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Scalar Nonet, Semileptonic K Decays, and Spectral-Function Sum Rules

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In this paper we show that if one takes $\epsilon(700)$, the broad isoscalar s-wave dipion resonance for which there exists considerable evidence, along with the as yet not firmly established $\eta_V(1071)$ and $\pi_V(1016)$ and a K_V (perhaps the one at 1080), as forming a scalar nonet, then the widths and branching ratio for the decay of these resonances into two pseudoscalar mesons are quite consistent with observation. In addition, the mixing of ϵ and η_V explains the puzzling smallness of the branching ratio $(\eta_V\to\pi\pi)/(\eta_V\to K\bar{K})$, and simultaneously accounts for the K_{e4} axial-vector form factors. Finally, these scalar resonances are found to fit quite well in the framework of current algebra and spectral-function sum rules stemming from $SU(3)\otimes SU(3)$ symmetry.

THERE have been many speculations about the possible existence of scalar-meson multiplets.[1–3] However, there is considerable evidence to support the existence of a broad isoscalar s-wave dipion resonance at $\sim$700 MeV (henceforth referred to as ϵ). In a previous paper[4] it was shown how such a resonance can provide agreement with

(a) Adler's sum rule for pion-pion scattering,
(b) backward pion-nucleon dispersion relations,
(c) the K_1-K_2 mass difference,
(d) Malamud and Schlein's "phase shifts" for pion-pion scattering.

In this paper we show that if one takes $\epsilon(700)$ along with the as yet not firmly established $\eta_V(1071)$, $\pi_V(1016)$, and a K_V (perhaps the one at 1080) as forming a scalar nonet, then the widths and branching ratios for the decay of these resonances into two pseudoscalar mesons is quite consistent with observation. In addition, the mixing of ϵ and η_V explains the puzzling smallness[2] of the branching ratio $(\eta_V\to\pi\pi)/(\eta_V\to K\bar{K})$, and simultaneously accounts for the K_{e4} axial-vector form factors. Finally, these scalar resonances are found to fit quite well in the framework of current algebra and spectral-function sum rules.

Using the symbols S_1 for the $SU(3)$ singlet and S_8, π_V, and K_V for the $(I,Y)=(0,0)$, $(1,0)$, and $(\frac{1}{2},1)$ members of the octet of scalar resonances, the physical η_V and ϵ are given by the combinations

$$\eta_V=S_8\sin\theta+S_1\cos\theta\,,\tag{1a}$$

$$\epsilon=S_8\cos\theta-S_1\sin\theta\,.\tag{1b}$$

The $SU(3)$-symmetric effective interaction Lagrangian is

$$L=g_{\epsilon\pi\pi}\epsilon\boldsymbol{\pi}\cdot\boldsymbol{\pi}+g_{\epsilon K\bar{K}}\epsilon\bar{K}K+g_{\eta_V\pi\pi}\eta_V\boldsymbol{\pi}\cdot\boldsymbol{\pi}+g_{\eta_V\bar{K}K}\eta_V\bar{K}K-\tfrac{1}{10}(\sqrt{30})g\boldsymbol{\pi}_V\cdot(\bar{K}\boldsymbol{\tau}K)+(\sqrt{\tfrac{1}{5}})g\boldsymbol{\pi}_V\boldsymbol{\pi}\eta+\tfrac{1}{10}(\sqrt{30})g(\bar{K}_V\boldsymbol{\tau}K\cdot\boldsymbol{\pi}+\bar{K}\boldsymbol{\tau}K_V\cdot\boldsymbol{\pi})\,,\tag{2}$$

where $\boldsymbol{\pi}$ ($\boldsymbol{\pi}_V$) and K (K_V) designate the π (π_V) isovectors and K (K_V) isospinors, respectively, and

$$g_{\epsilon\pi\pi}=(2\sqrt{2})^{-1}g_0\sin\theta+(\sqrt{5})^{-1}g\cos\theta\,,\tag{3a}$$

$$g_{\epsilon K\bar{K}}=(2\sqrt{2})^{-1}g_0\sin\theta-(2\sqrt{5})^{-1}g\cos\theta\,,\tag{3b}$$

$$g_{\eta_V\pi\pi}=-(2\sqrt{2})^{-1}g_0\cos\theta+(\sqrt{5})^{-1}g\sin\theta\,,\tag{3c}$$

$$g_{\eta_V K\bar{K}}=-(2\sqrt{2})^{-1}g_0\cos\theta-(2\sqrt{5})^{-1}g\sin\theta\,.\tag{3d}$$

