# $K^*(890)$  Photoproduction in the Regge-Pole Model<sup>†</sup>

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In this paper the Regge-pole model is applied to  $K^*(890)$  photoproduction. Conspiracy relations, kinematic constraints, factorization, and the question of when to set  $m<sub>z</sub> = 0$  are discussed. Predictions are obtained, in the limit of large s and small t, for  $d\sigma/d\Omega$ , the density matrix, and the K\*-decay angular distribution.

#### L INTRODUCTION

ECKNTLV much interest has been shown in the application of Regge poles to high-energy processes.<sup>1</sup>  $K^*(890)$  photoproduction is an interesting reaction to consider here; the particles all have spin, and the masses are all unequal.

In this paper we apply the Regge-pole model to the process

$$
\gamma + \rho \to K^* + Y, \tag{1}
$$

In this paper we apply the Regge-pole model to the<br>
process<br>  $\gamma + p \rightarrow K^* + Y$ , (1) where  $\lambda = a - b$  and  $\mu = c - d$ ,  $\tilde{f}^{\mu}$  is then defined by<br>
where  $K^*$  is the 1<sup>-</sup> meson at 890 MeV, and *Y* is either  $\vec{f}_{cd;ab}{}^t = \vec{f}_{$ a  $\Lambda$  or a  $\Sigma$  hyperon. Kinematic singularities are separated out of the helicity amplitudes, and the questions of conspiracy relations, kinematic constraints, factorization, and when to set  $m_{\gamma}=0$  are investigated. We investigate the values of  $t$  where constraints may arise and discover that the leading amplitudes for the process (1) are not involved in any constraint relations at  $t=0$  or at  $t=(m_p-m_Y)^2$ . We also discuss why the crossing matrix should yield the same conspiracy relations as those obtained from invariant amplitudes. Predictions for the large-s and small-t behavior of  $d\sigma/d\Omega$ , the density matrix  $\rho$ , and the K<sup>\*</sup>-decay angular distribution are obtained; no data are as yet available for comparison with these predictions.

The plan of this paper is as follows: In Sec. II, we define our kinematic-singularity-free amplitudes and find the dominant contributions in the limit of large s and small t; Sec.III discusses possible modifications due to conspiracy relations, kinematic constraints, factorization, and  $m_{\gamma} \rightarrow 0$ . In Sec. IV, we obtain our predictions for  $d\sigma/d\Omega$ , the density matrix, and the  $K^*$  angular distribution.

### II. t-CHANNEL HELICITY AMPLITUDES

In this section we shall investigate the  $t$ -channel helicity amplitudes associated with  $K^*(890)$  photoproduction. We shall construct t-channel helicity amplitudes that are free of kinematical singularities; these will then be Reggeized. In separating out the kinematic singularities we shall follow the method of  $W \text{ang}^2$ ; possible modifications will be discussed in Sec. III.

Each *t*-channel helicity amplitude  $f_{cd;ab}$ <sup>t</sup> contains factors of  $\sin \frac{1}{2}\theta_t$  and  $\cos \frac{1}{2}\theta_t$  arising from a partial-wave expansion in terms of  $d$  functions; these are separated out according to

$$
f_{cd;ab}{}^{t} = (\sin^1_2 \theta_t)^{|\lambda - \mu|} (\cos^1_2 \theta_t)^{|\lambda + \mu|} \tilde{f}_{cd;ab}{}^{t}, \qquad (2)
$$

where  $\lambda = a - b$  and  $\mu = c - d$ .  $\tilde{f}^{\ell \pm}$  is then defined by

$$
\bar{f}_{cd;ab}{}^t \pm \bar{f}_{-c-d;ab}{}^t = \bar{K}_{cd;ab}{}^-(t) \tilde{f}_{cd;ab}{}^{\pm}(t,s) , \qquad (3)
$$

where  $\bar{K}$  contains the kinematic singularities.  $\tilde{f}^t$  is the according to

kinematic-singularity-free amplitude which is Reggeized  
according to  

$$
\tilde{f}_{cd;ab}t(t,s) \rightarrow \gamma(t) \left( \frac{1 \pm e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)} \right) \left( \frac{s}{s_0} \right)^{\alpha(t)-M}, \qquad (4)
$$

where  $M = max\{|\lambda|, |\mu|\}$ . The factor M would be absent in the spinless case; in the case with spin it arises because some of the powers of s are absorbed by separating out the  $\sin \frac{1}{2}\theta_t$  and  $\cos \frac{1}{2}\theta_t$  factors before Reggeization. The rest of the powers of s are assumed to contribute full strength in our unequal-mass case (i.e. , we are assuming the action of daughter trajectories when writing  $s^{\alpha(t) - \tilde{M}}$ ).

