residue has been factored[16] into the form

$$\beta_i^{xy}(t)=\beta_{ix}(t)\beta_{iy}(t). \tag{20}$$

$q_{x\bar{x}}$ is the center-of-mass momentum of the $x\bar{x}$ system in the t channel and is given by

$$4(q_{x\bar{x}}^2+m_x^2)=t=4(q_{y\bar{y}}^2+m_y^2). \tag{21}$$

The total cross section $\sigma_{\text{tot}}(x,y)$ is given by the optical theorem

$$\sigma_{\text{tot}}(x,y)=\frac{1}{2\sqrt{(sq_{xy})}}\,\text{Im}A_{xy}(s,0), \tag{22}$$

where q_{xy} is the center-of-mass momentum in the s channel:

$$s^{1/2}=(q_{xy}^2+m_x^2)^{1/2}+(q_{xy}^2+m_y^2)^{1/2}. \tag{23}$$

As pointed out by Frautschi, Gell-Mann, and Zachariasen,[17] one would expect $\beta_{ix}(t)$ to behave like $q_{x\bar{x}}^{\alpha_i}(t)$. We therefore follow them and introduce a dimensionless "reduced" residue $b_{ix}(t)$, defined by

$$\beta_{ix}(t)/q_{x\bar{x}}^{\alpha_i}=b_{ix}(t)/s_i^{\alpha_i/2}, \tag{24}$$

where s_i is a scale parameter of dimension energy squared and $b_{ix}(t)$ is expected to be slowly varying in t for suitable a choice of s_i. As discussed by Desai[18] on the basis of nonrelativistic Regge theory we would expect s_i to be related to the radius of interaction in the t channel by $s_i=(1/R_i)^2$, where R_i is the interaction radius. From nonrelativistic considerations[19] R_i is related to $d\alpha/dt$ by the equation

$$\frac{d\alpha_i}{dt}=\frac{1}{4}\frac{1}{2\alpha_i+1}R_i^2=\frac{1}{4(2\alpha_i+1)}\frac{1}{s_i}. \tag{25}$$

This would yield $s_i\sim(1/2m_\pi)^2$ when the observed vector trajectory slopes are used.

Equation (19) is now given by

$$A_{xy}(s,t)=-\pi^{3/2}\sum_i\frac{\Gamma(\alpha_i+\frac{1}{2})}{\Gamma(\alpha_i+1)}(2\alpha_i+1)\times\left(\frac{1\pm e^{-i\pi\alpha_i}}{2\sin(\pi\alpha_i)}\right)b_{ix}(t)b_{iy}(t)(s/s_i)^{\alpha_i(t)}, \tag{26}$$

where $b_{ix}(t)$ is quite analogous to the reduced widths in compound-nucleus theories. In these theories the level widths have momentum dependences at threshold given by $(qR)^L$, where R is the channel radius, and the reduced widths are defined with this factor removed.

In our analysis of Eq. (26) we will consider total cross sections which give us $\text{Im}A_{x,y}(s,0)$. Also, we consider coupling constants involving vector particles, which give $A(s,t)$ in the neighborhood of the pole at $t=m_i^2$. m_i is the mass of the vector meson on the ith trajectory. Consequently, we will be able to evaluate

$$b_{ix}(m_i^2)b_{iy}(m_i^2)/s_i$$

as well as

$$b_{ix}(0)b_{iy}(0)/s_i^{\alpha_i(0)}.$$

Assuming that the "reduced widths" b are independent of t, we can solve for them and for s_i. Putting these results back into Eq. (26), we then have the complete s and t dependence of $A_{xy}(s,t)$ which can be checked against the s,t dependence found in various analyses of reactions where $t<0$. This comparison is made in Sec. V and it is verified that the $A(s,t)$ fould in reaction studies when continued to $t>0$ do in fact agree with $A(s,t)$ formulated in this paper when one considers empirical $A(s,t)$ functions which use $\alpha_i(t)$ such that $\alpha_V(M_V^2)=1$.