There are three parameters g_0, g, and θ to be determined by fitting any three widths for the decay of the scalar resonances into two pseudoscalars. We take the ϵ, η_V, and π_V to be the more firmly established members of the nonet. The existence of a K_V at 1080 MeV is still in doubt and its width is unknown. Fitting to the total widths[5] $\Gamma(\pi_V)=25\pm5$, $\Gamma(\eta_V)=80\pm15$, and $\Gamma(\epsilon)\simeq380$ MeV, we find

$$g_0\simeq8.1m_\epsilon\,,\tag{4a}$$

$$g\simeq3.6m_\epsilon\,,\tag{4b}$$

$$\theta\simeq41^0\,,\tag{4c}$$

where we have taken $m_\epsilon=700$ MeV. These values of the parameter predict the branching ratios:

$$\Gamma(\eta_V\to\pi\pi)/\Gamma(\eta_V\to K\bar{K})\simeq0.6\,,\tag{5}$$

$$\Gamma(\pi_V\to\eta\pi)/\Gamma(\pi_V\to K\bar{K})\simeq2\,.\tag{6}$$

The experimental knowledge of the first branching ratio[5] consists of only an upper bound of 2.5, which is compatible with our result. However, this upper bound would be hard to understand without mixing: if η_V were an unmixed member of the $SU(3)$ octet, this branching ratio would turn out to be $\sim$15.6, and if on the other hand one assigned it to be an unmixed $SU(3)$ singlet one would obtain a branching ratio of $\sim$2.6. The branching ratio for the π_V is experimentally found[5] to be less that 3, again consistent with our assignment. Using the Gell-Mann–Okubo mass formula for the unmixed octet, we predict the mass of K_V to be $\sim$960 MeV with a width $\Gamma_{K_V}\simeq44$ MeV. There have been

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[1] M. Gell-Mann, Physics 1, 63 (1964).
[2] L. E. Evans and T. Fulton, Phys. Rev. 168, 1706 (1968).
[3] K. Watanabe, Nuovo Cimento 51A, 551 (1967).
[4] B. Dutta-Roy and I. R. Lapidus, Phys. Rev. 169, 1537 (1968).
[5] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. 40, 77 (1968).

several evidences[6] for a shoulder in the $K\pi$ spectrum on the tail of the $K^*(890)$ peak [in the neighborhood of 1 BeV, quoted as $K_V(1080)$ in the Rosenfeld tables.[5]] Though the evidence for such a resonance so far is very tentative, further experiments in this difficult region [difficult because of the proximity to the $K^*(890)$ peak] could clarify the situation regarding the existence of the predicted K_V.

A further check of this assignment may be found in a simple model for the K_{e4} decay. We consider the process $K^+ \to \pi^+\pi^-e^+\nu$ to be described by the matrix element of the strangeness-changing axial-vector current

$$\langle\pi^+(p_+)\pi^-(p_-)|A_\mu{}^K|K^+(p)\rangle = (f_+/m_K)(p_++p_-)_\mu + (f_-/m_K)(p_+-p_-)_\mu\,, \quad (7)$$

where f_+ and f_- are dimensionless form factors. The term $f_3(p-p_+-p_-)_\mu$ and the vector-current contributions give unimportant corrections to the K_{e4} rate. We use the partially conserved axial-vector current (PCAC) hypothesis for the strangeness-changing current in the form

$$\partial_\mu A_\mu{}^K(x) = m_K{}^2 f_K \phi_K(x)\,, \quad (8)$$

where ϕ_K is the K-meson field and f_K defined by

$$(2p_0)^{1/2}\langle 0|A_\mu{}^K|K^+(p)\rangle = ip_\mu f_K\,. \quad (9)$$

This enables us to relate K_{e4} to the process $K^+K^- \to \pi^+\pi^-$. Assuming dominance to $K^+K^- \to \pi^+\pi^-$ by the ϵ, η_V, and ρ poles, we find

$$f_+ = \frac{m_K f_K}{(p-p_+-p_-)\cdot(p_++p_-)} \times\left(\frac{g_{\epsilon K^+K^-}g_{\epsilon\pi^+\pi^-}}{(p_++p_-)^2+m_\epsilon{}^2}+\frac{g_{\eta_V K^+K^-}g_{\eta_V\pi^+\pi^-}}{(p_++p_-)^2+m_{\eta_V}{}^2}\right), \quad (10)$$

$$f_- = \frac{2m_K f_K g_{\rho K^+K^-}g_{\rho\pi^+\pi^-}}{(p_++p_-)^2+m_\rho{}^2}\,, \quad (11)$$

where we have neglected the lepton pair energy in their center-of-mass system. The variations of f_+ and f_- with respect to the dipion energy is not very rapid. The ρ contributes only to f_- and the ϵ and η_V only to f_+ because of the antisymmetry and symmetry, respectively, under the exchange of the two pions.