We note that for small t and a given  $\alpha(t)$ , the highest power of s occurs when  $M=0$ . (The  $\sin \frac{1}{2}\theta_t$  and  $\cos \frac{1}{2}\theta_t$ factors do not contribute any powers of s here, since we are dealing with an unequal-mass case,  $m_{\gamma} \neq m_{K^*}$ . Thus we expect the  $M=0$  amplitudes to dominate. [An estimate of a typical range of  $|t|$  is as follows: Table I gives <sup>a</sup> list of the "parity-conserving" helicity amplitudes, together with their low-t behavior (obtained from the prescription of Wang<sup>2</sup>). The last four amplitudes in the table have been given an extra factor of  $t$ (we are assuming evasion at  $t=0$  for these amplitudes; see the following section for more details). The last column of Table I gives the contribution of each amplitude to the differential cross section. One can now estimate when to expect dominance of the  $M=0$ amplitudes: The contributions of the helicity amplitudes to  $d\sigma/d\Omega$  (see Table I) contain factors of  $s^{-2}$  or

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<sup>~</sup> For a summary of recent work on Regge poles, see L. Bertocchi, Rapporteur's talk at the 1967 Heidelberg Conference, references therein (unpublished).

and <sup>2</sup> L.-L. C. Wang, Phys. Rev. 142, 1187 (1966).<br><sup>3</sup> D. Z. Freedman and J. Wang, Phys. Rev. 153, 1596 (1967).

Amplitude	Dominant parity	Low- $t$ behavior	$16\pi^2$ s $p_i$ do Contribution to - — (omitting factors of $s^{2\alpha}$ ) $d\Omega$ Þſ
$f_{{11:34}}$ + $f_{{-1-1:34}}$ + $f_{11:14}$ ' – $f_{-1-1:14}$ ' $f_{10:11} + f_{-10:11}$ $f_{10:11} - f_{-10:11}t$ $f_{1-1;\frac{1}{2}\frac{1}{2}}+f_{-11;\frac{1}{2}\frac{1}{2}}$ $f_{1-1;\frac{1}{2}}i^t-f_{-11;\frac{1}{2}}i^t$ $f_{{11:\mathbf{i}-\mathbf{i}}}{\dagger} + f_{-1-1:\mathbf{i}-\mathbf{i}}{^t}$ $f_{{11:\,1\rightarrow\,1}}{}^{t}-f_{-1-1:\,1\rightarrow\,1}{}^{t}$ $f_{{10:\frac{1}{2}-1}}$ t $+f_{-10:\frac{1}{2}-1}$ t $f_{10:\{-1}}t - f_{-10:\{-1\}}t$ $f_{1-1:\frac{1}{2}-\frac{1}{2}}t+f_{-11:\frac{1}{2}-\frac{1}{2}}t$ $f_{1-1}$ , $f_{-1}$ , $f_{-11}$ , $f_{-1}$	$(-1)^{J}$ $(-1)^{J+1}$ $(-1)^{J+1}$ $(-1)^{J}$ $(-1)^{J}$ $(-1)^{J+1}$ $(-1)^{J}$ $(-1)^{J+1}$ $(-1)^{J}$ $(-1)^{J+1}$ $(-1)^{J+1}$ $(-1)^{J}$	$\left[t-(m_p-m_Y)^2\right]^{-1/2}$ $t^{-1/2}$ $t^{-1/2} [t-(m_p-m_Y)^2]^{1/2}$ $t^{-1} \lceil t - (m_n - m_Y)^2 \rceil$ $t^{-1}t-(m_n-m_Y)^2$ <sup>1/2</sup> $t^{-1} \lceil t - (m_n - m_Y)^2 \rceil^{1/2}$ $t^{-1/2}$ $\left[1-(m_p-m_Y)^2\right]^{1/2}$ $t^{-1/2} [t-(m_p-m_Y)^2]^{1/2}$ $t^{-1/2}$ [ $t-(m_p-m_Y)^2$ ] <sup>1</sup>	$\left[1-(m_p-m_Y)^2\right]^{-1}$ $s^{-2}$ <sub>7</sub> $t^{-1}$ sin <sup>2</sup> $\theta_t$ $s^{-2}\frac{1}{4}t^{-1}\left[t-(m_{p}-m_{Y})^{2}\right]$ <sup>1</sup> $\sin^{2}\theta_{t}$ $s^{-2}$ <sub>16</sub> $t^{-2}$ [ $t-(m_n-m_Y)^2$ ] <sup>2</sup> $\sin^4\theta_t$ $s^{-2}$ <sub>16</sub> $t^{-2}$ [ $t-(m_p-m_Y)^2$ ] <sup>1</sup> sin <sup>4</sup> $\theta_t$ $s^{-2}t^{-1}[t-(m_p-m_Y)^2]$ <sup>1</sup> sin <sup>2</sup> $\theta_t$ $s^{-2}$ <sup>+1</sup> $\sin^2\theta_t$ $s^{-2}\times\{\frac{1}{4}\lceil t-(m_p-m_Y)^2\rceil^1(1+\cos^2\theta_t)+\frac{1}{4}(1+\cos^2\theta_t)$ +Re $\cos\theta_i[t-(m_p-m_Y)^2]^{1/2}$ $s^{-4}\times\{\frac{1}{16}t^{-1}[t-(m_n-m_Y)^2]^1\sin^2\theta_t(1+\cos^2\theta_t)$ $+\frac{1}{16}t^{-1}\left[t-(m_v-m_Y)^2\right]^2\sin^2\theta_t(1+\cos^2\theta_t)$ $+\frac{1}{4}$ Re cos $\theta_t$ sin <sup>2</sup> $\theta_t t^{-1} [t-(m_p-m_Y)^2]^{3/2}$ .

