# Effect of the Existence of an Elementary Quark on High-Energy Scattering*†

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An elementary quark, if supposed to exist and to couple in quark-antiquark pairs to ordinary two-particle channels, gives rise to a Regge trajectory which would dominate high-energy scattering at large momentum transfer. A simple model theory embodying these features is presented. Specifically, the model assumes a single scalar-scalar pair to represent the ordinary two-particle channel which is observed, coupled to a spinor-spinor channel of large mass, and simple pole-type potentials approximated in the effective-range method. The Regge trajectory and subtraction generated by this coupling are computed, and arguments are given concerning their contribution to the cross section. Comparison with proton-proton scattering suggests that if such an elementary particle exists, its mass must be greater than about 10 GeV.

## I. INTRODUCTION AND OUTLINE

ATTEMPTS to verify the existence of the quark by its external production in high-energy collisions have to date not met with success.[1] This paper suggests an idea, exemplified here in a simple model, whereby the existence of a massive quark, at least as an internal particle in a collision process, might be made observable. This idea takes its roots in the study of the asymptotic behavior, as generated by the leading Regge singularities of the amplitude, of the scattering of "ordinary" particles (having integral baryonic and electric charge). It is possible that the concept of the elementary quark may be confirmed and better defined should this idea prove useful in explaining ordinary scattering experiments in this high-energy region.

Let us consider a scattering amplitude whose leading Regge trajectories fall below angular momentum $J=0$ for the negative of the squared momentum transfer $t$, less than some given (negative) $t_{(0)}$: The amplitude will thus fall asymptotically to zero for large values of the squared energy $s$. This is apparently an adequate description of a system of "ordinary" composite particles. The same is true of such a system into which are coupled elementary particles of infinite mass, the infinity of the mass effectively negating the coupling. If, however, the mass is finite, coupling to the elementary-particle channel is effected, and there appears a new Regge pole which alters this description. Let us assume these elementary particles to be our ideal of elementarity: the quarks, conforming to the fundamental spin representation, $J=\frac{1}{2}$, and to the fundamental unitary spin representation $\{\mathbf{3}\}$, thus coupling in quark-antiquark pairs to ordinary particle channels. We continue to assume, as the production experiments and the nonrelativistic models suggest, that the quark mass is quite large.

There is a critical difference of character between this new type of trajectory, generated by the introduction of elementary particles, and that type which is already familiar in high-energy scattering, and it is this difference which forms the basis of the idea of this work. The familiar Regge poles fall below $J=0$ for $-t$ large enough—perhaps they fall indefinitely[2]—whereas this new type of trajectory is bounded from below by $J=s_1+s_2-1$, where $s_1$ and $s_2$ are the spins of the elementary particles; i.e., $J=0$ for the quark pair. Why this is so will be argued immediately below, but the consequence is that the new trajectory is the dominant one and parhaps can be seen as actually dominating—its strength or residue being here the point of attention—for energy and momentum transfer large enough. If the behavior of experimental cross sections can be seen to conform with predictions generated by this new trajectory's presence, properties of the elementary particles, the quarks, in particular the mass, can be obtained. If such modification of present Regge theory is not necessary to obtain conformity at the energies and momentum transfers now attainable, we can at least place a lower bound on that mass.

The reason for this difference between the two types of trajectory is, of course, bound to the divergent concepts of the elementary and the composite particle. The presence of a pair of elementary particles gives rise to a term in the partial-wave amplitude proportional to the Kronecker delta $\delta_{s_1+s_2-n,J}$, where $n$ is a positive integer.[3] To see this we first note that, for $J$ a given (half-) integral angular momentum, states having helicity $|M|\leq J$, called sense states, are physical states at this $J$; whereas states having $|M|>J$, nonsense states, are unphysical there. In continuing from high $J$, where all states are physical sense states, we first encounter such unphysical nonsense states at the (half-)integer $J=s_1+s_2-1$ and subsequently at all (half-) integers $J=s_1+s_2-n$, the nonsense (half-)integers. At these values of $J$, the problem of the physical amplitude merely omits such states from consideration, via the Kronecker delta, whereas that of the Regge

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[1] H. Kasha, L. B. Leipuner, T. P. Wangler, J. Alspectro, and R. K. Adair, Phys. Rev. **154**, 1263 (1967); and references therein.
[2] S. Mandelstam, in *Elementary Particle Physics*, edited by G. Takeda and A. Fujii (W. A. Benjamin, Inc., New York, 1967), Part II; Phys. Rev. **166**, 1539 (1968).
[3] S. Mandelstam, Phys. Rev. **137**, 949 (1965).

amplitude, continued to this $J$ value, retains them. A difference between the two solutions is ensured by the Regge continuation's having fictitious elementary particles [the Castillejo-Dalitz-Dyson (CDD) poles] replacing the nonsense states, which are not at all present in the physical partial wave. An immediate consequence of the existence of these Kronecker-delta singularities is a "hardening" of the asymptotic behavior, the requirement of a subtraction when $s_1+s_2-1\geqq 0$. Out of such a singularity moves a Regge trajectory—of this new type—as the coupling to the channel is turned on, approaching as $t\to\infty$ the nonsense (half-)integer from which it emerges, and in the simplest case bounded below by it.

Particles composite in nature will not display such singularities. When a particle can be treated as composite, the Kronecker singularity vanishes in the same way that Mandelstam showed the Amati-Fubini-Stanghellini anomalous cuts to vanish. Thus no subtractions are required if no particles interacting in a system are elementary, and the asymptotic behavior is softened. Any Regge poles which exist are not bounded below and we have the familiar nearly linear trajectories well known in high-energy physics. But just how to treat an external particle as a composite one is not known. Thus in present theoretical schemes, a scalar-scalar theory has a single subtraction and a pole rising from $J=0$. So in the model to be presented here, we have chosen to work with a scalar-scalar theory, one not well realized in practice, but one having its leading trajectory falling through $J=0$, if only to $J=-1$. Had we attempted to work with a theory of a pair of ordinary spinors comparable with the experimentally well-known nucleon-nucleon scattering, already there would be a pole covering the one of particular interest, that new one derived from the coupling of this channel to the quark-antiquark spinor pair, which rises from and remains near to $J=0$.

In Sec. II we will carry out the procedures of calculation entailed in a model which exhibits the above features. This model and those procedures are briefly outlined in the following. We take, as our only "ordinary" particle channel in the scattering, a pair of scalars (because of the difficulty mentioned above) of equal mass, retaining only a minimum number of parameters in this naive model. These are to be coupled to a quark-antiquark pair, represented by much heavier spin-$\frac{1}{2}$ particles in two helicity channels. Simple potentials are chosen and approximated by an effective-range type method. The $N/D$ method, which now reduces from integral equations to numerical equations, is used to obtain the partial-wave amplitude, whose poles in $J$, $\alpha_i(t)$, and reduced residues in the ordinary particle channels, $\gamma_i(t)$, are found. Returning to the scattering amplitude by a Sommerfeld-Watson transformation, we obtain the large-energy amplitude and cross section in the crossed ($s$) channel. In the concluding part of Sec. II, the subtraction term is discussed. Finally, in Sec. III, comparison of the behavior of this imagined scalar pair scattering with experimental nucleon-nucleon scattering, for example, places an order-of-magnitude lower limit on the mass of the quark.