The b's can be evaluated in terms of the coupling constants when t takes on values corresponding to particle masses. For example, in the case of the $\pi^+\pi^+$ elastic scattering amplitude continued to $t=m_\rho^2$, we know that the amplitude is determined by the ρ-pole term alone and $\alpha_\rho(m_\rho^2)=1$. The contribution of the ρ pole (we follow the discussion of Ref. 17), at large s, is

$$A_{\pi^+\pi^+}(s,t)=2sg_{\rho\pi\pi}^2/(t-m_\rho^2). \tag{27}$$

The Regge formula gives, in the neighborhood of $t=m_\rho^2$,

$$A_{\pi^+\pi^+}(s,t)=\frac{3\pi}{2}\frac{s}{t-m_\rho^2}\frac{b_{\rho\pi}^2(m_\rho^2)}{\alpha'(m_\rho^2)s_\rho}. \tag{28}$$

Consequently, we have

$$b_{\rho\pi}^2(m_\rho^2)=\frac{4}{3\pi}\alpha'(m_\rho^2)g_{\rho\pi\pi}^2s_\rho. \tag{29}$$

Similarly, for the no-spin-flip amplitude in the πN elastic scattering amplitude we find that

$$b_{\rho\pi}(m_\rho^2)b_{\rho N}(m_\rho^2)=\frac{4\alpha'(m_\rho^2)}{3\pi}g_{\rho NN}g_{\rho\pi\pi}s_\rho. \tag{30}$$

In general we conclude that

$$b_{\rho x}(m_\rho^2)=(4\alpha'(m_\rho^2)s_\rho/3\pi)^{1/2}g_{\rho xx}, \tag{31a}$$

and for the ω trajectory that

$$b_{\omega x}(m_\omega^2)=(4\alpha'(m_\omega^2)s_\omega/3\pi)^{1/2}g_{\omega xx}, \tag{31b}$$

where the coupling constants are defined in Sec. III.

[16] M. Gell-Mann, Phys. Rev. Letters **8**, 263 (1962); V. N. Gribov and I. Y. Pomeranchuck, *ibid.* **8**, 343 (1962).
[17] S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. **126**, 2204 (1962).
[18] B. R. Desai, Phys. Rev. **138**, B1174 (1965).
[19] See Eq. (4–22) in Ref. 15.

In order to discuss total cross sections, we substitute Eq. (26) into Eq. (22) and write

$$\sigma_{\mathrm{tot}}(x,y)=\sum_i \frac{b_{ix}(0)b_{iy}(0)}{2q_{xy}\sqrt{s}}C_i\left(\frac{s}{s_i}\right)^{\alpha_i(0)}, \tag{32}$$

where

$$C_i=\pm\tfrac{1}{2}\pi^{3/2}\frac{\Gamma[\alpha_i(0)+\frac{1}{2}]}{\Gamma[\alpha_i(0)+1]}[2\alpha_i(0)+1]. \tag{33}$$

The plus sign goes with positive signature, or equivalently, for the trajectories studied here, with those trajectories even under charge conjugation.

We will discuss the trajectories which are odd under charge conjugation. Following Barger and Olsson[20] we isolate the contributions of the ρ and the ω trajectories by considering combinations

$$\Delta_{xy}\equiv\sigma_{\mathrm{tot}}(\bar{x},y)-\sigma_{\mathrm{tot}}(x,y). \tag{34}$$

Using the behavior under charge conjugation and isospin rotations, we introduce the residue parameters B. $B_{ix}\equiv b_{ix}(0)$.

$$\begin{aligned}
&B_{\omega K}=B_{\omega K^+}=-B_{\omega K^-},\\
&B_{\omega N}=B_{\omega p}=-B_{\omega\bar{p}}=B_{\omega n}=-B_{\omega\bar{n}},\\
&B_{\rho N}=B_{\rho p}=-B_{\rho n},\\
&B_{\rho\pi}=B_{\rho\pi^+}=-B_{\rho\pi^-},\\
&B_{\rho K}=B_{\rho K^+}=-B_{\rho K^-}.
\end{aligned} \tag{35}$$