TABLE I. The low-t behavior and contribution to  $d\sigma/d\Omega$  of the helicity amplitudes in  $K^*$  photoproduction. Evasion at  $t=0$  has been assumed for the last four amplitudes in the table.

 $s^{-4}$  (when M $\neq$ 0) times functions of t and  $\theta_t$ . When  $s=10$  and  $|t_{\min}|<|t|<0.7$ , for example, and if the dominant parity is  $(-1)^J$ , then these functions of t<br>and  $\theta_t$  are  $\leq 10$ , whereas  $s^{-2} = 10^{-2}$ . Thus, for  $P = (-1)^J$ amplitudes, the kinematic factors times  $s^{-M}$  are the largest when  $M=0$ , for  $s=10$  and |t| in the range where we might expect Reggeism to be a valid model  $(|t| \lesssim 0.7$  BeV). Exactly the same conclusion holds for  $P = (-1)^{J+1}$  amplitudes.

Next we apply parity conservation<sup>4</sup> to the  $\gamma K^*$ -Reggeon vertex and expand in partial waves; we find that exchange of natural-parity trajectories  $[P=(-1)^{J}]$  can contribute to  $\tilde{f}^{t+}$ <sub> $\lambda=\mu=0$ </sub>, but not to  $\tilde{f}^{\mu\nu}$ <sub> $\lambda=\mu=0$ </sub>. Exchange of unnatural-parity trajectories (such as those associated with the  $K$  and  $K_A$  mesons) would contribute to  $\tilde{f}^t$   $\lambda = \mu = 0$ , but the trajectory values  $\alpha(t)$  lie lower than those for natural-parity exchange. (The Wang kinematic factor<sup>2</sup> for  $\tilde{f}^{\mu\nu}{}_{\lambda=\mu=0}$  is  $\lceil t - (m_p - m_Y)^2 \rceil^{-1/2}$ ; this pole is assumed cancelled by evasion at  $t = (m_p - m_Y)^2$ .) We hence neglect exchange of unnatural-parity trajectories. Thus the amplitudes we shall need are those  $\tilde{f}^{t+}$  amplitudes having  $\lambda = \mu = 0.5$ Using parity<sup>4</sup> to reduce the number of independent amplitudes, we are left with the following amplitudes at large  $s$  and small  $t$ :

$$
f_{11;\frac{1}{2}\frac{1}{2}} = f_{-1-1;-\frac{1}{2}-\frac{1}{2}} \stackrel{\text{def}}{=} f_{-1-1;\frac{1}{2}\frac{1}{2}}^t
$$
  
=  $f_{11;-\frac{1}{2}-\frac{1}{2}} \stackrel{\text{def}}{=} \frac{1}{2} \overline{K}_{11;\frac{1}{2}\frac{1}{2}}^t \cdot (t) \tilde{f}_{11;\frac{1}{2}\frac{1}{2}}^t \cdot (5)$ 

where

$$
\tilde{f}_{11;\frac{1}{2}i}^{t+} \to \gamma(t) \bigg( \frac{1 \pm e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)} \bigg) \bigg( \frac{s}{s_0} \bigg)^{\alpha(t)}, \tag{6}
$$

<sup>4</sup> M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) 7, 404 (1959).

and  $\bar{K}_{11;\frac{1}{2}}(t)$  is a kinematical factor. Using the prescription given by Wang for the unequal-mass case (and setting  $m_{\gamma} \rightarrow 0$  in this result), we find that

$$
\overline{K}_{11:\frac{1}{2}\frac{1}{2}}(t) = \left[t - (m_p + m_Y)^2\right]^{-1/2}(t - m_K \epsilon^2)^{-2}.
$$
 (7)

Actually, some caution is needed with respect to this form of  $\bar{K}^+$ , and before proceeding further, we shall discuss possible modifications of the factor  $\bar{K}^+$  due to conspiracy relations, kinematic constraints, factorization, and the question of when to set  $m_{\gamma}=0$ .

