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Renormalizable Theory of the Weak Interactions*

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A class of renormalizable field theories of the weak interactions is investigated. These theories, of the type first suggested by Kummer and Segrè, postulate that the weak interactions are mediated by the simultaneous exchange of two scalar particles. It is found that although the leptonic and semileptonic weak interactions can be fairly easily described by a two-scalar-exchange mechanism, the structure of observed nonleptonic weak processes presents serious obstacles to the success of such a theory. Nevertheless, two particular models are discussed which may correctly describe not only the leptonic and semileptonic weak interactions but also nonleptonic weak processes.

I. INTRODUCTION

The presently observed weak interactions can be successfully described, as is well known, by the phenomenological Lagrangian

$$\mathcal{L}_{\text{ph}}=\tfrac{1}{2}G(J_\mu{}^h+J_\mu{}^l)^*(J_\mu{}^h+J_\mu{}^l)\,, \tag{1}$$

where $J_\mu{}^h$ and $J_\mu{}^l$ are the conventional hadronic and leptonic weak currents. In order to construct a complete theory, this Lagrangian, or one modified by the addition of an intermediate vector boson W,

$$\mathcal{L}_{\text{wk}}=g_1W_\mu{}^*(J_\mu{}^h+J_\mu{}^l)+g_1W_\mu(J_\mu{}^h+J_\mu{}^l)^*\,, \tag{2}$$

can be treated as a field-theoretic Lagrangian. The resulting field theory is nonrenormalizable and therefore plagued by serious difficulties impeding the calculation of all but the lowest order terms of an expansion in powers of G (or g_1). In the following we will attempt to circumvent these difficulties by adopting a fundamental Lagrangian basically very different from (1) or (2). In particular, a class of Lagrangian field theories will be discussed which not only agree to lowest order with the known leptonic and semileptonic weak interactions but are also renormalizable.

The type of theory to be discussed postulates that the weak interactions are mediated by the simultaneous exchange of two heavy scalar bosons.[1] Thus, for example, neutron β decay is described by diagrams of the type found in Fig. 1. We show that if certain general conditions are satisfied by the weak couplings of these exchanged bosons to the hadrons and leptons, then in first approximation the usual form (1) for the leptonic and semileptonic weak interactions[2] is guaranteed. These conditions can be satisfied by couplings consistent with renormalizable field theory. Additional requirements are placed on the theory by our knowledge of the nonleptonic weak interactions. However, the many difficulties associated with a field-theoretic treatment of the strong interactions prevent a useful general analysis. We will be content with a qualitative investigation of two simple models. These models must specify not only the details of the weak couplings but also part of the structure of the strong interactions.

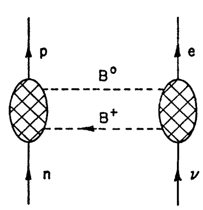

Fig. 1. An example of the type of diagram that describes neutron β decay in the two-scalar-exchange theory.

The number of different scalar bosons which can be exchanged in pairs can of course be large. For simplicity we will limit the discussion to theories containing N distinct scalar bosons which can be exchanged, all with the same mass M. These appear in a weak Lagrangian of the form

$$\begin{aligned}\mathcal{L}_{\text{wk}}&=\mathcal{L}_{\text{wk}}{}^l+\mathcal{L}_{\text{wk}}{}^h\\&=g\sum_{i=1}^{N}B_i(S_i{}^l+S_i{}^h)\,,\end{aligned} \tag{3}$$

where the B_i are N Hermitian fields corresponding to the bosons which can be exchanged. The local operators $S_i{}^l(x)$ and $S_i{}^h(x)$ are composed of leptonic and hadronic operators, respectively, and thus commute to zero order in g. $S_i{}^l(x)$ and $S_i{}^h(x)$ transform like scalar functions under the proper Lorentz group. The coupling constant g must be sufficiently small that perturbation theory is applicable.

In Sec. II it is shown that if the operators $S_i{}^l(x)$ are required to have a certain general form, then: (i) $\mathcal{L}_{\text{wk}}{}^l$ given in (3) will, to lowest order, correctly describe muon decay and all other known features of the low-energy leptonic weak interactions; (ii) $\mathcal{L}_{\text{wk}}{}^l$ will generate a renormalizable field theory. In order to be concrete, we discuss in detail a simple model for $\mathcal{L}_{\text{wk}}{}^l$. This model requires the introduction of two heavy neutral leptons

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[1] That such a theory may, in fact, be able to describe the observed weak interactions was first suggested and discussed by W. Kummer and G. Segrè, Nucl. Phys. 64, 585 (1965). The author is indebted to F. E. Low for bringing that article to his attention. Throughout this paper a "scalar" particle will be a particle with zero spin, but it may not have a definite parity.

[2] The leptonic, semileptonic, and nonleptonic weak interactions are weak processes involving only leptons, leptons and hadrons, and only hadrons, respectively.

L_e^0 and L_μ^0 as well as the intermediate scalar bosons $B^\pm$ and B^0. Additions to the anomalous magnetic moments of the charged leptons and corrections of order $(m_\mu/M)^2$ to the μ decay amplitude are computed. These set a lower limit on M, the heavy boson's mass,

$$M \geq 2 \text{ BeV}. \tag{4}$$

In addition, the lowest-order parity-violating corrections to the electromagnetic vertex of the charged leptons are computed.

In analogy with the weak couplings proposed for the leptons, the hadronic operators S_i^h are assumed to satisfy certain general requirements specified in Sec. III A. Here it is shown that, to order g^4 and zero order in $(m_N/M)^2$, these conditions imply that the low-energy semileptonic weak interactions are described by the phenomenological Lagrangian given in (1). m_N is the nucleon mass. Unfortunately, correction terms of order g^6 or $g^4(m_N/M)^2$ cannot, in general, be computed unless further assumptions are made. In Sec. III B these general considerations are illustrated by the discussion of a particular model. In this model, the operators $S_i^h(x)$ satisfy the requirements proposed in Sec. III A and are written explicitly in terms of an SU_3 triplet of spinor fields and an additional field corresponding to a new heavy fermion.

The nonleptonic weak interactions are discussed in Sec. IV. The general requirements placed on the theory in Secs. II and III are not sufficient to determine the nonleptonic weak interactions—the explicit form of the operators $S_i^h(x)$ must be given. Even when this detailed information has been supplied, only rough qualitative estimates of the weak processes of interest can be made. A general analysis of all possibilities is hence unrewarding, and we will instead discuss two simple models qualitatively. Both of these models assume that the weak hadronic current J_λ^h can be written as a sum of bilinear products of an SU_3 triplet of spinor fields. The first model, requiring a total of six new particles and their antiparticles, is the simplest. However, this model is so constrained by the smallness of parity violation in nuclear transitions (which are proportional to $1/M$) and strangeness-changing weak processes (which increase as M^2) that it may well be inconsistent with present experimental information. The second model, involving eleven new weakly interacting particles and their antiparticles, may allow the intermediate boson masses to be quite large ($M\sim30$ BeV or perhaps even greater), and is very likely consistent with all present experimental knowledge. This model, however, requires two relatively light neutral leptons (with masses about 1 BeV) whose weak couplings are exactly like those of the neutrinos ν_μ and ν_e if the Cabbibo angle θ_C is replaced by $\theta_C-\frac{1}{2}\pi$.

II. LEPTONIC PROCESSES

A. General Discussion

We will now show that the operators $S_i^l(x)$, appearing in the leptonic part of the Lagrangian

$$\mathcal{L}_{\text{wk}}{}^l = g\sum_{i=1}^{N} B_i S_i{}^l, \tag{5}$$

can be so restricted that (i) if taken to lowest order $\mathcal{L}_{\text{wk}}{}^l$ correctly describes the low-energy leptonic weak interactions, and (ii) $\mathcal{L}_{\text{wk}}{}^l$ generates a renormalizable theory. If $\mathcal{L}_{\text{wk}}{}^l$ is to yield a renormalizable theory, the operators $S_i{}^l$ must be constructed of bilinear products of spin-$\frac{1}{2}$ fields—without derivatives.[3] The $V-A$ structure of muon decay can be assured to zero order in $(m_\mu/M)^2$ if we require each lepton field ψ to appear only in the form $(1+\gamma_5)\psi$. M is the mass of the heavy bosons B_i and is assumed to be much larger than the muon mass m_μ. We will therefore require $S_i{}^l$ to have the form

$$S_i{}^l = \sum_{\psi=\mu,e,\nu_\mu,\nu_e} (4\pi)^{1/2} \sum_{n=1}^{N'} [C_i{}^n(\psi)\bar{\psi}(1-\gamma_5)L^n + C_i{}^n(\psi)^*\bar{L}^n(1+\gamma_5)\psi], \tag{6}$$

where μ, e, ν_μ, ν_e are the muon, electron, muon-neutrino, and electron-neutrino fields, respectively. The $4NN'$-complex constants $C_i{}^n(\psi)$ should have magnitude less than, or of the order of, one. The operators L^n are the field operators (conventionally normalized) for N' distinct and presently unobserved spin-$\frac{1}{2}$ particles with masses M_n. In addition to the couplings specified by (5) and (6), the L^n have only electromagnetic interactions. We will assume the existence of separately conserved, additive electron ($\mathbf{L}_e$) and muon ($\mathbf{L}_\mu$) lepton numbers. Conservation of $\mathbf{L}_e$ and $\mathbf{L}_\mu$ and the requirement that the bosons B_i have zero lepton number implies

$$\sum_{n=1}^{N'} C_i{}^n(\psi)^* C_j{}^n(\psi') = 0 \tag{7}$$

f ψ and ψ' carry different lepton numbers.

The Lagrangian described by (5)–(7) correctly predicts all known features of the leptonic weak interactions if a simple relation between g, $C_i{}^n(\psi)$, and G is satisfied, and if M is sufficiently large. To prove this we need only calculate the amplitude for the decay[4]

$$\mu^- \to e^-\nu_\mu\bar{\nu}_e. \tag{8}$$

To order g^4 and first order in $(m_\mu/M)^2$, the amplitude

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[3] A. Salam, Phys. Rev. **82**, 217 (1951); **84**, 426 (1951); P. T. Matthews and A. Salam, Rev. Mod. Phys. **23**, 311 (1951).

