Meson Bootstraps for Unnatural-Parity States

M. BISHARI, H. R. RUBINSTEIN, A. SCHWIMMER, AND G. VENEZIANO
Weizmann Institute of Science, Rehovot, Israel
(Received 17 May 1968)

In this paper we study the possibility of bootstrapping the unnatural-parity meson states, using finite-energy sum rules. Previous work on this bootstrap scheme was confined to natural-parity states. We find a large number of sum rules involving particles which are not firmly established, with results that are in good agreement with experiment (10%). The equations, though not mathematically consistent, are such that the two sides (Regge and resonances) are equal when t is varied over a large region, and we find several mass formulas, including $m_H^2=2m_\rho^2-\frac{3}{2}m_\pi^2$, $m_B=m_{A_2}-m_\pi$, $2m_B^2=0.5\ \mathrm{BeV}^2+2m_\delta^2+m_\omega^2+m_\pi^2$, and relations between coupling constants.

INTRODUCTION

In the last year, significant progress has been made in the understanding of the relationship between analyticity and Regge asymptotics. After the fundamental paper by De Alfaro, Fubini, Furlan, and Rossetti on superconvergence,[1] it was realized that saturation of superconvergence relations by means of a few one-particle states led to contradictions. Two important developments soon took place: (a) the realization of the importance of the Regge tail[2] and (b) the generalization of the equations for trajectories with high intercept.[3] In particular, Horn and Schmid emphasized the importance of the change in the limit of integration.

In previous work,[4,5] some reactions of the form $P+P\to P+J$ (where P is a pseudoscalar meson and J is a natural-parity state) are considered. The advantages of such a class of reactions has been duly emphasized in Ref. 5. The agreement with experiment is very good in spite of the very rough approximations involved.

For completeness we repeat some of the main points involved in the method, making this paper self-contained. For more details we refer the reader to the aforementioned works.

Here we are concerned with reactions of the form

$$A+B\to A+I\,,$$

where A and B are now scalar or pseudoscalar mesons and I is a vector or axial-vector state.

The introduction of external unnatural-parity states permits coupling to unnatural-parity trajectories, and in this way we hope to learn about their properties. By keeping our interest in inelastic reactions we avoid many problems[6] arising from the $I=0$ trajectories. The main limitations of our work result from the uncertain situation concerning some states from the experimental point of view. Our calculations, however, support both the existence and the assignment of the $H(990, I^G=0^-, J^P=1^+)$, the $\delta(950, I^G=1^-, J^P=0^+)$, and the $B(1210, I^G=1^-, J^P=1^+)$.

In Sec. 1 we study the reactions of the form $P+P\to P+A$ both in the $SU(3)$ limit and in two interesting $SU(2)$ examples: $\pi\pi\to\pi H$ and $\pi\eta\to\pi B$.

In Sec. 2 we study reactions involving the δ meson. We feel that the absence of the other members of the scalar octet at the present time does not justify the $SU(3)$ generalization.

1. REACTIONS OF THE FORM $P+P\to P+A$

We consider first the reaction $P_1+P_2\to P_3+A$, where P_i are members of the pseudoscalar meson octet and the indices label their four-momenta. A is the axial-meson octet of negative charge conjugation whose nonstrange members are presumably H and B. The scattering amplitude can be written as

$$T=ie_\mu[A(\nu,t)(p_2{}^\mu+p_3{}^\mu)+B(\nu,t)(p_2{}^\mu-p_3{}^\mu)]\,,\quad (1)$$

where $\nu=s-u$, $t=(p_2-p_3)^2$, and e_μ is the polarization vector of the vector meson. $SU(3)$ invariance, combined with charge-conjugation restrictions, allows for the following set of $SU(3)$ independent channels:

I, 8_f—8_d; II, 8_d—8_f; III, 10—10; and IV, $\overline{10}$—$\overline{10}$. (2)

The analyticity equations with the assumed form of Regge behavior read[5]:

$$\int_0^{\bar\nu}\nu^m\,\mathrm{Im}A^{(n)}(\nu,t)d\nu=\frac{\beta(t)}{\alpha-n+m+1}\left(\frac{\bar\nu}{\nu_0}\right)^{\alpha-n}\bar\nu^{m+1}\,,\quad (3)$$

where n is the minimum helicity flip in the t channel, ν_0 is the scale factor, and $\alpha(t)$ is the leading trajectory function. The Regge residue function $\beta(t)$ is assumed to be of the form

$$\beta(t)=c/\Gamma(\alpha-n+1)\,.\quad (4)$$

This prescription contains all the ingredients required by analyticity, and the only dynamical assumption