### III. CONSPIRACY RELATIONS, KINEMATIC CONSTRAINTS, FACTORIZATION, AND  $M_{\gamma} \rightarrow 0$

There are several points to be discussed in connection with the factor  $\overline{K}_{11;\frac{1}{2}i}$  (*t*). The general form of  $\overline{K}$  has been derived by Wang,<sup>2</sup> who examined the singularities in the crossing matrix relating s- and *t*-channel helicity amplitudes. In addition, however, the questions of conspiracy relations, kinematic constraints, factorization, and how to treat  $m_{\gamma} \rightarrow 0$  may arise. Consideration of these points could lead to a modified  $K$ .

One can ask at which points to expect special conditions on the *t*-channel helicity amplitudes. In  $\bar{N}N$ scattering, angular momentum conservation applied at  $cos\theta_s = \pm 1$  leads to conspiracy relations,<sup>6</sup> but in the unequal-mass case (i.e.,  $m_{\gamma} \neq m_{K*}$ ) there are no such relations at  $\cos\theta_s = \pm 1$ . (This has been shown by Högaasen and Salin<sup>7</sup>; the proof depends on the fact that for unequal masses,  $\cos\theta_{s} = \pm 1$  implies  $\cos\theta_{t} = \pm 1$ . Angular momentum conservation then says that each  $f_{\lambda \neq \mu}t$  must vanish at  $\cos\theta_s = \pm 1$ , and no *relations* between  $f^{\prime}$ s arise.)

As pointed out to the author by Dr. Frank Henyey, keeping<br>only amplitudes with  $M = 0$  requires care. We assume the absence of poles near the forward direction in those  $M \neq 0$  amplitudes not required to vanish by angular momentum conservation: In the<br>forward direction  $P = (-1)^J$  exchange can also contribute to<br> $f_{10;\frac{1}{J} \to \frac{1}{J} + \hat{J}_{-10;\frac{1}{J} \to \frac{1}{J}}$ . This amplitude has no kinematic singularity at<br> $t = (m_p$ 

<sup>&</sup>lt;sup>6</sup> D. V. Volkov and V. N. Gribov, Zh. Eksperim. i Teor. Fiz.<br>44, 1068 (1963) [English transl.: Soviet Phys.—JETP 17, 720  $(1963)$ ]

<sup>`&</sup>lt;sup>7</sup>H. Högaasen and Ph. Salin, CERN Report No. TH788 (unpublished).

Conspiracy relations among parity-conserving helicity amplitudes can still arise at  $t=0$  when all the masses are unequal, but no such relations occur for  $\lambda = 0$  or  $\mu = 0$ . which is the case of interest here. This can be seen as  $follows<sup>8</sup>$ : The Wang prescription<sup>2</sup> allows a maximum singularity  $(1/\sqrt{t})^{\lceil \lambda - \mu \rceil}$  at  $t=0$  in the individual singularity  $(1/\sqrt{t})^{|\lambda-\mu|}$  at  $t=0$  in the individual<br>amplitude  $\bar{f}_{cd;ab}$ . Thus  $\bar{f}_{cd;ab}$  and  $\bar{f}_{-c-d;ab}$  would in general have *different* maximum singularities at  $t=0$ , and  $\vec{f}_{cd,ab}$   $\stackrel{i}{\neq} \vec{f}_{-c-d;ab}$  would be allowed the larger of these two singularities. One can then show' that constraint equations between  $\bar{f}_{cd;ab}{}^t + \bar{f}_{-c-d;ab}{}^t$  and  $\bar{f}_{cd,ab}t - \bar{f}_{-c-d,ab}t$  would arise at  $t=0$ . But if  $\lambda=0$  or  $\mu=0$  (the case of interest here), the maximum singular- $\mu=0$  (the case of interest here), the maximum singularity at  $t=0$  is the *same* for  $\tilde{f}_{cd;ab}{}^t$  and  $\tilde{f}_{-c-d;ab}{}^t$  and thus no constraint equations involving these parity-conserving helicity amplitudes arise at  $t=0$ .

In general, we might also expect special relations between helicity amplitudes to arise at those points where the helicity becomes undefined.<sup>9</sup> One can define the helicity four-vector  $n_3(p_i)$  for a two-particle state by the conditions  $n_3 \cdot n_3 = -1$  and (in the c.m. system)  $n_3 \cdot p_i > 0$ <sup>9</sup>:

$$
n_3(p_i) = -\left(\frac{m_i{}^2 P - (p_i \cdot P) p_i}{\frac{1}{2} m_i [t - (m_1 + m_2)^2]^{1/2} [t - (m_1 - m_2)^2]^{1/2}}\right)
$$

where  $P = p_1 + p_2$ . It is evident that troubles arise when t is at a threshold or pseudothreshold  $t=(m_1\pm m_2)^2$ , and constraint conditions can occur between t-channel amplitudes at precisely these points. Another way of seeing that constraint cnoditions arise at  $t=(m_i\pm m_j)^2$ seeing that constraint cnoditions arise at  $t = (m_i \pm m_j)$ <br>has been discussed by Jackson and Hite,<sup>10</sup> who note that in a special basis system certain amplitudes vanish at precisely these points.

