PHYSICAL REVIEW VOLUME 176, NUMBER 5 25 DECEMBER 1968

# Meson-Baryon and Baryon-Baryon Scattering in Reggeized $U(6)\otimes U(6)$ Theory

R. DELBOURGO* AND HARUN-AR RASHID†
*International Atomic Energy Agency, International Centre for Theoretical Physics, 34001 Trieste, Italy*
(Received 20 June 1968)

A complete set of rules is presented for evaluating the master meson-trajectory exchange contribution to high-energy scattering between mesons ($M$) and baryons ($B$) in $U(6)\otimes U(6)$ theory. The practical implementation of these rules is illustrated in considerable detail for quark-quark scattering as well as for the $\mathbf{35}_W$-exchange contribution to $MB$ and $BB$ charge-exchange amplitudes.

## 1. INTRODUCTION

RECENTLY a $U(6)\otimes U(6)$ Reggeization program has been developed[1] during the course of the last year with the aim of combining Regge-pole ideas with the attractive results of supermultiplet schemes in the hope that the merger will serve to systematize the possible choices of Regge parameters in current fits to the experimental data. The program has been clearly outlined in a series of recent publications and the rules for calculating Regge-pole amplitudes have been stated for a few simple cases involving quarks and supersinglets. In this paper we shall give the complete set of rules for calculating the master meson trajectory contributions to high-energy scattering of the physical $(6,\bar{6})$ and $(56,1)$ multiplets and shall show explicitly how these rules are applied.

From the outset our work is purposely cast into the $M$-function framework of $U(6,6)$ theory[2] in order to avoid crossing and other complications of the canonical approach and especially in order to take account of symmetry-breaking effects (resulting from physical mass shifts) in the simplest possible way. Just as the Regge $J$-pole contribution to an $M$ function may be viewed as the continuation from integer to complex $J=\alpha(t)$ of the invariant amplitudes (occurring in the $M$-function expansion) corresponding to spin $J$ exchange, so we can obtain the master meson $N$-pole contribution to the $U(6,6)$ $M$ function for scattering of $U(6)\otimes U(6)$ states by exchanging an elementary meson field of quark number $N$ and continuing to complex $N=\alpha(t)$. "Sense-choosing" factorization is thus built into the formalism from the very beginning.[3] The residue functions for the meson trajectory are thereby related to the invariant couplings of the exchanged multiplet $(W_N,\bar{W}_N)$ with the external particle multiplets, like $M=(6,\bar{6})$ and $B=(56,1)$. The rules needed to construct these $U(6)_W$-preserving couplings are provided in Sec. 2, where the question of $N$ signature and its relation to charge conjugation for meson couplings is also discussed. The vertex-progatator-vertex product that represents the pole contribution to the amplitude in question is then rewritten as a product of derivatives of the basic rotation function $(|\mathbf{q}|\,|\mathbf{q}'|)^N C_N{}^3(\hat{\mathbf{q}}\cdot\hat{\mathbf{q}}')$ which describes the scattering of $U(6)\otimes U(6)$ supersinglets—the analog of the solid harmonic $(|\mathbf{q}|\,|\mathbf{q}'|)^J P_J(\hat{\mathbf{q}}\cdot\hat{\mathbf{q}}')$ which occurs in ordinary Regge theory. Section 3 is concerned with expanding such derivatives $\partial^{r+r'}C_N/\partial q^r\partial q'^{r'}$ as a series of kinematic functions $\mathcal{C}_i$ (products of $q$ and $q'$) multiplied into invariant functions $\mathcal{C}_i$ ($r+r'$ derivatives of $C_{N+R}{}^3$). To familiarize the reader with the use of these Reggeization rules we have treated the example of quark-quark scattering in Sec. 4 as this is the first nontrivial case which contains all the significant ideas. In Sec. 5 we have pursued the physical program by considering the $\mathbf{35}_W$ exchange contribution to high-energy $MB$ and $BB$ scattering.

## 2. INVARIANT COUPLINGS OF THE MESON TRAJECTORY

The degenerate class[4] of $U(6)\otimes U(6)$ representations to which we assign the physical particles means that we can only encounter auxiliary $U(6,6)$ tensor fields of the character $\Phi_{(A_1\cdots A_N)}{}^{(B_1\cdots B_N)}$ and $\Psi_{(A_1\cdots A_{n+3})}{}^{(B_1\cdots B_N)}$ subject to the usual Bargmann-Wigner equations[2] for projecting the free-particle states. Consider now the coupling of three such free fields. Since we wish to preserve the $U(6)_W$ subgroup in the spirit of supermultiplet theory, we shall only allow derivative (kineton) couplings out of the large number of possible Lorentz-invariant couplings. The total number of couplings which we wish to consider then directly equals the number of $U(6)_W$ scalars that can be formed out of the $W$-multiplet components contained in the $U(6)\otimes U(6)$ states. To see how this works in practice, we shall construct the effective Lagragian couplings in momentum space for a few important cases. The construction for other cases will then be rather obvious.

---

* Imperial College, London, England.
† International Centre for Theoretical Physics, Trieste, Italy. On leave of absence from University of Islamabad, Pakistan.

[1] Abdus Salam and J. Strathdee, Phys. Rev. Letters **19**, 339 (1967); R. Delbourgo, M. A. Rashid, Abdus Salam, and J. Strathdee, Phys. Rev. **170**, 1477 (1968); R. Delbourgo, Abdus Salam, and J. Strathdee, *ibid.* **172**, 1727 (1968).

[2] R. Delbourgo, M. A. Rashid, Abdus Salam, and J. Strathdee, in *Proceedings of the International Seminar in High-Energy Physics and Elementary Particles, Trieste, 1965* (International Atomic Energy Agency, Vienna, 1965). This is a review article of $U(6,6)$ theory.

[3] H. Jones and M. Scadron, Nucl. Phys. **B4**, 267 (1968), have used the same technique for conventional $J$ Reggeization.

[4] Y. Dothan, M. Gell-Mann, and Y. Ne'eman, Phys. Letters **17**, 148 (1965).