To evaluate f_-, we determine $g_{\rho\pi^+\pi^-}$ from the ρ decay and use universality [or $SU(3)$] to obtain $g_{\rho K^+K^-}$ to get

$$f_- \simeq 1.1\,. \quad (12)$$

To evaluate f_+, we determine the relevant coupling constants using $SU(3)$ and the values of g, g_0, and θ obtained from the widths [Eqs. (4a)–(4c)] to get

$$f_+ \simeq 1.2\,. \quad (13)$$

The finite width of the ϵ effectively suppresses the ϵ contribution by $\sim 10\%$. The above determinations of f_- and f_+ are in very good agreement with their phenomenological determinations,[7] namely,

$$f_- = 1.34 \pm 0.30\,, \quad (14)$$

$$f_+ = 1.19 \pm 0.13\,. \quad (15)$$

In this connection it may be pointed out that if the ϵ were an unmixed singlet, following the above method we would have obtained $f_+ \simeq 3$ in contradiction with experiment. Thus we conclude that the $\epsilon-\eta_V$ mixing supresses in a natural way both the $\pi\pi$ decay mode of η_V (extremely favored by phase space) and also the coupling $\epsilon K\bar{K}$ as required to explain the K_{e4} decay rate.

Another theoretical approach to the properties of these scalar mesons is provided by the current-algebra approach. Matsuda and Oneda,[8] by saturating the matrix element of the charge current commutator $[A^{\pi^+}, V_0{}^{K^0}(x)] = -A_0{}^{K^+}(x)$ between the vacuum and a K^- state by one-particle states in the frame where the momentum $|\mathbf{q}| \to \infty$, using pion PCAC and assuming unsubtracted dispersionr elations for the form factors F_+ and F_- in K_{l3} decay, obtain

$$G_{K^*}g_{K^*K\pi} = m_{K^*}{}^2\left[\frac{f_K}{f_\pi} - \frac{f_S{}^K g_{K_V K\pi}}{m_{K_V}{}^2 - m_K{}^2}\right], \quad (16)$$

where f_π and $f_S{}^K$ are defined by

$$(2p_0)^{1/2}\langle 0|A_\mu{}^\pi|\pi(p)\rangle = if_\pi p_\mu\,, \quad (17)$$

$$(2p_0)^{1/2}\langle 0|V_\mu{}^K|K_V(\mathrm{p})\rangle = if_S{}^K p_\mu\,, \quad (18)$$

and G_{K^*} by

$$2p_0\langle 0|V_\mu{}^K|K^*(p,\epsilon)\rangle = G_{K^*}\epsilon_\mu\,, \quad (19)$$

where ϵ_μ is the polarization of the K^*. The left-hand side of sum rule (16) may be estimated by assuming K^* dominance[9] of the F_+ form factor in K_{l3} decay to give

$$G_{K^*}g_{K^*K\pi}/m_{K^*}{}^2 = -F_+(0)\,. \quad (20)$$

Taking $F_+(0) = -1$, which is the $SU(3)$ value, considered reliable as a consequence of the Ademallo-Gatto theorem, and using Eq. (20), sum rule (16) reduces to

$$g_{K_V{}^0K^+\pi^-} = \left(\frac{f_K}{f_\pi} - 1\right)\frac{m_{K_V}{}^2 - m_K{}^2}{f_S{}^K}\,. \quad (21)$$

To estimate $[(f_K/f_\pi)-1]/f_S{}^K$, we use the spectral function sum rules[10] obtained from $SU(3)\otimes SU(3)$

[6] W. De Baere *et al.*, Nuovo Cimento **51A**, 401 (1967); J. M. Bishop *et al.*, Phys. Rev. Letters **16**, 1069 (1966); B. C. Shen *et al.*, *ibid.* **17**, 726 (1966).

[7] F. A. Berends, A. Donnachie, and G. C. Oades, Phys. Letters **26B**, 109 (1967).

[8] S. Matsuda and S. Oneda, Phys. Rev. **169**, 1172 (1968).

[9] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **18**, 761 (1967).

