Virtual-Photon–Pion Scattering, Hard-Pion Current Algebra, and Dispersion Relations*

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It is shown that the $SU(2)\times SU(2)$ current-algebra results for three- and four-point functions are broadly consistent with dispersion relations and the present saturation scheme, provided one writes once-subtracted dispersion relations for some of the form factors occurring in the virtual-photon–pion scattering amplitude.

RECENTLY Chanda, Mohapatra, and Okubo (CMO)[1] have obtained low-energy sum rules for virtual-photon-pion scattering (four-point function in the forward direction), utilizing both current algebra[2] and dispersion relations. Saturating the sum rules by low-lying intermediate states, they obtained results inconsistent with those given by sum rules of vertex functions.[3] We know that the latter have led to good results for the decay widths of both vector and axial-vector mesons.[3] CMO concluded that the inconsistency observed by them arose most likely because of incomplete saturation. We would like to raise a few points regarding their calculation.

(i) CMO worked in the soft-pion limit; now we can obtain hard-pion four-point-function sum rules, utilizing the recently developed technique of Ward identities.[4]

(ii) CMO assumed unsubtracted dispersion relations.[5] This may be justified on the basis of the high-energy behavior given by the Regge pole exchanged in the t channel. However, this is true only if one neglects the possible Regge cut contribution coming from the two-ρ exchange.[6] Furthermore, we notice that the present problem of virtual-photon–pion scattering is intimately related to the problem of the π^+-π^0 electromagnetic mass difference. It has been clearly established that a hard-pion current-algebra calculation of this mass difference[7] is logarithmically divergent, and hence requires a cutoff. Now, we adopt the viewpoint that a cutoff in the π^+-π^0 mass-difference calculation is reflected as requiring a subtraction in the virtual-photon–pion scattering problem.

Thus, we aim to show in this paper that the hard-pion $SU(2)\times SU(2)$ current-algebra results for three- and four-point functions are consistent with dispersion relations and with the present saturation scheme (of including up to spin-1 states) within the limits of the current-algebra approach,[8] provided that one introduces a subtraction in the dispersion relations for the form factors F_1 and F_2 (defined below) occurring in the virtual-phonon–pion scattering amplitude.

We define[9]

$$(2k_0V)i\int d^4x\, e^{iq\cdot x}[\langle\pi^+,k|T(V_\mu{}^3(x),V_\nu{}^3(0))|\pi^+,k\rangle - (\pi^+\to\pi^0)]$$
$$\equiv F_{\mu\nu}(k,q)=\delta_{\mu\nu}F_1+q_\mu q_\nu F_2+(q_\mu k_\nu+k_\mu q_\nu)F_3 + i(q_\mu k_\nu-k_\mu q_\nu)F_4+k_\mu k_\nu F_5\,, \quad (1)$$

where k and q are the four-momenta of the pion and the photon, respectively, and the F_i's are functions of q^2 and ν with $\nu=-k\cdot q$. Crossing symmetry and current conservation lead to the following relations:

$$F_1+q^2F_2=\nu F_3\,,\quad q^2F_3=\nu F_5\,,\quad F_4=0\,. \quad (2)$$

Utilizing the Ward identities for $SU(2)\times SU(2)$ algebra of currents, crossing symmetry, vector-current conservation, smooth dependence on the momentum, and saturation by π, A_1, and ρ states only, Schnitzer and Weinberg and Gerstein and Schnitzer[4] have recently obtained quite general expressions for three- and four-point functions, respectively. It is interesting to note

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[1] R. Chanda, R. N. Mohapatra, and S. Okubo, Phys. Rev. 170, 1344 (1968).

[2] These authors (hereafter referred to as CMO) mainly used the spectral functions sum rules obtained by S. Weinberg, Phys. Rev. Letters 18, 507 (1967).

[3] H. J. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967); T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters 19, 1067 (1967); S. G. Brown and G. B. West, *ibid.* 19, 812 (1967); R. Arnowitt *et al.*, *ibid.* 19, 1085 (1967).

[4] H. J. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967); I. S. Gerstein and H. J. Schnitzer, *ibid.* 170, 1638 (1968).

