Nonetheless, the close agreement with the results of Fujii[16] for the choice $m=\frac{1}{2}M_n$ suggests an important qualitative observation. It has long been known that the $Y_0^*(1405)$ resonance contributes significantly to $K\bar{N}$ scattering lengths. Thus, von Hippel and Kim,[19] in applying unitarity corrections to $\text{Re}a_c(\bar{K}N)$, specifically note the importance of the Y_0^*, and incorporate its effects in a forward dispersion relation. In our formulation, the Y_0^* must appear as a virtual bound state in the $\bar{K}N$ channel, as a result of our input forces. The close agreement of our results which those of Fujii[16] and of Akiba and Capps,[17] whose analyses include the effects of the Y_0^*, suggests that the existence of this resonance close to threshold is consistent with the results of current algebra when corrected to include the effects of unitarity.

ACKNOWLEDGMENTS

One of us (D.B.) would like to thank R. Dutt and R. N. Chaudhuri for many useful discussions. We would also like to thank Dr. S. H. Patil for useful comments.

PHYSICAL REVIEW VOLUME 176, NUMBER 5 25 DECEMBER 1968

Relation of Charged-K Decay to CP Violation

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(Received 26 August 1968)

An $SU(3)$ model is described which relates the rate of the charged-K decay $K^+\to\pi^++\pi^0$ to the magnitude of CP violation. A weak-interaction Lagrangian for nonleptonic processes is constructed from octet currents in a Cabibbo frame under the hypothesis that the CP-conserving processes are described predominantly by an octet-dominant interaction Lagrangian, and that the CP-violating processes enter solely through the 27-dimensional representation of $8\otimes8$. Three types of octet currents are used: L_μ, which is linear in the field operators, and J_μ and K_μ, which are bilinear and possess opposite C-transformation properties. It is shown that if only $I=2$ contributions enter into the 27 portion of the Lagrangian, then, approximating $|K_S\rangle$ and $|K_L\rangle$ by the CP eigenstates $|K_1\rangle$ and $|K_2\rangle$, the observed rate of the charged-K decay and $\theta=0.26$ for the Cabibbo angle fix $|\eta_{+-}|=5.38\times10^{-3}$ and $|\eta_{00}|=3.12\times10^{-3}$, of the proper order of magnitude. Introducing the experimentally observed departure of $|K_S\rangle$ and $|K_L\rangle$ from CP eigenstates and a renormalization constant for the K current yields $|\eta_{+-}|=1.97\times10^{-3}$ and $|\eta_{00}|=2.92\times10^{-3}$, in good agreement with experiment.

I. INTRODUCTION

THERE have been various approaches[1] to an explanation of CP violation since it was first seen in the charged-pion decay of the K^0-$\bar{K}^0$ system by Christenson, Cronin, Fitch, and Turlay.[2] These theories have been phenomenological in nature and in some cases have temporarily been in disagreement with the experimental evidence for the decay rates of the K_L and K_S modes into two charged and two uncharged pions. Specifically, data[3] had indicated that $|\eta_{+-}|\neq|\eta_{00}|$, and thus models which utilized CP-invariant weak-interaction Lagrangians were in apparent conflict with experiment. Recent data[4] indicate a smaller magnitude for $A[K_L\to(\pi\pi)_{I=2}]$. Within the existing experimental error $|\eta_{00}|$ may be equal to $|\eta_{+-}|$, again giving substance to such models as the superweak theory.[5]

It is the purpose of this paper to propose a rather straightforward model within the $SU(3)$ symmetry scheme which relates the rate of the charged K decay, $K^+\to\pi^++\pi^0$, to the magnitude of CP violation. We propose an ordinary symmetric coupled current-current theory within the $SU(3)$ framework. The CP violation is introduced by an octet vector current which, under charge conjugation, transforms into itself and thus