(*i*) (*1,1*)-(*1,1*)-($W_N,\bar{W}_N$):

$$\mathcal{L}_1=H\Phi(\tfrac{1}{2}p+q)\Phi(\tfrac{1}{2}-q)q_{B_1}{}^{A_1}\cdots q_{B_N}{}^{A_N}\Phi_{(A_1\cdots A_N)}{}^{(B_1\cdots B_N)}(p)\,.$$

(*ii*) (*6,1*)-(*1,*$\bar{6}$)-($W_N,\bar{W}_N$):

$$\mathcal{L}_2=\bar{\Psi}^B(\tfrac{1}{2}p+q)[G_0\delta_B{}^Aq_{B_1}{}^{A_1}+G_1\delta_B{}^{A_1}\delta_{B_1}{}^A]\Psi_A(\tfrac{1}{2}p-q)q_{B_2}{}^{A_2}\cdots q_{B_N}{}^{A_N}\Phi_{(A_1\cdots A_N)}{}^{(B_1\cdots B_N)}(p)\,.$$

(*iii*) (*56,1*)-(*1,56*)-($W_N,\bar{W}_N$):

$$\begin{aligned}\mathcal{L}_3=\bar{\Psi}^{(ACE)}(\tfrac{1}{2}p+q)[&G_0\delta_A{}^B\delta_C{}^D\delta_E{}^Fq_{B_1}{}^{A_1}q_{B_2}{}^{A_2}q_{B_3}{}^{A_3}+G_1\delta_A{}^{A_1}\delta_{B_1}{}^B\delta_C{}^D\delta_E{}^Fq_{B_2}{}^{A_2}q_{B_3}{}^{A_3}\\&+G_2\delta_A{}^{A_1}\delta_{B_1}{}^B\delta_C{}^{A_2}\delta_{B_2}{}^D\delta_E{}^Fq_{B_3}{}^{A_3}+G_3\delta_A{}^{A_1}\delta_{B_1}{}^B\delta_C{}^{A_2}\delta_{B_2}{}^D\delta_E{}^{A_3}\delta_{B_3}{}^F]\Psi_{(BDF)}(\tfrac{1}{2}p-q)q_{B_4}{}^{A_4}\cdots q_{B_N}{}^{A_N}\Phi_{(A_1\cdots A_N)}{}^{(B_1\cdots B_N)}(p)\,.\end{aligned}$$

(*iv*) (*6,6*)-(*6,6*)-($W_N,\bar{W}_N$):

$$\begin{aligned}\mathcal{L}_4=\Phi_A{}^B(\tfrac{1}{2}p+q)[&(H_0\delta_B{}^C\delta_D{}^A+H_0'q_B{}^Aq_D{}^C)q_{B_1}{}^{A_1}q_{B_2}{}^{A_2}+H_2\delta_A{}^{A_1}\delta_{B_1}{}^B\delta_C{}^{A_2}\delta_{B_2}{}^D\\&+(H_1q_B{}^A\delta_{B_1}{}^C\delta_D{}^{A_1}+H_1''q_D{}^C\delta_B{}^{A_1}\delta_{B_1}{}^A+H_1'\delta_B{}^C\delta_D{}^{A_1}\delta_{B_1}{}^A+H_1'''\delta_D{}^A\delta_B{}^{A_1}\delta_{B_1}{}^C)q_{B_2}{}^{A_2}]\,,\\&\Phi_C{}^D(\tfrac{1}{2}p-q)q_{B_3}{}^{A_3}\cdots q_{B_N}{}^{A_N}\Phi_{(A_1\cdots A_N)}{}^{(B_1\cdots B_N)}(p)\,.\end{aligned}$$

All such effective Lagrangians are automatically parity conserving because parity is of the $U(6)\otimes U(6)$ operations; however, they are not necessarily invariant, as they stand, under particle-antiparticle ($\mathcal{C}$) conjugation. To see the consequences of imposing $\mathcal{C}$ invariance on the $\mathcal{L}$ we recall[2] that the action of charge conjugation on field operators is given by

$$\mathcal{C}\Phi_{(A_1\cdots A_N)}{}^{(B_1\cdots B_M)}(x)\mathcal{C}^{-1}=(C^{-1})^{B_1D_1}\cdots(C^{-1})^{B_MD_M}\Phi_{(D_1\cdots D_M)}{}^{(C_1\cdots C_N)}(x)C_{C_1A_1}\cdots C_{C_NA_N}\,,\tag{1}$$

where $C$ is the usual charge-conjugation matrix having the property $(C^{-1})^{AB}k_B{}^CC_{CD}=-k_D{}^A$. Applying this to $\mathcal{L}$ we discover that mesons with odd quark number $N$ cannot couple to supersinglets, which could also have been deduced on the basis of Bose statistics. On the other hand, $\mathcal{C}$ conjugation gives no restrictions on $N$ values in $\mathcal{L}_2$ and $\mathcal{L}_3$. Finally, for $\mathcal{L}_4$ we find from Bose statistics and charge conjugation that $H_1$ $(=H_1'')$, $H_0$, $H_0'$ and $H_2$ are associated with even $N$ whereas $H_1'=\pm H_1'''$ according as $N$ is odd or even. We see therefore that $N$ signature arises in $U(6)\otimes U(6)$ for meson couplings. Taking all these constraints into account we may summarize vertices (*i*)-(*iv*) most elegantly as follows:

$$\mathcal{L}_1=\mu^{1-N}h^{(+)}\Phi(\tfrac{1}{2}p+q)\Phi(\tfrac{1}{2}p-q)\Phi^{(N)}(p,q)\,,\tag{2}$$

$$\mathcal{L}_2=m^{-N}\bar{\Psi}^B(\tfrac{1}{2}p+q)\Psi_A(\tfrac{1}{2}p-q)\left(g_0\delta_B{}^A+mg_1\frac{\partial}{\partial q_A{}^B}\right)\Phi^{(N)}(p,q)\,,\tag{3}$$

$$\begin{aligned}\mathcal{L}_3=m^{-N}\bar{\Psi}^{BDF}(\tfrac{1}{2}p+q)\Psi_{ACE}(\tfrac{1}{2}p-q)\bigg(&g_0\delta_B{}^A\delta_D{}^C\delta_F{}^E+mg_1\frac{\partial}{\partial q_A{}^B}\delta_D{}^C\delta_F{}^E\\&+m^2g_2\frac{\partial^2}{\partial q_A{}^B\partial q_C{}^D}\delta_F{}^E+m^3g_3\frac{\partial^3}{\partial q_A{}^B\partial q_C{}^D\partial q_E{}^F}\bigg)\Phi^{(N)}(p,q)\,,\end{aligned}\tag{4}$$

$$\begin{aligned}\mathcal{L}_4=\mu^{1-N}\Phi_A{}^D(\tfrac{1}{2}p+q)\Phi_C{}^B(\tfrac{1}{2}p-q)\bigg[&h_0{}^{(+)}\delta_B{}^A\delta_D{}^C+\mu^{-1}h_0{}^{(+)}q_D{}^Aq_B{}^C+\mu^2h_2{}^{(+)}\frac{\partial^2}{\partial q_C{}^D\partial q_A{}^B}+h_1{}^{(+)}\left(q_D{}^A\frac{\partial}{\partial q_C{}^B}+q_B{}^C\frac{\partial}{\partial q_A{}^D}\right)\\&+h_1{}^{(-)}\left(\delta_B{}^A\frac{\partial}{\partial q_C{}^D}+\delta_D{}^C\frac{\partial}{\partial q_A{}^B}\right)+h_1'{}^{(+)}\left(\delta_B{}^A\frac{\partial}{\partial q_C{}^D}-\delta_D{}^C\frac{\partial}{\partial q_A{}^B}\right)\bigg]\Phi^{(N)}(p,q)\,,\end{aligned}\tag{5}$$

where

$$\Phi^{(N)}(p,q)\equiv q_{A_1}{}^{B_1}\cdots q_{A_N}{}^{B_N}\Phi_{(B_1\cdots B_N)}{}^{(A_1\cdots A_N)}(p)\,.\tag{6}$$