## II. THE MODEL

In the first part (A) of this section, we will set up the problem, proceed through the solution of the $N/D$ equations in the $t$ channel, and comment briefly on the Sommerfeld-Watson transformation. In the second (B), we shall find the equations describing the $t$ channel poles, and their residues and contributions to the asymptotic $s$-channel amplitude. In the third (C), the subtraction term will be discussed.

### A. $N/D$ Solution

#### *1. Unitarity, the Right-Hand Cut*

To begin, we write down the $t$-channel unitarity equation for the amplitude $M$, normalized appropriately:

$$\frac{M_{ba}-M_{ba}^{\dagger}}{2i}=\sum_c\int\frac{d\Omega\, c}{4\pi}M_{bc}^{\dagger}\times\left(\frac{2p_c}{t^{1/2}}\theta[t-(M_{c1}+M_{c2})^2]\right)M_{ca}\,,\quad\text{(2A1)}$$

in terms of which the cross section is

$$\frac{d\sigma_{ba}}{d\Omega_b}=\frac{p_b}{p_a}\left|\frac{M_{ba}}{E}\right|^2.\quad\text{(2A2)}$$

Most notations are the customary ones. The particle channels in the problem to be considered are three:

(1) a pair of "ordinary" particles which in the model are likened to two pions, having no spin, mass $\mu$, and three-momentum $q$, but which are to be compared in the final analysis with some particles whose scattering is better known, for instance, the nucleons;

(2) a helicity-zero state; and

(3) a helicity-one state, of a pair of spin-$\frac{1}{2}$ antiparticle conjugates of mass $M$ and three-momentum $p$ which are treated as nucleon-antinucleon in the model and which are finally comparable with the very heavy quarks.

We are concerned here only with the dynamical aspects of the model; no isotopic spin (or unitary spin) considerations are put forward.

The angular momentum projection of $M$ in helicity states is

$$m_{ba}{}'^{J}=\frac{1}{2}\int_{-1}^{1}dz\, d_{\lambda_a\lambda_b}{}^J(\theta)M_{ba}(t,z)\,.\quad\text{(2A3)}$$

For spinless particles, the factors needed to remove threshold kinematic singularities are well known:

$$m_{11}{}^{J}=q^{-2J}m_{11}{}'^{J}=\frac{1}{2\pi}\int_{s_0}^{\infty}ds\,(q^2)^{-J-1}Q_J\left(1+\frac{s}{2q^2}\right)[M_{11}]_s. \quad (2A4)$$

For particles with spin, further kinematic singularities must be removed. From Frazer and Fulco we find[4]

$$m_{21}{}'^{J}=\frac{2}{p}\left(-p^2A_J+\frac{M}{2J+1}(pq)[(J+1)B_{J+1}-JB_{J-1}]\right) \quad (2A5a)$$

and

$$m_{31}{}'^{J}=\frac{2E}{p}(pq)\frac{[J(J+1)]^{1/2}}{2J+1}(B_{J-1}-B_{J+1}). \quad (2A5b)$$

Note that certain linear combinations of these amplitudes are those of defined parity and orbital angular momentum, not these quantities themselves. Writing $a_J=(pq)^{-J}A_J$, etc., for the projected invariant amplitudes with threshold kinematics removed, we see that

$$m_{21}{}^{J}=\frac{p}{M}(pq)^{-J}m_{21}{}'^{J}=2\left(-\frac{p^2}{M}a_J+\frac{1}{2J+1}\times[(J+1)(pq)^2b_{J+1}-Jb_{J-1}]\right) \quad (2A6a)$$

and

$$m_{31}{}^{J}=\frac{p}{E}(pq)^{-J}m_{31}{}'^{J}=2\frac{[J(J+1)]^{1/2}}{2J+1}\times[b_{J-1}-(pq)^2b_{J+1}] \quad (2A6b)$$

are free of kinematic singularities, and that in terms of these amplitudes the unitarity equation becomes an equation of the discontinuity across the right-hand cut:

$$\frac{m_{ba}{}^{J+}-m_{ba}{}^{J-}}{2i}=\sum_c m_{bc}{}^{J\pm}\rho_c{}^{J}m_{ca}{}^{J\mp},\quad\text{or}\quad [m^{-1}]_R=-\rho^J, \quad (2A7a)$$

where $\rho_c{}^J$ is a diagonal matrix with elements

$$(\rho^1,\rho^2,\rho^3)=\left(\frac{q^{2J+1}}{E}\theta(t-4\mu^2),\frac{M^2p^{2J-1}}{E}\theta(t-4M^2),\; p^{2J-1}E\theta(t-4M^2)\right). \quad (2A7b)$$

### 2. *Potential, the Left-Hand Cut*

The potential is chosen in the usual simplistic fashion: double-spectral contributions are ignored, and only the poles are saved (see Fig. 1):

$$[M_{11}]_s=(g\mu)^2\pi\delta(s-\mu^2), \quad (2A8a)$$

[4] W. R. Frazer and J. R. Fulco, Phys. Rev. 117, 1603 (1960).

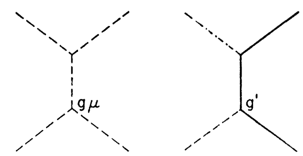


FIG. 1. The potential chosen in the model.

$$[B]_s=(g')^2\pi\delta(s-M^2). \quad (2A8b)$$

We have assumed a scalar type of coupling for the imagined pions, with coupling constant $g\mu$ ($\mu$ inserted for convenience). In the coupling of the spin-$\frac{1}{2}$ particles to the spinless one, we assume that the coupling constant $g'$ defined in analogy with that of nucleons to the pion remains of order $M^0=1$ as the mass $M$ becomes large; how to modify this assumption for other behavior in this mass parameter is obvious. It is also quite easy to modify the assumption made in the interest of economy of parameters that the masses of the exchanged particles are the same as those of the external "pion" and "nucleon."

The corresponding integrals over the left-hand cuts, the potential in the $N/D$ method, are therefore

$$\{m_{11}{}^J\}_L=g^2\tfrac{1}{2}\mu^2(q^2)^{-J-1}Q_J(z_\pi), \quad (2A9a)$$

$$\{m_{21}{}^J\}_L=g'^2(pg)^{-J}zQ_J(z), \quad (2A9b)$$

and

$$\{m_{31}{}^J\}_L=-g'^2\frac{1}{[J(J+1)]^{1/2}}(pq)^{-J}(z^2-1)Q_J{}'(z), \quad (2A9c)$$

where

$$z_\pi=1+\frac{\mu^2}{2q^2}\quad\text{and}\quad z=\frac{p^2+q^2+M^2}{2pq}. \quad (2A9d)$$

Now each part of the potential is approximated by a pole, a familiar scheme called the effective-range approximation. The method we have chosen in to find the position and residue of the pole for all $J$ by fitting at the channel-1 threshold ($q^2\to 0$) to the value and derivative of the original form. For examples, see Table I. So we have that

$$\tfrac{1}{2}\mu^2(q^2)^{-J-1}Q_J(z_\pi)$$

TABLE I. Original forms and approximations of the potential.

| | Original form | Approximating form |
|---|---|---|
| | $\frac{1}{2}\mu^2(q^2)^{-J-1}Q_J(z_\pi)$ | $\dfrac{a}{t-t_a}$ |
| $(\ )\vert_{q^2=0}$ | $\frac{1}{2}(\mu^2)^{-J}\pi^{1/2}\dfrac{\Gamma(J+1)}{\Gamma(J+\frac{3}{2})}$ | $\dfrac{a}{4\mu^2-t_a}$ |
| $\dfrac{d}{dt}(\ )\vert_{q^2=0}$ | $\frac{1}{2}(\mu^2)^{-J}\pi^{1/2}\dfrac{\Gamma(J+1)}{\Gamma(J+\frac{3}{2})}\left(-\dfrac{J+1}{2\mu^2}\right)$ | $-\dfrac{a}{(4\mu^2-t_a)^2}$ |

is approximated by

$$\frac{2\pi^{1/2}\Gamma(J+1)}{(\mu^2)^{J-1}\Gamma(J+\frac{3}{2})}\frac{\frac{1}{2}}{J+1}\frac{1}{4\mu^2\{x-1+[\frac{1}{2}/(J+1)]\}},$$

where $x=t/4\mu^2$. The other parts of the potential are approximated in a similar fashion.