[4] The masses M_n must be so chosen that order g^2 decays of the sort $\mu\to\nu+L^i+L^j$ cannot occur.

for the transition (8) in the limit $m_e=0$ is given by

$$\frac{g^4}{M^2}\left[\left(a_1+a_2\frac{m_\mu^2}{M^2}+a_3\frac{p_\mu\cdot k_e}{M^2}+a_4\frac{p_\mu\cdot p_e}{M^2}\right)\right.$$
$$\times\bar{U}_e\gamma^\lambda(1+\gamma_5)U_{\nu_e}\bar{U}_{\nu_\mu}\gamma^\lambda(1+\gamma_5)U_\mu+\frac{1}{M^2}\bar{U}_e p_\mu(1+\gamma_5)$$
$$\times U_{\nu_e}\bar{U}_{\nu_\mu}(1-\gamma_5)[d_1(p_e-k_e)+d_2m_\mu]U_\mu$$
$$\left.+O(m_\mu^4/M^2M_n^2)+O(m_\mu^4/M^4)\right], \quad (9a)$$

where

$$a_1=\sum_{nn'}\Big(\sum_{ij}C_i^n(\nu_\mu)C_j^n(\mu)^*[C_j^{n'}(e)C_i^{n'}(\nu_e)^*$$
$$-C_i^{n'}(e)C_j^{n'}(\nu_e)^*]\Big)\frac{M^2}{M_n^2-M_{n'}^2}$$
$$\times\left(\frac{M^2(M_n^2-M_{n'}^2)}{(M^2-M_n^2)(M^2-M_{n'}^2)}-\frac{M_n^4}{(M^2-M_n^2)^2}\ln(M^2/M_n^2)\right.$$
$$\left.+\frac{M_{n'}^4}{(M^2-M_{n'}^2)^2}\ln(M^2/M_{n'}^2)\right). \quad (9b)$$

The a_i and d_i depend only on $C_i^n(\psi)$, M, and M_n, and are given in Appendix A. p_μ, p_e, k_μ, and k_e are the four-momenta[5] of the muon, electron, muon neutrino, and electron neutrino, respectively, while U_μ, U_e, U_{ν_μ}, and U_{ν_e} are the corresponding spinors. Thus, if M is sufficiently small that the corrections in (9) of order m_μ^2/M^2 can be neglected, and if

$$g^4a_1/M^2=G/\sqrt{2}\,, \quad (10)$$

then the Lagrangian given by (5) and (6) correctly describes muon decay. This Lagrangian also allows, in general, all other lepton-number-conserving four-lepton interactions, such as

$$\bar{\nu}_e+e^-\to\bar{\nu}_e+e^-,$$
$$\nu_\mu+\nu_e\to\nu_\mu+\nu_e, \quad (11)$$
$$e^-+e^-\to e^-+e^-, \quad \text{etc.}$$

to occur to order g^4. The amplitudes for these processes can be easily computed and have a form analogous to that found for muon decay.

B. A Particular Model

Characteristic of this type of two-scalar exchange theory is the possibility of measurable effects of order g^2. If we consider only processes involving known leptons, then the only measurable g^2 effects are corrections to electromagnetic vertices and lepton propagators. Such corrections can be computed explicitly for the general theory described by (5) and (6). However, for clarity, we will investigate these g^2 effects for a specific, simple model. In this model there are two intermediate bosons B^+ and B^0, and their antiparticles, B^- and $\bar{B}^0$. (Here it is inconvenient to deal with Hermitian fields.) In addition, there are two heavy, neutral leptons L_e^0 and L_μ^0, one with electron lepton number and the other with muon lepton number. These leptons are assumed to have the same mass M as $B^\pm$ and B^0. In terms of these fields, the weak Lagrangian is given by

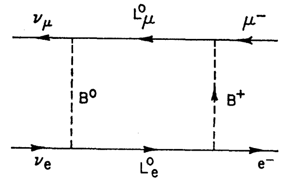

FIG. 2. The Feynman diagram representing the amplitude for muon decay as predicted by the model described in Sec. II B.

$$\mathcal{L}_{\text{wk}}{}^l=(4\pi)^{1/2}g\sum_{l=e,\mu}[B^-\bar{l}(1-\gamma_5)L_l^0+B^0\bar{\nu}_l(1-\gamma_5)L_l^0$$
$$+\text{Hermitian conjugate}]. \quad (12)$$

With this explicit choice of the parameters $C_i^n(\psi)$ we can considerably simplify the expression in (9) for the muon decay amplitude. The resulting amplitude is represented by the Feynman diagram in Fig. 2 and, with the approximations and notation of (9), is given by

$$\frac{1}{3}\frac{g^4}{M^2}\left[\bar{U}_e\gamma^\lambda(1+\gamma_5)U_{\nu_e}\bar{U}_{\nu_\mu}\gamma^\lambda(1+\gamma_5)U_\mu\right.$$
$$\times\left(1+\frac{1}{20}\frac{m_\mu^2}{M^2}-\frac{1}{10}\frac{p_\mu\cdot k_e}{M^2}\right)+\frac{1}{10M^2}\bar{U}_e(1-\gamma_5)p_\mu U_{\nu_e}$$
$$\left.\times\bar{U}_{\nu_\mu}(1-\gamma_5)(2m_\mu-p_e)U_\mu\right]. \quad (13)$$

This expression is certainly consistent with the observed properties of muon decay, if we require that

$$\tfrac{1}{3}(g^4/M^2)=G/\sqrt{2} \quad (14)$$

and that $M\geq 10m_\mu$.

In such a simple model we can easily compute the g^2 corrections to the lepton propagators and the electromagnetic vertex of the charged leptons. The "self-energy" contribution represented in Fig. 3(a) alters the bare propagator $i/(p-m_l^0)$ so that, to order g^2 and first order in p^2/M^2, the full inverse propagator of the leptons μ, e, ν_μ, and ν_e is given by

$$p-m_l^0+\frac{g^2}{4\pi}[\ln(\Lambda^2/M^2)-\tfrac{3}{2}-\tfrac{1}{6}(p^2/M^2)]p(1+\gamma_5), \quad (15)$$

where Λ is a regulator mass introduced into the F^0 propagator by the Pauli-Villars technique, and terms

[5] Throughout this paper we use $\mathbf{p}=-ip^\lambda\gamma^\lambda$, where γ^μ, $1\leq\mu\leq4$, are the standard Dirac matrices; p^1, p^2, p^3 are the spatial components of the four-vector p^λ and $p^4=ip^0$ is its imaginary fourth component; $(p)^2=(p^1)^2+(p^2)^2+(p^3)^2-(p^0)^2$. There is no distinction made between Lorentz indices appearing as subscripts or superscripts. For a vector operator V_μ, $V_\mu^*=V_\mu^\dagger(1-2\delta_{\mu4})$.

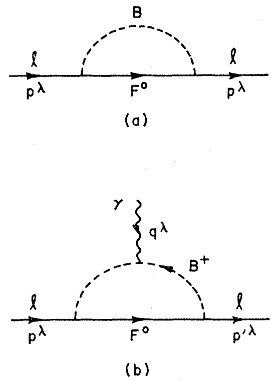

FIG. 3. Feynman diagrams representing corrections of order g^2 to (a) the lepton propagator, and (b) the electromagnetic vertex of the charged leptons, as computed in the model described in Sec. II B.

of order M^2/Λ^2 or higher have been dropped. The bare mass m_l^0 is, of course, zero in the case of the muon or electron neutrino; $m_{\nu_e}{}^0=m_{\nu_\mu}{}^0=0$. The full lepton propagator can be put in a more conventional form if it is written in terms of the renormalized lepton field operators:

$$\psi_{\rm ren}(x)=\left\{1+\frac{g^2}{8\pi}\left[\ln(\Lambda^2/M^2)-\tfrac{3}{2}+\tfrac{1}{2}\frac{m_l^2}{M^2}+\gamma_5\left(\ln(\Lambda^2/M^2)-\tfrac{3}{2}+\tfrac{1}{6}\frac{m_l^2}{M^2}\right)\right]\right\}\psi(x) \quad (16)$$

instead of the original lepton fields $\psi=\mu,\ e,\ \nu_\mu,\ \nu_e$. In terms of these renormalized fields, the inverse lepton propagator becomes

$$p-m_l+\frac{g^2}{24\pi}\left(\frac{(2m_l+p)(p-m_l)^2}{M^2}-p\gamma_5\frac{p^2+m_l^2}{M^2}\right), \quad (17)$$

where, to order g^2 and m_l^{02}/M^2, the physical lepton mass m_l is given by

$$m_l=m_l^0\left[1-\frac{g^2}{4\pi}\left(\ln(\Lambda^2/M^2)-\tfrac{3}{2}+\tfrac{1}{6}\frac{(m_l^0)^2}{M^2}\right)\right]. \quad (18)$$

Similarly, if one writes the bare vertex function for the charged leptons in terms of the renormalized fields $\psi_{\rm ren}(x)$ and adds to it the g^2 corrections represented by Fig. 3(b), then the finite renormalized, proper vertex function $\Gamma_{{\rm ren},l}{}^\lambda(p'-p)$ results:

$$\Gamma_{{\rm ren},l}{}^\lambda(p',p)=i\gamma^\lambda\left\{1-\frac{g^2}{8\pi}\left[\frac{1}{6}\frac{q^2+p^2+p'^2+6m_l^2}{M^2}+\gamma_5\left(\frac{q^2+p^2+p'^2+2m_l^2}{M^2}\right)\right]\right\}+\frac{q^2}{24\pi}\frac{pp'^\lambda+p'p^\lambda}{M^2}(1+\gamma_5), \quad (19)$$

where p and p' are the four-momenta of the incoming and outgoing charged leptons, respectively, and $q_\lambda=p_\lambda'-p_\lambda$. The anomalous magnetic moment in units of $e/2m_l$ implied by (19) is

$$\mu_a=-(g^2/6\pi)(m_l^2/M^2). \quad (20)$$

This correction is largest in the case of the muon, but $M\geq 2$ BeV is sufficiently small that its presence would not alter the present agreement between the measured value of $g_\mu-2$ for the muon and the predictions of quantum electrodynamics.[6]

III. SEMILEPTONIC PROCESSES

A. General Discussion

In order to extend our two-scalar-exchange theory to include the semileptonic weak interactions, we must specify, at least in part, the hadronic operators S_i^h which appear in the weak Lagrangian

$$\mathcal{L}_{\rm wk}=g\sum_i B_i(S_i^l+S_i^h). \quad (3)$$

This can be done most naturally by exploiting the analogy between the S_i^h and the leptonic operators S_i^l, which have already been partially determined by Eq. (6). Let

$$F_{ij}{}^x(q,\beta,\alpha)=+i\int e^{iq\cdot y}d^4y\langle\beta|T(S_i^x(0)S_j^x(y))|\alpha\rangle, \quad (21)$$

where $|\alpha\rangle$ and $|\beta\rangle$ are two states containing known particles and $x=l$ or h. In terms of the functions $F_{ij}{}^l(q,\alpha,\beta)$ and the notation introduced in the expression (9), the amplitude for μ^- decay is given by

$$\frac{1}{2}\frac{g^4}{(2\pi)^4}\sum_{ij}\int d^4q\frac{F_{ij}{}^l(q,\beta_1,\alpha_1)F_{ij}{}^l(-q,\beta_2,\alpha_2)}{[q^2+M^2][(q+p_\mu-k_\mu)^2+M^2]}, \quad (22)$$

where $|\beta_1\rangle$, $|\alpha_1\rangle$, $|\beta_2\rangle$, and $|\alpha_2\rangle$ are states of one muon neutrino, one negative muon, an electron and an anti-electron neutrino, and the vacuum, respectively. As was demonstrated in Sec. II, the amplitude (22) can be accurately evaluated for the case $m_\mu^2/M^2\ll 1$ if only the large q_λ limit of $F_{ij}{}^l(q,\beta,\alpha)$ is used,

$$\lim_{q_\lambda\gg m_\mu}F_{ij}{}^l(q,\beta,\alpha)=\sum_{n=1}^{N'}q^\sigma\frac{\langle\beta|V_\sigma{}^l(n,i,j)|\alpha\rangle}{M_n{}^2+q^2}, \quad (23a)$$