[10] S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 139 (1967).

saturated by scalar, pseudoscalar, vector, and axial-vector particles to obtain

$$f_K/f_\pi=[2(1-m_\rho^2/m_{K_A}^2)]^{1/2}\simeq 1.1G\,, \qquad (22)$$

$$f_S^K/f_\pi=[2(1-m_\rho^2/m_{K^*}^2)]^{1/2}\simeq 0.73\,. \qquad (23)$$

Using Eqs. (22) and (23), Eq. (21) yields

$$g_{K_V^0K^+\pi^-}\simeq 1.5m_\epsilon\,. \qquad (24)$$

On the other hand, if we use $f_K/f_\pi=1.28$ as obtained experimentally,[11] we obtain

$$g_{K_V^0K^+\pi^-}\simeq 2.6m_\epsilon\,. \qquad (25)$$

These values are to be compared with the value for this coupling constant obtained from the decay widths of the scalar resonances, namely,

$$g_{K_V^0K^+\pi^-}\simeq 2m_\epsilon\,. \qquad (26)$$

In spite of the sensitivity of the factor $(f_K/f_\pi)-1$ to the values of f_K/f_π used and possible deviations of the value of $F_+(0)$ from that given by $SU(3)$ symmetry, the agreement may be taken as an indication that the scalar nonet proposed by us could fit quite well into the framework of current algebra and spectral-function sum rules.

The previous considerations also enable us to obtain the value of the parameter ξ in K_{l3} decay. Following the usual procedure, we have

$$F_+(s)=G_{K^*}g_{K^{*0}K^-\pi^+}/(m_{K^*}^2-s)\,, \qquad (27)$$

$$F_-(s)=-\left(\frac{m_K^2-m_\pi^2}{m_{K^*}^2}\right)F_+(s)+\frac{f_S^K g_{K_VK^+\pi^-}}{m_{K_V}^2-s}\,, \qquad (28)$$

so that

$$\xi\equiv\frac{F_-(0)}{F_+(0)}=-\frac{m_K^2-m_\pi^2}{m_{K^*}^2}+\frac{f_s^K g_{K_VK^+\pi^-}}{m_{K_V}^2}\,. \qquad (29)$$

[11] For the experimental situation regarding f_K/f_π see the rapporteur's talk by N. Cabibbo, in *Proceedings of the Thirteenth Annual International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967).

Using Eq. (21), we thus obtain

$$\xi=-\frac{m_K^2-m_\pi^2}{m_{K^*}^2}+\left(\frac{f_K}{f_\pi}-1\right)\frac{m_{K_V}^2-m_K^2}{m_{K_V}^2}\,. \qquad (30)$$

Depending on whether we use the value for f_K/f_π determined from the sum rules or from experiment, we obtain for ξ the values $\xi=-0.16$ and $\xi=-0.08$, respectively. Unfortunately, the experimental situation regarding ξ is very confused[12]; our results are not inconsistent with the data.

It may also be mentioned that $\epsilon(700)$ in its mixing with η_V is analogous to $f'(1514)$ in its mixing[13] with $f(1260)$ and if $\epsilon(700)$ and $f'(1514)$ are taken to lie on a Regge trajectory a linear plot indicates that $\alpha(0)=-0.5$. It is interesting to note that it has been found by Olsson[14] on the basis of imposing restraints from continuous moment sum rules that the usual two-pole Regge vacuum exchange model is inadequate to account for asymptotic behavior, and a third trajectory with negative intercept $\alpha_{P''}(0)=-0.5$ is needed.

In conclusion, there is strong indication for the existence of a scalar nonet containing $\epsilon(700)$, $\eta(1070)$, $\pi_V(1016)$, and K_V. This assignment has been found to give agreement with the K_{e4} form factors. Also these particles are in general agreement with $SU(3)\otimes SU(3)$ spectral-function sum rules.[15] To clarify the situation, careful measurements of the widths and branching ratio of these resonances must be made and the existing anomaly in the $K\pi$ spectrum in the $K_V(1080)$ region should be reexamined for evidence of the strange member K_V of our nonet.

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[12] For a discussion of the experimental situation see B. G. Kenny, Phys. Rev. Letters **20**, 1217 (1968); **20E**, 1466 (1968).

[13] S. L. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 329 (1965).

[14] M. G. Olsson, University of Wisconsin Report No. C00-167, 1968 (unpublished).

[15] It is interesting to note that L. N. Chang and Y. C. Leung [Phys. Rev. Letters **21**, 122 (1968)] by analyzing K_{l3} decays on the basis of chiral symmetry obtain $m_{K_V}=1050$ MeV and obtain small negative values for ξ. This seems very similar to our conclusion regarding m_{K_V} and ξ.