In summary, we have approximated the potential by

$$\{m^J\}_L=V^J(t)=G\left[\frac{a_1}{t-t_1}\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}+\frac{a_2}{t-t_2}\begin{bmatrix}0&1&0\\1&0&0\\0&0&0\end{bmatrix}+\frac{a_3}{t-t_3}\begin{bmatrix}0&0&1\\0&0&0\\1&0&0\end{bmatrix}\right]=G\sum_{i=1}\frac{a_i v_i}{t-t_i},\qquad(2A10a)$$

where

$$G=4\mu^2g^2\left(\frac{1}{2}\frac{\pi^{1/2}\Gamma(J+1)}{\mu^{2J}\Gamma(J+\frac{3}{2})}\right);\qquad(2A10b)$$

$$a_1=\frac{\frac{1}{2}}{J+1},\quad x_1=\frac{t_1}{4\mu^2}=1-\frac{\frac{1}{2}}{J+1}=\frac{2J+1}{2(J+1)};\qquad(2A10c)$$

$$a_2=\left(\frac{g'}{g}\right)^2\frac{(\frac{3}{2}+J)\lambda^2}{(J+1)(J+2)+(J^2-2)\lambda^2},$$

$$x_2=\frac{t_2}{4\mu^2}=1-\frac{(\frac{3}{2}+J)\lambda^2}{(J+1)(J+2)+(J^2-2)\lambda^2};\qquad(2A10d)$$

and

$$a_3=\left(\frac{g'}{g}\right)^2\frac{(\frac{3}{2}+J)\lambda^2}{J[J+1+(3J+5)\lambda^2]}\left(\frac{J+1}{J}\right)^{1/2},$$

$$x_3=\frac{t_3}{4\mu^2}=1-\frac{(\frac{3}{2}+J)\lambda^2}{J[J+1+(3J+5)\lambda^2]};\qquad(2A10e)$$

and where $\lambda=\mu/M$ is the anticipated small ratio of the masses.

### *3. ND Solution*

We have now discussed the quantities which are to be put into the $N/D$ method for $m^J$: Eqs. (2A7) and (2A10), where $m^J=ND^{-1}$. The primary purpose in approximating the potential terms by poles is that the integral equation for $N$ now reduces to a numerical equation. Inserting Eq. (2A10a) into this equation for $N$,

$$N(t)=V(t)+\frac{1}{\pi}\int_R dt'\frac{V(t')-V(t)}{t'-t}\rho(t')N(t'),\qquad(2A11)$$

we may write the solution

$$N(t)=G\sum_{i=1}^{3}\frac{a_i\nu_i}{t-t_i},\qquad(2A11')$$

with the numerical system of equations for the matrices $\nu_i$,

$$\nu_i=v_i(1-\sum_{j=1}^{3}\rho_{ij}a_j\nu_j)\text{ or }\sum_{j=1}^{3}(\delta_{ij}+v_i\rho_{ij}a_j)\nu_j=v_i.\qquad(2A12)$$

This is a system of $3\times3$ matrices in the pole index, the elements of which are $3\times3$ in the channel number. We define here a matrix whose symmetrical elements in the pole subscript space are

$$\rho_{ij}=\frac{G}{\pi}\int_R dt'\frac{\rho(t')}{(t'-t_i)(t'-t_j)}=\frac{1}{t_i-t_j}\left(\frac{G}{\pi}\int_R dt'\frac{\rho(t')}{t'-t_i}-\frac{G}{\pi}\int_R dt'\frac{\rho(t')}{t'-t_j}\right)=\frac{\rho_i-\rho_j}{t_i-t_j},\qquad(2A13a)$$

whose diagonal elements in the channel number space are

$$(\rho_{ij})_{\alpha\beta}=\delta_{\alpha\beta}\rho_{ij}{}^{\alpha}=\delta_{\alpha\beta}\frac{\rho_i{}^{\alpha}-\rho_j{}^{\alpha}}{t_i-t_j},\qquad(2A13b)$$

where

$$(\rho_i)_{\alpha\beta}=\delta_{\alpha\beta}\rho_i{}^{\alpha}=\delta_{\alpha\beta}\frac{G}{\pi}\int_R dt'\frac{\rho^{\alpha}(t')}{t'-t_i}.\qquad(2A13c)$$

Note that in the numbers $\rho_{ij}{}^{\alpha}$ or $\rho_i{}^{\alpha}$, the subscripts denote the index of the approximating pole and the superscripts, the channel number. Explicitly these quantities are

$$\rho_i{}^1=-4\mu^2\frac{g^2}{\pi}\frac{\pi}{\sin\pi J}F(1,-J;\tfrac{3}{2};x_i),\qquad(2A14a)$$

$$\rho_i{}^2=+4\mu^2\frac{g^2}{\pi}\frac{\pi}{\sin\pi J}\lambda^{-2J}\left(\frac{J}{J+\frac{1}{2}}\right)\times F(1,1-J;\tfrac{3}{2};x_i\lambda^2),\qquad(2A14b)$$

and

$$\rho_i{}^3=-4\mu^2\frac{g^2}{\pi}\frac{\pi}{\sin\pi J}\lambda^{-2J}\left(\frac{J}{J+\frac{1}{2}}\right)\times F(1,-J;\tfrac{1}{2};x_i\lambda^2).\qquad(2A14c)$$

$F(a,b;c;x)$ is the hypergeometric function.

The system (2A12), which is $9\times9$, is immediately reduced to a $5\times5$ system by virtue of the fact that four rows of $v_i$ are zero, and is solved in terms of the $3\times3$ determinant $d$,

$$d=\begin{vmatrix}1+a_1\rho_{11}{}^1 & -a_2{}^2\rho_{12}{}^1\rho_{22}{}^2 & -a_3{}^2\rho_{13}{}^1\rho_{33}{}^3\\ a_1\rho_{12}{}^1 & 1-a_2{}^2\rho_{22}{}^1\rho_{22}{}^2 & -a_3{}^2\rho_{23}{}^1\rho_{33}{}^3\\ a_1\rho_{13}{}^1 & -a_2{}^2\rho_{23}{}^1\rho_{22}{}^2 & 1-a_3{}^2\rho_{33}{}^1\rho_{33}{}^3\end{vmatrix},\qquad(2A15)$$

and its cofactor matrix elements $d_{ij}$ [the determinant derived from $d$ in this form by setting the $i$th row and

