Regge-Pole Eikonal Theory of Small-Angle Pion-Nucleon Scattering*

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A quantitative model for high-energy, small-momentum-transfer pion-proton elastic scattering and charge-exchange data (both differential cross sections and polarizations) is presented. The model is based on P' and ρ Regge poles in the optical potential, together with a Pomeranchon whose residue is proportional to the product of electromagnetic form factors of pion and proton. Features not present in the Regge-pole approach, such as the crossover effect and nonzero charge-exchange polarization, appear automatically in this model. Specific properties of our ρ and P' poles are: (a) both choose nonsense; (b) constant reduced residues; (c) linear trajectories; (d) zero helicity flip for P'; and (e) helicity-nonflip P' residue given by exchange degeneracy (and symmetry, e.g., from quark model) in terms of ρ residue. The elastic scattering fit is satisfactory for all momenta greater than 6 GeV/c and all squared momentum transfers less than 0.5 GeV^2/c^2. All available charge-exchange data above 5 GeV/c are fitted well, as are total cross sections and real parts of amplitudes in the forward direction.

I. INTRODUCTION

It has been pointed out[1] that the outstanding difficulties with simple-minded Regge-pole analysis of high-energy scattering amplitudes, e.g., requirements of some form of conspiracy (for reactions involving spin) and difficulties in reconciling crossover effects, are removed if one uses an approach[2] in which the t-channel poles are regarded as an effective optical potential. We have explicitly carried out the calculations using the eikonal formulation[1,2] for pion-proton elastic scattering and charge-exchange amplitudes, yielding differential cross sections and polarizations, with no more parameters than in a pure pole approach, and with results that agree reasonably well with experiment (including the charge-exchange polarization) at sufficiently high energy.

The calculations begin with assumptions on the poles involved, as in the usual Regge analysis. A Fourier-Bessel transform of the poles is performed, and the resulting function is treated as the optical phase shift. After exponentiation the inverse Fourier-Bessel transform yields the scattering amplitude.

We make the following assumptions:

(a) The Pomeranchon is effectively a fixed pole with residue given by the product of pion and proton form factors. This leads to an asymptotic amplitude identical in form to that of Durand and Lipes[3] or of Chou and Yang.[4] We effectively fit the pion-charge radius to match the highest-energy elastic scattering data.

(b) The ρ and P' $(=f_0)$ trajectories are straight lines, with intercepts and slopes close to those obtained from a linear extrapolation from physical meson masses, assuming exchange degeneracy of (ρ,A_2) and (P',ω) trajectories. The precise values for trajectory parameters are determined by fitting the data.

(c) The moving pole residues, after required factors of α are extracted, are constants.

(d) Exchange degeneracy for the residues is assumed, together with $SU(3)$ with pure F for the charge couplings of the vector and tensor trajectories; this yields the requirement that the ρ as well as the P' nonflip residue have a factor of α. (In the P' case, such a factor is required to prevent a ghost; but if exchange degeneracy is not assumed, there is no reason for such a factor for the ρ.) In other words, both the ρ and the P' choose nonsense. These assumptions (and the usual mixing angle for vector mesons) also require that the value of the P' nonflip residue be three times the ρ nonflip residue, thus eliminating another parameter.

(e) The helicity-flip P' residue is assumed to be zero (as well as that of the Pomeranchon).

The fitted values of the trajectory parameters are

$$\alpha_\rho(0)=0.55\,,\quad \alpha_{P'}(0)=0.45\,,$$
$$\alpha_\rho'=0.8\,,\qquad \alpha_{P'}'=1.0\,.$$

