ing to contractions, there is also a leading curve corresponding to the uncontracted diagram. The curve bears a relationship to the two-Reggeon branch points similar to that of an anomalous threshold to normal thresholds. In particular, it touches the two-Reggeon branch points.

The asymptotic behavior of the production amplitude turns out to be controlled by the leading curve. It has the form $T\sim s^J/\ln s$, where the exponent J depends on $q_3'^2$ as well as on $q_1'^2$ and $q_2'^2$. This is quite different from the type of behavior that emerges from double-Reggeon exchange and has no analog in two-body scattering. The identification of such behavior experimentally would be an important support for the relevance of a Reggeon calculus.

Finally, it was noted that a necessary condition for the appropriateness of the definition adopted in this paper for the multi-partial-wave amplitude is the asymptotic simplicity of the analytic structure of the production amplitude. Such simplicity does seem to emerge from Gribov's analysis applied to production amplitudes.

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Two-Boson-Exchange Effects in Nucleon-Nucleon Scattering

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The two-pion-exchange contribution to nucleon-nucleon scattering is studied at scattering energies of 95 and 310 MeV through a partial-wave analysis of the exact relativistic scattering matrix. Additional two-boson-exchange effects are also studied at these energies corresponding to the $\pi+\eta$ and $\pi+\sigma$ exchange processes, with η the pseudoscalar resonance and σ the scalar resonance. It is found that the two-pion-exchange (TPE) phase parameters are large compared with one-pion-exchange (OPE) phase parameters for low values of L, and OPE+TPE is a reasonable representation of the phenomenological phases for the lower energy when $L\geqslant 3$ and for the higher energy when $L\geqslant 5$. The $\pi+\eta$ effect is found to be small compared with the pion-theoretical effects, but the $\pi+\sigma$ effect is large for a light scalar resonance if it couples strongly with the nucleon.

1. INTRODUCTION

THE description of the nucleon-nucleon interaction by resonance models consists of some compensation for the core of the interaction and contributions from the virtual exchange of a single π meson, an η pseudoscalar resonance, the ω and ρ vector resonances, and scalar resonances.[1] Despite the fact that the two-pion-exchange (TPE) mass is less than any established resonance mass, resonance models either exclude the TPE effect or simulate it by some approximation—perhaps by the introduction of scalar resonances whose existences have not yet been conclusively established experimentally. Moreover, the effect of the virtual exchange of a pion and η resonance together has not been considered even though the mass exchanged is less than a single vector resonance mass. Finally, if a light scalar resonance σ is used in a resonance model, the $\pi+\sigma$ effect should also be considered. It would therefore seem of interest to evaluate the TPE, $\pi+\eta$, and $\pi+\sigma$ contributions to nucleon-nucleon scattering.

An exact determination of the relativistic scattering operator for nucleon-nucleon scattering due to two-pion exchange by Gupta has been available since 1960.[2] It includes the total contribution of the pion-nucleon pseudoscalar interaction through the fourth order in the pion-nucleon coupling constant. A nonrelativistic approximation is also presented from which a potential is derived, and by using this potential Breit *et al.*[3] obtained two-pion-exchange phase parameters.

In a later work, Gupta, Haracz, and Kaskas obtained the relativistic scattering matrix corresponding to the TPE scattering operator and evaluated it at nucleon

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[1] R. A. Bryan and G. L. Scott, Phys. Rev. **135**, B434 (1964); A. E. S. Green and R. D. Sharma, Phys. Rev. Letters **14**, 390 (1965); A. Scotti and D. Y. Wong, Phys. Rev. **138**, B145 (1965); J. S. Ball, A. Scotti, and D. Y. Wong, *ibid.* **142**, 1000 (1966); R. A. Bryan and R. A. Arndt, *ibid.* **150**, 1299 (1966); R. A. Arndt, R. A. Bryan, and M. H. MacGregor, *ibid.* **152**, 1490, (1966); A. E. S. Green, T. Sawada, and R. D. Sharma, *Isobaric Spin In Nuclear Physics* (Academic Press Inc., New York, 1966); R. A. Bryan and B. L. Scott, Phys. Rev. **164**, 1215 (1967); R. D. Sharma and A. E. S. Green, Nucl. Phys. **B3**, 33 (1967). These references may be consulted for additional work on resonance models.