All coupling constants (whose intrinsic $N$ dependence has been suppressed in the formulas) have been rendered dimensionless by introducing mass scales $\mu$ and $m$ for mesons and baryons. The $\pm$ superscripts on them refer to the $N$ signatures with which they are associated. When these do not appear it is implied that both signatures contribute.

The remaining subscripts (0,1,2,3) are connected with the $W$ spin representation coupling of the master meson trajectory (viz. **1**, **35**, **405**, **2695**), the precise connection being stated in Sec. 3. The values of these coupling constants are of course known at integer $N$ from the actual decay rates of the physical meson resonances.

Coming now to the evaluation of the $(W_D,\bar{W}_N)$-exchange pole diagram, we have simply to combine the vertices at each end and insert a free-particle propagator for a meson of quark number $N$. Only products of couplings with the same signature need be taken. We shall now demonstrate that every scattering process can be expressed as a product of derivatives applied to the basic rotation function $C_N{}^3(\hat{\mathbf{q}}\cdot\hat{\mathbf{q}}')$.

To begin with, consider singlet-singlet scattering

$$T=h^{(+)2}\mu^{2-2N}\Delta_N(p;\,q,q')\,, \tag{7}$$

with

$$\begin{aligned}\Delta_N(p;\,q,q')&\equiv q'_{A_1'}{}^{B_1'}\cdots q'_{A_N}{}^{B_N'}\langle\Phi_{B_1'\cdots B_N'}{}^{A_1'\cdots A_N'}(-p)\Phi_{(A_1\cdots A_N)}{}^{(B_1\cdots B_N)}(p)\rangle q_{B_1}{}^{A_1}\cdots q_{B_N}{}^{A_N}\\&=(|\mathbf{q}|\,|\mathbf{q}'|)^N C_N(\hat{\mathbf{q}}\cdot\hat{\mathbf{q}}')/(p^2-M_N{}^2)\,,\end{aligned} \tag{8}$$

where

$$\hat{\mathbf{q}}\cdot\hat{\mathbf{q}}'=-q\cdot q'+q\cdot pq'\cdot p/M^2=|\mathbf{q}|\,|\mathbf{q}'|\,\cos\theta\,. \tag{9}$$

Next we treat the case of quark-quark scattering

$$(6,1)_{A,\,-\frac{1}{2}p+q}+(6,1)_{B',\,\frac{1}{2}p+q'}\to(\bar{6},1)_{\frac{1}{2}p+q}{}^{B}+(\bar{6},1)_{-\frac{1}{2}p+q'}{}^{A'}\,,$$

which is described by a 4-index $M$ function,

$$T_{BA'}{}^{AB'}=\left(g_0\delta_B{}^A+mg_1\frac{\partial}{\partial q_A{}^B}\right)\left(g_0\delta_A{}^{B'}+mg_1\frac{\partial}{\partial q_{B'}{}^{A'}}\right)m^{-2N}\Delta_N(p;\,q,q')\,. \tag{10}$$

Similarly the $M$ functions for more complicated reactions can be cast as factorized products of $\Delta_N$ derivatives; to take another example, meson-baryon scattering mediated by $(W_N,\bar{W}_N)$ exchange is characterized by a 10-index $M$ function

$$(56,1)_{ACE,\,\frac{1}{2}p+q}+(6,\bar{6})_{B',\,-\frac{1}{2}p+q'}{}^{C'}\to(\overline{56},1)_{-\frac{1}{2}p+q}{}^{BDF}+(\bar{6},6)_{D',\,\frac{1}{2}p+q'}{}^{A'}\,,$$

namely,

$$\begin{aligned}T_{A'C'BDF}{}^{B'D'ACE}=\Bigg[&h_0{}^{(+)}\delta_{A'}{}^{B'}\delta_C{}^{D'}+\mu^{-1}h_0{}^{(+)\prime}q'_{C'}{}^{B'}q'_{A'}{}^{D'}+\mu^2h_2{}^{(+)}\frac{\partial^2}{\partial q'_{D'}{}^{C'}\partial q'_{B'}{}^{A'}}\\&+h_1{}^{(+)}\left(q'_{C'}{}^{B'}\frac{\partial}{\partial q'_{D'}{}^{A'}}+q'_{A'}{}^{D'}\frac{\partial}{\partial q'_{B'}{}^{C'}}\right)+\mu h_1{}^{(-)}\left(\delta_{A'}{}^{B'}\frac{\partial}{\partial q'_{C'}{}^{D'}}+\delta_{C'}{}^{D'}\frac{\partial}{\partial q'_{B'}{}^{A'}}\right)+\mu h_1{}'^{(+)}\left(\delta_{A'}{}^{B'}\frac{\partial}{\partial q'_{C'}{}^{D'}}-\delta_{C'}{}^{D'}\frac{\partial}{\partial q'_{B'}{}^{A'}}\right)\Bigg]\\&\times\left(g_0\delta_B{}^A\delta_D{}^C\delta_F{}^E+mg_1\delta_D{}^C\delta_F{}^E\frac{\partial}{\partial q_A{}^B}+m^2g_2\delta_F{}^E\frac{\partial^2}{\partial q_A{}^B\partial q_C{}^D}+m^3g_3\frac{\partial^3}{\partial q_A{}^B\partial q_C{}^D\partial q_E{}^F}\right)\mu^{1-N}m^{-N}\Delta_N\,,\end{aligned} \tag{11}$$

where signature projections are explicitly indicated, and so on. Using formulas (2) to (5), the reader has all the equipment needed to construct master meson pole contributions to other processes.