$j$th column to zero, except for the $(i,j)$ element, which is set to one]. Thus it is found that

$$\nu_1=\frac{1}{d}\begin{bmatrix} d_{11}+d_{21}+d_{31} & (a_2/a_1)d_{12} & (a_3/a_1)d_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{2A16a}$$

$$\nu_2=\frac{1}{d}\begin{bmatrix} -a_2\rho_{22}{}^2(d_{12}+d_{22}+d_{32}) & d_{22} & (a_3/a_2)d_{23} \\ d_{12}+d_{22}+d_{32} & (d-d_{22})/(a_2\rho_{22}{}^2) & -d_{32}/(a_3\rho_{33}{}^3) \\ 0 & 0 & 0 \end{bmatrix}, \tag{2A16b}$$

and

$$\nu_3=\frac{1}{d}\begin{bmatrix} -a_3\rho_{33}{}^3(d_{13}+d_{23}+d_{33}) & (a_2/a_3)d_{32} & d_{33} \\ 0 & 0 & 0 \\ d_{13}+d_{23}+d_{33} & -d_{23}/(a_2\rho_{22}{}^2) & (d-d_{33})/(a_3\rho_{33}{}^3) \end{bmatrix}. \tag{2A16c}$$

The denominator function $D(t)$ is found by merely integrating:

$$D(t)=1-\frac{1}{\pi}\int_R dt'\frac{\rho(t')}{t'-t}N(t'), \tag{2A17}$$

which is

$$D(t)=1-\sum_{i=1}^{3} a_i\rho_{0i}\nu_i \tag{2A17'}$$

$$=\frac{1}{d}\begin{bmatrix} d-a_1\rho_{10}{}^1(d_{11}+d_{21}+d_{31}) +a_2{}^2\rho_{20}{}^1\rho_{22}{}^2(d_{12}+d_{22}+d_{32}) +a_3{}^2\rho_{30}{}^1\rho_{33}{}^3(d_{13}+d_{23}+d_{33}) & -a_2(\rho_{10}{}^1d_{12}+\rho_{20}{}^1d_{22}+\rho_{30}{}^1d_{32}) & -a_3(\rho_{10}{}^1d_{13}+\rho_{20}{}^1d_{23}+\rho_{30}{}^1d_{33}) \\ -a_2{}^2\rho_{20}{}^2(d_{12}+d_{22}+d_{32}) & d-(\rho_{20}{}^2/\rho_{22}{}^2)(d-d_{22}) & (a_3/a_2)(\rho_{20}{}^2/\rho_{22}{}^2)d_{23} \\ -a_3{}^2\rho_{30}{}^3(d_{13}+d_{23}+d_{33}) & (a_2/a_3)(\rho_{30}{}^3/\rho_{33}{}^2)d_{32} & d-(\rho_{30}{}^3/\rho_{33}{}^3)(d-d_{33}) \end{bmatrix}, \tag{2A17''}$$

where we use the subscript 0 to denote $t_i=t=t_0$. Its determinant is found to be

$$|D|=\bar{D}/d,$$

$$\bar{D}=\begin{vmatrix} 1+a_1(\rho_{11}{}^1-\rho_{10}{}^1) & -a_2{}^2(\rho_{12}{}^1-\rho_{20}{}^1)(\rho_{22}{}^2-\rho_{20}{}^2) & -a_3{}^2(\rho_{13}{}^1-\rho_{30}{}^1)(\rho_{33}{}^3-\rho_{30}{}^3) \\ a_1(\rho_{12}{}^1-\rho_{10}{}^1) & 1-a_2{}^2(\rho_{22}{}^1-\rho_{20}{}^1)(\rho_{22}{}^2-\rho_{20}{}^2) & -a_3{}^2(\rho_{23}{}^1-\rho_{30}{}^1)(\rho_{33}{}^3-\rho_{30}{}^3) \\ a_1(\rho_{13}{}^1-\rho_{10}{}^1) & -a_2{}^2(\rho_{23}{}^1-\rho_{20}{}^1)(\rho_{22}{}^2-\rho_{20}{}^2) & 1-a_3{}^2(\rho_{33}{}^1-\rho_{30}{}^1)(\rho_{33}{}^3-\rho_{30}{}^3) \end{vmatrix}. \tag{2A18}$$

In order to eliminate factors of $d$, we define

$$N=\mathfrak{N}/d, \tag{2A19}$$

and $\mathfrak{D}^{-1}$ to be the cofactor matrix of $D$; i.e., $D^{-1}=\mathfrak{D}^{-1}/|D|$; hence

$$D=(\bar{D}/d)\mathfrak{D} \tag{2A17'''}$$

and

$$m^J=\mathfrak{N}\mathfrak{D}^{-1}/\bar{D}. \tag{2A20}$$

If $\lambda$ is small, or zero, due to large, or infinite, quark mass, mass, and $J$ is not near zero, a good approximation is $a_2=a_3=0$. Then

$$\bar{D}=1+a_1(\rho_{11}{}^1-\rho_{10}{}^1) \tag{2A18'}$$

and

$$(\mathfrak{N}\mathfrak{D}^{-1})_{11}=\frac{G}{4\mu^2}\frac{a_1}{x-x_1}. \tag{2A21a}$$

If $\lambda$ is small, and if $J$ is near zero, the approximation to be made is $a_2=0$, and in this case

$$\bar{D}=\begin{vmatrix} 1+a_1(\rho_{11}{}^1-\rho_{10}{}^1) & -a_3{}^2(\rho_{13}{}^1-\rho_{30}{}^1)(\rho_{33}{}^3-\rho_{30}{}^3) \\ a_1(\rho_{13}{}^1-\rho_{10}{}^1) & 1-a_3{}^2(\rho_{33}{}^1-\rho_{30}{}^1)(\rho_{33}{}^3-\rho_{30}{}^3) \end{vmatrix}$$

and

$$(\mathfrak{N}\mathfrak{D}^{-1})_{11}=\left[\frac{G}{4\mu^2}\frac{a_1}{x-x_1}-a_3{}^2(\rho_{33}{}^3-\rho_{30}{}^3)\left(\frac{1}{x-x_3}+\frac{a_1(\rho_{33}{}^1-\rho_{13}{}^1)}{x-x_1}+\frac{a_1(\rho_{11}{}^1-\rho_{13}{}^1)}{x-x_3}\right)\right]. \tag{2A21b}$$

This approximation of large quark mass is taken in the following as an assumption to be verified by the final results, in comparison with experiment, and will be seen then to be justified.

### 4. *Sommerfeld-Watson Transform*

The procedure to be followed from here on is well known. One solves the equation $\bar{D}=0$ for the Regge poles, $J=\alpha_i(t)$, and their reduced residues,

$$\gamma_i(t)=\frac{(\mathfrak{N}\mathfrak{D}^{-1})_{11}}{\partial\bar{D}/\partial J}\bigg|_{J=\alpha_i(t)}, \qquad (2A22)$$

so that by the Sommerfeld-Watson transform the amplitude is, for large $s$,

$$M=\sum_i -2\pi^{1/2}\frac{\Gamma(\alpha_i+\frac{3}{2})}{\Gamma(\alpha_i+\frac{1}{2})}\frac{\gamma_i(-s)^{\alpha_i}}{\sin\pi\alpha_i}.$$

A simplifying factorization is made by defining

$$\gamma_i=\left[\frac{1}{2}\frac{\pi^{1/2}}{\mu^{2\alpha_i}}\frac{\Gamma(\alpha_i+1)}{\Gamma(\alpha_i+\frac{3}{2})}\right]\bar{\gamma}_i, \qquad (2A23)$$

this factor being included in $\mathfrak{N}$ via the factor in $G$, from Eq. (2A10b). With this inserted,

$$M=\sum_i-\frac{\pi}{\sin\pi\alpha_i}\bar{\gamma}_i\left(-\frac{s}{\mu^2}\right)^{\alpha_i}=\sum_i-\pi\bar{\gamma}^i\left(\frac{s}{\mu^2}\right)^{\alpha_i}(\cot\pi\alpha_i+1)$$
$$=\sum_i M_i. \qquad (2A24)$$