[1] V. De Alfaro, S. Fubini, G. Furlan, and C. Rossetti, Phys. Letters **21**, 576 (1966).
[2] M. Ademollo, H. R. Rubinstein, G. Veneziano, and M. A. Virasoro, Nuovo Cimento, **51A**, 227 (1967).
[3] K. Igi, Phys. Rev. Letters **9**, 76 (1962); A. A. Logunov, L. D. Soloviev, and A. N. Tavkhelidze, Phys. Letters **24B**, 181 (1967); R. Dolen, D. Horn and C. Schmid, Phys. Rev. Letters **19**, 402 (1967); K. Igi and S. Matsuda, *ibid.* **18**, 625 (1967); R. Gatto, *ibid.* **18**, 803 (1967).
[4] M. Ademollo, H. R. Rubinstein, G. Veneziano, and M. A. Virasoro, Phys. Rev. Letters **19**, 1402 (1967); Phys. Letters **27B**, 99 (1968). Related works: S. Mandelstam, Phys. Rev. **166**, 1539 (1968); P. G. O. Freund, Phys. Rev. Letters **20**, 235 (1968); C. Schmid, *ibid.* **20**, 628 (1968).
[5] M. Ademollo, H. R. Rubinstein, G. Veneziano, and M. A. Virasoro, Phys. Rev. **176**, 1904 (1968).
[6] It was suggested recently by H. Harari [Phys. Rev. Letters **20**, 1395 (1968)] that a significant background exists only in reactions to which the Pomeranchuk can couple.

based on simplicity is to assume c to be practically constant in our region of t. (It could have been any entire function.) It is perhaps important to emphasize that this simplest choice can only be achieved if one uses perturbative amplitudes since the use of an helicity decomposition implies restrictions on the residue functions (conspiracy conditions) that cannot be fulfilled by such a simple form.

Our parametrization, as in the case of the natural-parity states, is carried through by means of ν and not s. At this point our choice differs from Mandelstam and Schmid's for example, and gives a rather different t dependence.

The limit of integration is chosen halfway between the last resonance included and the first one left out. This choice is not unique, but its *a posteriori* justification is based mainly on the detailed agreement of the t dependence of Eq. (3).

The saturation problem is attacked as in the previous cases: the resonance side of Eq. (3) is approximated by narrow resonances and the Regge side by one leading trajectory. Third-spectral-function effects are consistently ignored. Channels I and II are controlled respectively by the vector and tensor trajectories while the absence of particles in other than the **8** and **1** representations implies no contribution in the 10 and $\overline{10}$ channels. As we take sum rules that behave the same way (approximately) at high energy, we get the following set of equations:

$$\int_0^{\nu} \nu \,\mathrm{Im}A_{8_fd}d\nu = C_V^{(A)}\Gamma^{-1}(\alpha_V)\times(\alpha_V+1)^{-1}\left(\frac{\bar{\nu}}{\nu_A}\right)^{\alpha_V+1}\nu_A^2, \quad (5)$$

$$\int_0^{\nu} \mathrm{Im}B_{8_fd}d\nu = C_V^{(B)}\Gamma^{-1}(\alpha_V+1)\times(\alpha_V+1)^{-1}\left(\frac{\bar{\nu}}{\nu_B}\right)^{\alpha_V+1}\nu_B, \quad (6)$$

$$\int_0^{\nu} \mathrm{Im}A_{8_fd}d\nu = C_T^{(A)}\frac{\Gamma^{-1}(\alpha_T)}{\alpha_T}\left(\frac{\bar{\nu}}{\nu_B}\right)^{\alpha_T}\nu_B, \quad (7)$$

$$\int_0^{\nu} \nu \,\mathrm{Im}A_{10,\overline{10}}d\nu = 0, \quad (8)$$

$$\int_0^{\nu} \mathrm{Im}B_{10,\overline{10}} = 0. \quad (9)$$

The choice of sum rules is fixed by the crossing properties of the amplitudes that can be read directly from (1).

The resonant side of the foregoing equations receive contributions from the scalar, vector, and tensor octets. Using the narrow-resonance approximation, one can evaluate these contributions by use of standard techniques (see Ref. 5). The definition of the coupling constants is given by extrapolation of the residue functions from the physical region to positive t, at the mass of the particle. This value is not in the region where the sum rule is evaluated, and we have assumed that the extrapolation is possible.

The couplings are easily expressed in terms of the residue at the pole.