[6] The lower limit of 2 BeV for M is obtained by requiring that $|\mu_a|$ be less than the difference between the present experimental value of $g_\mu-2$ and the predictions of quantum electrodynamics. For the experimental value we use $\frac{1}{2}(g_\mu-2)=(11\ 666\pm5)\times10^{-7}$ from J. Bailey *et al.* (unpublished), presented by F. Farley at the International Symposium on Electron and Photon Interactions at High Energies, 1967 (unpublished). The value of $(g_\mu-2)/2$ predicted by quantum electrodynamics is taken to be $11\ 655.2\times10^{-7}$ (just the α and α^2 contributions); H. Suura and E. Wichmann, Phys. Rev. **105**, 1930 (1957); A. Petermann, *ibid.* **105**, 1931 (1957); C. Sommerfield, *ibid.* **107**, 328 (1957); H. Elend, Phys. Letters **21**, 720 (1966). The value of μ_a given in (21) does *not* have the right sign to explain the discrepancy between these two numbers. The value for μ_a given in Eq. (6) differs from that estimated by W. Kummer and G. Segrè (Ref. 1), because of an error in their calculation.

where the vector operator $V_\sigma{}^l(n,i,j)$ is given by

$$V_\sigma{}^l(n,i,j)=-2i\sum_{\psi,\psi'=e,\mu,\nu_e,\nu_\mu}4\pi[C_i{}^n(\psi)C_j{}^n(\psi')^*-C_j{}^n(\psi)C_i{}^n(\psi')^*]\bar{\psi}\gamma^\sigma(1+\gamma_5)\psi'. \quad (23b)$$

Thus, let us assume in analogy that[7]

$$\lim_{q_\lambda\gg m_N}F_{ij}{}^h(q,\beta,\alpha)=\sum_{n=1}^{N''}q^\sigma\frac{\langle\beta|V_\sigma{}^h(n,i,j)|\alpha\rangle}{(M_n{}^h)^2+q^2}. \quad (24)$$

Here, m_N is the nucleon mass, and $|\alpha\rangle$ and $|\beta\rangle$ are known hadronic states with masses and (relative four-momentum)^2 of the order of the nucleon mass.

For large M, the amplitude for the semileptonic reaction

$$\alpha_l+\alpha_h\rightarrow\beta_l+\beta_h \quad (25)$$

can be computed to lowest order in g and $m_N{}^2/M^2$ in terms of the operators $V_\sigma{}^l$ and $V_\sigma{}^h$:

$$A(\alpha_l+\alpha_h\rightarrow\beta_l+\beta_n)$$
$$=\frac{1}{2}\frac{g^4}{(2\pi)^4}\int d^4q\sum_{ij}\frac{F_{ij}{}^h(q,\beta_h,\alpha_h)F_{ij}{}^l(-q,\beta_l,\alpha_l)}{[q^2+M^2][(q+p_{\beta l}-p_{\alpha l})^2+M^2]}$$
$$=\frac{1}{2}\frac{g^4}{(8\pi)^2}\sum_{i,j,n,n'}A_{n'n}\langle\beta_h|V^h(n',j,i)|\alpha_n\rangle\times\langle\beta_l|V^l(n,i,j)|\alpha_l\rangle+O(m_N{}^2/M^2), \quad (26a)$$

where

$$A_{n'n}=\frac{M^2}{[(M_{n'}{}^h)^2-M^2](M_n{}^2-M^2)}-\frac{M_n{}^4\ln(M^2/M_n{}^2)}{(M_n{}^2-M^2)^2[M_n{}^2-(M_{n'}{}^h)^2]}+\frac{(M_{n'}{}^h)^4\ln[M^2/(M_{n'}{}^h)^2]}{[(M_{n'}{}^h)^2-M^2]^2[M_n{}^2-(M_{n'}{}^h)^2]}. \quad (26b)$$

The states α_l, β_l and α_h, β_h are composed of known leptons and hadrons, respectively, and are assumed to have masses and (relative four-momentum)^2 of the order of m_N. The structure displayed in Eq. (26) is similar to the vector-vector coupling observed in the semileptonic weak interactions. The amplitude given by (26) can be made to agree with the usual phenomenological description of semileptonic processes if we require that

$$\sum_{n,n',i,j}\langle\beta_h|V^h(n',j,i)|\alpha_h\rangle\langle\beta_l|V^l(n,i,j)|\alpha_l\rangle\times A_{n'n}\frac{g^4}{(8\pi)^2}\times\tfrac{1}{2}=(G/\sqrt{2})\langle\beta_h|J^h|\alpha_n\rangle\langle\beta_l|J^{l*}|\alpha_l\rangle+(G/\sqrt{2})\langle\beta_h|J^{h*}|\alpha_n\rangle\langle\beta_l|J^l|\alpha_l\rangle, \quad (27)$$

and that the term $O(m_N{}^2/M^2)$ in Eq. (26a) is negligible. A stringent upper limit on this term of order $m_N{}^2/M^2$ derives from the near equality of the experimentally determined vector coupling constants[8] in β decay (g_β) and μ decay (g_μ):

$$g_\mu/g_\beta=1.012\pm0.002. \quad (28)$$

Since it is natural to demand that the symbol G appearing in (10) and (27) represents the same number in both equations, the presence of the term $O(m_N{}^2/M^2)$ in (26a) will act to alter the equality of g_β and g_μ. If we require that this term contribute less than 1% to g_β and, for example, estimate its size by computing the effect of a similar term given explicitly in (13) for μ decay (in which we replace m_μ by m_N), then we conclude that

$$M\geq 3m_N. \quad (29)$$

In general, the term $O(m_N{}^2/M^2)$ in Eq. (26) not only produces corrections to the amplitudes of semileptonic weak processes allowed by the phenomenological Lagrangian (1) but also permits other, presently unobserved, semileptonic transitions to occur. In particular, the small experimental upper bounds on the rates for strangeness-changing decays producing a neutral lepton pair such as

$$K_2{}^0\rightarrow\mu^+\mu^-,$$
$$K^+\rightarrow\pi^+e^+e^-, \quad (30a)$$
$$K^+\rightarrow\pi^+\nu\bar{\nu}, \quad (30b)$$

put severe restrictions on this correction term. The rates predicted for these decays, which are of course forbidden to zero order in $m_N{}^2/M^2$ by Eqs. (24), (26), and (27), depend on the details of the theory and must be discussed separately for each particular model.

B. A Particular Model

As we have seen, Eqs. (24), (26), and (27), together with certain restrictions on the term $O(m_N{}^2/M^2)$ in Eq. (26), are sufficient to predict correctly the observed structure of the semileptonic weak interactions. These conditions are apparently very general and may well be satisfied by a variety of models. In order to illustrate the implications of conditions (24), (26), and (27), let us discuss in detail a simple, two-boson-exchange model for the leptonic and semileptonic weak interactions. The model is an extension of the one investigated in Sec. II

[7] It is interesting to note that limits of this type (in the case $M_k{}^h\ll q^\lambda$) can be derived from the equal-time commutators of the operators $S_i{}^h(x)$ using techniques developed by J. D. Bjorken, Phys. Rev. **148**, 1467 (1966). This possibility was pointed out by F. E. Low and M. L. Goldberger (private communication).

[8] See, for example, T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. **15**, 381 (1965).

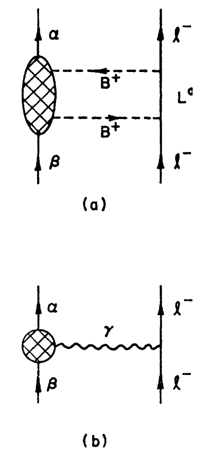

FIG. 4. Feynman diagrams representing (a) the additional weak coupling of the charged leptons to the hadrons predicted by the model described in Sec. III B, and (b) the competing electromagnetic process.

and described by the Lagrangian $\mathcal{L}_{\text{wk}}{}^l$ given by (12) and (14). We assume that the observed hadronic weak current $J_\mu{}^h$ can be formed of bilinear products of a SU_3 triplet of fields $\mathcal{P}(x)$, $\mathcal{N}(x)$, and $\lambda(x)$ [9]:

$$J_\mu{}^h(x)=i[\cos\theta_C\,\bar{\mathcal{N}}(x)+\sin\theta_C\,\bar{\lambda}(x)]\gamma^\mu(1+\gamma_5)\mathcal{P}(x)\,, \quad (31)$$

where $\theta_C\simeq 0.25$ is the usual Cabbibo angle. The fields $\mathcal{P}$, $\mathcal{N}$, and λ are assumed to carry one, zero, and zero units of electric charge, respectively; $\mathcal{P}$ and $\mathcal{N}$ make up a conventional isodoublet and have the same strangeness, while λ belongs to an isosinglet and has one unit less strangeness than $\mathcal{P}$ and $\mathcal{N}$. The total weak Lagrangian is given by

$$\begin{aligned}\mathcal{L}_{\text{wk}}=\mathcal{L}_{\text{wk}}{}^l+(4\pi)^{1/2}g[&B^+\bar{\mathcal{P}}(1-\gamma_5)F^0\\ &+B^0(\bar{\mathcal{N}}\cos\theta_C+\bar{\lambda}\sin\theta_C)(1-\gamma_5)F^0\\ &+\text{Hermitian conjugate}]. \quad (32)\end{aligned}$$

An additional heavy fermion F^0 of mass M with only weak interactions has been introduced. Thus, this model contains five new particles

$$F^0,\ L_\mu{}^0,\ L_e{}^0,\ B^+,\ \text{and}\ B^0, \quad (33)$$

and their antiparticles.