Further progress necessitates knowledge about the derivatives $\partial^{r+r'}C_N/\partial q^r\partial q'^{r'}$. Once these have been found (in Sec. 3) in the form $\sum_i \mathcal{K}_i(q,q')C_{N+R_i}{}^{(r+r')}$, the Reggeization prescription can be straightforwardly applied; it consists in making the replacement

$$\frac{g_N{}^{(\pm)}g_N{}'^{(\pm)}C_{N+R}{}^{(r+r')}}{t-M_N{}^2}\to\frac{\gamma^{(\pm)}(\alpha,t)\gamma'^{(\pm)}(\alpha,t)(1+e^{i\pi\alpha})C_{\alpha+R}{}^{(r+r')}}{\sin\pi_a} \tag{12}$$

in the usual way to the invariant-amplitude components appearing in the $M$ function. Detailed examples are provided in Secs. 4 and 5, where the crucial question of $U(6)\otimes U(6)$ symmetry breaking due to physical mass shifts is also discussed as well as the problem of $\sqrt{t}$ singularities as $t\to 0$.

### 3. GENERALIZED $U(6)\otimes U(6)$ REPRESENTATION FUNCTIONS

This section is devoted to the invariant expansion of the derivatives $\partial^{r+r'}C_N/(\partial q_B{}^A)^r\cdot(\partial q'_{A'}{}^{B'})^{r'}\cdot$, but before we tackle this problem, a few remarks about the relationship between these derivative functions and the functions $d_{WW'}{}^N(\theta)$ of the canonical approach are in order.[5] The main thing to grasp is that a one-one correspondence exists between the two formulations, $M$-function and canonical. Thus there is a direct connection between couplings (2) to (5) and the $U(6)_W$ symmetric vertices (Clebsch-Gordan coefficients); suppose for instance that we have incoming nucleon $(56,1)_{\frac{1}{2}p-q}$ and an incoming antinucleon $(1,\overline{56})_{\frac{1}{2}p+q}$, as is well known, these provide the $U(6)_W$ representations **1**, **35**, **405**, and **2695** which couple to the self-same components contained in the $(W_N,\overline{W}_N)$-exchanged meson, and the effective Lagrangian $\mathcal{L}_3$ demonstrates just this fact, for $g_0$ relates to **1**, $g_1$ to $\mathbf{1}\oplus\mathbf{35}$, $g_2$ to $\mathbf{1}\oplus\mathbf{35}\oplus\mathbf{405}$, and $g_3$ to $\mathbf{1}\oplus\mathbf{35}\oplus\mathbf{405}\oplus\mathbf{2695}$, so that the total $\mathbf{1}_W$ residue is given by a linear combination of $g_0$, $g_1$, $g_2$, and $g_3$ whereas the $\mathbf{2695}_W$ residue arises from $g_3$ alone.

As we already know, supersinglet scattering provides the function $C_N{}^3(\cos\theta)$ and since $\mathbf{1}_W$ representations can ever appear at the vertices, we immediately deduce that this is proportional to $d_{w=1,\,w'=1}{}^N(\theta)$ in the canonical representation, the analog of $P_J(\cos\theta)$ of the rotation group. On the other hand, more complicated functions like $d_{35,35}{}^N(\theta)$ can be recovered from the $g_1{}^2$ terms in Eq. (10) by taking appropriate projections of $\partial^2C_N/\partial q\partial q'$ as we shall illustrate below. In any case, regardless of whether or not we establish connections between the derivitive functions and the $d_{WW'}{}^N(\theta)$, expressions like (10) and (11) give the entire answer for the pole-dominated scattering amplitudes. They have the added advantage over the canonical forms that crossing properties are trivially stated and that symmetry-breaking prescriptions are more readily formulated in $M$-function terms.

Returning to the expansion problem itself, we must first examine the detailed representation for the basic function[1] in $U(\nu)\otimes U(\nu)$ theory ($\nu=6$ in our case):

$$C_N{}^{\frac{1}{2}\nu}(\hat{\mathbf{q}}\cdot\hat{\mathbf{q}}')=\Big(\sum_{B\text{ perms}}\hat{q}_{A_1}{}^{B_1}\cdots\hat{q}_{A_N}{}^{B_N}\Big)\times\Gamma_{-q'B_1}{}^{A_1}\cdots\Gamma_{-q'B_N}{}^{A_N}/N!,\quad(13)$$

where

$$\Gamma_{\mp k}\equiv(M\mp p)\hat{k}(M\pm p)/4M^2=\Gamma_\mp\hat{k}\Gamma_\pm\quad(14)$$

and $\hat{k}$ is to be understood as the unit four-vector $k/|\mathbf{k}|$ so that between $\Gamma_-$ and $\Gamma_+$, $\hat{q}_A{}^B$ reduces to $\hat{\mathbf{q}}\cdot(\boldsymbol{\gamma})_A{}^B$ in the rest frame ($\mathbf{p}=0$). We proceed to build up the derivatives $\partial^{r+r'}C_N/\partial q^r\partial q'^{r'}$ step by step.

[5] R. Delbourgo, J. Math. Phys. (to be published).

With a single derivative one first establishes by inspection that $\partial C_N/\partial\hat{q}$ must reduce to the linear combination $\mathcal{C}_1\Gamma_{-q'}+\mathcal{C}_2\Gamma_{-q}$, where $\mathcal{C}_1$ and $\mathcal{C}_2$ are invariant functions of $\hat{\mathbf{q}}\cdot\hat{\mathbf{q}}'$ which can be determined from the pair of relations,

$$\hat{q}_A{}^B\partial C_N/\partial\hat{q}_A{}^B=NC_N,\quad \hat{q}'_A{}^B\partial C_N/\partial\hat{q}_A{}^B=(N+\nu-1)C_{N-1}.$$

A simple calculation gives $\mathcal{C}_1$ and $\mathcal{C}_2$ and the formula

$$\nu\partial C_N/\partial\hat{q}_A{}^B=\Gamma_{-q'}C_N'-\Gamma_{-q'}C_{N-1}'.\quad(15a)$$

Obviously also

$$\nu\partial C_N/\partial\hat{q}'_A{}^B=\Gamma_{+q}C_N'-\Gamma_{+q'}C_{N-1}'.\quad(15b)$$

We are now in a position to determine the double-derivative expansions. Again the complexion of kinematic terms arising in $\partial^2C_N/\partial\hat{q}_A{}^B\partial\hat{q}_C{}^D$ is obtained by inspection to be a linear combination of the three terms

$$(\Gamma_{-q'B}{}^A\Gamma_{-q'D}{}^C+\Gamma_{-q'D}{}^A\Gamma_{-q'B}{}^C),$$
$$(\Gamma_{-qB}{}^A\Gamma_{-q'D}{}^C+\Gamma_{-qD}{}^A\Gamma_{-q'B}{}^C+\Gamma_{-q'B}{}^A\Gamma_{-qD}{}^C+\Gamma_{-q'D}{}^A\Gamma_{-qB}{}^C),$$
$$(\Gamma_{-qB}{}^A\Gamma_{-qD}{}^C+\Gamma_{-qD}{}^A\Gamma_{-qB}{}^C),$$

and, in the same way as before, the invariant functions which they multiply can be deduced from the pair of equations

$$\hat{q}_C{}^D\frac{\partial^2C_N}{\partial\hat{q}_A{}^B\partial\hat{q}_C{}^D}=(N-1)\frac{\partial C_N}{\partial\hat{q}_A{}^B},$$

$$\hat{q}'_D{}^C\frac{\partial^2C_N}{\partial\hat{q}_A{}^B\partial\hat{q}_C{}^D}=(N+\nu-1)\frac{\partial C_{N-1}}{\partial\hat{q}_A{}^B}.$$