We note in passing that for a pole whose position and residue are small and of the same order of magnitude, the contribution to $M$ is

$$M_i=-\frac{\bar{\gamma}_i}{\alpha_i}-\bar{\gamma}_i[\ln(s/\mu^2)+i\pi]. \qquad (2A24')$$

By crossing, $M$ is the Reggeized asymptotic amplitude for scattering in the crossed ($s$) channel, for which

$$\frac{d\sigma}{dt}=\frac{4\pi}{sq_s^2}|M|^2. \qquad (2A25)$$

## B. Regge Poles, Residues, and Asymptotic Contributions

### 1. *Uncoupled Case*

We first study the case of infinite quark mass for which $M=\infty$, or $\lambda=a_2=a_3=0$. The pole to be found in this case will, of course, be present in the finite mass case, but perturbed by the small but finite $\lambda$. Here, setting Eq. (2A18′) to zero, we have the equation of the trajectory $J=\alpha_{\rm I}(x)$. Explicitly, this is

$$0=1+\frac{g^2}{\pi}\frac{\pi}{\sin\pi\alpha_{\rm I}}\frac{\frac{1}{2}}{\alpha_{\rm I}+1}$$
$$\times\left(\frac{F(1,-\alpha_{\rm I};\frac{3}{2};x)-F(1,-\alpha_{\rm I};\frac{3}{2};x_1)}{x-x_1}-F'(1,-\alpha_{\rm I};\tfrac{3}{2};x_1)\right), \qquad (2B1)$$

with $x_1=x_1(\alpha_{\rm I})=1-[\frac{1}{2}/(\alpha_{\rm I}+1)]$. Since $F(1,-\alpha;\frac{3}{2};x)$ does not take a closed form except for $\alpha$ equal to any half-integer, for simplicity we shall find the pole positions in $x$ for $\alpha_{\rm I}=-1,-\frac{1}{2}$, and 0 and interpolate.

At $\alpha_{\rm I}=-1$, $x$ approaches $-\frac{1}{2}/(\alpha_{\rm I}+1)$. (2B2)

At $\alpha_{\rm I}=-\frac{1}{2}$, Eq. (2B1) becomes

$$1+\left(\frac{1}{3}+\frac{1}{g^2}\right)x=F(1,\tfrac{1}{2};\tfrac{3}{2};x)=\frac{1}{2x^{1/2}}\ln\left(\frac{1+x^{1/2}}{1-x^{1/2}}\right). \qquad (2B3)$$

Thus, numerically or graphically, one can find $x_{(-1/2)}$, the pole position in energy for angular momentum $\alpha_{\rm I}=-\frac{1}{2}$ for any coupling $g$. At this limits, $x_{(-1/2)}=1$ for $g=0$ and $x_{(-1/2)}=0$ for $g=\infty$.

At $\alpha_{\rm I}=0$, the 0/0 form is easily reduced, since

$$F(1,-\alpha;\tfrac{3}{2};x)=1-\tfrac{2}{3}\alpha xF(1,1;\tfrac{5}{2};x)$$
$$=1-2\alpha[1-\mu(x)] \qquad (2B4)$$

near $\alpha=0$, and

$$\mu(x)=\left(\frac{1-x}{-x}\right)^{1/2}\ln[(-x)^{1/2}+(1-x)^{1/2}]$$
$$=\left(\frac{1-x}{x}\right)^{1/2}\arcsin(x^{1/2}). \qquad (2B5)$$

Equation (2B1) at $\alpha_{\rm I}=0$ is thus

$$\tfrac{1}{4}\pi+\left(1-\tfrac{1}{2}\pi-\frac{1}{(g^2/\pi)}\right)(x-\tfrac{1}{2})=\mu(x). \qquad (2B6)$$

Again a numerical or graphical solution is easily obtained for $x_{(0)}$. The range in $x$ of the pole along $J=0$ with $g$ varying is from $\frac{1}{2}$ at $g=0$ to 1 at $g^2/\pi=1$, and there is no solution for $g$ smaller than this.

Assembling the information from these three values of $\alpha_{\rm I}$ [for $(g^2/\pi)\geqslant 1$] and interpolating, we take for the Regge pole the form

$$x=x_{\rm I}(J)=\frac{a_{\rm I}}{J+1}+b_{\rm I}+c_{\rm I}(J+1), \qquad (2B7)$$

where

$$a_{\rm I}=-\tfrac{1}{2},\quad b_{\rm I}=\tfrac{3}{2}-x_{(0)}+2x_{(-1/2)},$$
$$\text{and}\quad c_{\rm I}=2[x_{(0)}-x_{(-1/2)}]-1. \qquad (2B7')$$

Inversely, we have

$$J=\alpha_{\rm I}(x)=-1+\frac{x-b_{\rm I}+[(x-b_{\rm I})^2-4a_{\rm I}c_{\rm I}]^{1/2}}{2c_{\rm I}}. \qquad (2B8)$$

Thus $\bar{D}$ may be written

$$\bar{D}=\frac{x-x_{\rm I}(J)}{x_{\rm I}(J)-x_{\rm I}(J)}, \qquad (2B9)$$

since $\bar{D}=0$ on the trajectory $x=x_{\rm I}(J)$, and $\bar{D}=1$ at $x=x_{\rm I}(J)$. Hence one may compute

$$\partial\bar{D}/\partial J\,|_{J=\alpha_{\rm I}}$$

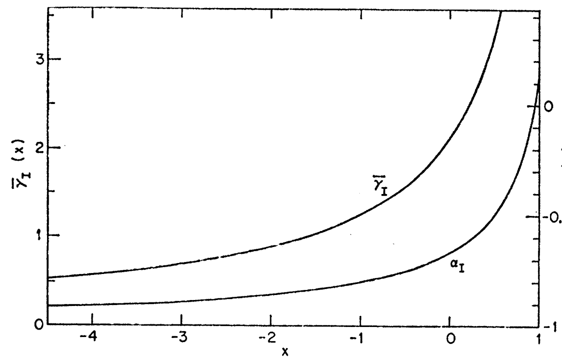


FIG. 2. $\alpha_{\mathrm{I}}(x)$ and $\bar{\gamma}_{\mathrm{I}}(x)$ for $g^2/\pi=2$.

and from Eq. (2A21a) $(\mathfrak{N}\mathfrak{D}^{-1})_{11}$, in order to find $\bar{\gamma}_{\mathrm{I}}$ by Eq. (2A22):

$$\bar{\gamma}_{\mathrm{I}}=\frac{\frac{1}{2}\pi(g^2/\pi)}{[(x-b_{\mathrm{I}})^2-4a_{\mathrm{I}}c_{\mathrm{I}}]^{1/2}}. \qquad (2\mathrm{B}10)$$

Therefore, for the uncoupled problem, the asymptotic behavior of the amplitude in the $s$ channel is

$$M=-\pi\bar{\gamma}_{\mathrm{I}}(s/\mu^2)^{\alpha_{\mathrm{I}}}(\cot\pi\alpha_{\mathrm{I}}+i)=M_{\mathrm{I}}. \qquad (2\mathrm{B}11)$$

For $(g^2/\pi)<1$, we have

$$J=\alpha_{\mathrm{I}}(x)=-1+a_{\mathrm{I}}/(x-b_{\mathrm{I}}), \qquad (2\mathrm{B}8')$$

with

$$a_{\mathrm{I}}=-\tfrac{1}{2} \quad \text{and} \quad b_{\mathrm{I}}=1+x_{(-1/2)}, \qquad (2\mathrm{B}8'')$$

and

$$\bar{\gamma}_{\mathrm{I}}=\frac{\frac{1}{2}\pi(g^2/\pi)}{x-b_{\mathrm{I}}}. \qquad (2\mathrm{B}10')$$

In Fig. 2 we have drawn $\alpha_{\mathrm{I}}(x)$ and $\bar{\gamma}_{\mathrm{I}}(x)$ [Eqs. (2B8) and (2B10)] arbitrarily picking the value $(g^2/\pi)=2$.