II. SPECIFIC EQUATIONS OF MODEL

The specific expressions used in our model are as follows: For each reaction (π^+, π^-, or charge exchange)

$$\frac{d\sigma}{dt}=\frac{\pi}{k^2s}(|G_+|^2+|G_-|^2)\,,$$

$$P=\frac{2\,\mathrm{Im}(G_+G_-^*)}{|G_+|^2+|G_-|^2}\,.$$

These G's are obtained from the expressions below, which refer to isospin eigenamplitudes, by using the appropriate Clebsch-Gordan coefficients:

$$G_+=ikW\cos(\tfrac{1}{2}\theta)\int_0^\infty bdb\,J_0(b\Delta)[1-e^{i\chi_0(s,b)}\cos\chi_f(s,b)]\,,$$

$$G_-=kW\int_0^\infty bdb\,J_1(b\Delta)e^{i\chi_0(s,b)}\sin\chi_f(s,b)\,.$$

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[1] R. C. Arnold, Argonne National Laboratory Report No. ANL-HEP 6804, 1968 (unpublished).
[2] R. C. Arnold, Phys. Rev. 153, 1523 (1967).
[3] L. Durand, III, and R. Lipes, Phys. Rev. Letters 20, 637 (1968).
[4] T. T. Chou and C. N. Yang, Phys. Rev. Letters 20, 1213 (1968).

In these expressions,

$$\Delta=\sqrt{(-t)}\,,\quad W=\sqrt{s}\,,$$

$$\chi_0(s,b)=\frac{1}{kW}\int_0^\infty \Delta d\Delta\, J_0(b\Delta)\frac{G_+{}^B(s,\Delta)}{\cos(\frac{1}{2}\theta)}\,,$$

$$\chi_f(s,b)=\frac{1}{kW}\int_0^\infty \Delta d\Delta\, J_1(b\Delta)G_-{}^B(s,\Delta)\,,$$

where the G^B's are the pole approximations to the amplitudes of definite isospin:

$$G_+{}^B=G_+{}^P+G_+{}^{P'}+G_+{}^\rho\,,$$
$$G_-{}^B=G_-{}^\rho\,.$$

Using a dipole ansatz for the product of pion and proton form factors, we take the form of the Pomeranchon contribution as

$$G_+{}^P=iCkW[\mu^4/(\mu^2-t)^2]^2\,;$$

the parameters are adjusted to fit the highest-energy scattering data. For P',

$$G_+{}^{P'}=-\alpha b_1{}^{P'}\left(\frac{s}{s_0}\right)^\alpha\frac{1+e^{-i\pi\alpha}}{\sin\pi\alpha}\,,\quad \alpha=\alpha_{P'}(t)$$

and for ρ,

$$G_+{}^\rho=\alpha b_1{}^\rho\left(\frac{s}{s_0}\right)^\alpha\frac{1-e^{-i\pi\alpha}}{\sin\pi\alpha}\,,$$

$$G_-{}^\rho=\frac{\Delta\alpha}{2M}(b_1{}^\rho-b_2{}^\rho)\left(\frac{s}{s_0}\right)^\alpha\frac{1-e^{-i\pi\alpha}}{\sin\pi\alpha}\,,\quad \alpha=\alpha_\rho(t)\,.$$

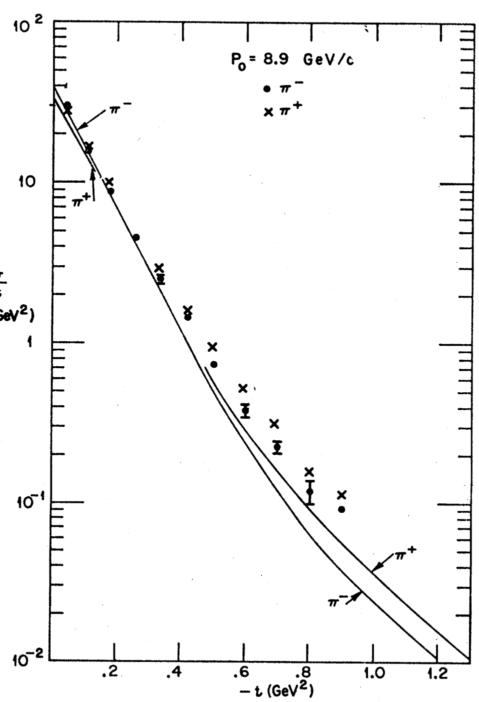

FIG. 1. Differential cross sections for $\pi^\pm$-p elastic scattering around 9 GeV/c.