[2] S. N. Gupta, Phys. Rev. **117**, 1146 (1960).

[3] G. Breit, K. E. Lassila, H. M. Ruppel, and M. H. Hull, Jr., Phys. Rev. Letters **6**, 138 (1961).

scattering energies of 95 and 310 MeV.[4] The $\pi+\eta$ and $\pi+\sigma$ scattering matrices were derived and evaluated at these same scattering energies by Barker, Gupta, and Haracz.[5,6] These relativistic scattering matrices are further studied here. To avoid the core effects of the nucleon-nucleon interaction, a partial-wave analysis is made and the S waves excluded. This is done by obtaining phase parameters from a field-theoretical quantity called the W matrix.

2. RELATIONSHIP BETWEEN THE W MATRIX AND PHASE PARAMETERS

The field-theoretical scattering operator S can be expanded in powers of the boson-nucleon coupling constants so that the expansion is unitary to all orders by expressing S in terms of a Hermitian operator K as[7]

$$S=(1-\tfrac{1}{2}iK)/(1+\tfrac{1}{2}iK). \tag{1}$$

The operator K is expanded as

$$K=\sum_{n=1}^{\infty} K(n), \tag{2}$$

where $K(n)$ contains coupling constants to the order n. Thus, if the series (2) is terminated and substituted into (1), a unitary approximation to S is obtained. If the incident nucleons have propagation four-vectors p and q and the scattered nucleons p' and q', respectively, the W matrix in the center-of-mass (c.m.) system, where $\mathbf{q}=-\mathbf{p}$, $\mathbf{q}'=-\mathbf{p}'$, and $p_0=q_0=p_0'=q_0'$, is defined by

$$\begin{aligned} W&=\sum_{n=1}^{\infty} W(n),\\ K(n)&=(1/c\hbar)(2\pi)^4\delta(p-p'+q-q')\\ &\quad :[\psi_L^{-*}(\mathbf{p}')\psi_L^{-*}(\mathbf{q}')W(n)\psi_L^{+}(\mathbf{q})\psi_L^{+}(\mathbf{p})]:. \end{aligned} \tag{3}$$

The real matrix $W(n)$ is a function of the scattering energy, scattering angle, and contains coupling constants to the order n. The ψ_L are large-component Pauli spinors.[8]

Phase parameters are usually defined in terms of the M matrix[9] which is related to a direct expansion of the scattering operator S in powers of coupling constants. Thus, if S and M are expanded as

$$S=\sum_{n=1}^{\infty} S(n),\quad M=\sum_{n=1}^{\infty} M(n), \tag{4}$$

$M(n)$ is related to $S(n)$. However, if these expansions are terminated, the resulting approximation to S is not unitary. Since effects through the fourth order in boson-nucleon coupling constants are the main concern of this work, it would seem that a unitary approximation to the scattering operator from these effects might be more desirable than a nonunitary approximation, hence the W-matrix approach will be used. It may be of interest to note that the relationship between corresponding terms of W and M through the fourth order are

$$\begin{aligned} M(2)&=-(p_0/4\pi c\hbar)W(2),\\ \mathrm{Re}M(4)&=-(p_0/4\pi c\hbar)W(4), \end{aligned} \tag{5}$$

where "Re" means "the real part."

In accordance with the notation of the Yale group for phase parameters,[10] let the singlet and triplet phase parameters be related to the parameters α as

$$\begin{aligned} \alpha_L&=(1/2i)[\exp(2iK_L)-1],\\ \alpha_L{}^L&=(1/2i)[\exp(2i\delta^L{}_L)-1],\\ \alpha_J{}^L&=(1/2i)\{[1-\rho_J{}^2]^{1/2}[\exp(2i\theta^L{}_J)]-1\},\\ \alpha_J&=\tfrac{1}{2}\rho_J\exp[i(\theta^{J-1}{}_J+\theta^{J+1}{}_J)], \end{aligned} \tag{6}$$