The following five combinations of cross sections share the feature that each depends on a single vector trajectory:

$$\Delta_{Kp}+\Delta_{Kn}=B_{\omega K}B_{\omega N}\frac{(s/s_\omega)^{\alpha_\omega}}{m_N p}\pi^{3/2}\times\frac{\Gamma(\alpha_\omega+\frac{1}{2})}{\Gamma(\alpha_\omega+1)}(2\alpha_\omega+1), \tag{36a}$$

$$\Delta_{pp}+\Delta_{pn}=B_{\omega N}B_{\omega N}\frac{(s/s_\omega)^{\alpha_\omega}}{m_N p}\pi^{3/2}\times\frac{\Gamma(\alpha_\omega+\frac{1}{2})}{\Gamma(\alpha_\omega+1)}(2\alpha_\omega+1), \tag{36b}$$

$$\Delta_{\pi p}=\tfrac{1}{2}B_{\rho N}B_{\rho\pi}\frac{(s/s_\rho)^{\alpha_\rho}}{m_N p}\pi^{3/2}\times\frac{\Gamma(\alpha_\rho+\frac{1}{2})}{\Gamma(\alpha_\rho+1)}(2\alpha_\rho+1), \tag{36c}$$

$$\Delta_{Kp}-\Delta_{Kn}=B_{\rho N}B_{\rho K}\frac{(s/s_\rho)^{\alpha_\rho}}{m_N p}\pi^{3/2}\times\frac{\Gamma(\alpha_\rho+\frac{1}{2})}{\Gamma(\alpha_\rho+1)}(2\alpha_\rho+1), \tag{36d}$$

$$\Delta_{pp}-\Delta_{pn}=B_{\rho N}B_{\rho N}\frac{(s/s_\rho)^{\alpha_\rho}}{m_N p}\pi^{3/2}\times\frac{\Gamma(\alpha_\rho+\frac{1}{2})}{\Gamma(\alpha_\rho+1)}(2\alpha_\rho+1), \tag{36e}$$

where m_N is the target nucleon mass and p is the laboratory momentum of the projectile. We have used the notation $\alpha_i\equiv\alpha_i(0)$.

In Table II, we present the values of the first four combinations based on the experimental data of Galbraith *et al.*,[21] for momentum ranging from 6 to 18 BeV/c. The experimental errors for the fifth combination (36e) is larger than the mean values. Hence (36e) has not been used in the present analysis. It is clear that the ratio of (36b) and (36a) will give us the ratio $B_{\omega N}/B_{\omega K}$ and, similarly, the ratio of (36c) and (36d) will give us the ratio $B_{\rho\pi}/2B_{\rho K}$. These are presented in columns 4 and 7 of Table II. From Eqs. (31) and (14) we see that

$$b_{\omega N}(m_\omega^2)/b_{\omega K}(m_\omega^2)=g_{\omega NN}/g_{\omega KK}=3F_{\omega NN}(0)/F_{\omega KK}(0), \tag{37}$$
$$b_{\rho\pi}(m_\rho^2)/2b_{\rho K}(m_\rho^2)=g_{\rho\pi\pi}/2g_{\rho KK}=F_{\rho\pi\pi}(0)/F_{\rho KK}(0).$$

If we assume $b(t)$ is independent of t, then we obtain the same ratios for the B's:

$$B_{\omega N}/B_{\omega K}=3F_{\omega NN}(0)/F_{\omega KK}(0)\simeq3, \tag{38a}$$

$$B_{\rho\pi}/2B_{\rho K}=F_{\rho\pi\pi}(0)/F_{\rho KK}(0)\simeq1. \tag{38b}$$

We are assuming here that the form factors are close to unity at zero momentum transfer. This point is discussed further by Kroll *et al.*[6] In particular, refer to their equations (7.9) and (7.10). Earlier discussions of this point are given in Ref. 2, cf. their equations (6.3) and (6.4).

The experimental data of Galbraith *et al.*[21] have been fitted to a Regge-pole formula by Barger, Olsson, and Sarma[22] using a least-squares fitting procedure, where the coupling of the vector Regge trajectories to the hadrons was constrained to be invariant under $SU(3)$. Thus, in their analysis $B_{\rho\pi}$ was restricted to be equal to $2B_{\rho K}$. They found that $B_{\omega n}/B_{\omega K}=3.4\pm0.7$. This is implicit in Eq. (9) of Ref. 22. In a recent paper[23] Barger and Olsson have shown that $B_{\rho\pi}/2B_{\rho K}\simeq1$ without the constraint of $SU(3)$ symmetry. The purpose of presenting an analysis of the experimental data in this paper is to show that the conclusion regarding the coupling of the ω trajectory is also independent of any assumption concerning $SU(3)$ symmetry.