[1] T. T. Wu and C. N. Yang, Phys. Rev. Letters 13, 380 (1964). See also N. Byers, S. W. MacDowell, and C. N. Yang, in *Proceedings of the Seminar in High-Energy Physics and Elementary Particles, Trieste, 1965* (International Atomic Energy Agency, Vienna, 1965), pp. 955–980; C. N. Yang, in Proceedings of the Argonne International Conference on Weak Interactions, 1965 [Argonne National Laboratory Report No. ANL-7130, pp. 29–39 (unpublished)]; L. Wolfenstein, Nuovo Cimento 42, 17 (1966); J. S. Bell and J. Steinberger, in *Proceedings of the Oxford International Conference on Elementary Particles, 1965* (Rutherford High-Energy Laboratory, Chilton, Berkshire, England, 1966), pp. 195–222; T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. 16, 471 (1966); B. Martin and E. de Rafael, Phys. Rev. 162, 1453 (1967); T. N. Truong, Phys. Rev. Letters 13, A358 (1965).

[2] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964).

[3] J. W. Cronin, in Proceedings of the Rochester Conference on High Energy Physics, 1967 (unpublished); J. W. Cronin *et al.*, Phys. Rev. Letters 18, 25 (1967); J. M. Gaillard *et al.*, *ibid* 18, 20 (1967).

[4] V. Fitch *et al.*, Bull. Am. Phys. Soc. 13, 16 (1968); D. F. Bartlett *et al.*, Phys. Rev. Letters 21, 558 (1968); M. Banner *et al.*, *ibid.* 21, 1107 (1968).

[5] L. Wolfenstein, Phys. Rev. Letters 13, 562 (1965).

does not interact with the electromagnetic field.[6] We assume that this current, because it does not couple to the electromagnetic field, does not contribute to an octet-dominant Lagrangian. Thus, the hypothesis is that the CP-conserving processes are described predominantly by an octet-dominant interaction Lagrangian and the CP-violating processes enter solely through the 27-dimensional representation of $8\otimes 8$.

The particular construction of the currents and the interactions will be discussed in Sec. II. In Sec. III we discuss the Cabibbo transformation as applied to the matrix elements. The method of parametrization by utilizing the non-CP eigenstate $K^+\to\pi^++\pi^0$ to fix the degree of CP violation is considered in Sec. IV, together with relevant computations. Finally, in the concluding section, we present the theoretical results for the amplitudes and consider the effect of the nonorthogonality between $|K_L\rangle$ and $|K_S\rangle$ together with some theoretical considerations of the conclusions.

II. INTERACTION LAGRANGIAN

A. Definition of Currents

The nonet field currents of pseudoscalar mesons are defined in the form

$$J_\mu{}^{(k)}=+\tfrac{1}{2}i\,\mathrm{Tr}\lambda^{(k)}[P,\partial_\mu P]_-\,, \tag{1}$$

$$K_\mu{}^{(k)}=-\tfrac{1}{2}\,\mathrm{Tr}\lambda^{(k)}[P,\partial_\mu P]_+\,, \tag{2}$$

$$L_\mu{}^{(k)}=+\tfrac{1}{2}\,\mathrm{Tr}\lambda^{(k)}\partial_\mu P\,, \tag{3}$$

where

$$P_\alpha{}^\beta=\frac{1}{\sqrt{2}}\sum_{k=0}^{8}P^{(k)}(\lambda^{(k)})_\alpha{}^\beta.$$

The $P^{(k)}$ define the pseudoscalar nonet and the $\lambda^{(k)}$ are as usually defined.[7]

Under the operation of charge conjugation[8] it follows that

$$CJ_\mu{}^{(k)}C^{-1}=-\epsilon^{(k)}J_\mu{}^{(k)}\,, \tag{4}$$

$$CK_\mu{}^{(k)}C^{-1}=+\epsilon^{(k)}K_\mu{}^{(k)}\,, \tag{5}$$

$$CL_\mu{}^{(k)}C^{-1}=+\epsilon^{(k)}L_\mu{}^{(k)}\,, \tag{6}$$

where $(\lambda^{(k)})_\alpha{}^\beta=\epsilon^{(k)}(\lambda^{(k)})_\beta{}^\alpha$ defines the $\epsilon^{(k)}$. Since we are primarily interested in an effective three-particle weak interaction of pseudoscalar mesons, the bilinear and linear currents will be combined. We shall formulate the effective Lagrangian in a Cabibbo frame,[9] i.e., the physical Lagrangian for nonleptonic weak decay is regarded as arising by the unitary transformation $e^{2i\theta F_7}$ on a Lagrangian for which the change in strangeness is zero, where we take $\theta=\theta_A=0.26$. For the effective Lagrangian we assume that the current-current interaction is of a symmetric nature, and that the octet portion of the interaction is CP-invariant. The basic interaction Lagrangian in the Cabibbo frame must then take the general form