Having thus discussed where one can expect constraint conditions, we next turn to the question of constructing them. One way of deriving constraint conditions is to express the invariant scalar amplitudes for the process in terms of linear combinations of  $t$ channel helicity amplitudes. Since the scalar amplitudes have no poles in t, one then derives certain conditions on linear combinations of t-channel helicity amplitudes. These are the conspiracy relations. The point to be noted is that the only property of the scalar amplitudes that is used is that they have no poles in  $t$ . Thus one could have started with any other complete set of s-channel amplitudes having no poles in  $t$ , and the results would have been exactly the same. Hence it is equally valid to start with  $f^*$  amplitudes. Since these are related to the  $f^{\prime}$ 's by crossing, one can thus obtain the conspiracy relations by examining the crossing matrix at the values of  $t$  in question. This approach has been investigated in detail by Cohen-Tannoudji et al.<sup>9</sup>

We now turn to explicit construction of the desired constraint relations for  $K^*$  photoproduction. We need those relations which involve  $\lambda = \mu = 0$  helicity amplitudes (these are the relevant amplitudes for large s, as noted in Sec. II); the point of interest is  $t=t_0$  $=(m_p-m_Y)^2$ . The other three threshold or pseudothreshold points for this reaction involve much larger values of t and are thus not needed for a study of the behavior at small  $t$ . Following the method involving the crossing matrix<sup>9</sup> as illustrated by Högaasen and Salin,<sup>6</sup> we discover that there are indeed constraint relations between several t-channel helicity amplitudes at  $t = t_0$ , but none of these relations involves  $\bar{f}_{cc,aa}t$ at  $t = t_0$ , but none of these relations involves  $f_{c,aa}$ <br> $+\bar{f}_{-c-c;aa}$  amplitudes, i.e., those amplitudes which contain the leading s behavior (as discussed above) are not involved in any conspiracy or constraint relations.

This result can be partially understood in the follow-I ins result can be partially understood in the following way: The  $\bar{f}_{cc;aa}^i + \bar{f}_{-c-c;aa}^i$ 's cannot be linearly related to other  $\bar{f}$ ''s whose leading s behavior is also governed by natural-parity  $[P=(-1)^{J}]$  exchanges at  $t=t_0$ , since the leading powers of s would be different. governed by natural-parity  $[P = t_0$ , since the leading powers of<br>On the other hand,  $\tilde{f}_{ce;aa}t + \tilde{f}_{-oc-c}$ ;<br>to a polynomial in s times other t cannot be related to a polynomial in s times other  $\bar{f}^{\nu}$ s, since none of the  $\bar{f}^{\nu}$ s has any kinematic singularities in s.

Thus in finding the kinematical factor  $\bar{K}_{ce; aq}$ <sup>+</sup> we can completely avoid the question of possible extra factors due to conspiracy or constraint relations. We next note that factorization of residue functions will also not yield any new information. The residue factors  $\bar{K}$  factor automatically at thresholds and pseudothresholds, and for our unequal-mass  $\lambda = \mu = 0$  helicity amplitudes there are no factors of t in  $\bar{K}$  for the relevant processes, so factorization with respect to these pieces is automatically satisfied.

To find the kinematical factors, we thus gain no new information from constraint conditions or from factorization. The only remaining uncertainty in finding  $\bar{K}_{cc;aa}$ <sup>+</sup> arises from the question when to put  $m_{\gamma}=0$ . One could put  $m_{\gamma}=0$  in Wang's general prescription<sup>2</sup> for  $\bar{K}_{cc, a\alpha}$ <sup>+</sup>, or one could put  $m_{\gamma}=0$  in the crossing matrix for  $K_{cc;\bm{aa^+}}$ , or one could put  $m_\gamma\!=\!0$  in the crossing matrix<br>and derive a modified prescription for  $\bar{K}_{cc;\bm{aa^+}}$ .<sup>11</sup> The end result for the two cases can in general differ in the net power to which  $(t-m_{K^*}^2)$  should be raised. Since for small t this factor is smooth, the exact power need not concern us, and we will simply use Wang's prescription and set  $m_{\gamma} \rightarrow 0$  at the end. We thus obtain the result

$$
\bar{K}_{11;\frac{1}{2}+}(t) = \left[t - (m_p + m_Y)^2\right]^{-1/2} \left[t - m_K \cdot \frac{2}{3}\right]^{-2}.
$$
 (7)