We have, thus, experimentally verified the universality ratio for $b_{\omega N}(0)/b_{\omega K}(0)$ and $b_{\rho\pi}(0)/b_{\rho K}(0)$. We emphasize that this supports the particular form of universality assumed in Eq. (6).

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TABLE II. Comparison of the predictions (38a) and (38b) with experiment. See text for fuller explanation.

Momentum (BeV/c)	(36a)	(36b)	$B_{\omega N}/B_{\omega K}$	(36c)	(36d)	$B_{\rho\pi}/2B_{\rho K}$
6	11.4±0.5	35.6±4.4	3.1±0.4	2.3±0.35	2.6±0.5	0.9 ±0.25
8	8.7±0.5	33.7±4.4	3.9±0.5	2.4±0.35	3.9±0.5	0.62±0.12
10	8.5±0.5	...	...	1.7±0.35	1.9±0.5	0.9 ±0.3
12	7.2±0.5	25.7±4.2	3.6±0.7	1.7±0.35	1.4±0.5	1.2 ±0.3
14	6.8±0.5	24.8±4.2	3.7±0.7	1.5±0.35	1.4±0.5	1.1 ±0.3
16	7.6±0.7	23.0±4.2	3.0±0.7	1.7±0.35	1.0±0.7	1.7 ±1.3
18	7.1±1.3	26.8±9.0	3.8±1.3	1.5±1.35	0.7±1.3	2.1 ±4.2

V. EVALUATION OF b_{ix} AND s_i PARAMETERS

In this section we will assume b_{ix} is independent of t and evaluate it and s_i in terms of the coupling constants and the cross-section data. Combining Eqs. (29)–(31) we find that

$$\left(\frac{s_\rho}{\mathrm{BeV}^2}\right)^{1-\alpha_\rho}=\frac{3}{64\pi^{3/2}}\frac{\Gamma(\alpha_\rho+1)}{\Gamma(\alpha_\rho+\frac{3}{2})}\left(\frac{g_{\rho\pi\pi}g_{\rho NN}}{4\pi}\right)^{-1}\left(\frac{s}{\mathrm{BeV}^2}\right)^{1-\alpha_\rho}\frac{2\Delta_{\pi p}}{\alpha_\rho'}$$
$$=\frac{3}{64\pi^{3/2}}\frac{\Gamma(\alpha_\rho+1)}{\Gamma(\alpha_\rho+\frac{3}{2})}\left(\frac{g_{\rho KK}g_{\rho NN}}{4\pi}\right)^{-1}\left(\frac{s}{\mathrm{BeV}^2}\right)^{1-\alpha_\rho}\frac{\Delta_{K\rho}-\Delta_{KN}}{\alpha_\rho'}, \tag{39a}$$

$$\left(\frac{s_\omega}{\mathrm{BeV}^2}\right)^{1-\alpha_\omega}=\frac{3}{64\pi^{3/2}}\frac{\Gamma(\alpha_\omega+1)}{\Gamma(\alpha_\omega+\frac{3}{2})}\left(\frac{g_{\omega NN}^2}{4\pi}\right)^{-1}\left(\frac{s}{\mathrm{BeV}^2}\right)^{1-\alpha_\omega}\frac{\Delta_{p\rho}+\Delta_{\rho N}}{\alpha_\omega'}$$
$$=\frac{3}{64\pi^{3/2}}\frac{\Gamma(\alpha_\omega+1)}{\Gamma(\alpha_\omega+\frac{3}{2})}\left(\frac{g_{\omega KK}g_{\omega NN}}{4\pi}\right)^{-1}\left(\frac{s}{\mathrm{BeV}^2}\right)^{1-\alpha_\omega}\frac{\Delta_{K\rho}+\Delta_{KN}}{\alpha_\omega'}. \tag{39b}$$

As already verified, Eqs. (39a) and (39b) are essentially independent of s for the data shown in Table II. Hence average values were used in the evaluation of s_i. The values of $b_{\rho\pi\pi}$ and $b_{\omega NN}$ were found using Eq. (31) and the values of $g_{\rho\pi\pi}$ and $g_{\omega NN}$ discussed in Sec. III.