We combine Eqs. (5), (6), (8), and (9) to isolate the vector part. Then the equations reads:

$$\frac{(t+m_V^2+\Sigma_V)}{\alpha_V'}\left(\frac{\beta_V^A}{q_V^2}[2(2t+\Sigma_V)\theta_V-q_V^2]+\beta_V^B\frac{2t+\Sigma_V}{\nu_B}\right)=\frac{\beta_V^A\nu_A^2}{2\Gamma(\alpha_V+2)}\left(\frac{\bar{\nu}}{\nu_A}\right)^{\alpha_V+1}, \quad (10)$$

$$\frac{1}{\alpha_V'}\left(\frac{\beta_V^A}{q_V^2}[2(2t+\Sigma_V)\theta_V+3q_V^2]+\beta_V^B\frac{(2t+\Sigma_V)}{\nu_B}\right)=-\frac{\beta_V^B\nu_B}{2\Gamma(\alpha_V+2)}\left(\frac{\bar{\nu}}{\nu_B}\right)^{\alpha_V+1}, \quad (11)$$

$$g_S\frac{m_A^2}{q_S}(m_S+t+\Sigma_S)+\frac{(m_T^2+t+\Sigma_T)}{\alpha_T'}\left[\frac{\beta_T^A\theta_T}{\alpha_T'\nu_Aq_T^2}\left((2t+\Sigma_T)^2-\frac{m_T^2-3m_p^2}{3m_T^2}q_T^2\right)\right.$$
$$+\frac{\beta_T^A}{\alpha_T'\nu_Aq_T^2}((t+m_A^2-m_p^2)\theta_T-2m_A^2(m_A^2-m_p^2))(2t+\Sigma_T)$$
$$\left.+\frac{\beta_T^B}{2\nu_B^2\alpha_T'}\left((2t+\Sigma_T)^2-\frac{m_T^2-4m_p^2}{3m_T^2}q_T^2\right)\right]=\frac{\beta_V^A\alpha_V}{2\Gamma(\alpha_V+2)}\left(\frac{\bar{\nu}}{\nu_A}\right)^{\alpha_V+1}, \quad (12)$$

$$g_S\frac{m_A}{q_S}+\frac{\beta_T{}^A}{\alpha_T{}'\nu_A q_T{}^2}\left[\left((2t+\Sigma_T)^2\theta_T-\frac{m_T{}^2-4m_p{}^2}{3m_T{}^2}q_T{}^2\right)+[\theta_T+(m_p{}^2+\Sigma_T)(\theta_T-2m_A{}^2)-8m_A{}^2m_p{}^2](2t+\Sigma_T)\right]$$
$$+\frac{\beta_T{}^B}{2\nu_B\alpha_T{}'}\left((2t+\Sigma_T)^2-\frac{(m_T{}^2-4m_p{}^2)}{3m_T{}^2}q_T{}^2\right)=-\frac{\beta_V{}^B\nu_B}{2\Gamma(\alpha_V+2)}\left(\frac{\bar{\nu}}{\nu_B}\right)^{\alpha_V+1},\quad(13)$$

where ν_A and ν_B are scale factors $\Sigma_i=m_i{}^2-m_A{}^2-3m_P{}^2$, $\theta_i=(m_i{}^2+m_A{}^2-m_P{}^2)$, and $q_X{}^2=[m_X{}^2-(m_A-m_P)^2]\times[m_X{}^2-(m_A+m_p)^2]$. Equations (10) and (11) are of the pure bootstrap type, a situation analogous to the case of the $PP\rightarrow PV$. As stated before, we impose on our solutions the condition that the two sides of the equations be equal to each other in a given interval of t.

To find a solution, we impose the conditions obtained by the reactions already solved.[4] This implies $\alpha'=(2m_V{}^2-3m_p{}^2)^{-1}$. We require that the equations are exactly satisfied at $\alpha=0$ and $\alpha=-1$. This transforms the equations into an algebraic set and the *a posteriori* justification of the correctness of the procedure is given by verifying good agreement for the whole region in between. The choice of this interval is of course not arbitrary; it is determined by demanding that the first neglected state (3^-) gives a negligible contribution in the whole range.