Let us now complete our model so that it satisfies equations (24), (26), and (27). Not being able to specify consistently the detailed properties of the operators $\mathcal{P}$, $\mathcal{N}$, and λ, we can only *assume* that the following formal derivation of (24), (26), and (27) leads to a correct conclusion. It is convenient to relabel the operators $S_i{}^x$ and the functions $F_{ij}{}^x(q,\alpha,\beta)$ so that $S_+{}^x$, $S_-{}^x$, and $S_0{}^x$ are the operators coupled to the bosons B^+, B^-, and B^0, respectively, and

$$F_{ab}{}^x(q,\alpha,\beta)=i\int e^{iq\cdot y}d^4y\langle\alpha|\,T(S_a{}^x(0)S_b{}^x(y))\,|\beta\rangle \quad (34)$$

[9] M. Gell-Mann, Phys. Letters **8**, 214 (1964). G. Zweig, CERN Reports No. 8182/TH 401 and No. 8419/TH 412, 1964 (unpublished).

for $a,b,=+,\ -,\ 0$ and $x=h,\ l$. To lowest order in g the fields F^0 and $\bar{F}^0$ appearing in (34) are noninteracting and, using Wick's theorem, can be replaced by a fermion propagator. If this is done, for example, in the expression for $F_{+0}{}^h(q,\alpha,\beta)$, one obtains

$$\begin{aligned}F_{+0}{}^h(q,\alpha,\beta)=\frac{4}{(2\pi)^3}\int d^4k\int e^{-ik\cdot y}d^4y\langle\alpha|\bar{\mathcal{P}}(0)(\boldsymbol{q}+\boldsymbol{k})(1+\gamma_5)\\ \times[\mathcal{N}(y)\cos\theta_C+\lambda(y)\sin\theta_C]|\beta\rangle\frac{1}{(q+k)^2+M^2}. \quad (35)\end{aligned}$$

Furthermore, if it is assumed that the integral over k_ρ converges rapidly and receives negligible contribution for $k_\rho\gg m_N$, then for $q_\rho\gg m_N$ we can neglect k_ρ with respect to q_ρ. Consequently,

$$\begin{aligned}\lim_{q_\rho\gg m_N}F_{+0}{}^h(q,\alpha,\beta)&=\frac{-i8\pi q^\sigma}{q^2+M^2}\\ &\times\langle\alpha|\bar{\mathcal{P}}(0)\gamma^\sigma(1+\gamma_5)[\mathcal{N}(0)\cos\theta_C+\lambda(0)\sin\theta_C]|\beta\rangle\\ &=-\frac{8\pi q^\sigma}{q^2+M^2}\langle\alpha|J_\sigma{}^{h*}|\beta\rangle. \quad (36)\end{aligned}$$

Likewise,

$$\lim_{q_\rho\gg m_N}F_{-0}{}^h(q,\alpha,\beta)=-\frac{8\pi q^\sigma}{q^2+M^2}\langle\alpha|J_\sigma{}^h|\beta\rangle\,,$$

$$\lim_{q_\rho\gg m_N}F_{++}{}^h=\lim_{q_\rho\gg m_N}F_{00}{}^h=\lim_{q_\rho\gg m_N}F_{--}{}^h=0\,, \quad (37)$$

$$\lim_{q_\rho\gg m_N}F_{+-}{}^h(q,\alpha,\beta)=-\frac{i8\pi q^\sigma}{q^2+M^2}\langle\alpha|\bar{\mathcal{P}}\gamma^\sigma(1+\gamma_5)\mathcal{P}|\beta\rangle. \quad (38)$$

The vanishing of $F_{00}{}^h(q,\alpha,\beta)$ for $q_\rho\gg m_N$ follows from the equality of B^0 and $B^{0\dagger}$. Thus, except for Eq. (38),[10] this model reproduces conditions (24), (26), and (27) and therefore (to order g^4 and zero order in $m_N{}^2/M^2$) agrees with the conventional phenomenological description of the leptonic and semileptonic weak interactions.

This model is sufficiently concrete to allow a more complete discussion of the unobserved decays (30). This will be done in Appendix B.

IV. NONLEPTONIC WEAK PROCESSES

We now turn our attention to the nonleptonic weak interactions. Unlike the leptonic and semileptonic weak

[10] Equation (38) introduces a new weak coupling represented in Fig. 4(a) which is usually negligible compared to the competing electromagnetic process shown in Fig. 4(b). For energies considerably less than M, the form given by Eq. (34) for $F_{+-}{}^h$ leads to an effective Lagrangian of the form

$$(G/\sqrt{2})\sum_{l=e,\mu}\bar{l}\gamma^\lambda(1+\gamma_5)l\bar{\mathcal{P}}\gamma^\lambda(1+\gamma_5)\mathcal{P}.$$

This parity-violating weak coupling does not have the damping at large (momentum transfer)^2 which is found in the competing electromagnetic process, because of the additional photon propagator.

processes, the observed nonleptonic weak reactions have not yet been described by a systematic, detailed, phenomenological theory. Lacking such a phenomenological theory, we use field theory to specify directly the operators $S_i^h(x)$ of Eq. (3), which describe the hadron-intermediate boson coupling. Because of the large number of possibilities and the difficulties of calculation, we will not attempt a general analysis of the restrictions placed on the operators $S_i^h(x)$ by the observed nonleptonic weak processes. Instead, we discuss qualitatively the properties of the nonleptonic weak interactions predicted by two simple models. These models are distinguished by the number of different types of intermediate bosons and the mechanism for strangeness violation. In both of these models the weak couplings of the hadrons are specified in terms of an SU_3 triplet of hadronic field operators $\mathcal{P}$, $\mathfrak{N}$, and λ, as was done in Sec. III B. The Lagrangian for each model is so chosen that (i) Eqs. (6), (9), and (10) of Sec. II A are satisfied, and (ii) the arguments and assumptions given in Sec. III B can be used to derive conditions (24), (26), and (27); thereby assuring the usual current $\times$ current form for low-energy (energy$\ll M$) leptonic and semileptonic weak processes to lowest order in g and m_N^2/M^2. The models are discussed in order of increasing complexity.

Before these two models are described, the nonleptonic weak interactions predicted by the very simple model investigated in Sec. III B should be examined. Perhaps the most serious difficulty of this model is the size predicted for strangeness violation. A simple self-energy loop of order g^2, as shown in Fig. 5, produces strangeness violation. The unrenormalized amplitude for a $\Delta S=1$ transition $A \rightarrow C$ is given by

$$4 \sin\theta_C \cos\theta_C \frac{g^2}{(2\pi)^7}\int d^4q \int d^4k \int e^{ik\cdot x} d^4x \times \frac{\langle C|T(\mathfrak{N}(x)(\boldsymbol{k}+\boldsymbol{q})(1+\gamma_5)\lambda(0))|A\rangle}{(q^2+M^2)[(k+q)^2+M^2]}, \quad (39)$$

where the same techniques used to derive Eq. (35) have been employed here. If we introduce a regulator term with mass Λ into the B^0 propagator and expand to lowest order in m_N^2/M^2, assuming the integrand vanishes quickly for $k^\rho > m_N$, we obtain

$$\frac{g^2}{4\pi} \sin\theta_C \cos\theta_C \langle C|T\left(\mathfrak{N}(x)\gamma^\mu(1+\gamma_5)\frac{\partial\lambda(x)}{\partial x^\mu}\right)|A\rangle \times[\ln(\Lambda^2/M^2)-\tfrac{1}{2}]. \quad (40)$$

Because of the logarithmic dependence on the cutoff Λ, we cannot calculate such a transition amplitude in renormalized perturbation theory. However, it is reasonable to assume that a rough estimate can be obtained by replacing $\ln(\Lambda^2/M^2)$ by one. If the hadronic matrix element is also estimated to be of order one in appropriate units, we obtain a $\Delta S=1$ transition strength of

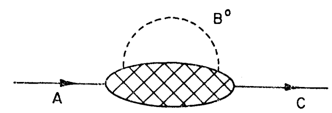

FIG. 5. The Feynman diagram representing the amplitude for a nonleptonic $\Delta S=1$ transition $A \rightarrow C$ as computed in the model described in Sec. III B.

$$\sin\theta_C \frac{g^2}{4\pi} \sim 3\times 10^{-4} \frac{M}{m_N} \sin\theta_C, \quad (41)$$

which is almost certainly too large to correctly predict the observed rates of nonleptonic, strangeness-changing decays.

A. Two-Boson Model

In this model the leptonic part of the Lagrangian is the same as that introduced in Sec. II B. However, in addition to the two bosons $B^\pm$ and B^0 and the two heavy leptons L_e^0 and L_μ^0, two heavy fermions F^0 and $F^{0\prime}$ are introduced. The complete weak interaction Lagrangian for this model is[11]

$$\mathcal{L}_{\rm wk} = \mathcal{L}_{\rm wk}^l + \mathcal{L}_{\rm wk}^h = (4\pi)^{1/2} g \sum_{l=e,\mu} [\bar{l}(1-\gamma_5)L_l^0 B^- + \bar{\nu}_l(1-\gamma_5)L_l^0 B^0 + \text{Hermitian conjugate}] + (4\pi)^{1/2} g[\bar{\mathcal{P}}(1-\gamma_5)(F^0\cos\theta_C + F^{0\prime}\sin\theta_C)B^+ + \bar{\mathfrak{N}}(1-\gamma_5)F^0 B^0 + \bar{\lambda}(1-\gamma_5)F^{0\prime}B^0 + \text{Hermitian conjugate}], \quad (42)$$

where the hadronic fermion fields $\mathcal{P}$, $\mathfrak{N}$, and λ are members of an SU_3 triplet and identical to those described in Sec. III B.

The two fermions F^0 and $F^{0\prime}$ are introduced to avoid nonleptonic strangeness violation of order g^2 (of the type discussed above). In a theory containing only two types of scalar intermediate bosons $B^\pm$ and B^0, $\mathcal{L}_{\rm wk}^h$ cannot be invariant under a strangeness-gauge transformation because, for example, both the K^+ and π^+ pseudoscalar mesons must be able to make virtual transitions to the B^+B^0 state. However, any amplitude for a transition between hadronic states can, to order g^2, be divided into parts, each involving either $B^\pm$ or B^0 but not both. The absence of $\Delta S=1$ terms of order g^2 can then be naturally assured by so constructing the $B^\pm$- and B^0-dependent parts of $\mathcal{L}_{\rm wk}^h$ that each is separately invariant under a (different) strangeness-gauge transformation. The above Lagrangian has this property because of the presence of the *two* heavy fermions.

[11] This model was first proposed by M. L. Goldberger and F. E. Low (private communication).

The Lagrangian (42) is so chosen that, except for the coupling of neutral currents of charged leptons, the usual phenomenological description of the leptonic and semileptonic weak interactions is predicted. In fact, using the same techniques and assumptions employed in the derivation of Eqs. (9) and (35)–(38), we can describe the low-energy (energy$\ll M$) leptonic and semileptonic effects of (42) to order g^4 and zero order in m_N^2/M^2 by the phenomenological Lagrangian

$$\mathcal{L}_{\rm ph}=(G/\sqrt{2})\{-J^{h*}J^l-J^{l*}J^h+J^{l*}J^l -J_\mu^{h'}J_\mu^{l'}+\tfrac{1}{2}J^{l'}J^{l'}\}\,, \quad (43a)$$

which is consistent with present experimental knowledge. The currents J_μ^h and J_μ^l appearing in (43a) are the usual hadronic and leptonic weak currents, while

$$J_\lambda^{l'}=i\bar{e}(1-\gamma_5)\gamma^\lambda e+i\bar{\mu}(1-\gamma_5)\gamma^\lambda\mu\,,\qquad J_\lambda^{h'}=i\bar{\mathcal{P}}(1-\gamma_5)\gamma^\lambda\mathcal{P}. \quad (43b)$$

1. $\Delta S=0$. Having completely specified the model, let us first discuss strangeness-conserving nonleptonic transitions. Using the techniques employed in obtaining Eq. (35), the unrenormalized amplitude for the strangeness-conserving transition $A\to C$ can be derived:

$$\frac{4g^2}{(2\pi)^7}\sum_{\psi=\mathcal{P},\mathcal{N},\lambda}\int d^4q\int d^4k\int e^{ik\cdot x}d^4x \times\frac{\langle C|T(\bar\psi(x)(\boldsymbol{q}+\boldsymbol{k})(1+\gamma_5)\psi(0))|A\rangle}{(q+k)^2+M^2} \times\left(\frac{1}{q^2+M^2}-\frac{1}{q^2+\Lambda^2}\right), \quad (44)$$

where a regulator term of mass Λ has been added to the $B^\pm$ and B^0 propagators. If we *assume* that the integrand falls off very rapidly as the hadronic momentum k^ρ increases above m_N, then to first order in m_N^2/M^2 this amplitude reduces to

$$\frac{g^2}{4\pi}\int d^4k\int\frac{e^{ik\cdot x}}{(2\pi)^4}d^4x \times\sum_{\psi=\mathcal{P},\mathcal{N},\lambda}\langle C^{\,\psi}T(\bar\psi(x)\gamma^\mu k^\mu(1+\gamma_5)\psi(0))|A\rangle \times[\ln(\Lambda^2/M^2)-\tfrac{1}{2}-\tfrac{1}{6}(k^2/M^2)]. \quad (45)$$