We find that

$$s_\rho^{1/2}\simeq186\ \mathrm{MeV},$$
$$s_\omega^{1/2}\simeq290\ \mathrm{MeV},$$
$$b_{\rho\pi\pi}\simeq1.02,$$
$$\tfrac{1}{3}b_{\omega nn}\simeq0.62.$$

It is interesting to note that Van Royen and Weisskopf[8] in constructing a quark-antiquark wave function for the vector mesons demand that $|\psi_V(0)|^2=m_V m\pi^2/2$. Since this is an s wave function we find an approximate radius from the condition

$$\tfrac{4}{3}\pi R_V^3|\psi_V(0)|^2\simeq1,$$

which yields

$$R_V\simeq(1/m_\pi)[(\tfrac{3}{2}\pi)(m_\pi/m_V)]^{1/3}.$$

The resulting values $R_\omega\approx R_\rho\approx0.62$ F are consistent with the scaling lengths obtained by us, which are $s_\rho^{-1/2}\sim1$ F, $s_\omega^{-1/2}\sim0.7$ F.

VI. COMPARISON WITH OTHER EVALUATIONS OF $A(s,t)$

Rather than compare various evaluations of $A(s,t)$ directly, it is more convenient to take out the factors $\xi(t)s^{\alpha(t)}$ which they all have in common. Here

$$\xi(t)=(e^{-i\pi\alpha}-1)/\sin\pi\alpha\sim2/\pi\alpha'(t)(t-m_V^2) \tag{40}$$

in the neighborhood of a vector-meson pole at $\alpha=1$. We consequently define a new function which is real and dimensionless,

$$G_V^2(t)/4\pi=A_V(s,t)/4\pi^2\xi(t)[s\alpha'(t)]^{\alpha(t)}. \tag{41}$$

In this equation, $A_V(s,t)$ is the contribution of a single vector trajectory to a given process. In the neighborhood of a vector pole at $\alpha(m_V^2)=1$ for a process $x+y\to x+y$,

$$G_V^2(m_V^2)/4\pi=g_{Vxx}g_{Vyy}/4\pi.$$

This result can be checked explicitly for $\pi\pi$ scattering by referring to Eq. (27) where $A_{\pi\pi}(s,t)$ is given in the neighborhood of the pole at $t=m_\rho^2$. In terms of our parameters for $x+y\to x+y$,

$$\frac{G_V^2(t)}{4\pi}=\frac{1}{8\pi^{1/2}}\frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)}(2\alpha+1)\frac{b_{Vx}b_{Vy}}{(s_{V\alpha}')^\alpha}, \tag{42}$$

where we make the assumption[17] that b_{Vx}, b_{Vy} are

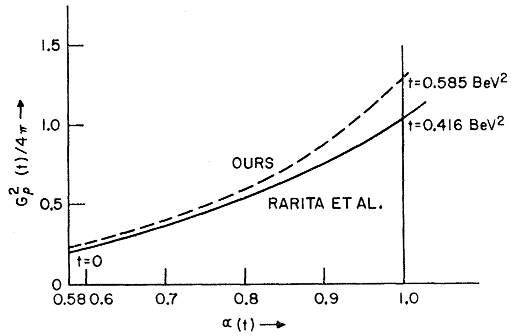

FIG. 1. Plot of $G_\rho^2(t)/4\pi$ against $\alpha(t)$.

independent of t. At $\alpha=1$ this reduces to

$$G_V^2(m_V^2)=\tfrac{3}{4}\pi[b_{Vx}b_{Vy}/s_i\alpha'(m_V^2)]$$

and gives back the relations in Eq. (31).

At $t=0$ and using the high-energy approximation $2s^{1/2}q_{xy}\sim s$,

$$G_V^2(0)/4\pi=s\sigma_V(s)/4\pi^2[s\alpha_V'(0)]^{\alpha_V(0)}\,, \tag{43}$$

where $\sigma_V(s)$ is the magnitude of the contribution of the trajectory in question to the total cross section.