The solution demands[7] $\nu_A=\nu_B=4m_V{}^2$, a mass formula $m_A{}^2=2m_V{}^2-\frac{3}{2}m_p{}^2$, and the ratio of the couplings g_T and g_L of the vertex VPA given by

$$g_L/g_T=(3m_V{}^4+m_A{}^4+4m_p{}^4-7m_V{}^2m_p{}^2-5m_A{}^2m_p{}^2)/2m_Am_V(2m_V{}^2-3m_p{}^2),\quad(14)$$

where the couplings are defined by the effective Lagrangian

$$\mathcal{L}=[g_Lp_\mu q_\lambda+(g_T/m_H{}^2)(\epsilon^{\lambda\alpha\beta\gamma}\epsilon^{\mu\alpha'\beta'\gamma}p_\alpha q_\beta p_{\alpha'}q_{\beta'})]e_\lambda{}^He_{\mu'}{}^\rho.$$

To check the solution we go to physical processes. We start with $\pi\pi\rightarrow\pi H$. By going through the same procedure as before, we find that the first two equations are identical to the ones obtained in $SU(3)$ except of course for the appearance of physical masses. The trajectory involved is in this case the ρ trajectory. The mass formula is well verified for the H and π (to 10%) and we predict $g_L/g_T=1$. This predicts s-wave distributions for H decay and is in agreement with the values deduced from the $\pi\rho$ sum rules.[8]

Another interesting reaction is $\pi+\eta\rightarrow\pi+B$. The equations can be easily written down. The main differences are (a) that scalar intermediate states can contribute, and (b) that the high-energy behavior is dominated by the A_2 trajectory. The logical candidate for the scalar particle is the $\delta(962)$; however, its coupling to the external states can be neglected.[9]

The resulting equations read (neglecting scalar states and saturating with A_2)

$$\frac{3\cos^2\theta^A-S_{\pi\eta}{}^2S_{\pi B}{}^2}{3\cos^2\theta^B-S_{\pi\eta}{}^2S_{\pi B}{}^2}=\frac{\cos\theta^A}{\cos\theta^B}\frac{\cos\theta^A(m_{A_2}{}^2+m_B{}^2-m_\pi{}^2)-S_{\pi B}{}^2(m_{A_2}{}^2+m_\eta{}^2-m_\pi{}^2)}{\cos\theta^B(m_{A_2}{}^2+m_B{}^2-m_\pi{}^2)+S_{\pi B}{}^2(3m_{A_2}{}^2+m_\pi{}^2-m_\eta{}^2)},\quad(15)$$

where

$$S_{xy}=[m_{A_2}{}^2-(m_x+m_y)^2]^{1/2}[m_{A_2}{}^2-(m_x-m_y)^2]^{1/2},$$

and $\cos\theta_{A,B}$ are the values of $\cos\theta_s$ (where θ_s is the scattering angle in the s channel) for which the A and B amplitudes, respectively, vanish.

The solution to these equations is not unique. However, the following solution exists: $S_{\pi B}=0$, hence $m_{A_2}=m_B+m_\pi$ (notice linear masses), and we predict $g_1/g_2\approx0$, where g_1 and g_2 are the $A_2B\pi$ coupling constants defined by the effective Lagrangian:

$$\mathcal{L}(A_2B\pi)=\epsilon^{*\mu\nu}(A_2)[g_1p_\nu{}^{(\pi)}g_{\mu\sigma}+g_2p_\mu{}^{(\pi)}p_\nu{}^{(\pi)}p_\sigma{}^{(\pi)}]\epsilon^\sigma(B).\quad(16)$$

Improvement of the t Dependence

If the method is successful as a step-by-step approximation as in the $PPPV$ case, one expects that adding more resonances should enlarge the region where the t dependence is well satisfied. If one looks to the first iteration of $\pi\pi-\pi H$ in which only the ρ drives the resonant side, it can be seen that for positive t the agreement deteriorates very badly very soon. In this section we study whether inclusion of further states (3^-) improves the agreement. As discussed in Ref. 5, if the Regge amplitude and the resonant side satisfy a local average, then every extra piece of

[7] Notice that our definition of ν differs by a factor of 4 with respect to that of Ref. 5. Such a factor appears also in the values we found for the scale factors.

[8] The spin-parity of the H was established supposing a pure s-wave decay. Moreover, there are indications from other sum rules that the H has indeed a pure s-wave decay. M. Bishari and A. Schwimmer, Nucl. Phys. (to be published).