### 2. Coupled Case

Now we shall consider the case when $M$ is not infinite, but is large and finite, $\lambda\ll 1$. Let us note the following general point before continuing on to the details. The elements of $\bar{D}$ in its second column as written in Eq. (2A18) containing $a_2^2(\rho_{22}^2-\rho_{20}^2)$ are proportional to $\lambda^8$ ($\lambda^4$ near $J=-1$) and those in the third column containing $a_3^2(\rho_{33}^3-\rho_{30}^3)$ are proportional to $\lambda^8$ ($\lambda^8/J^3$ near $J=0$). Thus the pole of the uncoupled case, $\alpha_{\mathrm{I}}(x)$, is only negligibly perturbed, and this is also a pole of the coupled case, except near $J=0$, $x=x_{(0)}$ where the solution is, as yet, undetermined. A second zero of $\bar{D}$ occurs for $J$ near enough to zero that terms containing $a_3^2(\rho_{33}^3-\rho_{30}^3)$ become considerable. Terms containing $a_2^2(\rho_{22}^2-\rho_{20}^2)$ remain always negligibly small and we write, as in Eq. (2A18′),

$$\bar{D}=\begin{vmatrix}\bar{D}_1 & \bar{D}_{13}\\ \bar{D}_{31} & \bar{D}_3\end{vmatrix}=\bar{D}_1\left(\bar{D}_3-\frac{\bar{D}_{13}\bar{D}_{31}}{\bar{D}_1}\right).$$

The zero of $\bar{D}/\bar{D}_1=\bar{D}_3-\bar{D}_{13}\bar{D}_{31}/\bar{D}_1$, $\alpha_{\mathrm{III}}(x)$, will be treated below, and it is a pole of the coupled case, except near $x=x_{(0)}$, where apparently the two poles will cross each other, for $(g^2/\pi)\gtrsim 1$. Actually, the poles will not cross each other in this anticipated fashion, but break and rejoin near $x=x_{(0)}$, $J=0$, so as to avoid crossing while passing very close by.[5] Since in what follows we are concerned only with the region somewhat below this point, we may take, as the approximate poles of the coupled problem, $\alpha_{\mathrm{I}}(x)$ and $\alpha_{\mathrm{III}}(x)$, defined as the zeros of $\bar{D}_1$ and $\bar{D}/\bar{D}_1$.

With the approximations $\lambda\ll 1$ and $|J|\ll 1$ inserted to lowest order into $\bar{D}$, we now find $\alpha_{\mathrm{III}}$ as the zero of $\bar{D}\bar{D}_1$:

$$\bar{D}_1=1+\frac{g^2}{\pi}\left(\frac{\mu_0-\mu_1}{x_0-x_1}-\mu_1'\right), \qquad (2\mathrm{B}12\mathrm{a})$$

$$\bar{D}_3=1+\left(\frac{g'^2}{\pi}\right)\lambda^{-2J}6\frac{\lambda^8}{J^3}(x_0-x_3)\times\left(\frac{\mu_0-\mu_3}{x_0-x_3}-\mu_3'\right), \qquad (2\mathrm{B}12\mathrm{b})$$

$$\bar{D}_{13}\bar{D}_{31}=\left(\frac{g^2}{\pi}\right)\left(\frac{g'^2}{\pi}\right)\lambda^{-2J}6\frac{\lambda^8}{J^3}(x_0-x_3)\left(\frac{\mu_0-\mu_3}{x_0-x_3}-\frac{\mu_1-\mu_3}{x_1-x_3}\right)\times\left(\frac{\mu_0-\mu_1}{x_0-x_1}-\frac{\mu_3-\mu_1}{x_3-x_1}\right), \qquad (2\mathrm{B}12\mathrm{c})$$

where

$$\mu_1=\mu(x_1),\ \text{etc.,} \quad \text{and} \quad \mu_1'=\frac{d}{dx}\mu(x)|_{x=x_1},\ \text{etc.,}$$

and

$$x_1=\tfrac{1}{2} \quad \text{and} \quad x_3=1-(3\lambda^2/2J).$$

From this it is apparent that $J=\alpha_{\mathrm{III}}(x)$ will be of order $\lambda^{8/3}$, that is, not only are $\alpha_{\mathrm{III}}$ and $\lambda$ both small, but as well, $|\alpha_{\mathrm{III}}|\ll\lambda^2$, and $x_3$ is large and negative. Inserting this further approximation, we have, to lowest order in $J/\lambda^2$,

$$x_3=-3\lambda^2/2J,$$

$$\mu_3=-\tfrac{1}{2}\ln(J/6\lambda^2),$$

and

$$\mu_3'=-J/3\lambda^2,$$

and for $|x_0|\ll|x_3|$,

$$\frac{\mu_0-\mu_3}{x_0-x_3}=\frac{J}{3\lambda^2}\left[\ln\left(\frac{J}{6\lambda^2}\right)+2\mu_0\right].$$

Writing $J/\lambda^{8/3}=j$, we have as the numerically solvable equation for $j_{\mathrm{III}}(x)=\lambda^{-8/3}\alpha_{\mathrm{III}}(x)$, good to order $\lambda^{8/3}$,

$$\bar{D}/\bar{D}_1=1+\left(\frac{g'^2}{\pi}\right)^2\frac{1}{j_{\mathrm{III}}(x)}\times[\ln(j_{\mathrm{III}}^3(x))+h(x)]=0, \qquad (2\mathrm{B}13\mathrm{a})$$

[5] J. Hartle and C. E. Jones, Phys. Rev. 140, B90 (1965).