Thus a straightforward computation gives

$$\nu(\nu+2)\frac{\partial^2C_N}{\partial\hat{q}_A{}^B\partial\hat{q}_C{}^D}=\sum_{\text{distinct perm}}\big[\Gamma_{-q'B}{}^A\Gamma_{-q'D}{}^CC_N''-\Gamma_{-qB}{}^A\Gamma_{-q'D}{}^CC_{N-1}''+\Gamma_{-qB}{}^A\Gamma_{-qD}{}^CC_{N-2}''\big].\quad(16)$$

The mixed derivative $\partial^2C_N/\partial\hat{q}_A{}^B\partial\hat{q}'_{B'}{}^{A'}$ is more complicated to handle because of the greater complexity of kinematic terms, namely the six terms,

$$\Gamma_{+A'}{}^A\Gamma_{-B}{}^{B'},\quad(\Gamma_{+q'qA'}{}^A\Gamma_{-B}{}^{B'}+\Gamma_{+A'}{}^A\Gamma_{-qq'B}{}^{B'}),$$
$$\Gamma_{+q'qA'}{}^A\Gamma_{-qq'B}{}^{B'},\quad\Gamma_{-q'B}{}^A\Gamma_{+qA'}{}^{B'},$$
$$(\Gamma_{-q'B}{}^A\Gamma_{+q'A'}{}^{B'}+\Gamma_{-qB}{}^A\Gamma_{+qA'}{}^{B'}),\quad\Gamma_{-qB}{}^A\Gamma_{+qA'}{}^{B'},$$

where

$$\Gamma_{\pm kk'}\equiv(M\pm p)\hat{k}(M\mp p)\hat{k}'(M\pm p)/8M^3.\quad(17)$$

The determination of the invariant functions which multiply these follows the previous steps and gives us the formula

$$\nu(\nu+2)\frac{\partial^2 C_N}{\partial\hat{q}_A{}^B\partial\hat{q}_{B'}{}^{A'}}=\Gamma_{+A'}{}^A\Gamma_{-B}{}^{B'}C_{N+1}''+(\Gamma_{-q'B}{}^A\Gamma_{+qA'}{}^{B'}$$

$$-\Gamma_{+qq'A'}{}^A\Gamma_{-B}{}^{B'}-\Gamma_{+A'}{}^A\Gamma_{-qq'B}{}^{B'})C_N''$$

$$+(\Gamma_{+q'qA'}{}^A\Gamma_{-qq'B}{}^{B'}-\Gamma_{-q'B}{}^A\Gamma_{+q'A'}{}^{B'}$$

$$-\Gamma_{-qB}{}^A\Gamma_{+qA'}{}^{B'})C_{N-1}''+\Gamma_{-qB}{}^A\Gamma_{+q'A'}{}^{B'}C_{N-2}''. \quad (18)$$

The underlying structure and simplicity of the general expansion

$$\frac{\partial^{r+r'}C_N}{\partial\hat{q}_A{}^B\partial\hat{q}_C{}^D\cdots\partial\hat{q}_{B'}{}^{A'}\partial\hat{q}'_{D'}{}^{C'}\cdots};$$

$$=\sum_i \mathcal{K}_i(q,q')_{BD\cdots A'C'\cdots}{}^{AC\cdots B'D'\cdots} \quad (19)$$

should be apparent from Eqs. (16) and (18). We shall in fact now state the rules for writing down directly the $\mathcal{K}_i$ and $\mathcal{C}_i$. Their correctness can be checked explicitly case by case and is left to the reader.

A. The number of kinematic terms $\mathcal{K}_i$ in general equals the number of $U(3)\otimes U(3)$ components of $(W_r,\bar{W}_r)$ on one side which overlap with $(W_{r'},\bar{W}_{r'})$ on the other side, since $U(3)\otimes U(3)$ is the conserved coplanar subgroup. For example, that there are six kinematic terms occurring in $\partial^2C/\partial q\partial q'$ is evident from the reduction

$$(6,\bar{6})=(1,1)+(1,8)+(8,1)+(1,1)+(3,\bar{3})+(\bar{3},3).$$

B. The $2(r+r')$-indexed quantities

$$\mathcal{K}_{iBD\cdots A'C'\cdots}{}^{AC\cdots B'D'\cdots}$$

are built up from products of the eight types of tensor

$$\Gamma_{+A'}{}^A,\quad \Gamma_{+q'qA'}{}^A,\quad \Gamma_{-B}{}^{B'},\quad \Gamma_{-q'qB}{}^{B'},$$
$$\Gamma_{-qB}{}^A,\quad \Gamma_{-q'B}{}^A,\quad \Gamma_{+qA'}{}^{B'},\quad \Gamma_{+q'A'}{}^{B'},$$

with distinct symmetric permutations taken over indices of like type ($AB\cdots$, $CD$, $\cdots$ and, separately, $A'B'\cdots$, $C'D'\cdots$). Observe carefully the crisscrossing of indices. This is most important for understanding what follows.

C. The function $\mathcal{C}_i(\cos\theta)$ which a given $\mathcal{K}_i$ multiplies equals $\pm C_{N+R_i}{}^{(r+r')}(\cos\theta)$, where $R_i$ is given by the complexion of the $\mathcal{K}_i$. Indeed to see how the character of $\mathcal{K}_i$ determines $R_i$, take $r\geqslant r'$ for definiteness. Then,

$R_1=r$ when $\mathcal{K}_1\sim(\Gamma_+)^{r'}(\Gamma_-)^{r'}(\Gamma_{+q'})^{r-r'}$, $R_2=R_3=r-1$ when

$$\mathcal{K}_2\sim[(\Gamma_+)^{r'}(\Gamma_-)^{r'-1}\Gamma_{-q'q}+(\Gamma_+)^{r'-1}(\Gamma_-)^{r}\Gamma_{+qq'}](\Gamma_{+q'})^{r-r'},$$

and $\mathcal{K}_3\sim(\Gamma_+)^{r'}(\Gamma_-)^{r'}(\Gamma_{+q'})^{r-r'-1}\Gamma_{+q}$,

and so on down to

$$R_f=-r-r'\quad\text{when}\quad \mathcal{K}_f\sim(\Gamma_{-q})^r(\Gamma_{+q'})^{r'}.$$

Finally the $\pm$ sign in front of $C_{N+R}$ is fixed by the number of times that $\Gamma_{+q'q}$, $\Gamma_{-q}$, and $\Gamma_{+q'}$ appear; it is $+$ when they occur in even numbers, $-$ when they occur in odd numbers.