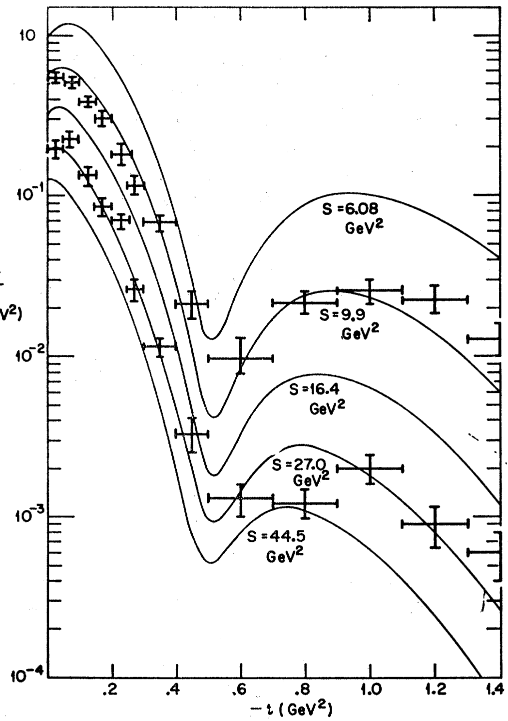

FIG. 2. Differential cross sections for π^--p charge-exchange scattering.

As discussed above, we impose the exchange-degeneracy constraint $b_1{}^{P'}=3b_1{}^\rho$ and we find this to be acceptable. The ratio of helicity flip to nonflip for the ρ was fitted to the shape of the charge-exchange cross section, and the best results were obtained with $b_2{}^\rho\cong 6b_1{}^\rho$.

The scale parameter s_0 was a free parameter, and we found over-all optimal fit with $s_0=0.30$ GeV^2. The strength of the ρ term was $b_1{}^\rho=0.78$. Good agreement with data above 20 GeV was achieved with the Pomeranchon parameters

$$C=5.35\ \text{GeV}^{-2}\,,\quad \mu=1.1\ \text{GeV}\,.$$

This choice leads to no diffraction zero for $-t$ less than 1.5 GeV^2; this may be compared with the p-p analyses, which (with use of the empirical electromagnetic proton form factor) give a zero in the asymptotic cross section at about 1 GeV^2. The difference lies mainly in the smaller pion radius, reflected in the value of 1.1 GeV mentioned above compared to 0.84 GeV in the proton-proton case.

As discussed at some length by Chiu and Finkelstein,[5] who did a similar calculation (with drastically simplified assumptions for the poles) to obtain the shape of pp and $p\bar{p}$ differential cross sections at high energies, this kind of model can be regarded as a calculation of the moving cuts in the angular-momentum plane in terms of the poles and the Pomeranchon, whose origin is obscure.

[5] C. Chiu and J. Finkelstein, Nuovo Cimento **57**, 649 (1968).

III. RESULTS

When the results of this calculation are compared with available data, a generally good agreement is noted for sufficiently high energy and small momentum transfers. This agreement specifically includes the polarization observed in charge exchange and the π^--π^+ crossover effect, both of which are lacking in a simple minimal-pole approach.

The purposes of carrying out this calculation can be summarized as follows: (a) Are there any general features of the data which do not agree with this model? (b) How low in energy can the model be used without serious disagreement? We find that these two questions are closely related. The main disagreement is in the P' amplitude for $-t$ greater than 0.40 GeV^2. The sum of P' pole and cut contributions, our entire exchanged isospin-zero nonasymptotic amplitude, for such large momentum transfers has the wrong sign for its imaginary part. This results in two undesirable features: (a) The asymptotic cross sections in that momentum-transfer region are approached from below rather than from above, as indicated by the data, and the secondary maximum observed in elastic scattering around 3 GeV is not reproduced with sufficient magnitude. (b) The elastic polarizations are not so small as data indicate if one examines the region between 0.4 and 0.7 GeV^2 in $-t$ at energies around 5 GeV. If there were some mechanism to suppress entirely the P' for large momentum transfers, we would obtain better agreement with data.