Hence, $G_V^2(0)/4\pi$ and $G_V^2(m_V^2)/4\pi$ are empirically given in terms of $\sigma_V(s)$, $\alpha_V(0)$, $\alpha_V'(0)$ and the coupling constants. The behavior for $0<t<m_V^2$ goes as $2s^{1/2}q_{xy}\sim s$ $(2\alpha+1)[\Gamma(\alpha+\frac{1}{2})/\Gamma(\alpha+1)](s_i\alpha')^{-\alpha}$ in our model. This behavior is compared to two reaction analyses in Figs. 1 and 2. For the ρ trajectory we present $G_\rho^2(t)/4\pi$ derived from the recent analysis of Rarita, Riddell, Chiu, and Phillips.[24] [Their solution (1a) was used.] We did not use their ω-trajectory results since their $\alpha_\omega(t)$, when continued to positive t, missed the point $\alpha_\omega=1$ at $t=m_\omega^2$ by a large amount, and their value of $G_\omega^2/4\pi$ at $\alpha_\omega=1$ differed considerably from the approximately known values of the coupling constants.

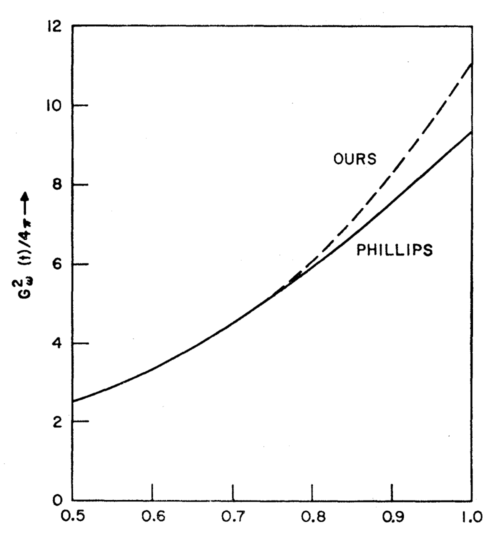

FIG. 2. Plot of $G_\omega^2(t)/4\pi$ against $\alpha(t)$.

The earlier analyses of Phillips[13] did constrain $\alpha_\omega(m_\omega^2)=1$ and we present his $G_\omega^2(t)/4\pi$ for the ω trajectory. The comparison is quite good when made in the region $0<t<m_V^2$. This lends support to the form assumed in Eq. (42). It should be pointed out that for $t<0$ the residues go through zero[24] and this characteristic is *not* a feature of the formula in Eq. (42).

VII. CONCLUSIONS

We have used the Frautschi, Gell-Mann, Zachariasen[17] model for the Regge residue $\beta(t)$ given by Eq. (24) where it is assumed that the reduced residue $b(t)$ is independent of t and a scaling parameter s_i was introduced. Using information about total cross sections, the value of b_{ix} were shown to be in the ratios given by the form of the universality hypothesis given in Eq. (6). The particular ratios are given in Eqs. (38a) and (38b). The scaling parameters indicate a characteristic length of the order of (0.7–1.0) F which is consistent with the observed slopes $(d\alpha/dt)$ as discussed in Sec. IV. The various methods for determining the ω coupling constant were discussed in Sec. III and it was verified that the universal value of g_ω coming from π^0 decay is consistent with the value of $g_{\omega NN}$ coming from the nucleon-nucleon scattering thus lending support to the universality hypothesis. The simple t dependence of $\beta(t)$ which follows from a t-independent reduced residue was found to be consistent with the phenomenological analyses of $\beta(t)$ for $0<t<m_V^2$ which appeared in Refs. 13 and 24.

We, therefore, conclude that the concept of a reduced residue slowly varying in t with an associated scale parameter is a physically meaningful one and quite useful for the study of coupling theories. In particular, the universality theory of couplings gains support through this type of analysis, and in the sense that the ρ meson is coupled to the I_z we conclude that the ω meson is coupled to $\frac{1}{2}(2N+Y)$. The explicit statement is given in Eq. (6).

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