Armed with these rules the reader may construct the $M$-function forms of the $d_{WW'}{}^N(\theta)$ for the degenerate class of $U(\nu)\otimes U(\nu)$ representations. In practice the only physically important ones involve three derivatives of each momentum at most. We have already given the explicit formulas for $\partial^2C/\partial q^2$ and $\partial^2C/\partial q\partial q'$ so the only other ones needed are $\partial^4C/(\partial q)^2(\partial q')^2$, $\partial^4C/(\partial q)^2(\partial q')^2$, and $\partial^6C/(\partial q)^3(\partial q')^3$ and these are straightforwardly obtained from the rules. Although they are certainly required in the general analysis of hadron scattering, we shall not make use of them in the following sections.

To close this section, it may be worthwhile to derive the precise connections between the $M$ function and the canonical $d(\theta)$. Specifically, suppose we require $d_{WW'}{}^N(\theta)$. Under $U(3)\otimes U(3)$, $\mathbf{35}_W$ decomposes as

$$\mathbf{35}=(1,1)+(8,1)+(1,8)+(3,\bar{3})+(\bar{3},3).$$

In $M$-function language this is translated into a statement about transformation properties of various $(\gamma T)$ matrices of the **143** representation of $U(6,6)$. Thus,

| | | |
|---|---|---|
| $\gamma_3T^i$, $\gamma_1T^i$ | belong to $(1,8)\oplus(8,1)$ | of $\mathbf{35}_W$, |
| $\gamma_1$ | belongs to $(1,1)$ | of $\mathbf{35}_W$, |
| $\gamma_2T^i$, $\gamma_2$, $\gamma_5T^i$ and $\gamma_5$ | belong to $(3,\bar{3})+(\bar{3},3)$ | of $\mathbf{35}_W$, |

while

| | | |
|---|---|---|
| $\gamma_3$ | belongs to $(1,1)$ | of $\mathbf{1}_W$. |

Correspondingly we can project out each conserved $U(3)\otimes U(3)$ piece by appropriate tracing with these matrices. In the simplest instance suppose we are interested in the $U(3)\otimes U(3)$ singlet piece of $\mathbf{35}_W$; the answer comes out easily as $(\gamma_1)_B{}^A(\gamma_1)_{A'}{}^{B'}\partial^2C/_B{}^A\partial\hat{q}'_{A'}{}^{B'}$ in the frame where $p$ is at rest and $q$, $q'$ are confined to the $XOZ$ plane. Hence from (18),

$$d_{35(1,1)35}{}^N(\theta)\propto C_{N+1}''-2\cos\theta\, C_N''-\cos^2\theta\, C_{N-1}''. \quad (20)$$

Other more complicated $d_{WW'}{}^N(\theta)$ are derivable from other tracings over the covariant $d(\theta)$.

## 4. QUARK-QUARK SCATTERING

Although unphysical, quark-quark scattering is the simplest nontrivial case which exhibits all the ramifications of the Reggeization scheme and for this reason it is a useful example for acquainting the reader with the techniques presented so far. In the $U(6)\otimes U(6)$ symmetry limit the amplitude for the reaction mediated by $(W_N,\bar{W}_N)$ exchange has already been written as formula (10). Nevertheless, by itself (10) cannot satisfy the (Fermi?) statistics of quarks; in order for the amplitude to do so one must add the pole contribution with quarks ($A'$ and $B$) interchanged. Since, however, one is interested in near forward high-energy scattering one may expect the inclusion of the crossed amplitude to be insignificant (it only serves to relate the forward to the backward amplitude) so we shall carry on using formula (10). It receives contributions from both $N$ signatures in $\mathbf{1}_W$ and $\mathbf{35}_W$ exchanges. As it happens, a split into the two signatures before Reggeization need not be carried out because it is a dynamical accident that even- and odd-signature trajectories approximately coincide, "meson exchange degeneracy."[6] If we are dealing with charge-exchange amplitudes in the $SU(3)$ generalized sense, the $\mathbf{1}_W$ contributions disappear ($g_0$ couplings are irrelevant); on the other hand, for elastic processes where $\mathbf{1}_W$ and $\mathbf{35}_W$ are important we may anticipate that even these are insufficient for a correct description of high-energy scattering and that they must be supplemented with the (fixed pole?) Pomeranchuk contribution.

By substituting relations (15a), (15b), and (18) into (10), we obtain the explicit form of the quark-quark scattering amplitude, which may be further simplified by making full use of the Bargmann-Wigner equations satisfied by the external spinors. We shall not write down the full expression as it does not teach us very much, but we shall rather focus our attention of the particular piece of it which describes the charge-exchange processes. The leading $\cos\theta_t$ ($\approx s/2m^2$) dependence is simply given by

$$T_{\text{c.e.}}=\frac{(g_1/2M)^2}{t-M^2}\bar{u}'(M+p)u \times\bar{u}(M-p)u'\left(\frac{|\mathbf{q}|\,|\mathbf{q}'|}{m^2}\right)^N C_{N+1}'' \quad (21)$$

at the elementary-particle pole; after Reggeization this assumes the form

$$T_{\text{c.e.}}=\frac{\gamma^2(t)}{\sin\pi\alpha(t)}\left(\frac{|\mathbf{q}|\,|\mathbf{q}'|}{m^2}\right)^\alpha C_{\alpha+1}'' \times\bar{u}'(\sqrt{t}+p)u\bar{u}(\sqrt{t}-p)u \quad (22)$$

predicting that

$$T_{\mathcal{P}\mathfrak{N}\to\mathfrak{N}\mathcal{P}}=T_{\mathcal{P}\lambda\to\lambda\mathcal{P}}=T_{\mathfrak{N}\lambda\to\lambda\mathfrak{N}}=\frac{\beta(\alpha+1)\alpha}{\sin\pi\alpha}\left(\frac{s}{2m^2}\right)^\alpha, \quad (23)$$

where $\alpha(t)$ ($\approx 0.5+t$ in units of GeV/$c^2$) is the master meson trajectory function.

[6] This has been exploited by R. C. Arnold [Phys. Rev. **162**, 1334 (1967)] in the $l$-Reggeized version of $(6,\bar{6})_l$ for the orbital-excitation theory.