This result for the kinematic factor  $\bar{K}$  can be checked by means of simple angular momentum and parity by means of simple angular momentum and parit<sub>i</sub><br>arguments.<sup>12</sup> As an example of the method, conside the point  $t=(m_p+m_Y)^2$ . Setting I (orbital angular momentum) for the  $N\bar{Y}$  system equal to zero, the possible  $N\overline{Y}$  states have  $J^P = 0^-$  or 1–. Since the  $N\overline{Y}$  system is coupled to a  $P = (-1)^J$  Reggeon in our model,

<sup>s</sup> S. Frautschi and L. Jones, Phys. Rev. 167, 1335 (1968).

<sup>&</sup>lt;sup>9</sup> G. Cohen-Tannoudji, A. Morel, and H. Navelet, Saclay Report, 1967 (unpublished).

<sup>&</sup>lt;sup>10</sup> J. D. Jackson and G. E. Hite, Phys. Rev. 169, 1248 (1968).

<sup>&</sup>quot;S. Frautschi and L. Jones, Phys. Rev. 163, <sup>1820</sup> (1967).

<sup>&</sup>lt;sup>12</sup> S. Frautschi and L. Jones, Phys. Rev. 164, 1918 (1967).

 $J^P$  is restricted to 1<sup>-</sup>. Now expand the helicity ampli- at  $\alpha_2^*(t) = 0$  (Chew ghost-eliminating mechanism<sup>15</sup>).<br>tudes in terms of partial waves: Thus we set tudes in terms of partial waves:

$$
f_{11;\frac{1}{2}\frac{1}{2}}\mathbf{f} + \dot{f}_{-1-1;\frac{1}{2}\frac{1}{2}}\mathbf{f} = \sum_{J} F_{11;\frac{1}{2}\frac{1}{2}} J d_{00} J(\cos\theta_i).
$$

The F<sup>J</sup>'s have the threshold behavior  $q_N\bar{r}^l$ , and  $\cos\theta_t$ is proportional to  $1/q_N\bar{y}$ . Hence, we deduce the behavior

$$
F^{J}d_{00}J(\cos\theta_{t})\propto q_{N}\overline{Y}^{l}(1/q_{N}\overline{Y})^{J}=(q_{N}\overline{Y})^{-1}
$$

i.e., near  $t = (m_p + m_Y)^2$  the kinematic factor  $\bar{K}^+(t)$ goes as  $q_{NY}^{-1} \propto [t-(m_p+m_Y)^2]^{-1/2}$ , in agreement with (7). For t near the other threshold or pseudothreshold points, analogous arguments go through [the  $\bar{Y}$  is treated as having positive parity at  $t=(m_p-m_Y)^2$ , and one obtains exactly the Wang kinematic factor (7).

## IV. CROSS SECTION, DENSITY MATRIX, AND  $K^*$ -DECAY ANGULAR DISTRIBUTION

The differential cross section in the c.m. frame can be written

$$
\frac{d\sigma}{d\Omega} = \frac{p_f}{4\pi^2 s p_i} \frac{1}{4} \sum |f^s|^2,
$$

where the sum goes over the s-channel helicity amplitudes. Orthogonality of the crossing matrix $13$  then gives

$$
\frac{d\sigma}{d\Omega} = \frac{p_f}{4\pi^2 s p_i} \frac{1}{4} \sum |f^i|^2, \tag{8}
$$

where the sum is now over all  $t$ -channel helicity amplitudes. We take the limit of large s and use the results (5), (6), and (7) of the preceding sections. Thus

$$
\frac{d\sigma}{d\Omega}\Big|_{\text{c.m.}} \xrightarrow{\text{Inrg } e \ s} \frac{p_f}{16\pi^2 s p_i} |t - (m_p + m_Y)^2|^{-1} [t - m_{K^*}^2]^{-4}
$$

$$
\times \Big| \sum_i \gamma_i(t) \frac{1 + e^{-i\pi \alpha_i(t)}}{\sin \pi \alpha_i(t)} \Big(\frac{s}{s_0}\Big)^{\alpha_i(t)} \Big|^2. \tag{9}
$$

The sum is taken over the  $K^*(1^-;890)$  and  $K_V(2^+;$ 1420) trajectories. '4