[9] The small width of the δ ($\sim$5 MeV) puts an upper limit on its possible couplings to $\pi\eta$. The neglect of δ led to consistent results for other sum rules. F. Gilman and H. Harari, Phys. Rev. **165**, 1803 (1968).

amplitude added must compensate for the Regge part by itself. (This behavior turns out to be so in the case of $\pi\pi \to \pi\omega$.) The new contributions to the two invariant amplitudes are (in $\pi\pi \to \pi H$)

$$A_{(3^-)}=\frac{\beta^A}{\alpha_\rho' \nu_A^2}\left[\frac{5}{2}\frac{(m_R^2+m_H^2-m_\pi^2)(2\alpha+\Pi)^3}{q_R\alpha_\rho'^3}-\frac{3(m_R^2-4m_\pi^2)(m_R^2+m_H^2-m_\pi^2)(2\alpha+\Pi)}{2m_R^2\alpha_\rho'}\right.$$
$$\left.-\frac{m_R^2-4m_\pi^2}{10m_R^2}\left(\frac{5m_R^2(2\alpha+\Pi)^2}{\alpha_\rho'(m_R^2-4m_\pi^2)q_R}-1\right)\left(q_R-\frac{(m_R^2+m_H^2-m_\pi^2)}{\alpha_\rho'}(2\alpha+\Pi)\right)\right]$$
$$+\frac{\beta^B}{\nu_A^3\alpha_\rho'}\left(\frac{5}{6\alpha_\rho'^3}(2\alpha+\Pi)^3-q_R\frac{(m_R^2-4m_\pi^2)(2\alpha+\Pi)}{2\alpha_\rho' m_R^2}\right) \quad (16)$$

and

$$B_{(3^-)}=-A_{(3^-)}-\frac{2}{5}\frac{\beta^A(m_R^2-4m_\pi^2)}{m_R^2\nu_A^2\alpha_\rho'}q_R\left(\frac{5m_R^2(2\alpha+\Pi)^2}{(m_R^2-4m_\pi^2)\alpha_\rho'^2 q_R}-1\right), \quad (17)$$

where

$$\Pi=\alpha_\rho'(3m_\rho^2-3m_\pi^2-m_H^2)$$

and other symbols have been already defined. We extract a function that is constant (to 10%) within the interval of interest to convert the system into an algebraic one. This function is

$$\frac{2}{\Gamma(\alpha+4)}\left(\frac{\alpha+\frac{11}{2}}{1.8}\right)^{\alpha+1}\simeq 1. \quad (18)$$

We have changed slightly our choice of ν_A and ν_B to maximize the smoothness of Eq. (18). The result implies

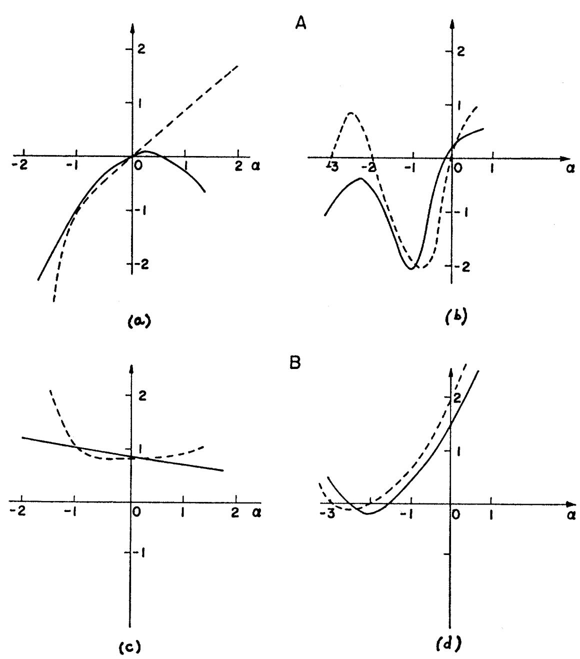

FIG. 1. (a) The t dependence of the two sides of the sum rule for the A amplitude after saturation with ρ. The dashed line represents the Regge contribution and the solid line the resonance contribution. (b) The same as (a) after saturation with R. (c) The same as (a) for the B amplitude. (d) The same as (b) for the B amplitude.

$\nu_A\alpha'=\nu_B\alpha'=1.8$. We plot the two sides of the sum rule in Fig. 1 and find reasonable agreement for

$$\beta_B/\beta_A=-6,$$

and the best mass for H comes out to be 1100 MeV. We defer the detailed discussion of the result for Sec. 3.

2. REACTIONS INVOLVING THE δ MESON

In this section we study reactions involving the δ meson, presumed to be a scalar object. There are two different types of reactions which involve this particle, as is shown below. Here we only study the $SU(2)$ limit because (a) there is little evidence for the other members of an $SU(3)$ multiplet, and (b) the $SU(3)$ restrictions are less powerful in this case since the external octets are not identical.

$$\delta+\pi \to \delta+\omega$$

With the parity angular-momentum restrictions implied by this reaction, the s and u channels can only couple to states of $P=(-)^{J+1}$ and $I^G=1^+$. The t channel is dominated by trajectories having $P=(-)^J$, and hence the logical candidate is the trajectory that for positive t materializes in the ρ meson.