$$\mathcal{L}'=G\{(L_\mu\otimes J_\mu)_{8;Y=0,I,I_z=0}+\alpha[L_\mu\otimes(J_\mu+aK_\mu)]_{27;Y=0,I,I_z=0}\}\,, \tag{7}$$

where G is the over-all effective coupling constant. The parameters a and α are to be determined in Sec. IV, the latter by comparing the rates of the charged and the short-lived K modes. Since K_μ is a nonconserved current, the parameter a will be regarded as a renormalization constant whose value is not far removed from unity.

We have, however, the added complication that the processes in the Cabibbo frame in which $\Delta S=0$ are those for which $I=0$ or $I=1$ in the octet, and $I=0$, 1, 2 in the **27**. For the physical $\Delta S=1$ processes the $I=0$, 1 in the Cabibbo-transformed octet are merely constant multiples of one another[10] and thus serve only to adjust the over-all coupling constant. On the other hand, in the 27-dimensional representation, the $I=0$, 1, 2 lead to different possible descriptions of the physical processes of interest. We shall show, however, that only if the Cabibbo transformation of **27** is removed in isospin maximally from the transformed octet, i.e., **27**; $Y=0$, $I=2$, $I_z=0$, can one obtain a CP-violating effect as small as is experimentally observed.

III. MATRIX ELEMENTS AND THE CABIBBO TRANSFORMATION

To perform the Cabibbo unitary transformation of (7) it will be necessary to consider the transformation equations for an octet[11] and for a 27-plet. For the processes of interest here, viz., for charged and uncharged K decay into two pions, we require only the $\Delta S=\pm1$, $\Delta Q=0$ components in the physical frame. Thus, retaining only the $\Delta S=\pm1$ components, the transformations for the 27-plet components of (7) are[12] of the form

$$(27;0,2,0)'=-\sin2\theta\{\tfrac{1}{6}(\sqrt{5})\sin^2\theta[(27;1,\tfrac{1}{2},-\tfrac{1}{2})+(27;-1,\tfrac{1}{2},\tfrac{1}{2})]+(1-\tfrac{1}{3}\sin^2\theta)[(27;1,\tfrac{3}{2},-\tfrac{1}{2})+(27;-1,\tfrac{3}{2},\tfrac{1}{2})]\}\,, \tag{8}$$

[6] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. **139**, B1650 (1965), and, e.g., R. Bowen *et al.*, Phys. Letters **24B**, 206 (1967); M. Gormley *et al.*, Phys. Rev. Letters **21**, 399 (1968).

[7] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[8] The charge conjugation is defined so that $CP_\alpha{}^\beta C^{-1}=+P_\beta{}^\alpha$.

[9] N. Cabibbo, Phys. Rev. Letters **12**, 62 (1964).

[10] It is easily shown that

$$\{e^{2i\theta F_7}[\sqrt{3}(L_\mu\otimes J_\mu)_{8;0,1,0}+(L_\mu\otimes J_\mu)_{8;0,0,0}]e^{-2i\theta F_7}\}_{\Delta S=1}=0.$$

[11] J. J. De Swart, Rev. Mod. Phys. **35**, 916 (1963), and Ref. 9.