where K_L is the singlet phase shift, $\delta^L{}_L$ the uncoupled triplet phase shift, $\theta^L{}_J$ the coupled phase shift, and ρ_J the coupling parameter. If the parameters α and the phase parameters are expanded in powers of the coupling constants as

$$\begin{aligned} \alpha=\sum_{n=1}^{\infty}\alpha(n),\quad \delta^L{}_L=\sum_{n=1}^{\infty}\delta^L{}_L(n),\quad \theta^L{}_J=\sum_{n=1}^{\infty}\theta^L{}_J(n),\\ \rho_J=\sum_{n=1}^{\infty}\rho_J(n), \end{aligned} \tag{7}$$

and terms of like power equated in Eq. (6), the result to the fourth order is

$$\begin{aligned} K_L(2)&=\alpha_L(2),\quad K_L(4)=\mathrm{Re}\alpha_L(4),\\ \delta_L{}^L(2)&=\alpha_L{}^L(2),\quad \delta_L{}^L(4)=\mathrm{Re}\alpha_L{}^L(4),\\ \theta^L{}_J(2)&=\alpha^L{}_J(2),\quad \theta^L{}_J(4)=\mathrm{Re}\alpha^L{}_J(4),\\ \rho_J(2)&=2\alpha_J(2),\quad \rho_J(4)=2\,\mathrm{Re}\alpha_J(4). \end{aligned} \tag{8}$$

The fourth-order corrections to the phase parameters are thus related to the real parts of $\alpha(4)$, and the $\mathrm{Re}\alpha(4)$ are in turn related to the spin matrix elements

[4] S. N. Gupta, R. D. Haracz, and J. Kaskas, Phys. Rev. **138**, B1500 (1965).

[5] B. M. Barker, S. N. Gupta, and R. D. Haracz, Phys. Rev. **142**, 1144 (1966).

[6] B. M. Barker, S. N. Gupta, and R. D. Haracz, Phys. Rev. **161**, 1411 (1967).

[7] See Ref. 2 for a discussion of the operator K and its relationship to the various orders of S and the effective-interaction operator W.

[8] For the relativistic reduction of matrix elements involving the ordered products of Dirac spinors to those involving Pauli spinors, see S. N. Gupta, Phys. Rev. **122**, 1923 (1961).

[9] The relationship between phase parameters and the M matrix is given by R. A. Bryan and R. A. Arndt, Phys. Rev. **150**, 1298 (1966).

[10] The Yale notation is similar to nuclear bar notation with $\delta_{J-1}=\theta^{J-1}{}_J$, $\bar{\delta}_{J+1}=\theta^{J+1}{}_J$, $\rho_J=\sin(2\bar{\epsilon}_J)$. See G. Breit and R. D. Haracz, in *High Energy Physics*, edited by E. H. S. Burhop (Academic Press Inc., New York, 1967), Vol. I, Chap. 2, p. 50.

of $W(4)$ by

$$\mathrm{Re}\alpha_L(4)=\beta_0{}^L(4),$$

$$\mathrm{Re}\alpha^L{}_L(4)=\frac{1}{2}\left[\frac{2}{L(L+1)}\beta_1{}^L(4)+\beta_2{}^L(4)-\frac{(L-1)(L+2)}{L(L+1)}\beta_3{}^L(4)\right],$$

$$\mathrm{Re}\alpha_{L-1}(4)=-\frac{1}{2L-1}\left(\frac{L-1}{L}\right)^{1/2}\left(\beta_1{}^L(4)+\tfrac{1}{2}L\beta_2{}^L(4)+\tfrac{1}{2}(L+2)\beta_3{}^L(4)+\beta_4{}^L(4)-L\beta_5{}^L(4)\right),$$

$$\mathrm{Re}\alpha^L{}_{L-1}(4)=\frac{1}{2L-1}\left(\frac{L-1}{L}\beta_1{}^L(4)+\tfrac{1}{2}(L-1)\beta_2{}^L(4)+\frac{(L-1)(L+2)}{2L}\beta_3{}^L(4)-\beta_4{}^L(4)+L\beta_5{}^L(4)\right), \quad (9)$$