All the above is in the limit of exact $U(6)\otimes U(6)$. To make contact with experiment (hypothetical though this case is) it is absolutely vital to take account of symmetry-breaking effects. If we draw the lessons of supermultiplet theory in the physical-decay region[2] it would seem that the most crucial role of $U(6)\otimes U(6)$ breaking is in shifting masses, leaving invariant coupling constants relatively unaffected. We shall apply this conclusion to the unphysical (Regge pole) region in $t$ by assuming that the trajectory functions are changed from the symmetry to the physical values but that residue relations are essentially unaltered. If this were not so, all vestige of the symmetry would be lost and there would be no point in Reggiezing supermultiplet theories in the first place.

These considerations can be taken over very simply to the leading contributions to the charge-exchange amplitudes which come from $\rho$ and $K^*$ exchange. If these vector-meson trajectories are [$SU(3)$] split to their physical values, predictions (23) are modified to

$$T_{\mathcal{P}\mathfrak{N}\to\mathfrak{N}\mathcal{P}}=\frac{\beta(\alpha_\rho+1)\alpha_\rho}{\sin\pi\alpha_\rho}\left(\frac{s}{2m_{\mathcal{P}}^2}\right)^{\alpha_\rho},$$

$$T_{\mathcal{P}\lambda\to\lambda\mathcal{P}}=T_{\mathfrak{N}\lambda\to\lambda\mathfrak{N}}=\frac{\beta(\alpha_{K^*}+1)\alpha_{K^*}}{\sin\pi\alpha_{K^*}}\left(\frac{s}{2m_{\mathcal{P}}m_\lambda}\right)^{\alpha_{K^*}}. \quad (24)$$

Of course this is not the end of the story, as apart from $SU(3)$ breaking, there is the $U(2)\otimes U(2)$ spin splitting which alters the $\rho$–$\pi$ trajectory spacing from the symmetry value of 1 to the physical value of about 0.5, i.e., $\alpha\to\alpha_V$ for the vector projection and $\alpha-1\to\alpha_P$ for the pseudoscalar projection. (But this is to lower order in $s$.) Algebraically the split is carried out via the decomposition

$$(M+p)_\alpha{}^{\alpha'}(M-p)_{\beta'}{}^{\beta}=V_{\alpha\beta'}{}^{\alpha'\beta}+P_{\alpha\beta'}{}^{\alpha'\beta},$$

where

$$P_{\alpha\beta'}{}^{\alpha'\beta}=\tfrac{1}{2}[(M+p)\gamma_5]_\alpha{}^\beta[(M-p)\gamma_5]_{\beta'}{}^{\alpha'}.$$

General $SU(3)$ mass splittings can likewise be incorporated by performing the appropriate projections and substitutions.

## 5. MESON-BARYON CHARGE-EXCHANGE SCATTERING

We finally make contact with physics by considering meson-baryon scattering. It is described in its entirety (including production processes of vector mesons and decuplet resonances) by expression (11) of Sec. 2. Let us concentrate on those pieces which describe charge-exchange processes, as it is for these amplitudes that the Regge-pole predictions have been most spectacular (and to which the Regge revival is due!). Now most charge-exchange processes are dominated by vector and tensor meson trajectory exchanges. In our terms these would be given by the negative $N$-signature piece (which comprises vector-meson exchange) which arises from the $h_1^{(-)}$ coupling in (11). Retaining only the $\mathbf{35}_W$-exchange contributions,[7] the charge-exchange processes will therefore be described by the amplitude

$$T_{\text{c.e.}}=\frac{g_1h_1^{(-)}\mu m}{p^2-M^2}\{\Phi(-\tfrac{1}{2}p+q'),\Phi(-\tfrac{1}{2}p-q')\}_{B'}{}^{A'}\frac{\partial}{\partial q'_{B'}{}^{A'}}$$
$$\times\bar{u}^{BCD}(-\tfrac{1}{2}p+q)u_{ACD}(-\tfrac{1}{2}p-q)$$
$$\times\frac{\partial}{\partial q_A{}^B}\left(\frac{|\mathbf{q}||\mathbf{q}'|}{m\mu}\right)^N C_N(\hat{\mathbf{q}}\cdot\hat{\mathbf{q}}'). \quad (25)$$

Upon substituting expansion formula (18) into (25), we have

$$T_{\text{c.e.}}=\frac{q_1h_1^{(-)}}{p^2-M^2}\{\Phi(-\tfrac{1}{2}p+q'),\Phi(-\tfrac{1}{2}p-q')\}_{B'}{}^{A'}$$
$$\times\bar{u}^{BCD}(-\tfrac{1}{2}p+q)u_{ACD}(-\tfrac{1}{2}p-q)\left(\frac{|\mathbf{q}||\mathbf{q}'|}{m\mu}\right)^{N-1}$$
$$\times[\Gamma_{+A'}{}^A\Gamma_{-B}{}^{B'}C_{N+1}''+\Gamma_{+q'qA'}{}^A\Gamma_{-qq'B}{}^{B'}C_{N-1}''$$
$$-(\Gamma_{+q'qA'}{}^A\Gamma_{-B}{}^{B'}+\Gamma_{+A'}{}^A\Gamma_{-qq'B}{}^{B'})C_N''],$$

whose leading behavior is

$$T_{\text{c.e.}}=\frac{g_1h_1^{(-)}}{p^2-M^2}\bar{u}^{BCD}(\Gamma_-\{\Phi,\Phi\}\Gamma_+)_B{}^A$$
$$\times u_{ACD}\left(\frac{|\mathbf{q}||\mathbf{q}'|}{m\mu}\right)^{N-1}C_{N+1}''. \quad (26)$$

Even at this stage (26) describes a host of charge-exchange processes, reactions like $\pi^+N^{*+}\to\pi^0N^{*++}$ and $\rho^0p\to\rho^+n$. We restrict our attention still further by focussing on $0^-\frac{1}{2}^+$ reactions mediated by $\mathbf{35}_W$-exchange Lairing $C=-1$. A little Dirac algebra simplifies (26) to

$$T_{PN'\to P'N}=\frac{g_1h_1^{(-)}}{p^2-M^2}\left(1+\frac{M}{2\mu}\right)\left(\frac{|\mathbf{q}||\mathbf{q}'|}{m\mu}\right)^{N-1}C_{N+1}''$$
$$\times(PP')_F\frac{iq_{\nu'}}{\mu}\bar{u}^{BCD}\left[\left(1+\frac{p'}{M}\right)\gamma_\nu T^i\right]_B^A u_{ACD}. \quad (27)$$