A typical result for elastic scattering is shown in Fig. 1, where the calculated curve for 8.9 GeV is shown together with data[6] at or near that momentum. This is not yet asymptotic, and the negative interference with the Pomeranchon generated by the P' and associated cuts is evident by the smaller-than-desired differential cross section around $-t=0.50$ GeV^2.

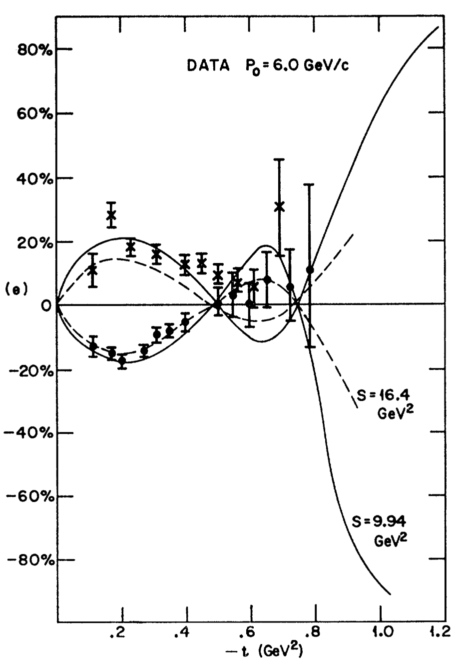

FIG. 3. Polarizations for $\pi^{\pm}$-p elastic scattering; calculations at 5 and 8 GeV/c; (typical) data at 6 GeV/c shown.

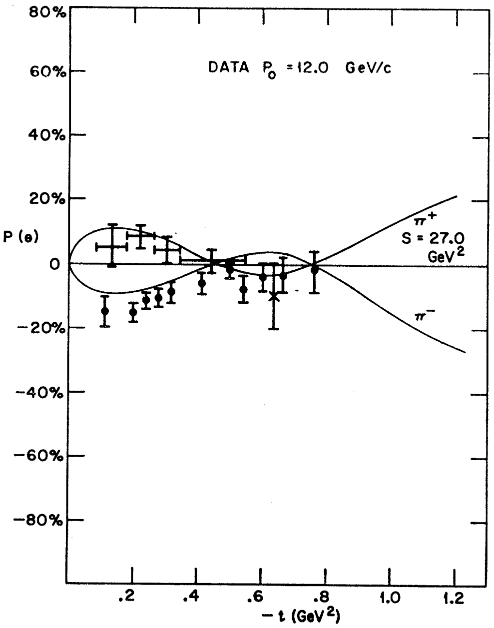

FIG. 4. Polarization for $\pi^{\pm}$-p elastic scattering; calculation at 13 GeV/c, compared with data at 12 GeV/c.

The charge-exchange differential cross-section data[7] are fitted quite well, as shown in Fig. 2. Only data for two energies are shown for clarity; the other energies also fit.

In Fig. 3 we show typical elastic polarization curves, together with data[8] at 6 GeV, which lies intermediate in energy between the calculated curves. In Fig. 4 the 12-GeV polarization data[8] are compared with our result at approximately the same energy. In both figures reasonable agreement is seen. The qualitative features that we see are similar to those obtained in a pure pole calculation where the P' is given a no-compensation mechanism. (Note, however, that our P' does not have that mechanism. It is the cuts in our model calculation that are responsible for the change in sign of the real part of the P'-plus-cut terms. Unfortunately, the imaginary part has only a simple zero, not a double zero, which leads to the undesired features previously mentioned.) Although our predicted polarization at 2.5 GeV agrees with π^- data, this is probably accidental, since we do not agree well near 5 GeV.