$$\mathrm{Re}\alpha_{L+1}(4)=\frac{1}{2L+3}\left(\frac{L+2}{L+1}\right)^{1/2}\left(\beta_1{}^L(4)-\tfrac{1}{2}(L+1)\beta_2{}^L(4)-\tfrac{1}{2}(L-1)\beta_3{}^L(4)+\beta_4{}^L(4)+(L+1)\beta_5{}^L(4)\right),$$

and

$$\mathrm{Re}\alpha^L{}_{L+1}(4)=\frac{1}{2L+3}\left(-\frac{L+2}{L+1}\beta_1{}^L(4)+\tfrac{1}{2}(L+2)\beta_2{}^L(4)+\frac{(L-1)(L+2)}{2(L+1)}\beta_3{}^L(4)+\beta_4{}^L(4)+(L+1)\beta_5{}^L(4)\right),$$

where

$$\beta_0{}^L(4)=-\frac{p_0|\mathbf{p}|}{8\pi ch}\int_0^\pi {}^0W_{00}(4)P_L\sin\theta d\theta,$$

$$\beta_1{}^L(4)=\frac{p_0|\mathbf{p}|}{2^{1/2}(4\pi ch)}L\int_0^\pi W_{01}(4)(\cos\theta P_L-P_{L-1})d\theta,$$

$$\beta_2{}^L(4)=-\frac{p_0|\mathbf{p}|}{4\pi ch}\int_0^\pi W_{11}(4)P_L\sin\theta d\theta,$$

$$\beta_3{}^L(4)=-\frac{p_0|\mathbf{p}|}{4\pi ch}\frac{L}{(L-1)(L+2)}\int_0^\pi W_{-11}(4)\left(\frac{2\cos\theta}{\sin^2\theta}(P_{L-1}-\cos\theta\, P_L)-(L+1)P_L\right)\sin\theta\, d\theta, \quad (10)$$

$$\beta_4{}^L(4)=\frac{p_0|\mathbf{p}|}{2^{1/2}(4\pi ch)}L\int_0^\pi W_{10}(4)(\cos\theta\, P_L-P_{L-1})d\theta,$$

and

$$\beta_5{}^L(4)=-\frac{p_0|\mathbf{p}|}{8\pi ch}\int_0^\pi W_{00}(4)P_L\sin\theta d\theta.$$

In these relations, the matrix elements are defined by ${}^0W_{00}={}^1\chi_0{}^\dagger W\,'\chi_0$ and $W_{-11}={}^3\chi_{-1}{}^\dagger W\,{}^3\chi_1$, etc., where χ are the usual singlet and triplet spinors. Further, $P_L=P_L(\cos\theta)$ is the Legendre polynomial with θ the c.m. scattering angle.

Thus, the first two orders of the phase parameters are related to the first two orders of W, or by Eq. (5) to the real part of the first two orders of M.[11]

3. TWO-PION-EXCHANGE EFFECTS

The relativistic W matrix for nucleon-nucleon scattering, assuming a pion-nucleon coupling constant of 14, was presented by Gupta, Haracz, and Kaskas in terms of coefficients of W similar to those used for the M matrix by Wolfenstein,[12] namely,

$$W=BS+iC\sin\theta(\sigma_n{}^{(1)}+\sigma_n{}^{(2)})+\tfrac{1}{2}G(\sigma_l{}^{(1)}\sigma_l{}^{(2)}+\sigma_m{}^{(1)}\sigma_m{}^{(2)})T+\tfrac{1}{2}H(\sigma_l{}^{(1)}\sigma_l{}^{(2)}-\sigma_m{}^{(1)}\sigma_m{}^{(2)})T+N\sigma_n{}^{(1)}\sigma_n{}^{(2)}T, \quad (11)$$

where σ_l, σ_m, and σ_n are components of the Pauli spin matrices along the directions of $\mathbf{p}'-\mathbf{p}$, $\mathbf{p}'+\mathbf{p}$, and $(\mathbf{p}'-\mathbf{p})\times(\mathbf{p}'+\mathbf{p})$, respectively, and S and T are spin singlet and triplet projection operators. The relationships between the coefficients B, C, G, H, and N, which are given at 95 and 310 MeV over a range of c.m. scattering angles between 0° and 180°, and the spin matrix elements of W are