Since the B meson, first contributor to the resonant side, is below threshold, we add the next member of the trajectory directly, as in Sec. 1. We have investigated, however, whether a solution exists for the B alone and found that this is the case (see below). We write our equations without explanation since the procedure is identical to the one Sec. 1.

The amplitude reads:

$$T(\nu,t)=\epsilon_{\mu\nu\alpha\beta}p_1{}^\mu q_2{}^\nu p_2{}^\alpha \epsilon^\beta(q_2)A(\nu,t). \quad (19)$$

The four-momenta are defined by $\delta(p_1)+\pi(q_1)\to\delta(p_2)+\omega(q_2)$ and $\nu=(p_1+p_2)(q_1+q_2)$. The corresponding sum rule is found immediately to be

$$(2m^2-\Sigma+t)+(2M_b^2-\Sigma+t)\left(\frac{1}{10\nu_2^2}\right)\times[\tfrac{5}{16}Y^2(M_b^2,t)-(p^2M_b)q^2(M_b)]=\frac{\alpha_\rho(t)\alpha_\rho'}{4\Gamma(\alpha_\rho+2)}\left(\frac{\bar\nu}{\nu_1}\right)^{\alpha_\rho-1}\bar\nu^2\theta, \quad (20)$$

where

$$\Sigma=2m_\delta^2+m_\omega^2+m_\pi^2,$$

$$Y(M_b^2,t)=2t+M_b^2-\Sigma+\frac{(m_\rho^2-m_\omega^2)(m_\rho^2-m_\pi^2)}{M_b^2}.$$

Here, M_b is the $B(3^+)$ mass, $p(M_b)$ and $q(M_b)$ are the center-of-mass (c.m.) momenta of the initial and final states, respectively, at the $B(3^+)$ pole, and ν_1, ν_2 are the scale factors of the ρ and B trajectories, respectively. Also, $\theta=g_{\rho\pi\omega}g_{\rho\delta\delta}/g_{B\delta\pi}g_{B\omega\delta}$ is a ratio of coupling constants. We assume our ρ trajectory as given from the previous work.[4] If we start by cutting the integration just after the B, we find that the sum rule at $\alpha=0$ reduces to $2m_B^2-\Sigma-0.5=0$. Thus, $m_B^2=1.49$ BeV^2 (experimentally, 1.46 BeV^2). Because of this interesting result the next member of the trajectory must also give a vanishing contribution at the dip point or nearby. This condition imposes a restriction on the mass of the 3^+ meson, and if we assume linear trajectories and impose the condition that α_ρ vanishes at $t=-0.5$, then we get the expression

$$\alpha_B(t)\approx t/m_B^2. \quad (21)$$

This is in agreement with the trajectory used by Barmawi[10] to fit high-energy data. We proceed as before. We cut the integration at the point halfway between the last resonance included and the first left out, and extract a function from the Regge side that equals one in the interval to 10% accuracy. We then get algebraic relations demanding equality of the coefficients of the powers of α on both sides. These restrictions demand (equating α and α^2 coefficients)

$$\nu_1=2.8\text{ BeV}^2,\quad \nu_2=2.6\text{ BeV}^2,\quad \theta=0.55. \quad (22)$$

The last ratio is also determined by equating the α^3 coefficients and turns out to give $\theta=0.45$ in satisfactory agreement with the previous number.

$$\pi+\delta\rightarrow\pi+\omega$$

In these reactions the same states are involved in the respective channels. However, there are two independent couplings $B\pi\omega$. We start by considering the contribution of the B alone.