In Fig. 5 the charge-exchange polarization calculated is compared with CERN data[9] at 5.9 and 11.2

[6] K. J. Foley *et al.*, Phys. Rev. Letters **11**, 425 (1963).
[7] P. Sonderegger *et al.*, Phys. Letters **20**, 75 (1966).
[8] M. Borghini *et al.*, Phys. Letters **24B**, 77 (1966).
[9] P. Bonamy *et al.*, Phys. Letters **23**, 501 (1966).

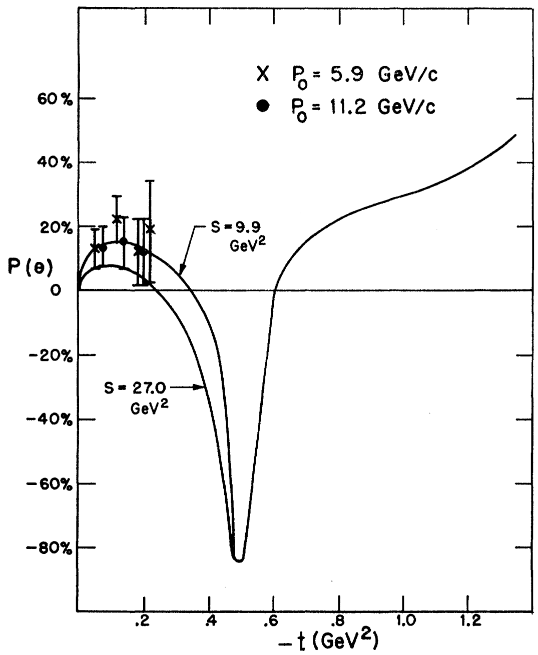

FIG. 5. Polarization for the reaction $\pi^- p \to \pi^0 n$; calculations for 5 and 13 GeV/c, compared with available data.

GeV/c. In the small-momentum-transfer region where data are available, we find agreement. Satisfactory results are obtained for small $-t$ as low as 3.5 GeV/c when compared with ANL data.[10] Thus we find our model for charge exchange to be quite satisfactory. The future observation of the striking negative spike in the charge-exchange polarization would be a dramatic success of our model, since, unlike many other models of this polarization, we have no adjustable parameters.

Although we do not present graphs, the total cross sections and ratio of real-to-imaginary parts of amplitudes at $t=0$ are also fitted satisfactorily with our model, for energies above 5 GeV.

It should be remaked that this large negative feature for charge-exchange polarization in the region of the cross-section minimum is not present if the ρ chooses sense instead of nonsense, i.e., if we drop the exchange-degeneracy restriction on the residues. Thus, other calculations such as that of Schrempp[11] and of Cohen-Tannoudji, Morel, and Navelet,[12] although somewhat similar in general spirit and yielding satisfactory agreement with high-energy charge-exchange polarization, show only a dip in polarization, and it remains positive (going large and positive at large $-t$) over the whole range considered here; they use a ρ that chooses sense.

Another feature which depends crucially on the ρ mechanism is the position of the crossover point, i.e., that t value at which $\mathrm{Im}(G_+)$ goes through zero for the charge-exchange amplitude (the ρ plus cuts), which is the point where the π^- and π^+ cross sections cross as a function of t. With the choosing-sense mechanism we found that this point was consistently 0.6 GeV^2 or larger, which is not adequate to explain this feature in the data. However, with our exchange-degenerate residues (choosing nonsense for ρ) the crossover point is between 0.3 and 0.4 GeV^2, which is satisfactory.

[10] D. D. Drobnis *et al.*, Phys. Rev. Letters **20**, 274 (1968).

[11] F. Schrempp, DESY Report No. 68/1, 1968 (unpublished).

[12] G. Cohen-Tannoudji, A. Morel, and H. Navelet, Nuovo Cimento **48**, 1075 (1967).