[5] See, e.g., H. Harari, Phys. Rev. Letters 17, 1303 (1966); 18, 319 (1967).

[6] I. J. Muzinich, Phys. Rev. Letters 18, 381 (1967); R. J. N. Phillips, Phys. Letters 24B, 342 (1967).

[7] I. S. Gerstein *et al.*, Phys. Rev. Letters 19, 1064 (1967); G. C. Wick and B. Zumino, Phys. Letters 25B, 479 (1967).

[8] Thus, e.g., in the present calculation, current algebra leads us to $G_{\omega\pi\gamma}=0$, whereas experimentally we know that $\Gamma(\omega\to\pi\gamma)=1.16\pm0.11$ MeV. This discrepancy arises since at no stage in the $SU(2)\times SU(2)$ current-algebra calculation for three- and four-point functions does the ω state contribute. We further notice that such a contribution is also absent in the phenomenological $SU(2)\times SU(2)$ chiral Lagrangian approach; see, e.g., D. A. Geffen and S. Gasiorowicz, Argonne National Laboratory Report, 1968 (unpublished).

[9] As mentioned in Ref. 1, this choice of the amplitude automatically removes the $I=0$ scalar term arising in the current-algebra approach, since such a term will contribute equally to $\gamma\pi^+$ and $\gamma\pi^0$ amplitudes.

that the latter lead to results identical with those obtained from the chiral Lagrangian approach.[10] Making use of the results of Gerstein and Schnitzer,[4] we obtain[11]

$$F_1(\nu,q^2)=-\frac{2m_\rho^2}{q^2+m_\rho^2}\left[m_\rho^2+\tfrac{1}{2}q^2-\frac{1}{4}\left(\frac{\nu_A(\nu^2\delta^2+q^4)-2\delta\nu^2q^2}{\nu_A^2-\nu^2}\right)\right], \quad (3a)$$

$$F_2(\nu,q^2)=\frac{2m_\rho^2}{q^2(q^2+m_\rho^2)^2}\left[m_\rho^2+\tfrac{1}{2}q^2-\frac{1}{4}\left(\frac{\nu_A[q^4-\nu^2q^2(1-\delta)^2/2m_\rho^2]-2\delta\nu^2q^2}{\nu_A^2-\nu^2}\right)+\frac{4m_\rho^2\nu^2}{q^4-4\nu^2}\left(1+\frac{q^2(1+\delta)}{4m_\rho^2}\right)^{2}\right], \quad (3b)$$

where $\nu_A=\frac{1}{2}(2m_\rho^2-m_\pi^2+q^2)$ and δ is a free parameter.[3,4] Expressions for F_3 and F_5 are readily obtained from Eqs. (2) and (3).

Now, we *assume* that for fixed q^2, F_i $(i=1,2)$ satisfy once-subtracted dispersion relations (OSDR):

$$F_i(\nu,q^2)-F_i(\nu_0,q^2)=\frac{\nu-\nu_0}{\pi}\int_{-\infty}^{\infty}d\nu'\frac{\operatorname{Im}F_i(\nu',q^2)}{(\nu'-\nu)(\nu'-\nu_0)}, \quad (4)$$

where ν_0 is the subtraction point. The ImF_i's are easily calculated by expanding the absorptive part on the left-hand side in Eq. (1), and saturating the intermediate states by π, A_1, and ω states only.[12] The couplings used are defined as

$$\langle\pi^+,k|V_\mu^3(0)|\pi^+,p\rangle=\frac{1}{V}\frac{1}{(4p_0k_0)^{1/2}}(p+k)_\mu F_\pi(q^2), \quad (5a)$$

$$\langle\pi^0,k|V_\mu^3(0)|\omega,p\rangle=\frac{i}{V}\frac{1}{(4p_0k_0)^{1/2}}G_{\omega\pi\gamma}(q^2)\epsilon_{\mu\alpha\beta\gamma}k_\alpha p_\beta\epsilon_\gamma{}^\omega(p), \quad (5b)$$