If we use the standard results of the $U(6,6)$ currents[2] $\bar{u}(\gamma T)u$, the amplitude can be reduced to the conventional form

$$T_{PN'\to NP'}=(PP')_F[A(\bar{N}N')_F+A_D(\bar{N}N')_D$$
$$+1/\mu\{B_F(\bar{N}\mathbf{q}'N')_F+B_D(\bar{N}\mathbf{q}'N)_D\}] \quad (28)$$

with $F$ and $D$ having the usual $SU(3)$ cannotations. The invariant amplitudes are found to be $[\epsilon\equiv(s-u)/4m\mu]$

$$A_D=-\frac{g_1h_1^{(-)}}{p^2-M^2}\left(1+\frac{M}{2\mu}\right)\left(\frac{|\mathbf{q}||\mathbf{q}'|}{m\mu}\right)^{\alpha-1}$$
$$\times\left(1+\frac{2m}{M}\right)(N+1)N\epsilon^N,$$

$$A_F=\left(-\frac{M}{2m}+\frac{2}{3}\right)A_D, \quad (29a)$$

$$B_D=\tfrac{3}{2}B_F=\frac{g_1h_1^{(-)}}{p^2-M^2}\left(1+\frac{M}{2\mu}\right)\left(1+\frac{2m}{M}\right)$$
$$\times\left(\frac{|\mathbf{q}||\mathbf{q}'|}{m\mu}\right)^{N-1}\left(1-\frac{M^2}{4m^2}\right)(N+1)N\epsilon^{N-1}. \quad (29b)$$

Hence the non-flip amplitude which contributes to the forward scattering cross section is (at the pole)

$$A'=A+\epsilon B/(1-t/4m^2),$$
$$=(PP')_F(\bar{N}N')_F\frac{g_1h_1^{(-)}}{p^2-M^2}\left(1+\frac{M}{2\mu}\right)\left(1+\frac{M}{2m}\right)$$
$$\times(N+1)N\epsilon^N, \quad (30)$$

i.e., *a product of charge F couplings at either vertex.* After Reggeization we should predict that the forward scattering amplitude is given by

$$T_{PN'\to NP'}(\theta=0)=[\beta(1-e^{-i\pi\alpha})/\sin\pi\alpha]$$
$$\times(PP')_F(\bar{N}N)_F(\alpha+1)\alpha(s/2m\mu)^\alpha, \quad (31)$$

which is the usual answer when one has vector-meson trajectory exchange.

Thus the perfect symmetry prediction ensuing from (31) would be typically that in the forward direction

$$T_{\pi^-p\to\pi^0n}=-2T_{\pi^-p\to K^0\Sigma^0},$$
$$=\frac{\beta(1-e^{-i\pi\alpha})}{\sin\pi\alpha}(\alpha+1)\alpha\epsilon^\alpha. \quad (32)$$

On the other hand, the mass-breaking prescriptions presented previously would modify (32) to

$$T_{\pi^-p\to\pi^0n}=\frac{2\beta(1-e^{-i\pi\alpha_\rho})}{\sin\pi\alpha_\rho}(\alpha_\rho+1)\alpha_\rho\left(\frac{s}{2m_Nm_\pi}\right)^{\alpha_\rho},$$

$$T_{\pi^-p\to K^0\Sigma^0}\approx-\frac{\beta(1-e^{-i\pi\alpha_{K^*}})}{\sin\pi\alpha_{K^*}}(\alpha_{K^*}+1)\alpha_{K^*}$$
$$\times\left(\frac{s}{m_N(m_\pi+m_K)}\right)^{\alpha_{K^*}},$$

and so on.

[7] In the canonical formulation we are retaining the $d_{35,35}{}^N(\theta)$ piece of the amplitude.

There are obviously a host of other consequences that we could tabulate concerning resonance production, etc. At this stage, however, one must mention a difficulty common to all Regge approaches; the problem of singularities at $t=0$. This concerns the Mandelstam analyticity of the invariant amplitudes after Reggeization, a problem which is common to all Reggeization procedures where high external spins are involved.

The difficulty has to do with the $M=\sqrt{t}$ singularities. The present formalism inherits these difficulties; to see them here note that in the Feynman pole, numerators that enter basically as positive and negative parity projections of $U(2)\otimes U(2)$

$$\tfrac{1}{2}(1\pm p/M)\delta(p^2-M^2)$$

and which produce typical coupling factors $(1+2m/M)$, etc., after contraction over external spin wave functions; these factors persist after Reggeization of the particular $U(2)\otimes U(2)$ multiplet. For instance even if $A_D$ in (29a) has the correct analyticity properties, $A_F$ does not, owing to the presence of the glaring $(M/2m)$ factor in the residue at the pole. One way to circumvent this trouble is the conspiracy theory which doubles the number of bosons in analogy with the Gribov doubling mechanism for fermion exchange. For instance let us introduce a $(\bar{6},6)$ partner to the original $(6,\bar{6})$ multiplet. We should thereby supplement

$$T=(gh/p^2-M^2)\bar{u}(1+\boldsymbol{p}/M)\boldsymbol{q}'uC_{N+1}'' \qquad (27')$$

with

$$T'=(g'h'/p^2-M'^2)\bar{u}(1-\boldsymbol{p}/M)\boldsymbol{q}'uC_{N'+1}'', \qquad (28')$$

giving the total Reggeized amplitude

$$T=\frac{\beta s^{\alpha-1}}{\sin\pi\alpha}\bar{u}\left(1+\frac{p}{\sqrt{t}}\right)q'u+\frac{\beta' s^{\alpha'-1}}{\sin\pi\alpha'}\bar{u}\left(1-\frac{p}{\sqrt{t}}\right)\boldsymbol{q}'u.$$

Providing then that we choose $\beta(0)=\beta'(0)$ and $\alpha(0)=\alpha'(0)$, the $\sqrt{t}\to 0$ singularity cancels and all is well. The fact that $(6,\bar{6})$ calls for new multiplets at $p\to 0$ should not come as a surprise in view of the expected $U(6,6)$ symmetry in that limit. We believe that a correct treatment of these difficulties will come by Reggeization of $U(6,6)$ rather than $U(6)\times U(6)$ in the manner of Toller. For non-spin-flip amplitudes (which correspond in the present formalism to $W$-spin-conserving amplitudes), the problem is trivial since the answer is essentially obtained by replacing $C_N{}^3$ with $C_N{}^{11/2}$. For spin-flip amplitudes the problem is much more complicated and is under further study at present. Assuming these difficulties can be satisfactorily overcome, we see no further snags to the Reggeization scheme.

## ACKNOWLEDGMENTS

The authors wish to thank Professor Abdus Salam and the members of the theoretical group at Imperial College. They also wish to thank the IAEA for hospitality at the International Centre for Theoretical Physics, Trieste.