It must be emphasized that the analysis to follow requires the convergence of this integral over k^ρ which is at best uncertain and may very well not occur. Thus, with this assumption, the effects of the Lagrangian (43) to order g^2 on low-energy processes involving only hadrons can be summarized by the following effective Hamiltonian density

$$H_{\rm eff}(x)=-\frac{g^2}{4\pi}\sum_{\psi=\mathcal{P},\mathcal{N},\lambda}\left\{\ln(\Lambda^2/M^2)-\tfrac{1}{2}+\frac{\Box_y^2}{6M^2}\right\} \times T\left(\frac{\partial}{\partial y_\mu}\bar\psi(y)\gamma^\mu(1+\gamma_5)\psi(x)\right)_{x=y}, \quad (46)$$

where the four-dimensional Laplacian is

$$\Box_y^2=\sum_{i=1}^{3}\frac{\partial^2}{\partial y_i^2}-\frac{\partial^2}{\partial y_0^2}.$$

The most significant feature of this effective Hamiltonian is its noninvariance under parity. In order to discuss the size of the parity violation evidenced by Eq. (46), it is necessary to introduce further assumptions about the structure of the strong interactions. For this purpose we will assume that the strong interactions can be described by a field-theoretic Lagrangian density of the form

$$\mathcal{L}_{\rm st}(x)=-\sum_{\psi=\mathcal{P},\mathcal{N},\lambda}\bar\psi(x)\left(\gamma^\mu\frac{\partial}{\partial x_\mu}+m\right)\psi(x)+\mathcal{L}_1(\psi)\,, \quad (47)$$

where the second term describes the interaction of the fields ψ and may include other fundamental fields. The expression within the curly brackets in (46) can be divided into two terms: a constant term, $\ln(\Lambda^2/M^2)-\frac{1}{2}$, and a term linear in $\Box^2$, $\Box^2/6M^2$. Since the constant term depends on the cutoff Λ, it requires renormalization and consequently cannot be computed using perturbation theory. As before, we will attempt to roughly estimate its size by replacing $\ln(\Lambda^2/M^2)$ by one. Thus, in general, one may expect this constant term in (46) to produce parity violation whose size is characterized by the dimensionless number

$$g^2/4\pi\simeq 3\times10^{-4}(M/m_N). \quad (48)$$

Even for $M=3$ BeV, the lower bound suggested by the discussion in Secs. II B and III A, the resulting 10^{-3} is most likely too large to be compatible with the small upper limits on the size of parity-violating effects in nuclear physics. However, let us assume that $\mathcal{L}_1(\psi)$ given in (47) is invariant to first order in δ under the replacement $\psi\to(1+\delta\gamma_5)\psi$, i.e.,

$$\mathcal{L}_1(\psi)=\mathcal{L}_1[(1+\delta\gamma_5)\psi]+O(\delta^a)\,, \quad (49)$$

where δ is a small real number. If the Lagrangian $\mathcal{L}_{\rm st}$ is written in terms of the field variables $\psi'=(1+\delta\gamma_5)\psi$, it becomes, to first order in δ,

$$\mathcal{L}_{\rm st}(x)=-\sum_{\psi=\mathcal{P},\mathcal{N},\lambda}\bar\psi'(x)\left(\gamma^\mu\frac{\partial}{\partial x_\mu}+m\right)\psi'(x) +\mathcal{L}_1(\psi')+2\sum_{\psi=\mathcal{P},\mathcal{N},\lambda}\delta\bar\psi'(x)\gamma^\mu\frac{\partial}{\partial x_\mu}\gamma_5\psi'(x). \quad (50)$$

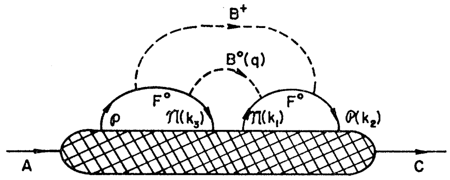

FIG. 6. A Feynman diagram representing one term of order g^4 in the strangeness-conserving, nonleptonic transition $A\rightarrow C$. The momenta enclosed in parentheses indicate the four-momentum carried by particular lines.

For $\delta=g^2[\ln(\Lambda^2/M^2)-\frac{1}{2}]/8\pi$ the term farthest to the right in Eq. (50) acts as a counter-term, cancelling that part of the constant term in H_{eff} which multiplies the γ_5. Thus, if the strong interactions are symmetric under the transformation described by (49) the constant term in H_{eff} has no parity-violating effects.

Parity violation produced by the term linear in $\Box^2$ found in H_{eff} cannot be so easily transformed away, but for low-energy processes (energy$\ll M$) may be quite small. If the size of the effects caused by this linear term is estimated by replacing $\Box^2$ by m_N^2 and computing the resulting coefficient of $\bar{\psi}(x)\gamma_5\gamma^\mu\partial\psi/\partial x_\mu$, one obtains

$$\frac{g^2}{24\pi}\times\frac{m_N^2}{M^2}=\frac{m_N}{M}\times 4\times 10^{-5}, \qquad (51)$$

which is 10^{-5} or less for

$$M\geq 4 \text{ BeV}. \qquad (52)$$

Thus, if the strong-interaction dynamics satisfies certain conditions, then parity violation in nonleptonic weak interaction of order g^2 depends inversely on M, and can be made sufficiently small for large M.

Finally, let us consider the nonleptonic, strangeness-conserving, weak interactions of order g^4. A typical term in the amplitude for such a nonleptonic transition $A\rightarrow C$ is represented in Fig. 6, and is given by

$$\frac{4g^4}{\pi^2}\int d^4q\prod_{i=1}^{3}\left[\int d^4k_i\int\frac{e^{ik_i\cdot x_i}}{(2\pi)^4}d^4x_i\right]\cos^2\theta_C\frac{\langle C|T(\mathcal{N}(x_3)(\boldsymbol{k}_3+\boldsymbol{q})(1+\gamma_5)\mathcal{P}(0)\bar{\mathcal{P}}(x_2)(\boldsymbol{k}_1+\boldsymbol{q})(1+\gamma_5)\mathcal{N}(-x_1))|A\rangle}{(q^2+M^2)[(q+k_1-k_2)^2+M^2][(q+k_1)^2+M^2][(q+k_3)^2+M^2]}. \qquad (53)$$

The convergence properties of the integrals over the momenta k_1, k_2, and k_3 carried by the hadron lines will determine whether this amplitude contains parity violation of magnitude g^4 or of magnitude $g^4m_N^2/M^2$. Unfortunately, the behavior of these integrals is essentially a matter for speculation. Renormalized perturbation theory for the strong interactions cannot be used for a guide, because it is inconsistent with assumptions that have already been made, e.g., Eq. (38).[12] Consequently, the size of the nonleptonic parity violation of order g^4 predicted by this model is uncertain and may or may not grow with increasing M.

2. $\Delta S=1$. The Lagrangian (42) was purposely chosen to eliminate $\Delta S\neq 0$ hadronic transitions of order g^2. Strangeness violation can occur to order g^4, and should have a magnitude in qualitative agreement with the observed strangeness-violating nonleptonic decays. To order g^4, the $\Delta S=1$ transition $A\rightarrow C$ is represented by Fig. 7(a), and is given by

$$\frac{4g^4}{\pi^2}\int d^4q\prod_{i=1}^{3}\left[\int d^4k_i\int\frac{e^{ik_i\cdot x_i}}{(2\pi)^4}d^4x_i\right]\cos\theta_C\sin\theta_C \times\frac{\langle C|T(\mathcal{N}(x_3)(\boldsymbol{q}+\boldsymbol{k}_3)(1+\gamma_5)\mathcal{P}(0)\bar{\mathcal{P}}(x_2)(\boldsymbol{q}+\boldsymbol{k}_1)(1+\gamma_5)\lambda(-x_1))|A\rangle}{(q^2+M^2)[(q+k_1-k_2)^2+M^2][(q+k_1)^2+M^2][(q+k_3)^2+M^2]}. \qquad (54)$$

Evaluation of the expression (54) is made difficult and uncertain by the presence of the product of four hadronic operators. The most singular part of the integrand in (54) for large k_i and q can perhaps be estimated by replacing the operator $\mathcal{P}(0)\bar{\mathcal{P}}(x_2)$ appearing in (54) by the Feynman propagator $S_F(-x_2)$ for a free particle of mass m_N, as is represented in Fig. 7(b).[13] If this is done and only terms linear in $k_1=k_3$ are kept, the resulting amplitude is

$$\frac{g^4}{4\pi^2}\cos\theta_C\sin\theta_C\langle C|\mathcal{N}(x)(1-\gamma_5)\gamma^\mu\frac{\partial}{\partial x_\mu}\lambda(x)|A\rangle \times[\tfrac{1}{4}\ln^2(\Lambda^2/M^2)-\tfrac{1}{6}\ln(\Lambda^2/M^2)-0.94], \qquad (55)$$

[12] It is interesting to note that sufficient convergence to insure a g^4/M^2 behavior for the order g^4, nonleptonic, parity-violating amplitude is not obviously inconsistent with the existence of a local algebra for the currents $\bar{\psi}\gamma_\mu\psi'$ and $\bar{\psi}\gamma_\mu\gamma_5\psi'$, where ψ and ψ' are members of the triplet of fields $\mathcal{P}$, $\mathcal{N}$, and λ. Such a local algebra may require g^4 behavior only for terms in the amplitude (54), where the product of two field operators $\psi(x)\bar{\psi}(y)$ is replaced by a fermion propagator $S_F(x-y)$. These terms contain only two hadronic operators and hence have the same structure as the amplitude of order g^2. Therefore, with the assumption of (49) (where δ must now depend on ψ) and the arguments given above, such terms will produce parity-violating effects of order g^4/M^2.

[13] In contrast with the assumptions made in previous sections, this procedure requires the relatively slow decrease of a particular hadronic amplitude as it transmits a larger and larger four-momentum. If no such singular behavior is present then the estimate given in (55) must be reduced by roughly a factor of m_N^2/M^2 and consequently no longer places an upper limit on M. (See, however, Ref. 12.)

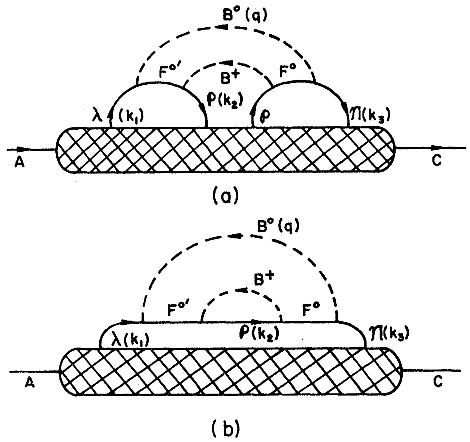

FIG. 7. Feynman diagrams representing (a) the complete amplitude of order g^4 for the $\Delta S=1$ nonleptonic transition $A\rightarrow C$, and (b) the approximation used to estimate the most singular part of that amplitude. The particular structure of (a) is that predicted by the model discussed in Sec. IV A.