$$\langle\pi^a,k|V^c(0)|A_1{}^b,p\rangle=\frac{-\epsilon^{abc}}{V(4p_0k_0)^{1/2}}\{[\delta_{\mu\alpha}(p^2-k^2)+(p+k)_\mu k_\alpha]C(q^2)+[\delta_{\mu\alpha}q^2+(p-k)_\mu k_\alpha]D(q^2)\}\epsilon_\alpha{}^{A_1}(p), \quad (5c)$$

with $q^2=(k-p)^2$ and a, b, c being the isospin indices. We find that whereas π and A_1 contribute to the π^+ term in Eq. (1), the only contribution to the π^0 term comes from ω state. Substituting the absorptive parts in Eqs. (4), we obtain

$$F_1(\nu,q^2)-F_1(\nu_0,q^2)=(\nu^2-\nu_0^2)\left(\frac{[(m_\pi^2-2m_\rho^2)C(q^2)+q^2D(q^2)]^2\nu_A}{(\nu_A^2-\nu_0^2)(\nu_A^2-\nu^2)}+G_{\omega\pi\gamma}{}^2(q^2)\frac{[m_\pi^2m_\omega^2-(\nu_\omega+m_\pi^2)^2]\nu_\omega}{(\nu_\omega^2-\nu_0^2)(\nu_\omega^2-\nu^2)}\right), \quad (6a)$$

$$F_2(\nu,q^2)-F_2(\nu_0,q^2)=(\nu^2-\nu_0^2)\left(\frac{8q^2F_\pi^2(q^2)}{(q^4-4\nu_0^2)(q^4-4\nu^2)}+\frac{\nu_A}{2m_\rho^2}\frac{\{\nu_A^2[D(q^2)-C(q^2)]^2+8m_\rho^2\nu_AC(q^2)[C(q^2)+D(q^2)]-2m_\rho^2q^2[C(q^2)+D(q^2)]^2\}}{(\nu_A^2-\nu_0^2)(\nu_A^2-\nu^2)}+G_{\omega\pi\gamma}{}^2(q^2)\frac{\nu_\omega m_\pi^2}{(\nu_\omega^2-\nu_0^2)(\nu_\omega^2-\nu^2)}\right), \quad (6b)$$

where $\nu_{A,\omega}=\frac{1}{2}(m_{A,\omega}{}^2-m_\pi^2+q^2)$. Substituting for the left-hand sides in Eqs. (6a) and (6b) from Eqs. (3a) and (3b), respectively, and comparing, we obtain

$$F_\pi(q^2)=\frac{m_\rho^2}{q^2+m_\rho^2}\left(1+\frac{q^2(1+\delta)}{4m_\rho^2}\right), \quad (7)$$

$$C(q^2)=\frac{\delta}{2\sqrt{2}}\frac{m_\rho}{q^2+m_\rho^2},\quad D(q^2)=\frac{(2-\delta)}{2\sqrt{2}}\frac{m_\rho}{q^2+m_\rho^2}, \quad (8)$$

$$\nu_\omega G_{\omega\pi\gamma}{}^2=0. \quad (9)$$

First, we observe that the results obtained above are independent of the choice of the subtraction point. Secondly, we see that they are identical with those obtained from hard-pion current-algebra calculations

[10] See, e.g., J. Schwinger, Phys. Letters **24B**, 473 (1967); S. Weinberg, Phys. Rev. **166**, 1568 (1968); J. Wess and B. Zumino, *ibid.* **163**, 1727 (1967); B. W. Lee and H. T. Nieh, *ibid.* **166**, 1507 (1968).

[11] Throughout this paper, we make use of both the spectral-function sum rules $g_\rho^2/m_\rho^2=g_{A_1}{}^2/m_{A_1}{}^2+f_\pi^2$ and $g_\rho^2=g_{A_1}{}^2$, and the KSRF relation $g_\rho^2=2m_\rho^2f_\pi^2$. For these, see Ref. 2 and the following: K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966); J. J. Sakurai, Phys. Rev. Letters **19**, 803 (1967).