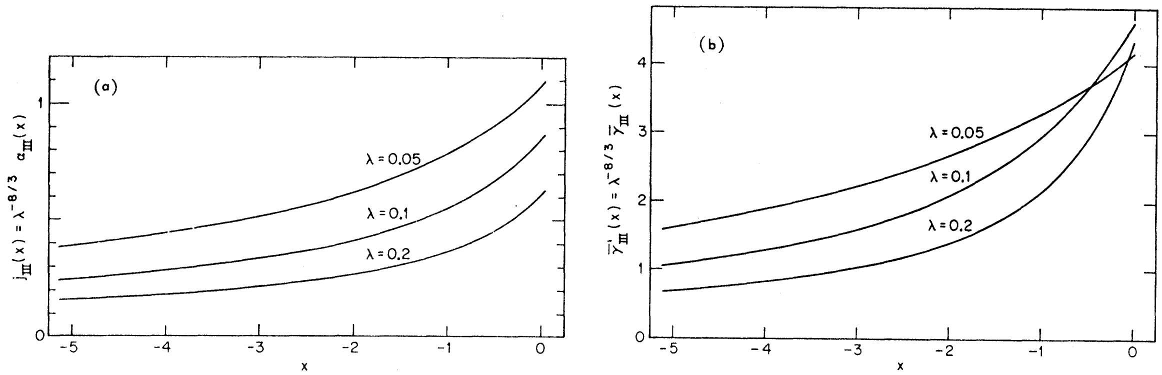


FIG. 3. (a) $j_{III}(x)$ for $g^2/\pi=2$, $g'^2/\pi=1$, and $\lambda=0.2$, 0.1, 0.05. (b) $\bar{\gamma}_{III}'(x)$ under the same conditions.

with

$$h(x)=3\left[\ln\tfrac{1}{6}(\lambda^{2/3})+2\mu_0+1-2\left(\frac{g^2}{\pi}\right)\frac{(\mu_0-\mu_1)^2/(x_0-x_1)}{\bar{D}_1(x)}\right]$$
$$=3\left(\ln\tfrac{1}{6}(\lambda^{2/3})+2\mu_1+1+2\frac{(\mu_0-\mu_1)[1-(g^2/\pi)\mu_1]}{\bar{D}_1}\right),\quad(2B13b)$$

valid for $|x_0|\ll\lambda^{-2/3}$ and below (or above) the region of the zero of $\bar{D}_1$, $x=x_0$. As $\lambda\to 0$,

$$j_{III}(x)\to(g'^2/\pi)^{2/3}\ln^{1/3}(1/\lambda^2)\to\infty$$

and

$$\alpha_{III}(x)\to(g'^2/\pi)^{2/3}\lambda^{8/3}\ln^{1/3}(1/\lambda^2)\to 0.$$

One quantity of interest in computing the residue of this pole is

$$\left.\frac{\partial\bar{D}}{\partial J}\right|_{J=\alpha_{III}}=\lambda^{-8/3}\left.\frac{\partial\bar{D}}{\partial j}\right|_{j=j_{III}}=\frac{3\lambda^{-8/3}\bar{D}_1(x)}{j_{III}(x)}\left(1+\frac{(g'^2/\pi)^2}{j_{III}^3(x)}\right).$$

The other is [from Eq. (2A21b)]

$$(\mathfrak{N}\mathfrak{D}^{-1})_{11}|_{J=\alpha_{III}}=\frac{\frac{1}{2}\pi(g^2/\pi)}{x-\frac{1}{2}}\left(\frac{1}{2}\frac{\pi^{1/2}}{\mu^{2\alpha_{III}}}\frac{\Gamma(\alpha_{III}+1)}{\Gamma(\alpha_{III}+\frac{3}{2})}\right)\times\left[1+\left(\frac{g'^2}{\pi}\right)^2\frac{1}{j_{III}^3}[\ln(j_{III}^3)+k]\right],$$

where, analogously to Eq. (2B13b),

$$k(x)=3\left(\ln\tfrac{1}{6}(\lambda^{2/3})+2\mu_1+1+2\frac{(x_0-x_1)[1-(g^2/\pi)\mu_1']}{(g^2/\pi)}\right).$$

Thus, in summary of the properties of this pole,

$$\bar{\gamma}_{III}(x)=\alpha_{III}\frac{\frac{1}{6}\pi(g^2/\pi)}{(x-\frac{1}{2})\bar{D}_1}\times\left(\frac{1+(g'^2/\pi)^2(k/j_{III}^3)+(g'^2/\pi)^2[\ln(j_{III}^3)/j_{III}^3]}{1+(g'^2/\pi)^2(1/j_{III}^3)}\right),\quad(2B14)$$

and $\alpha_{III}(x)=\lambda^{8/3}j_{III}(x)$, which is defined by Eq. (2B13a). The contribution of this pole to the Reggeized asymptotic amplitude in the $s$ channel is, by Eq. (2A24′),

$$M_{III}=-\frac{\bar{\gamma}_{III}}{\alpha_{III}}-\bar{\gamma}_{III}[\ln(s/\mu^2)+i\pi].\quad(2B15)$$

In Fig. 3, we have drawn $j_{III}(x)=\lambda^{-8/3}\alpha_{III}(x)$ and $\bar{\gamma}_{III}'(x)=\lambda^{-8/3}\bar{\gamma}_{III}(x)$ [from Eqs. (2B13a) and (2B14)], arbitrarily choosing the values $(g^2/\pi)=2$ and $(g'^2/\pi)=1$.

Finally we note that all approximations but one appear to be correct to order of magnitude in the mass $M$, or $\lambda$, and that one is that the behavior of the coupling $g'$, being unknown as a function of $\lambda$, was assumed to be constant. If we allow $(g'^2/\pi)$ to go to $(g'^2/\pi)\lambda^\delta$, in most instances $\lambda^{8/3}\to\lambda^{(8+\delta)/3}$, and all the structure of the model remains intact.

### C. Subtraction Term

Within the assumptions of the model we have now computed the Reggeized asymptotic amplitude in the $s$ channel,

$$M_r=M_I+M_{III}.\quad(2C1)$$

There yet remains to compute the corresponding physical amplitude whose square is proportional to the cross section of the physical process. These two cannot be the same; they differ by a constant in $s$, a subtraction term, engendered by a Kronecker-delta singularity in the $J$ plane at $J=0$[3]:

$$m_p=m_r+\delta_{J,0}(\Delta m).\quad(2C2)$$

To obtain the physical amplitude, one must omit from the Reggeized amplitude all contributions arising from terms in the potential connecting sense channels to nonsense at all nonsense (half-) integers—here $V_{13}=V_{31}$ and $V_{23}=V_{32}$ at $J=0$. If one attempts the calculation of $\Delta m$ within the confines of the model, it appears that the subtraction is zero, due to divergences encountered at $J=0$. This, however, cannot in fact be so, and we now argue that something can be known about the subtraction term, though the model denies us exact knowledge of it. The pole $\alpha_{\mathrm{III}}$ emerges from the Kronecker singularity as the coupling $g'^2/\pi$ or $\lambda$, is turned on, and it must exactly cancel it at zero coupling. A pole can cancel a delta function if its residue and position are limitingly small of the same order of magnitude and their ratio is equal (and opposite) to the coefficient of the delta function:

$$\lim_{\alpha_{\mathrm{III}}\sim\bar{\gamma}_{\mathrm{III}}\to 0}\left(\frac{\bar{\gamma}_{\mathrm{III}}}{J-\alpha_{\mathrm{III}}}\right)=-\frac{\bar{\gamma}_{\mathrm{III}}}{\alpha_{\mathrm{III}}}\delta_{J,0}\,, \qquad (2\mathrm{C}3)$$

and $\bar{\gamma}_{\mathrm{III}}/\alpha_{\mathrm{III}}$ is finite at zero coupling, of order $\lambda^0$. Thus the real term $\Delta m$ must equal $(\bar{\gamma}_{\mathrm{III}}/\alpha_{\mathrm{III}})+M_s$, where $M_s$, a function of $t$ only, is of finite positive order in the coupling $\lambda$. This leaves us with the physical amplitude

$$M_p=M_{\mathrm{I}}+M_s-\bar{\gamma}_{\mathrm{III}}[\ln(s/\mu^2)+i\pi]. \qquad (2\mathrm{C}4)$$

We can say no more of the subtraction term $M_s$, so let us proceed to see what can be said even in this ignorance.