We make the following choices for the residue functions and Regge trajectories:  $\gamma_1$ -(t) is assumed roughly constant, while  $\gamma_2+(t)$  is put proportional to  $\alpha_2+(t)$  in order to cancel the pole in

$$
\frac{1+e^{-i\pi\alpha_2+(t)}}{\sin\pi\alpha_2+(t)}
$$

$$
\gamma_1^{-}(t) = \tilde{\gamma}_1^{-}(t),
$$
  
\n
$$
\gamma_2^{+}(t) = \tilde{\gamma}_2^{+}(t)\alpha_2^{+}(t),
$$
\n(10)

where the  $\tilde{\gamma}$ 's are assumed slowly varying in *t*. The  $K^*(1^-)$  and  $K^*(2^+)$  trajectory functions are taken parallel to those of the *p* and *A*2. We take<sup>16</sup>  $\alpha_{\rho}(t) \cong t$  $K^*(1^-)$  and  $K^*(2^+)$  trajectory functions are taken<br>parallel to those of the  $\rho$  and A2. We take<sup>16</sup>  $\alpha_{\rho}(t) \cong t$  $+0.57$  and  $\alpha_{A_2}(t) \approx t + 0.35$ ; thus

$$
\alpha_1^{-}(t) \leq t + 0.37,
$$
  
\n
$$
\alpha_2^{(t)} \leq t + 0.02.
$$
\n(11)

To determine the  $\tilde{\gamma}_i$ 's appearing in the residue functions  $\gamma_i$ , we first examine A2 and  $\rho$  exchange in  $\rho^0$  photoproduction; then we obtain the corresponding residues in  $K^*$  photoproduction.

Since a photon does not couple to two  $\rho^{0}$ 's by C invariance, only the A2 contribution need be studied. Maheshwari<sup>17</sup> has used universality and vector dominance to estimate the A2 contribution to  $\rho^0$  photoproduction. Evaluating his results at  $t \approx 0$ , we find that

$$
\tilde{\gamma}_{A2} \cong -\frac{1}{2}\sqrt{\alpha}, \quad \gamma p \to \rho^0 p.
$$

Thus for  $\rho^0$  photoproduction  $\tilde{\gamma}_\rho=0$  and  $\tilde{\gamma}_{A2} \cong -\frac{1}{2}\sqrt{\alpha}$ . Using universality<sup>18</sup> to relate  $\tilde{\gamma}_{\rho}$  and  $\tilde{\gamma}_{A2}$  to  $\gamma_1$ - and  $\gamma_2$ + for  $K^*$  photoproduction, we obtain

$$
\begin{aligned}\n\tilde{\gamma}_1 &= 0, \\
\tilde{\gamma}_2 &= -\frac{1}{4}\sqrt{(2\alpha)}, \qquad \gamma p \to K^{*0}\Sigma^+, \\
\tilde{\gamma}_2 &= \frac{1}{4}\sqrt{\alpha}, \qquad \gamma p \to K^{*+}\Sigma^0, \\
\tilde{\gamma}_2 &= -(1/4\sqrt{3})\sqrt{\alpha}, \quad \gamma p \to K^{*+}\Lambda^0.\n\end{aligned} \tag{12}
$$

We note that  $g_{K^*(1^-) \ge N}$  and  $g_{K^*(1^-) \triangle N}$  are small,<sup>19</sup> so that  $\tilde{\gamma}_1$ - is probably small even though  $K^*$  exchange is not prohibited by  $C$  invariance.

Thus we obtain the prediction [using  $(9)$ ,  $(10)$ ,  $(11)$ , and (12) and setting  $s_0 \approx 1$  BeV<sup>2</sup>

$$
\frac{d\sigma}{d\Omega}\Big|_{\text{e.m.}}
$$
\n
$$
\frac{\text{large } s}{\text{small } t} \xrightarrow{\text{figure } s} \frac{p_f |\tilde{\gamma}|^2 \alpha^2(t) |1 + e^{-i\pi\alpha(t)}|^2 s^{2\alpha(t)}}{16\pi^2 s p_i |t - (m_p + m_Y)^2| \left[t - m_{K^*}^2\right]^4 \sin^2\pi\alpha(t)}
$$

where

$$
\quad\text{and}\quad
$$

$$
\begin{aligned}\n\tilde{\gamma}(t) &= -\frac{1}{4}\sqrt{(2\alpha)} \,, & \gamma p &\rightarrow K^{*0}\Sigma^{+} \\
&= \frac{1}{4}\sqrt{\alpha} \,, & \gamma p &\rightarrow K^{*+}\Sigma^{0} \\
&= -\left(\sqrt{\alpha}\right)/4\sqrt{3} \,, & \gamma p &\rightarrow K^{*+}\Lambda^{0}.\n\end{aligned}
$$

 $\alpha(t) = t + 0.02$ 

15 G. F. Chew, Phys. Rev. Letters 16, 60 (1966).<br><sup>16</sup> F. Cooper, Phys. Rev. Letters 20, 643 (1968).

<sup>17</sup> A. M. Maheshwari, Phys. Rev. 170, 1523 (1968). <sup>18</sup> The coupling at the K<sup>\*</sup> vertex is pure d by C invariance we take the pure f coupling given by universality (Ref. 16) at the barrier of coupling given by universality (Ref. 16) at

the baryon vertex.<br><sup>19</sup> H. Hogaasen and J. Hogaasen, Nuovo Cimento 40A, 560 (1965).