[12] Note that the ρ pole does not contribute because of G-parity considerations and that ϕ contribution is negligible since $G_{\phi\pi\gamma}\approx0$.

for three-point functions.[13] Since our results do not depend upon the subtraction constant, we may take, for simplicity, $\nu_0=0$. With this choice, from Eqs. (2) and (4) it at once follows that $F_3(\nu,q^2)$ satisfies an unsubtracted dispersion relation and that F_5 leads to a superconvergent relation.

To sum up, we have shown that the current-algebra results for virtual-photon–pion scattering are consistent with dispersion relations, provided that we write OSDR for the amplitudes F_1 and F_2. In other words, we have shown that hard-pion results for a four-point function in the forward direction can be easily obtained by writing OSDR, and by determining the subtraction constant from the soft-pion current algebra[14] for the four-point function and the dispersion integral from the hard-pion three-point function. The only anomaly is that with the neglect of the Regge cuts, it is expected[5] that F_1 and F_2 should satisfy unsubtracted dispersion relations. Thus, the algebra of currents and Regge pole theory (without cuts) lead us to different results. Similar calculations are in progress for πA_1 and $\pi\rho$ scattering amplitudes, and perhaps these will help in reaching a more definite conclusion.

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[13] For the first three sum rules, see papers quoted in Ref. 3. For the last sum rule, see Ref. 8 and D. G. Sutherland, Nucl. Phys. **B2**, 433 (1967).

[14] The first successful application of current algebra to determine the subtraction constant in OSDR was made by S. Okubo *et al.*, Phys. Rev. Letters **19**, 407 (1967) for the decay $K\rightarrow 2\pi$. Later, a similar approach in an N/D formalism was used by S. C. Bhargava, S. N. Biswas, K. C. Gupta, and K. Datta, *ibid.* **20**, 558 (1968) to calculate unitarity corrections to current-algebra results for s-wave πN scattering.

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Compton Scattering of Spin-One Particles

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Kinematic singularity- and zero-free invariant amplitudes for Compton scattering of spin-one particles are derived. Gauge invariance is automatically satisfied. A complete set of low-energy theorems is obtained. In addition to the well-known first-order theorems, higher-order theorems also exist. It is shown that up to second order in the photon energy the 12 independent amplitudes are determined by the static moments of the target particle plus four dynamic structure-dependent constants; up to third order they are determined by the static moments plus eight additional constants. Dispersion relations may be written down for the invariant amplitudes without any *ad hoc* subtractions. The asymptotic behavior of these amplitudes is carefully examined, leading to a systematic derivation of possible sum rules and superconvergence relations. Generalizations of previously known sum rules as well as new ones are obtained.

I. INTRODUCTION

RECENTLY, there has been much interest in studying the analytic structure of the scattering amplitude due to Lorentz transformation properties of the *external* particle states involving spin. Detailed analysis of these *kinematic* singularities and zeros of the scattering amplitude has led to physically interesting results. In particular, for processes involving photons these analytic properties are intimately related to gauge invariance. Detailed analysis of these analytic properties has helped in clarifying the meaning of the well-known low-energy theorems as well as in deriving new ones for various photon processes.[1]

The kinematic structure of the two-body helicity amplitudes has been extensively studied recently.[2] It is shown that regularized helicity amplitudes can be defined such that they are free of kinematic singularities. However, they must satisfy certain kinematic constraints which make their use in certain applications rather cumbersome. For example, if one wants to write down dispersion relations for these amplitudes, one must make *ad hoc* subtractions in order to ensure that these constraints are satisfied.

On the other hand, there exist invariant amplitudes which, if properly chosen, are free of both kinematic singularities and zeros[3] (constraints). In obtaining these

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[2] Y. Hara, Phys. Rev. **136**, B507 (1964); L. L. C. Wang, *ibid.* **142**, 1187 (1965); H. P. Stapp, *ibid.* **160**, 1251 (1967); G. Cohen-Tannoudji, A. Morel, and H. Navelet, Ann. Phys. (N. Y.) **46**, 239 (1968); J. P. Ader, M. Capdeville, and H. Navelet, Nuovo Cimento **55A**, 315 (1968). The last reference deals with two-body scattering amplitudes involving at least one massless particle.

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