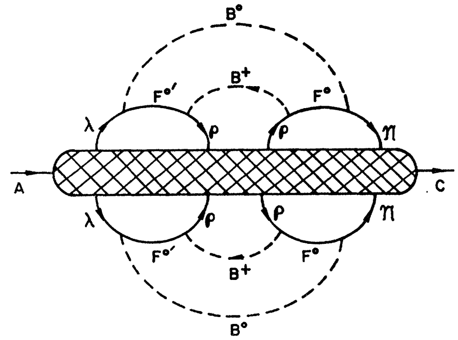

FIG. 8. The Feynman diagram representing the amplitude for the $\Delta S=2$ transition $A\rightarrow C$ as computed for the model described in Sec. IV A to order g^8.

where a regulator term of mass Λ has been added to the $B^\pm$ and B^0 propagators and the constant 0.94 appearing in the square brackets is the result of numerical integration. Although the amplitude (55) is cutoff-dependent and consequently requires renormalization, we may attempt to determine its size by replacing the quantity in square brackets by 1. If we require that the resulting coefficient of the operator $\bar{\mathfrak{N}}(x)(1-\gamma_5)\gamma^\mu(\partial/\partial x_\mu)\lambda(x)$ be less than $10^{-5}\sin\theta_C$, then

$$M\leq 4 \text{ BeV}. \tag{56}$$

The expression (55) is an estimate of only the leading $\Delta S=1$ amplitude for large M. There will, of course, be other less divergent contributions depending, for example, on the combination g^4/M^2. Thus the size of (55) can yield only an upper limit on M.

3. $\Delta S=2$. Finally, we investigate the size of $\Delta S=2$ transitions which, for this model, occur in order g^8. The amplitude for the $\Delta S=2$ transition $A\rightarrow C$ is represented by Fig. 8. As in the case of $\Delta S=1$ processes, the weak coupling constant g need not always occur in the combination g^4/M^2. In particular the presence of terms depending on g^8 or g^8/M^2 will place an upper limit on M. A very crude estimate of the diagram shown in Fig. 8 indicates that the inequality (56) is sufficient to insure $\Delta S=2$ transitions of magnitude consistent with the observed K_1-K_2 mass difference.[14]

This concludes the discussion of the two-boson model. The smallness of observed parity violation in $\Delta S=0$ processes and the weakness of strangeness-violating nonleptonic decays place conflicting requirements on M. These requirements may in fact be inconsistent, although the uncertain estimates (52) and (56) are not in disagreement. Since the estimate given by (56) is probably correct within a factor of three, this two-boson model must involve new particles of mass

$$M\lesssim 10 \text{ BeV}. \tag{57}$$

It must be remembered that even these qualitative conclusions depend critically on a number of assumptions about the strong interactions. In particular, the rapid convergence of certain integrals over hadronic momenta and a symmetry (49) of the strong-interaction Lagrangian were required in order to reduce the size of $\Delta S=0$ parity violation to an experimentally acceptable level. Finally, it must be emphasized that the restrictions (49) and (56) were derived from estimates of logarithmically divergent expressions which had been regularized. If one works strictly within the framework of renormalized perturbation theory, such divergent quantities should be replaced by *free* parameters which are determined from experiment. Had we adopted such an attitude, the troublesome terms giving rise to the restrictions (49) and (56) could be simply renormalized to zero. In the resulting theory the strong interactions would not have to satisfy (49), but the small size of $\Delta S=2$ transitions, (i.e., the K_1-K_2 mass difference) would still limit M to a value of a few BeV.[14]

B. Three-Boson Model

In this model the strength of strangeness violation is so weakened that the observed size of $\Delta S=1$ and $\Delta S=2$ transitions *does not* place an upper limit on the weak coupling constant g and a corresponding upper limit on M (in contrast with the two-boson model discussed above). This is brought about by two features of the theory. First the coupling between the intermediate scalar bosons and the hadrons is altered so that it is

[14] The estimate of the size of $\Delta S=2$ transitions which is referred to here was made by replacing the operator products $\mathcal{P}(x_i)\bar{\mathcal{P}}(x_j)$, which appear in the expression represented by Fig. 8, by the propagator $S_F(x_i-x_j)$ as was done above. Such mild momentum dependence of a hadronic amplitude is suggested by local current algebra, but is by no means certain. If a more convergent behavior were assumed for the integrals represented in Fig. 8 the $\Delta S=2$ amplitude would depend on g^8/M^4 and would place no upper limit on M.

strangeness conserving. This requires the existence of at least three types of intermediate bosons, which can be assigned strangeness quantum numbers. Second, two new neutral leptons e^* and μ^* are added with masses m_{e^*} and m_{μ^*} of the order of one BeV. These appear in the weak Lagrangian in such a way that strangeness is exactly conserved in the limit $m_{\mu^*}=m_{e^*}=0$. The weak-interaction Lagrangian for this model is

$$\mathcal{L}_{\rm wk}=(4\pi)^{1/2}g[B^+\bar{\mathcal{P}}(1-\gamma_5)F^0+B^0\bar{\mathfrak{N}}(1-\gamma_5)F^0+B^{0\prime}\bar{\lambda}(1-\gamma_5)F^0+\text{Hermitian conjugate}]$$
$$+(4\pi)^{1/2}g\sum_{l=e,\mu}[B^+\bar{l}(1-\gamma_5)L_l^0+B^0(\bar{\nu}_l\cos\theta_C+\bar{l}^*\sin\theta_C)(1-\gamma_5)L_l^0$$
$$+B^{0\prime}(\bar{\nu}_l\sin\theta_C-\bar{l}^*\cos\theta_C)(1-\gamma_5)L_l^0+\text{Hermitian conjugate}]+\mathcal{L}', \quad (58a)$$

where $B^\pm$, B^0, and $B^{0\prime}$ are the three intermediate scalar particles with mass M, $G=\sqrt{2}g^4/3M^2$, and the same symbol has been used to represent both particles and fields. $\mathcal{P}$, $\mathfrak{N}$, and λ are the three integrally charged fields that were discussed in Sec. III B. The heavy fermion F^0 and the heavy leptons L_e^0 and L_μ^0 are the same as those appearing in the model of Sec. III B and all have mass M.

Because the hadronic part of this Lagrangian must conserve strangeness, B^0 or $B^{0\prime}$ cannot be its own antiparticle. We will, therefore,[15] assume B^0, $B^{0\dagger}$, $B^{0\prime}$, and $B^{0\prime\dagger}$ are four independent fields. Consequently, the natural suppression found before for the decay (30b) and the low energy scattering $\nu+$ hadron $\rightarrow\nu+$ hadron$'$ does not occur in this model. These processes were suppressed in the models discussed previously because those models contained only one, self-conjugate, neutral intermediate boson. The final term $\mathcal{L}'$ has been added to the Lagrangian (58a) to reduce the rates for these two processes[16] so that they do not occur to lowest order in g and $(m_N/M)^2$:

$$\mathcal{L}'=(4\pi)^{1/2}g\sum_i\Bigg[(\sqrt{\tfrac{2}{3}})\mathcal{B}^\dagger\tau_i\bar{\Psi}(1-\gamma_5)F_i^0$$
$$-\frac{i}{\sqrt{3}}\mathcal{B}\tau_2\tau_i\bar{\Psi}(1-\gamma_5)F_i^0$$
$$+\text{Hermitian conjugate}\Bigg]. \quad (58b)$$

The τ_i are the usual 2×2 Pauli matrices, $\bar{\Psi}$, $\mathcal{B}$, and $\mathcal{B}^\dagger$ are 2×1, 1×2, and 1×2 matrices of field operators respectively,

$$\bar{\Psi}=\begin{pmatrix}\lambda\\-\bar{\mathfrak{N}}\end{pmatrix},\quad \mathcal{B}=(B^0,B^{0\prime}),$$
$$\text{and}\quad \mathcal{B}^\dagger=(B^{0\dagger},B^{0\prime\dagger}), \quad (58c)$$

and the F_i^0 are three new heavy fermions, all with mass, M. The combinations $\mathcal{B}^\dagger\tau_i\bar{\Psi}$ and $\mathcal{B}\tau_2\tau_i\bar{\Psi}$ represent the results of matrix multiplication; similarly, the usual matrix multiplication of juxtaposed spinor operators and Dirac matrices is implied. With the addition of $\mathcal{L}'$, the Lagrangian (58), together with the methods and assumptions used in Sec. III B to derive (35)–(38) predicts that low-energy (energy$\ll M$) leptonic and semileptonic processes can be described by the phe-

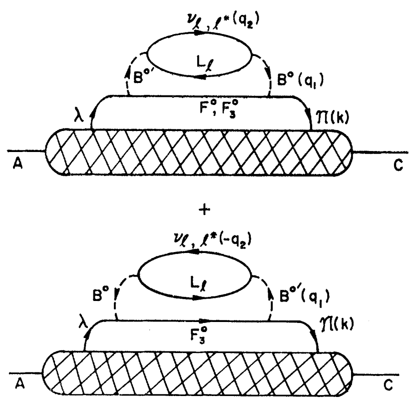

FIG. 9. The Feynman diagrams representing the amplitude for the $\Delta S=1$ transition $A\rightarrow C$ as computed for the model described in Sec. IV B to order g^4.

[15] We will demand that, except for the $l^*-\nu_l$ mass difference, the weak Lagrangian be invariant under the SU_3 subgroup generated by F_6, F_7, and $F_3-\sqrt{3}F_8$ in the notation of Gell-Mann, California Institute of Technology Synchrotron Laboratory, Report No. CTSL-200, 1961 (unpublished). This U-spin symmetry implies that the Cabbibo angle θ_c appearing in (58a) is not well defined unless the SU_3-violating part of the strong interactions is specified.