$$\begin{aligned}{}^0W_{00}&=B,\quad W_{00}=N+H\cos\theta,\\ W_{10}&=-W_{-10}=\sqrt{2}\sin\theta(C-\tfrac{1}{2}H),\\ W_{01}&=-W_{0-1}=-\sqrt{2}\sin\theta(C+\tfrac{1}{2}H),\\ W_{11}&=W_{-1-1}=\tfrac{1}{2}(G-H\cos\theta),\\ W_{1-1}&=W_{1-1}=\tfrac{1}{2}(G+H\cos\theta-2N).\end{aligned} \quad (12)$$

[11] The relationship between phase parameters and the various orders of field-theoretical quantities was clarified by Breit. See G. Breit, Ann. Phys. (N. Y.) 16, 346 (1961).
[12] L. Wolfenstein, Phys. Rev. 96, 1654 (1954).

TABLE I. One- and two-pion exchange phase parameters and Yale phenomenological phase parameters[a] at 95 and 310 MeV for the isosinglet state ($T=0$). The coupled and uncoupled phase shifts are in radians.

	95 MeV					310 MeV				
Phase	OPE	Δ(OPE)	TPE	Pion sum	Yale	OPE	Δ(OPE)	TPE	Pion sum	Yale
K_1	−0.2061	−0.0310	0.1043	−0.1328	−0.1954 (0.0194)	−0.2170	−0.1590	0.0251	−0.3509	−0.6798 (0.0910)
ρ_1	1.1065	0.0920	−0.9792	0.2193	0.0910 (0.0220)	1.9596	0.4598	−2.2900	0.1294	0.1859 (0.1321)
${}^3\theta^{D}{}_1$	−0.1851	−0.0016	0.0409	−0.1458	−0.2090 (0.0044)	−0.4759	−0.0035	0.1590	−0.3204	−0.4432 (0.0584)
${}^3\delta^{D}{}_2$	0.2435	0.0021	0.1326	0.3782	0.3155 (0.0078)	0.5582	0.0015	0.6337	1.1934	0.4602 (0.0860)
${}^3\theta^{D}{}_3$	−0.0320	−0.0003	−0.0787	−0.1110	0.0183 (0.0037)	−0.1066	−0.0017	−0.5340	−0.6423	0.0951 (0.0248)
K_3	−0.0397	−0.0003	0.0043	−0.0357	−0.0362 (0.0063)	−0.0747	−0.0007	0.0296	−0.0458	−0.0144 (0.0326)
ρ_3	0.1242	0.0010	−0.0022	0.1230	0.1120 (0.0085)	0.3140	0.0043	−0.0188	0.2995	0.2506 (0.0530)
${}^3\theta^{G}{}_3$	−0.0141	−0.0001	−0.0026	−0.0168	−0.0150 (0.0047)	−0.0612	−0.0011	0.0203	−0.0420	−0.0187 (0.0473)
${}^3\delta^{G}{}_4$	0.0346	0.0003	0.0064	0.0413	0.0370 (0.0058)	0.1231	0.0054	0.0222	0.1507	0.0704 (0.0481)
${}^3\theta^{G}{}_5$	−0.0041		−0.0031	−0.0072	−0.0044 (0.0035)	−0.0235	0.0002	−0.0162	−0.0395	0.0173 (0.0173)
K_5	−0.0089	−0.0001	0.0006	−0.0084	OPE	−0.0298	−0.0003	0.0035	−0.0266	−0.0374 (0.0288)
ρ_5	0.0238	0.0002	−0.0012	0.0228	OPE	0.0973	0.0001	−0.0018	0.0956	0.1058 (0.0223)
${}^3\theta^{I}{}_5$	−0.0020		−0.0005	−0.0025	OPE	−0.0146	0.0002	0.0025	−0.0119	OPE
${}^3\delta^{I}{}_6$	0.0068		−0.0005	0.0063	OPE	0.0395	−0.0034	0.0023	0.0384	OPE

[a] The $T=0$ phase parameters correspond to fit $(\text{Y-IV})_{pp+np}$ in Ref. 15. The number in parentheses under each phase parameter is the uncertainty in the phase parameter as given in Table VI of that paper. These uncertainties are obtained by parallel shifts of the phase-energy curves within specified energy intervals.