## III. COMPARISON OF THE MODEL WITH EXPERIMENT

To make a comparison of the results of this model with experiment, we allow that, of the terms contained in $M_p$, $M_{\mathrm{I}}$ represent the contribution of ordinary falling Regge trajectories as determined by present Regge fits; we do not attempt to allow that our $\alpha_{\mathrm{I}}$ pole as calculated above has any physical reality. The remaining terms we retain intact as representing in some fashion the contributions of the quark-induced pole and subtraction. When and if energies and momentum transfers are reached at which contributions as these becomes visible, that would be fitted to a theory similar to but more nearly correct than the model presented here, rendering a determination of the quark's mass and coupling.

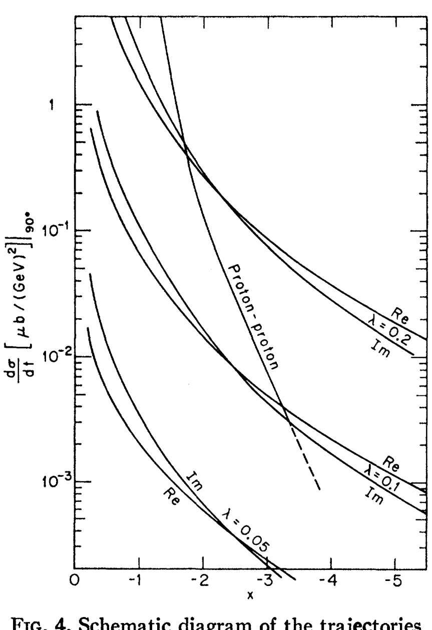


FIG. 4. Schematic diagram of the trajectories of proton-proton scattering.

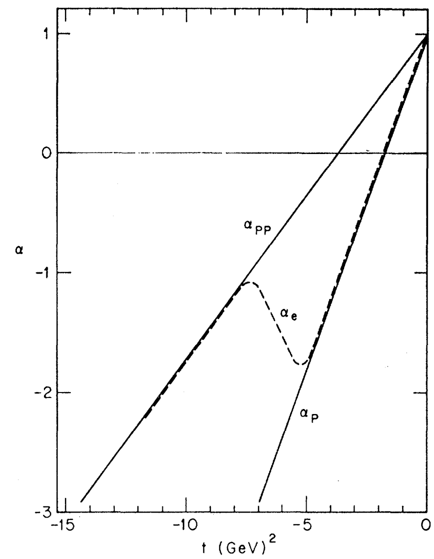


FIG. 5. Comparison of the model with experiment for proton-proton scattering at 90° in the center-of-mass system. The curve marked "proton-proton" is a close fit to the data of Akerlof *et al.*, the broken-line portion denoting preliminary results. The three pairs of curves are $d\sigma/dt|_{\mathrm{ImIII},90°}$ and $d\sigma/dt|_{\mathrm{ReIII},90°,M_s=0}$ for $\lambda=0.2, 0.1, 0.05$, with $g^2/\pi=2$, $g'^2/\pi=1$, and $\mu=m_{\mathrm{proton}}$.

The experimental situation with which we have chosen to compare this model in qualitative fashion is proton-proton scattering at large angles. Immediately after these considerations, a crudely quantitative comparison will be made with the data at 90° in the center-of-mass system. Huang and Pinsky[6] have observed that the effective trajectory, $\alpha_e$, may be analyzed into two parts (see the schematic diagram Fig. 4): the Pomeranchon $\alpha_P$, and the di-Pomeranchon cut $\alpha_{PP}$. The trajectory $\alpha_e$ falls from one to below zero as $\alpha_P$, then rises slightly to the $\alpha_{PP}$ trajectory, and then falls off again as the $\alpha_{PP}$ to the experimental limits on $-t$ in the present data. Thus, below $t\approx-3.7$, where all ordinary Regge poles (or cuts) have dropped below zero, $\alpha_{\mathrm{III}}$ is the conventional "*dominant*" pole—but not the *dominating* pole, since $\alpha_e$ shows no signs of rising again to zero, as we might expect if a pole at or near $J=0$ were actually to dominate. So with present data there is no reason to suppose that a quark, as we have described it, should exist; our task is not to estimate (to order of magnitude only) what is the least mass that would be compatible with the present data, supposing the quark exists.

Since the imaginary part of the scattering amplitude, $M_{\mathrm{Im}p}=M_{\mathrm{Im}}-i\pi\bar{\gamma}_{\mathrm{III}}$, is effectively $M_{\mathrm{Im\,I}}$, $\pi^2\bar{\gamma}^2_{\mathrm{III}}{}^2\ll M_{\mathrm{Im\,I}}{}^2\approx M_{\mathrm{Im}p}{}^2$. But $|M_p|^2\geqq M_{\mathrm{Im}p}{}^2$. Thus we derive

[6] K. Huang and S. Pinsky, M. I. T. Report (unpublished).

the inequality which must be satisfied:

$$\frac{d\sigma}{dt} \gg \frac{4\pi}{sq_s^2}\pi^2\bar{\gamma}_{\mathrm{III}}^2 \equiv \left.\frac{d\sigma}{dt}\right|_{\mathrm{ImIII}}. \tag{3.1a}$$

Similarly,

$$\frac{d\sigma}{dt} \gg \frac{4\pi}{sq_s^2}[\bar{\gamma}_{\mathrm{III}} \ln(s/\mu^2) - M_s]^2 \equiv \left.\frac{d\sigma}{dt}\right|_{\mathrm{ReIII}}. \tag{3.1b}$$

The latter is of little use until we can compute $M_s$, but in the following its right-hand side with $M_s=0$ will be displayed.

In Fig. 5 the data of Akerlof *et al.*[7] for proton-proton scattering at 90° in the center-of-mass system are compared with $d\sigma/dt|_{\mathrm{ImIII},90°}$ and $d\sigma/dt|_{\mathrm{ReIII},90°}$ (for $M_s=0$). We have arbitrarily chosen $g^2/\pi=2$, $g'^2/\pi=1$ and calculated these quantities at $\lambda=0.2$, 0.1, and 0.05, i.e., $M=5$, 10, and 20 times the mass of the proton. In order to satisfy the inequality [Eqs. (3.1)], $M$ would certainly have to be greater than about 10 GeV, and one may guess this to be a decent order-of-magnitude lower bound on this mass as may be gleaned from these data, under the many suppositions of the model.

## IV. CONCLUSION

It appears that whenever approximations are made on our model, they are expected to be correct to order of magnitude in the mass $M$ of the quark, except that the unknown behavior of the coupling constant $g'$ as a function of $\lambda$, was assumed to be constant. Thus we can expect our result to be correct only to order of magnitude, and none of the consequences of subtle details, like the misrepresentation of a spinor by a scalar, to be portrayed with any fidelity. It is in this spirit that the result $M \gtrsim 10$ GeV is stated.

It is not intended that the model presesented here be more than a naive first effort towards an analytic approach to the confrontation of idea and fact. The experimental facts, the data, are there; the idea may be well stated in its theoretical context: it is the mapping between the two worlds which is lacking, the model which must be improved. The minor improvements which could be made on this model are many, other similar models could be suggested, or more radical means of relating the idea to the data may be tried. The idea remains an attractive one: that there is a crucial difference of character between composite particles and elementary ones, that if the latter exist, observable effects must arise, and that these may be manifestations of the operation of higher symmetries.

[7] C. W. Akerlof, R. H. Hieber, A. D. Krisch, K. W. Edwards, L. D. Ratner, and K. Ruddick, Phys. Rev. **159**, 1138 (1967).

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