The scattering amplitude is now

$$T(\nu,t)=[A(\nu,t)P^\mu+B(\nu,t)Q^\mu]\epsilon_\mu(q_1), \quad (23)$$

where $P=p_1+p_2$, $Q=p_1-p_2$, $\pi(p_1)+\omega(q_1)\rightarrow\pi(p_2)+\delta(q_2)$, and the crossing properties of the invariant amplitudes are

$$A(\nu,t)=+A(-\nu,t),\quad B(\nu,t)=-B(-\nu,t), \quad (24)$$

leading to the sum rules

$$\int_0^{\bar\nu}\nu\,\mathrm{Im}A(\nu,t)d\nu=\frac{C_1\alpha_\rho}{\Gamma(\alpha_\rho+2)}\left(\frac{\bar\nu}{\nu_1}\right)^{\alpha_\rho-1}\bar\nu^2, \quad (25)$$

$$\int_0^{\bar\nu}\mathrm{Im}B(\nu,t)d\nu=\frac{C_0}{\Gamma(\alpha_\rho+2)}\left(\frac{\bar\nu}{\nu_1'}\right)^{\alpha_\rho}\bar\nu, \quad (26)$$

where

$$C_1=\tfrac{1}{2}\pi\alpha_\rho'(m_\rho^2)q_t^2(m_\rho)g_T^{\rho(\delta\omega)}g_{\rho(\pi\pi)},\quad C_0=\tfrac{1}{4}\pi\alpha_\rho'(m_\rho^2)\nu_1'\times\left(\frac{(m_\rho^2+m_\omega^2-m_\delta^2)}{2m_\rho^2}g_T^{\rho(\delta\omega)}+g_L^{\rho(\delta\omega)}\right)g_{\rho(\pi\pi)}, \quad (27)$$

and $\nu_1\nu_1'$ are the scale factors of the ρ trajectory in A and B, respectively. Also,

$$q_t^2(m_\rho)=[m_\rho^2-(m_\delta+m_\omega)^2][m_\rho^2-(m_\delta-m_\omega)^2]/4m_\rho^2.$$

By performing the algebra, we find for $\nu A^{(1)}$

$$\nu_B\{X(m_B^2,t)g_L^{B(\pi\omega)}-[4q_S^2(m_B^2+m_\delta^2-m_\pi^2)-(m_B^2+m_\omega^2-m_\pi^2)X(m_B^2,t)]g_T^{B(\pi\omega)}\}g_{B(\pi\delta)}=-(2/\pi)C_1\alpha_\rho/\Gamma(\alpha_\rho+2)(\bar\nu/\nu_1)^{\alpha_\rho-1}\bar\nu^2 \quad (28)$$

and for $B^{(1)}$

$$(X(m_B^2,t)g_L^{B(\pi\omega)}+[4q_S^2(3m_B^2+m_\pi^2-m_\delta^2)+(m_B^2+m_\omega^2-m_\pi^2)X(m_B^2,t)]/2m_B^2g_T^{B(\pi\omega)})g_{B(\pi\delta)}=-(2/\pi)C_0/\Gamma(\alpha_\rho+2)(\bar\nu/\nu_1')^{\alpha_\rho}\bar\nu, \quad (29)$$

where

$$X(m_B^2,t)=2t+m_B^2-\Sigma+(m_\omega^2-m_\pi^2)(m_\delta^2-m_\pi^2)/m_B^2,$$

$$\Sigma=2m_\pi^2+m_\omega^2+m_\delta^2,$$

and

$$q_S^2=[m_B^2-(m_\pi+m_\omega)^2][m_B^2-(m_\pi-m_\omega)^2]/4m_B^2.$$

[10] M. Barmawi, Phys. Rev. Letters **16**, 595 (1966).

Using the explicit formula of the B trajectory determined in the previous example, and cutting at the point corresponding to spin 2, we find by the same method as before

$$\nu_1=\nu_1'=1.9\text{ BeV}^2. \quad (30)$$

Using the condition on the dip of the $\nu_A^{(1)}$, we determine the ratio of the couplings to be

$$g_L{}^{B(\pi\omega)}/g_T{}^{B(\pi\omega)} \approx -1.19. \quad (31)$$

The linear and square terms in α give also conditions that read

$$g_L{}^{B(\pi\omega)}/g_T{}^{B(\pi\omega)} \approx -0.99$$

and

$$g_T{}^{B(\pi\omega)} g_{B(\pi\delta)}/g_T{}^{\rho(\delta\omega)} g_{\rho(\pi\pi)} \approx -2.8. \quad (32)$$

The results are again consistent to within 10% and they are of course testable by analyzing the angular distribution of B decay. This combination predicts a large amount of d wave to be present.