The fourth-order pion theoretical contributions to W correspond to diagrams (a) through (i) in Fig. 1 in the paper by Gupta.[13] After renormalization, the contributions from vertex and self-energy diagrams (c)–(e) produce W-matrix coefficients $B=-G=-H$ as does the one-pion-exchange diagram; hence these can be regarded as contributing a fourth-order correction to OPE and are denoted Δ(OPE). The crossed and uncrossed diagrams (a) and (b) are regraded as the TPE part of the W matrix.

The relativistic phase parameters, excluding S waves, are numerically obtained from the W-matrix coefficients of Gupta, Haracz, and Kaskas[14] by putting Eqs. (12) in (8), (9), and (10), and the results in radians are presented in Table I for isosinglet phase parameters and in Table II for isotriplet phase parameters. The one-pion exchange and recent Yale phenomenological phase parameters[15] are included in these tables as a standard for reference.

[13] See Ref. 2, p. 1147.

[14] The Δ(OPE) W-matrix coefficients are presented in Table I and the TPE W-matrix coefficients in Tables II and III of Ref. 4.

[15] R. E. Seamon, K. A. Friedman, G. Breit, R. D. Haracz, J. M. Holt, and A. Prakash, Phys. Rev. **165**, 1579 (1968). Isosinglet parameters are taken from Table III and isotriplet parameters from Table IV. The parameters at 95 MeV are obtained by a linear interpolation of the Yale values between 90 and 100 MeV.

The Δ(OPE) isosinglet phase parameters, shown in Table I, are small compared with OPE at a scattering energy of 95 MeV, although the coupling parameter ρ_1 is close to the Yale phenomenological value. At 310 MeV this fourth-order correction to OPE is comparable to OPE only for K_1 and ρ_1. In contrast to this, the TPE isosinglet phase parameters, also shown in Table I, are large compared with both the OPE and phenomenological values through D waves at 95 MeV and through G waves at 310 MeV.

The isotriplet phase parameters are shown in Table II, where it is again seen that Δ(OPE) is a small correction to OPE. Indeed, only ${}^3\delta^{P}{}_1$ at 310 MeV is significantly large. The TPE phase parameters are large through D waves at 95 MeV and through F waves at 310 MeV. It is interesting to note that although the TPE W-matrix coefficients are larger than OPE coefficients owing to the large coupling constant, the TPE phase parameters become smaller than OPE for large values of L.

It is further observed that the pion contributions through the fourth order in the pion-nucleon coupling constant, called pion sum in Tables I and II, show a

TABLE II. One- and two-pion exchange phase parameters and Yale phenomenological phase parameters[a] at 95 and 310 MeV for the isotriplet states ($T=1$).