[16] Sufficient suppression of the decay (30b) and the low-energy elastic scattering $\nu+$hadron $\rightarrow\nu+$hadrons can be obtained without the addition of $\mathcal{L}'$ if the equality of the B^0, $B^{0\prime}$, and B^+ coupling strengths is altered. For example, if the constant describing the weak coupling of neutral bosons B^0 and $B^{0\prime}$ is set equal to one-third of the charged boson's coupling constant [i.e., the Lagrangian (58a) is modified by multiplying the fields B^0 and $B^{0\prime}$ by $\frac{1}{3}$ wherever they appear], then the rates for these processes are reduced by a factor of 0.013 relative to the rates for the transitions $K^+\rightarrow\pi^0e^+\nu$ and $\nu_\mu+$hadron $\rightarrow\mu+$hadrons.

nomenological Lagrangian

$$\mathcal{L}_{\rm ph}=-\frac{G}{\sqrt{2}}\sum_{l=e,\mu}\{i\bar{\nu}_l\gamma^\lambda(1+\gamma_5)l[J_\lambda^{(0)}\cos\theta+J_\lambda^{(1)}\sin\theta]+i\bar{l}^*\gamma^\lambda(1+\gamma_5)l[J_\lambda^{(0)}\sin\theta-J_\lambda^{(1)}\cos\theta]$$
$$+\tfrac{1}{2}i\bar{l}\gamma^\lambda(1+\gamma_5)l[i\bar{\mathcal{P}}\gamma^\lambda(1+\gamma_5)\mathcal{P}]+\text{Hermitian conjugate}\}-\frac{G}{\sqrt{2}}\sum_{l,l'=e,\mu}[\bar{l}\gamma^\lambda(1+\gamma_5)\nu_l\bar{\nu}_{l'}\gamma^\lambda(1+\gamma_5)l'$$
$$+\bar{l}\gamma^\lambda(1+\gamma_5)l^*\bar{l}'^*\gamma^\lambda(1+\gamma_5)l'+\tfrac{1}{2}\bar{l}\gamma^\lambda(1+\gamma_5)l\bar{l}'\gamma^\lambda(1+\gamma_5)l'+\tfrac{1}{2}\bar{\nu}_l\gamma^\lambda(1+\gamma_5)\nu_l\bar{\nu}_{l'}\gamma^\lambda(1+\gamma_5)\nu_{l'}$$
$$+\bar{\nu}_l\gamma^\lambda(1+\gamma_5)l^*\bar{l}'^*\gamma^\lambda(1+\gamma_5)\nu_{l'}+\tfrac{1}{2}\bar{l}^*\gamma^\lambda(1+\gamma_5)l^*\bar{l}'^*\gamma^\lambda(1+\gamma_5)l'^*],\quad(59)$$

where the currents $J_\mu^{(0)}$ and $J_\mu^{(1)}$ are the strangeness-conserving and strangeness-changing hadronic weak currents so normalized that $J_\mu^h=J_\mu^{(0)}\cos\theta_C+J_\mu^{(1)}\sin\theta_C$. In addition to the usual terms found in (1) this Lagrangian describes the coupling of certain neutral leptonic currents and the leptonic and semileptonic weak interactions of the new neutral leptons e^* and μ^*. For m_{e^*} and m_{μ^*} greater than the K-meson's mass, the phenomonological Lagrangian (59) is consistent with experiment.

1. $\Delta S=0$. Strangeness-conserving weak processes and, in particular, $\Delta S=0$ parity violation are of the same sort as those predicted by the two-boson model described in Sec. IV A. Consequently, the discussion and rough estimate given previously in the paragraphs dealing with the two-boson model are applicable here.

2. $\Delta S=1$. Any strangeness-violating nonleptonic process $A\rightarrow C$ must involve the leptons virtually. To order g^4, the amplitude for such a transition is represented by Fig. 9 and has the value

$$\sum_{l=e,\mu}(-i)\frac{g^4}{6\pi^6}\int d^4q_1d^4q_2d^4k\int\frac{e^{ik\cdot x}}{(2\pi)^4}$$
$$\times d^4x\langle C|T(\bar{\mathfrak{N}}(x)(1-\gamma_5)(\boldsymbol{k}+\boldsymbol{q}_1)\lambda(0))|A\rangle$$
$$\times\frac{\text{trace}[(1+\gamma_5)\boldsymbol{q}_2(1+\gamma_5)(\boldsymbol{q}_1+\boldsymbol{q}_2)]}{(q_1^2+M^2)^2[(q_1+q_2)^2+M^2][(q_1+k)^2+M^2]}$$
$$\times\left(\frac{1}{q_2^2}-\frac{1}{q_2^2+m_{l^*}^2}\right)\sin\theta_C\cos\theta_C.\quad(60)$$

We will assume that the integral over the hadronic momentum k^ρ converges sufficiently rapidly that, to lowest order in m_N^2/M^2, k^ρ can be neglected with respect to q_1^ρ. If also only the lowest power in $m_{l^*}^2/M^2$ is retained, (60) reduces to

$$\sin\theta_C\cos\theta_C\frac{4g^4}{9\pi^2}\frac{m_{e^*}^2+m_{\mu^*}^2}{M^2}[\ln(\Lambda^2/M^2)-\tfrac{4}{3}]$$
$$\times\langle C|\bar{\mathfrak{N}}(x)(1-\gamma_5)\gamma^\mu\frac{\partial}{\partial x_\mu}\lambda(x)|A\rangle,\quad(61)$$

where a regulator term of mass Λ has been added to the L_l^0 propagator. The presence of the $\ln(\Lambda^2/M^2)$ in the expression (61) indicates that the amplitude must be renormalized and consequently cannot be computed within the framework of renormalized perturbation theory. We may, however, attempt to estimate its size by replacing the terms within the square brackets by one. If the resulting numerical coefficient of the matrix element $\langle C|\bar{\mathfrak{N}}(x)(1-\gamma_5)\gamma^\mu\partial/\partial x_\mu\lambda(x))|A\rangle$ is required to be less than $10^{-5}\sin\theta_C$, then

$$m_{e^*}^2+m_{\mu^*}^2\leq 10m_N^2\quad(62)$$

and no new restriction is placed on M.

3. $\Delta S=2$. It can be concluded directly from the structure of $\mathcal{L}_{\rm wk}$ given in (58) that $|\Delta S|=2$ nonleptonic transitions can occur only to order g^8 or higher and must also contain the factor $\sin^2\theta_C m_{l^*}^4/M^4$. Thus, for $m_{l^*}\simeq m_N$, such an amplitude will contain the factor $\sin^2\theta_C\, g^8m_N^4/M^4\simeq 10^{-10}\sin^2\theta_C$ and may consequently be expected to have the correct order of magnitude.

The model specified by the Lagrangian (58), the requirements (49) and (62), and various assumptions made throughout this section, is thus in qualitative agreement with the observed nonleptonic weak interactions. The size of the heavy mass M is no longer limited by the magnitude of observed nonleptonic strangeness violation. However, M probably cannot be arbitrarily large: (i) If perturbation theory is to be applicable, g cannot be very large; for g less than one, $M\leq 200m_N$. (ii) Although, under certain assumptions, $\Delta S=0$ parity violation of order g^2 *decreases* with increasing M, there may exist parity-violating terms of order g^4 which contain no factor of $1/M^2$ and will therefore *increase* with increasing M. The presence of such parity-violating terms of order g^4 might require M to be as small as a few BeV. However, as was discussed in part A of this section, the existence of such terms is not certain. (iii) The weak renormalizations of order g^2 of the vector coupling constants g_β and g_μ, appearing in β decay and muon decay, will in general be different. For large g^2, $g_\beta-g_\mu$ may be sufficiently large to conflict with the observed near equality of these coupling constants (28). We can roughly estimate the size of such renormalization effects by computing the order g^2 renormalization of the fundamental lepton–heavy-lepton–intermediate-boson vertex. In the case of the $e-L_e^0-B^+$ vertex, the weak corrections to the unrenormalized coupling constant g_0 are produced by the e

and L_e^0 wave function renormalization, and are given by

$$g_{\rm ren}=g_0\left[1-\frac{g^2}{\pi}[\tfrac{3}{4}\ln(\Lambda^2/M^2)-\tfrac{3}{8}]\right]. \tag{63}$$

For

$$M\leq 10 \tag{64}$$

this is less than a 2% correction if, as before, the cutoff-dependent part within the inner square brackets is replaced by 1. In addition to nine new heavy particles and their antiparticles with mass between perhaps 5 and 50 BeV, this model requires relatively light, weakly interacting, neutral leptons with masses of the order of 1 BeV whose weak coupling to the hadrons is identical with that of the neutrinos if the Cabbibo angle θ_C, found in the usual neutrino coupling, is changed to $\theta_C-\frac{1}{2}\pi$.

V. CONCLUSION

As Secs. II and III demonstrate, one can deduce the usual phenomenological form for low-energy leptonic and semileptonic processes from a two-scalar-exchange theory with only minor difficulty. The minimum number of new particles required is four, and at least the leptonic part of the interaction Lagrangian is renormalizable. However, the situation becomes much more difficult when nonleptonic processes are also considered. Under favorable circumstances the experimental absence of large parity violation in nuclear processes is consistent with the predictions of the models discussed, provided that some of the intermediate particles are quite heavy (with masses of at least 4 BeV and most likely larger). However, in the simple model discussed in Sec. IV A the large weak coupling constant associated with the heavy intermediate masses produces strangeness violation whose strength may well be too large to agree with experiment. The size of this strangeness violation can be reduced, if a more complicated model is adopted. Such a model was discussed in Sec. IV B and in the most favorable situation allows intermediate particle masses perhaps as large as 50 BeV. (It must be remembered that even these very qualitative conclusions rest critically on a number of unproven assumptions.) A natural and appealing model containing heavy intermediate particles ($M>10$ BeV) and predicting the correct size for $\Delta S=1$ and $\Delta S=2$ processes has not been found. Even in the absence of a really attractive model, this two-scalar-exchange theory serves as a useful example of a *complete* weak-interaction theory which reproduces the usual current $\times$ current form for low-energy weak phenomena but is so modified (or effectively cut off) at high energies that it is renormalizable.

Finally, let us summarize some of the possible consequences of such a two-scalar-exchange scheme:

(i) the production of new, heavy, weakly interacting fermions and bosons in high-energy reactions. In particular, these particles may be produced by very energetic neutrinos with cross sections considerably larger than those found at low energy for the reaction

$$\nu_\mu+n\rightarrow p+\mu^-. \tag{65}$$

Such neutrino production of new heavy particles may be most easily detected in underground experiments where the heavy charged particles could be observed directly or the muon flux produced by the chain of reactions

$$\begin{aligned}\nu_\mu+N&\rightarrow L_\mu+\Gamma_1,\\ L_\mu+N&\rightarrow \mu^-+\Gamma_2\end{aligned} \tag{66}$$

might be observed. The states Γ_1 and Γ_2 in the reactions (66) contain a combination of hadrons and new, heavy, weakly interacting particles, N is a nucleon state and L_μ is a new heavy lepton, carrying muon lepton number, of the type required by the specific models previously described. Although the model Lagrangians discussed in Secs. II B, III B, IV A, and IV B require that at least one of the heavy particles be stable, it is certainly possible to add additional weak couplings so that all the heavy particles are unstable.

(ii) significant deviations of weak amplitudes from the predictions of the usual phenomenological Lagrangian (1) for energies of the order of 50 BeV or lower.

(iii) two new leptons, e^* and μ^*, with masses of the order of the nucleon mass and greater than the K meson's mass. These particles are required by the model discussed in Sec. IV B. If their properties are correctly specified by this model, they are quite hard to detect, being neutral and having their cross section for production by neutrinos reduced by a factor of m_N^4/M^4 relative to that for the reaction (65).

(iv) weak electron-proton scattering, competing with the effects of electromagnetism at large (momentum transfer)^2.

(v) a possibly significant energy dependence in nonleptonic, strangeness-conserving parity-violating processes. The presence of the operator $\Box^2$ in Eq. (46) suggests that parity-violating effects may be enhanced in hadronic reactions at large energies or momentum transfers.