3. CONCLUSIONS

The problems attacked in this paper concern mainly the unnatural-parity states. However, they necessarily involve the normal-parity particles as well. As a result, the consistency of the lowest part of the spectrum of meson trajectories generated by this bootstrap procedure can be tested. The results are in agreement with experiment, if one is satisfied with 10% agreement. Of course these states are not completely well established and some of the couplings cannot yet be tested. Nevertheless the results seem to confirm the general features discovered in previous papers: (a) that saturation can be achieved with a low cutoff after a few resonances, and (b) that reasonable agreement can be achieved in a rather large region of t. One might wonder why the agreement is not as spectacular as in the $\pi\pi \to \pi\omega$ and $\pi\pi \to \pi A_2$ cases.[5] Though we cannot give a precise answer, we feel that it could be that the reactions considered in this paper have some continuum contributions, that the states neglected do contribute, or finally, that higher thresholds make the saturation by a few resonances less reliable.

ACKNOWLEDGMENTS

We would like to thank M. Ademollo and M. A. Virasoro for discussions and correspondence.

PHYSICAL REVIEW VOLUME 176, NUMBER 5 25 DECEMBER 1968

Interacting Rarita-Schwinger Field

GYAN MOHAN
Department of Physics, University of Denver, Denver, Colorado 80210
(Received 23 October 1967)

To avoid the mutual inconsistency between interaction, quantization, and subsidiary conditions of the Rarita-Schwinger spin-$\frac{3}{2}$ field, an additional subsidiary field is introduced in the spirit of the Stückelberg formalism. Relativity is violated in the time development of the subsidiary field, but no observable consequences seem to follow. The unwanted states are removed invariantly from the asymptotic states through a Gupta-Bleuler kind of subsidiary condition. Energy has a positive-definite expectation value in the physical states.

I. INTRODUCTION

THE sixteen-component Rarita-Schwinger field $\Psi_{\mu,\alpha}(x)$ transforms according to the reducible representation of the Lorentz group

$$D^{(\frac{1}{2},\frac{1}{2})}\otimes(D^{(\frac{1}{2},0)}\oplus D^{(0,\frac{1}{2})})=D^{(\frac{1}{2},0)}\oplus D^{(0,\frac{1}{2})}\oplus D^{(1,\frac{1}{2})}\oplus D^{(\frac{1}{2},1)}.$$

It carries two spins of $\frac{1}{2}$ and a spin of $\frac{3}{2}$. The spin-$\frac{1}{2}$ constituents may be taken as

$$i\partial_\mu \Psi_\alpha{}^\mu(x) \quad \text{and} \quad i\epsilon^{\mu\nu\lambda\rho}(\Sigma_{\nu\lambda})_{\alpha\alpha'}\partial_\mu\Psi_{\rho,\alpha'}(x),$$

where, in terms of the usual Dirac γ matrices, $\Sigma_{\mu\nu} = \frac{1}{2}i(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$. In the literature one often finds $\gamma^\mu\Psi_\mu$ in place of the latter spin-$\frac{1}{2}$ constituent. The advantage of our choice lies in the commutability of $\epsilon^{\mu\nu\lambda\rho}\Sigma_{\nu\lambda}\partial_\mu$ with the Dirac operator $i\gamma^\mu\partial_\mu - M$.

In order that the field may represent only the spin $\frac{3}{2}$, one might consider imposing the subsidiary conditions

$$i\partial_\mu\Psi^\mu = 0 \quad (1)$$

and

$$i\epsilon^{\mu\nu\lambda\rho}\Sigma_{\nu\lambda}\partial_\mu\Psi_\rho = 0. \quad (2)$$

These equations, although consistent for the noninteracting case, will conflict with the time development governed by an interaction. There also arises the question of consistency with the quantization postulate.[1] Consequently, the subsidiary condition is formulated in a different way as a selector of physical states from an extra-large Hilbert space.

II. FREE FIELD

We shall adopt a generalization of the Stückelberg formalism[2] to enunciate a suitable subsidiary condition. However, before proceeding further, we wish to examine a free Rarita-Schwinger field $\psi_{\mu,\alpha}(x)$ without the subsidiary condition. We assume the expansion into the annihilation and creation operators of all possible spins,

$$\psi_{\mu,\alpha}(x) = (2\pi)^{-3/2}\int d^3p \left(\frac{M}{p_0}\right)^{1/2} \times\sum_{r,s}\left[a_{r,s}(\mathbf{p})\epsilon_\mu{}^{(r)}(\mathbf{p})u_\alpha{}^{(s)}(\mathbf{p})e^{-ipx} + b_{r,s}{}^\dagger(\mathbf{p})\epsilon_\mu{}^{(r)}(\mathbf{p})v_\alpha{}^{(s)}(\mathbf{p})e^{ipx}\right], \quad (3)$$

[1] K. Johnson and E. C. G. Sudarshan, Ann. Phys. (N. Y.) 13, 126 (1961).