	95 MeV					310 MeV				
Phase	OPE	Δ(OPE)	TPE	Pion sum	Yale	OPE	Δ(OPE)	TPE	Pion sum	Yale
$^3\theta^{P}{}_2$	0.0370	0.0003	0.1600	0.1973	0.1912 (0.0016)	0.0952	0.0011	−0.1381	−0.0418	0.2764 (0.0098)
$^3\delta^{P}{}_1$	−0.3225	−0.0222	0.2976	−0.0471	−0.2286 (0.0021)	−0.6205	−0.1116	0.9821	0.2500	−0.4756 (0.0150)
K_2	0.0292	0.0003	0.0318	0.0613	0.0646 (0.0019)	0.0412	0.0010	0.2086	0.2508	0.1749 (0.0101)
ρ_2	−0.1081	−0.0009	0.0025	−0.1065	−0.0930 (0.0022)	−0.2227	−0.0034	0.0364	−0.1897	−0.0883 (0.0141)
$^3\theta^{F}{}_2$	0.0149	0.0001	0.0048	0.0198	0.0104 (0.0027)	0.0497	0.0004	0.0440	0.0941	0.0056 (0.0099)
$^3\delta^{F}{}_3$	−0.0286	−0.0002	0.0061	−0.0227	−0.0279 (0.0024)	−0.0808	−0.0007	0.0476	−0.0339	−0.0617 (0.0082)
$^3\theta^{F}{}_4$	0.0037		0.0021	0.0058	0.0052 (0.0014)	0.0159	0.0001	0.0083	0.0243	0.0498 (0.0052)
K_4	0.0062	0.0001	0.0009	0.0072	0.0062 (0.0009)	0.0156	0.0001	−0.0075	0.0082	0.0205 (0.0047)
ρ_4	−0.0176	−0.0002	−0.0002	−0.0180	OPE	−0.0562	−0.0005	0.0017	−0.0550	−0.0422 (0.0098)
$^3\theta^{H}{}_4$	0.0017		0.0008	0.0025	OPE	0.0096	0.0001	0.0045	0.0142	0.0134 (0.0062)
$^3\delta^{H}{}_5$	−0.0050		−0.0003	−0.0053	OPE	−0.0226	−0.0002	0.0046	−0.0182	−0.0281 (0.0074)
$^3\theta^{H}{}_6$	0.0006		0.0006	0.0012	OPE	0.0041		0.0018	0.0059	0.0092 (0.0041)
K_6	0.0014		0.0001	0.0015	OPE	0.0064		−0.0050	0.0014	OPE
ρ_6	−0.0037		0.0002	−0.0035	OPE	−0.0195	−0.0001		−0.0196	OPE

[a] The $T=1$ phase parameters for n-p scattering correspond to fit (Y-IV)$_{pp+np}$ in Ref. 15. The number in parentheses under each phase is the uncertainty in the phase as given in Table VI of that paper obtained by the parallel-shift method. The phase parameters appearing in Ref. 15 were obtained by interpolation from those at data-deck energies in the fit (Y-IV)$_{pp+np}$, and the authors estimate that the reproduction of these values are good to at least 0.4% but more frequently to 0.2% of the total range between maximum and minimum of the phase parameter within the energy range analyzed.

general agreement with the phenomenological phase parameters at 95 MeV with only $^3\theta^{D}{}_3$ and $^3\delta^{P}{}_1$ differing markedly, but they do not reproduce the low L phenomenological phase parameters at 310 MeV. At 310 MeV, many phase parameters are at variance to values of L of about four. However, even at 310 MeV, the pion-theoretical phase parameters are a reasonable approximation to the phenomenological phase parameters for $L>4$.

Additional corrections to these pion contributions are clearly necessary for values of $L\leqslant 4$ at higher energies.

TABLE III. Two-boson exchange phase parameters corresponding to $\pi+\eta$ and $\pi+\sigma$ at 95 and 310 MeV for the isosinglet and isotriplet states. The coupled and uncoupled phase shifts are in radians.

$T=0$	95 MeV		310 MeV		$T=1$	95 MeV		310 MeV	
phase	$\pi+\eta$	$\pi+\sigma$	$\pi+\eta$	$\pi+\sigma$	phase	$\pi+\eta$	$\pi+\sigma$	$\pi+\eta$	$\pi+\sigma$
K_1	−0.0247	−0.8399	−0.0308	−2.2545	$^3\theta^{P}{}_2$	0.1563	−0.0453	0.4465	−0.0918
ρ_1	0.2099	1.1436	0.4388	−2.1642	$^3\delta^{P}{}_1$	−0.0342	−1.1048	−0.0968	−2.2819
$^3\theta^{D}{}_1$	0.0008	−0.1274	0.0027	−0.6773	K_2	0.0004	0.0155	0.0023	0.0540
$^3\delta^{D}{}_2$	−0.0117	0.1478	−0.1447	0.6804	ρ_2	−0.0018	−0.0410	−0.0086	0.0453
$^3\theta^{D}{}_3$	0.0086	−0.0194	0.1905	−0.1089	$^3\theta^{F}{}_2$	0.0003	0.0018	0.0013	0.0236
K_3	0.0005	−0.0061	−0.0014	−0.0318	$^3\delta^{F}{}_3$	−0.0003	−0.0030	0.0022	−0.0341
ρ_3	0.0008	0.0112	0.0004	0.0107	$^3\theta^{F}{}_4$	0.0001	0.0004	−0.0036	0.0055
$^3\theta^{G}{}_3$	−0.0002	0.0001	0.0006	−0.0100	K_4	−0.0001	−0.0007	···	0.0021
$^3\delta^{G}{}_4$	0.0011	−0.0013	−0.0021	0.0176	ρ_4	0.0001	−0.0007	0.0002	−0.0027
$^3\theta^{G}{}_5$	···	−0.0002	0.0016	−0.0029	$^3\theta^{H}{}_4$	···	0.0002	−0.0005	0.0010
K_5	···	−0.0003	−0.0004	−0.0012	$^3\delta^{H}{}_5$	−0.0001	0.0003	0.0003	−0.0016
ρ_5	−0.0002	−0.0001	−0.0004	0.0023	$^3\theta^{H}{}_6$	−0.0001	0.0002	−0.0004	0.0004
$^3\theta^{I}{}_5$	···	−0.0001	0.0008	−0.0006	K_6	···	0.0001	0.0001	0.0002
$^3\delta^{I}{}_6$	−0.0004	−0.0008	−0.0006	0.0009	ρ_6	···	−0.0003	−0.0001	−0.0006