ACKNOWLEDGMENTS

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APPENDIX A

Definitions of the constants a_i and d_i which appear in the μ-decay amplitude (9a) are given below:

$$
\begin{aligned}
a_1 &= \sum_{nn'} D_{nn'}^{-} I_1(X_n,X_{n'})\,,\\
a_2 &= \tfrac{1}{3}\sum_{nn'} D_{nn'}^{-}[I_2(X_n,X_{n'})+I_4(X_n,X_{n'})]\,,\\
a_3+a_4 &= \tfrac{2}{3}\sum_{nn'} D_{nn'}^{-}[2I_2(X_n,X_{n'})-I_3(X_n,X_{n'})]\,,\\
a_3-a_4 &= \tfrac{2}{3}\sum_{nn'} D_{nn'}^{+} I_3(X_n,X_{n'})\,,
\end{aligned}
\tag{A1}
$$

$$
\begin{aligned}
d_1 &= \tfrac{1}{2}(a_3-a_4)\,,\\
d_2 &= 2a_2-\tfrac{1}{2}(a_3+a_4)\,,
\end{aligned}
\tag{A2}
$$

where

$$
\begin{aligned}
X_n &= M_n^2/M^2\,,\\
D_{nn'}^{\pm} &= \sum_{ij} C_i^{n}(\nu_\mu)C_j^{n}(\mu)^* \times[C_j^{n'}(e)C_i^{n'}(\nu_e)^* \pm C_i^{n'}(e)C_j^{n'}(\nu_e)^*]\,,
\end{aligned}
\tag{A3}
$$

and the functions $I_i(x,y)$ are given by

$$
\begin{aligned}
I_1(x,y) &= \frac{1}{(x-1)(y-1)}+\frac{1}{y-x}\left(\frac{y^2\ln y}{(y-1)^2}-\frac{x^2\ln x}{(x-1)^2}\right),\\
I_2(x,y) &= \frac{11}{6(x-1)(y-1)}+\frac{5(x+y)-4}{2(x-1)^2(y-1)^2}+\frac{(x-y)^2}{(x-1)^3(y-1)^3}+\frac{1}{y-x}\times\left(\frac{y^3\ln y}{(y-1)^4}-\frac{x^3\ln x}{(x-1)^4}\right),\\
I_3(x,y) &= \frac{1}{(x-1)^2(y-1)^2}+\frac{1}{(y-x)^2}\left(\frac{x^2}{(x-1)^2}+\frac{y^2}{(y-1)^2}\right)+\frac{x+y}{(y-x)^3}\left(\frac{x^2\ln x}{(x-1)^2}-\frac{y^2\ln y}{(y-1)^2}\right)-\frac{2}{(y-x)^2}\times\left(\frac{x^2\ln x}{(x-1)^3}+\frac{y^2\ln y}{(y-1)^3}\right),\\
I_4(x,y) &= \frac{y-3}{2(y-1)^2(x-1)^2}+\frac{x(xy+2y-3x^2)}{(x-1)^3(y-x)^2}+\frac{x^2\ln x}{(y-x)^2(x-1)^4}[2x+5x-3y-4]+\frac{2xy+y-3x}{(y-x)^3}\left(\frac{y^2\ln y}{(y-1)^3}-\frac{x^2\ln x}{(x-1)^3}\right).
\end{aligned}
\tag{A4}
$$

Equations (A2) and the expression (9a) imply that, to order m_μ^2/M^2, the muon decay amplitude predicted by any two-scalar-exchange model of the weak interactions will depend on only four parameters.[17] These four parameters a_1, a_2, a_3, and a_4 can be chosen to be real if time-reversal invariance is assumed. In contrast, five real parameters are necessary to specify the most general time-reversal-invariant amplitude for the decay (8), which (i) is at most quadratic in the external momenta, (ii) is consistent with the usual two-component description of the neutrinos, (iii) contains the approximation $m_e=0$, and (iv) allows the emission of only left-handed electrons. Thus the presence of only four parameters is a model independent prediction of the two-scalar-exchange theory. In the case $d_1=d_2=0$, the nonlocal effects in muon decay implied by the amplitude (9a) are identical to those found to order m_μ^2/M_W^2 in an intermediate vector boson theory of the weak interactions where $M_W=M\sqrt{2}/a_3$ is the vector boson's mass.

APPENDIX B

As was discussed in Sec. III A, the conditions (24), (26), and (27) on the hadronic operators $S_i^h(x)$ ensure the correct form for low-energy semileptonic weak interactions only to lowest order in g and zero order in m_N^2/M^2. In particular, processes involving a neutral lepton pair, such as

$$K^+\to\mu^+\mu^-\,, \tag{B1}$$

$$K^+\to\pi^+e^+e^-\,, \tag{B2}$$

$$K^+\to\pi^+\nu\bar{\nu}\,, \tag{B3}$$

$$\nu+\text{hadron}\to\nu+\text{hadrons}, \tag{B4}$$

may occur in higher orders. In this appendix we investigate the rates predicted for such transitions by the model discussed in Sec. III B. These reactions are forbidden to lowest order in m_N^2/M^2 in this model. However, the reactions (B3) and (B4) can occur to order Gm_N^2/M^2, while the decays (B1) and (B2) are allowed at sixth order in g and first order in m_N^2/M^2.

The amplitude for the decay (B3) to order g^4 is represented in Fig. 10 and is given by[18]

$$
\frac{4g^4}{\pi^2}\sin\theta_C\cos\theta_C\int d^4q\int d^4q'\frac{e^{-iq'\cdot x}}{(2\pi)^4}\bar{U}_\nu(1-\gamma_5)\times\left[\frac{\mathbf{q}+\mathbf{k}}{(q+k)^2+M^2}-\frac{\mathbf{q}+\mathbf{k}'}{(q+k')^2+M^2}\right]U_{\bar{\nu}}\times\frac{\langle\pi^+|T(\bar{\lambda}(0)(\mathbf{q}+\mathbf{q}')(1+\gamma_5)\mathfrak{N}(x))|K^+\rangle}{(q^2+M^2)[(q+q')^2+M^2][(q+k+k')^2+M^2]}, \tag{B5}
$$

[17] Our derivation of (9a) and (A2) contained the assumption that all the intermediate scalar bosons have the same mass M. In fact, the expresson (9a) and the equations (A2) are valid for the general case of unequal intermediate boson masses $M_{Bi}>M$.

[18] The significant cancellation between the two amplitudes represented in Fig. 10 was pointed out to the author by F. E. Low and M. L. Goldberger (private communication). This cancellation is, of course, evidence that the Eqs. (37) are satisfied.

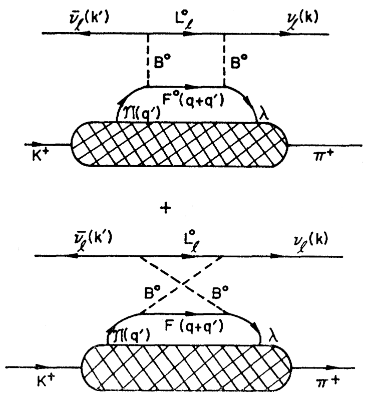

FIG. 10. The Feynman diagrams representing the amplitude for the decay $K^+\to\pi^+\nu\bar{\nu}$ as predicted by the model described in Sec. III B to order g^4.

where k and k' are the momenta of the neutrino and antineutrino, and U_ν and $U_{\bar{\nu}}$ are their respective spinors. If we assume that the integrand vanishes rapidly when the hadronic momentum q'^ρ increases beyond m_N and keep only the lowest nonvanishing order in m_N^2/M^2, this amplitude becomes

$$(g^4/15M^4)\sin\theta_C\cos\theta_C\xi_1 P\cdot(k-k')\bar{U}_\nu \boldsymbol{P}(1+\gamma_5)\bar{U}_{\bar{\nu}}, \quad \text{(B6a)}$$

where we have set

$$\begin{aligned}\langle\pi^+|\bar{\lambda}(x)\gamma^\rho(1+\gamma_5)(\partial/\partial x_\sigma)\mathfrak{N}(x)|K^+\rangle \\ =\xi_1P^\rho P^\sigma+\xi_2Q^\rho Q^\sigma+\xi_3P^\rho Q^\sigma \\ +\xi_4Q^\rho P^\sigma+\xi_5\varepsilon^{\rho\sigma\delta\epsilon}P^\delta Q^\epsilon+\xi_6\delta^{\rho\sigma}. \quad \text{(B6b)}\end{aligned}$$

The four-momentum P is the sum of the pion and kaon four-momenta, while Q is the difference. The amplitude (B6a) leads to a rate for the decay (B3), summed over the two types of neutrinos, given by

$$m_K\left(\frac{|\xi_1|^2G^2m_K^8\sin^2\theta_C\cos^2\theta_C}{9600\pi^3M^4}\right)\times\left(4\lambda^3\ln\lambda + \frac{1-\lambda^2}{15}(1-9\lambda+46\lambda^2-9\lambda^3+\lambda^4)\right), \quad \text{(B7)}$$

where $\lambda=m_\pi^2/m_K^2$. This expression implies a branching of

$$\frac{\Gamma(K^+\to\pi^+\nu\bar{\nu})}{\Gamma(K^+\to\text{all modes})}=5.7\times10^{-4}|\xi_1|^2\left(\frac{m_K}{M}\right)^4, \quad \text{(B8)}$$

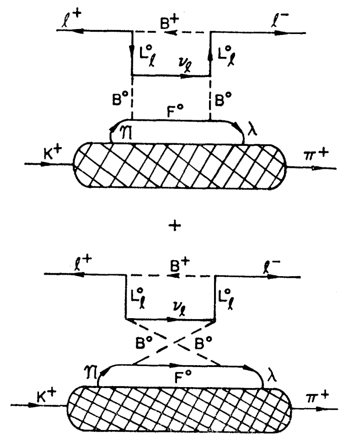

FIG. 11. The Feynman diagrams representing the amplitude to order g^6 for the decay $K^+\to\pi^+e^+e^-$ as predicted by the model described in Sec. III B.

which for $|\xi_1|=1$ and $M>2m_K$ lies below the present experimental upper limit[19] of 1.1×10^{-4}. We can expect a similar suppression (by about a factor of 100) of the cross section for the reaction (B4) relative to that for the allowed process

$$\nu_\mu+n\to\mu^-+p. \quad \text{(B9)}$$

The decays (B1) and (B2) can occur to order $g^2Gm_N^2/M^2$. The amplitude for the decay (B2) is represented by the diagram in Fig. 11. A rough estimate of this diagram suggests a branching ratio

$$\frac{\Gamma(K^+\to\pi^+e^+e^-)}{\Gamma(K^+\to\text{all modes})}=10^{-4}\left(\frac{m_K}{M}\right)^2, \quad \text{(B10)}$$

which is to be compared with the experimental upper limit[20] of 1.1×10^{-6}. Thus, although transitions of the type (B1)–(B4) are not forbidden by the model of Sec. III B they are sufficiently suppressed that no conflict with experiment arises.

Similar considerations apply to the models discussed in Secs. IV A and IV B.

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[19] D. Cline, thesis, University of Wisconsin, 1965 (unpublished). The author is indebted to W. J. Willis for informing him of the existence of such an upper limit.

[20] U. Camerini *et al.*, Phys. Rev. Letters **13**, 318 (1964).