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(13)

176

<sup>1&#</sup>x27;T. L. Trueman and G. C. Wick, Ann. Phys. (N. Y.) 26, <sup>322</sup>

<sup>(1964).&</sup>lt;br><sup>14</sup> As emphasized by Jackson and Hite (Ref. 10),  $d\sigma/d\Omega$  does<br>not have kinematic poles at  $t = m_{K^*}^2$  or  $t = (m_p + m_Y)^2$  (these poles<br>are cancelled when one includes constraint conditions between the amplitudes at these points). Since these two values of t are far from the physical region, however, empirical 6tting will give essentially the same results whether we incorporate the constraint conditions at these two points or not (Ref. 10).



FIG. 1. Theoretical prediction of  $(d\sigma/dt)_{t}/(d\sigma/dt)_{t_{\text{min}}}$  versus eoretical prediction of  $(d\sigma/dt)_t/(d\sigma/dt)$ <br>  $|t|$  for the process  $\gamma + p \rightarrow K^* + \Lambda$ ; s = 10.

The cross section for  $\gamma + \gamma \rightarrow K^* + \Lambda$  (divided by its value in the forward direction) is plotted in Fig. 1.

The density matrix may also be found in our formalism. The density matrix can be expressed in terms of t-channel helicity amplitudes<sup>20</sup> (we go to the  $K^*$  rest frame; the s axis is taken parallel to the incident photon momentum as seen in this frame):

$$
\rho_{mm'} = \sum_{a, b, d} f_{ma; db} t^* f_{m'a; db} t / \sum_{a, b, c, d} |f_{ca; db} t|^2. \tag{14}
$$

Inserting the result (5) into this expression, we obtain the predictions (for large  $s$  and small  $t$ )

$$
\rho_{11} = \rho_{-1-1} = \frac{1}{2}, \quad \rho_{00} = \rho_{m \neq m'} = 0. \tag{15}
$$

These results hold true when exchange of one Regge trajectory is assumed (as was done in this and the

preceding sections). If other trajectories are also permitted, then conspiracy relations arise at  $t=0$ , for example, between parity-conserving amplitudes with  $\lambda$  and  $\mu$  nonzero.<sup>8</sup> Such amplitudes would then have extra factors of  $(1/t)$  (if conspiracy is assumed); these extra factors could effectively restore the powers of s which were removed by the  $\sin \frac{1}{2}\theta_t$  and  $\cos \frac{1}{2}\theta_t$  terms in the definition of  $\tilde{f}^t$ . Thus  $\tilde{f}^{t\pm}$ 's with  $\lambda$  and  $\mu \neq 0$  could become as important as those  $\bar{f}^{t\pm}$ 's with  $\lambda=\mu=0$ . In particular (if  $M=1$  amplitudes are assumed important),  $\rho_{\pm 10}$ ,  $\rho_{0\pm 1}$ , and  $\rho_{00}$  would no longer be required to vanish, and could be of the same magnitude as  $\rho_{11}$  and  $\rho_{-1-1}$ .

The  $K^*$ -decay angular distribution has been written The K<sup>\*</sup>-decay angular distribution has been<br>in terms of  $\rho_{mm'}$ , by Gottfried and Jackson<sup>20,21</sup>:

$$
W(\theta,\phi) = (3/4\pi)(\rho_{00} \cos^2\theta + \rho_{11} \sin^2\theta - \rho_{1-1} \sin^2\theta \cos2\phi -\sqrt{2} \text{ Re}\rho_{10} \sin2\theta \cos\phi). \quad (16)
$$

Hence we directly obtain  $\lceil \text{from } (15) \rceil$  the prediction (for large s and small  $t$ )

$$
W(\theta,\phi) = (3/8\pi)\sin^2\!\theta\,,\tag{17}
$$

where  $\theta$  is the angle made with the z axis in the frame described above.

The only data on  $K^*$  photoproduction give an upper limit<sup>22</sup> of 0.1–0.05  $\mu$ b on the cross section for  $\gamma + p \rightarrow K^{0*}$  $+\Sigma^{+}$ , so no comparison with experiment can be made at the present time.

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<sup>~</sup>K. Gottfried and J. D. Jackson, Nuovo Cimento 33, <sup>309</sup> (1964).

**T.D. Jackson, Rev. Mod. Phys. 37, 484 (1965).**<br>
<sup>22</sup> K. Strauch, in Proceedings of the 1967 International Sym<br>
posium on Electron and Photon Interactions at High Energie (Stanford Linear Accelerator Center, Stanford, Calif. , 1967), p. 257.