4. ADDITIONAL TWO-BOSON-EXCHANGE EFFECTS

The pion-theoretical contribution to nucleon-nucleon scattering is supplemented in resonance models by employing the contributions from the boson resonances. The masses of these resonances exchanged between the nucleons is about 780 MeV or less. It would then be consistent to include all processes where the total exchanged mass is less than or equal to the above value.

This means the $\pi+\eta$ exchange contribution should be included since the combined mass is only 686 MeV. The relativistic W-matrix coefficients for the $\pi+\eta$ process was obtained by Barker, Gupta, and Haracz for scattering energies of 95 and 310 MeV by evaluating the crossed and uncrossed diagrams.[16] The phase parameters corresponding to this W matrix are presented in Table III, arbitrarily taking the value 14 for the η-nucleon coupling constant. In the case of the isosinglet phase parameters, it is seen that only ρ_1 is comparable to the corresponding OPE values at 95 MeV, while at 310 MeV only ρ_1, ${}^3\delta^D{}_2$, and ${}^3\theta^D{}_3$ are comparable to OPE values. For the isotriplet states, ${}^3\theta^P{}_2$ is the only large $\pi+\eta$ phase parameter, being larger than both OPE and TPE at 310 MeV. Hence, the $\pi+\eta$ contribution is small compared with the pion-theoretical results, and it would be negligible if the η-nucleon coupling constant were given the $SU(3)$ value of two.

Scalar resonances are somewhat peculiar since light scalar resonances have not been observed and yet they are essential to the success of one-boson-exchange resonance models. Since the TPE effect is not included in such models, the appearance of scalar resonances may well be a way to replace TPE. However, if a very light scalar resonance is included, with mass about 400 MeV, it is necessary to study the $\pi+\sigma$ effect as well. The W matrix for this effect corresponding to crossed and uncrossed diagrams was obtained by Barker, Gupta, and Haracz,[17] and the phase parameters representing this W matrix are included in Table III, arbitrarily taking the σ-nucleon coupling constant to be equal to 14. It is observed that the $\pi+\sigma$ phase parameters are comparable to TPE phase parameters. Thus, if a very light scalar resonance exists with a large coupling to the nucleon, the $\pi+\sigma$ effect is large.

5. DISCUSSION

It is evident from the results presented here that the two-boson-exchange contributions to the nucleon-nucleon interaction are large for the low-L phase parameters and cannot be neglected. Therefore, a more reasonable model for the two-nucleon interaction should include the contributions from the virtual exchange of one and two pions, the η resonance, the η resonance and a pion together, the ω and ρ vector resonances, and perhaps the scalar resonance and the σ and pion together. It is felt that an attempt should be made to obtain a fit to nucleon-nucleon scattering data without using the unobserved scalar resonance.

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[16] See Ref. 5, Fig. 1. The remaining fourth-order diagrams are not calculated since they proved to be small compared to the crossed and uncrossed diagrams in the pion case.

[17] See Ref. 6.