[12] The notation $(N:Y,I,I_z)$, where N is the dimensionality of the $SU(3)$ representation, is used.

$$(27;0,1,0)'=\sin 2\theta\{-\tfrac{1}{6}\sqrt{3}(2-5\sin^2\theta)[(27;1,\tfrac{1}{2},-\tfrac{1}{2})+(27;-1,\tfrac{1}{2},\tfrac{1}{2})] + \tfrac{1}{6}(\sqrt{15})(1-\sin^2\theta)[(27;1,\tfrac{3}{2},-\tfrac{1}{2})+(27;-1,\tfrac{3}{2},\tfrac{1}{2})]\}, \quad (9)$$

$$(27;0,0,0)'=\sin 2\theta\{(\tfrac{1}{3})(3-5\sin^2\theta)[(27;1,\tfrac{1}{2},-\tfrac{1}{2})+(27;-1,\tfrac{1}{2},\tfrac{1}{2})] + \tfrac{1}{3}(\sqrt{5})\sin^2\theta[(27;1,\tfrac{3}{2},-\tfrac{1}{2})+(27;-1,\tfrac{3}{2},\tfrac{1}{2})]\}. \quad (10)$$

It should be noted that the small value of the Cabibbo angle emphasizes the $\Delta I=\frac{3}{2}$ transitions in the $(27;I=2)$, while in the $(27;I=0)$ the $\Delta I=\frac{1}{2}$ transitions are emphasized.

We shall denote the two octets, m and n, which give rise in their direct product to the 27-plet, by the pseudoscalar octet particle symbols. Then the $\Delta S=\pm 1$, $\Delta Q=0$ components of the 27-plet are given by[13]

$$\begin{aligned}
(27;1,\tfrac{3}{2},-\tfrac{1}{2}) &= (\sqrt{\tfrac{1}{6}})(K_m{}^+\pi_n{}^-+\pi_m{}^-K_n{}^+)+(\sqrt{\tfrac{1}{3}})(K_m{}^0\pi_n{}^0+\pi_m{}^0K_n{}^0),\\
(27;1,\tfrac{3}{2},+\tfrac{1}{2}) &= (\sqrt{\tfrac{1}{6}})(K_m{}^-\pi_n{}^++\pi_m{}^+K_n{}^-)+(\sqrt{\tfrac{1}{3}})(\bar{K}_m{}^0\pi_n{}^0+\pi_m{}^0\bar{K}_n{}^0),\\
(27;1,\tfrac{1}{2},-\tfrac{1}{2}) &= (1/30)^{1/2}(K_m{}^+\pi_n{}^-+\pi_m{}^-K_n{}^+)-(1/60)^{1/2}(K_m{}^0\pi_n{}^0+\pi_m{}^0K_n{}^0)+(9/20)^{1/2}(\eta_m{}^0K_n{}^0+K_m{}^0\eta_n{}^0),\\
(27;1,\tfrac{1}{2},+\tfrac{1}{2}) &= (1/30)^{1/2}(K_m{}^-\pi_n{}^++K_m{}^-\pi_n{}^+)-(1/60)^{1/2}(\bar{K}_m{}^0\pi_n{}^0+\pi_m{}^0\bar{K}_n{}^0)+(9/20)^{1/2}(\eta_m{}^0\bar{K}_n{}^0+\bar{K}_m{}^0\eta_n{}^0).
\end{aligned} \quad (11)$$

Identifying the first octet with the linear current L_μ and the second octet with the bilinear currents J_μ and K_μ and performing a first-order perturbation calculation using the physical masses we obtain the contributions to the various decay amplitudes with the prescription of the interaction Lagrangian (7). The contributions to the effective first-order Lagrangian, where we have denoted the two CP eigenstates of the K^0-$\bar{K}^0$ system by $K_1{}^0=(K^0-\bar{K}^0)/i\sqrt{2}$ and $K_2{}^0=(K^0+\bar{K}^0)/\sqrt{2}$, are of the form

$$(L_\mu\otimes J_\mu)'_{27;0,2,0}=-(1/\sqrt{3})(m_{K^0}{}^2-m_{\pi^+}{}^2)\pi^-\pi^+K_1{}^0+(1/4\sqrt{3})(3+\cos 2\theta)(m_{K^0}{}^2-m_{\pi^+}{}^2)\pi^0\pi^0K_1{}^0 - (i/4\sqrt{3})[2(m_{K^+}{}^2-2m_{\pi^-}{}^2+m_{\pi^0}{}^2)+(3+\cos 2\theta)(m_{K^+}{}^2-m_{\pi^-}{}^2)](\pi^0\pi^-K^+-\pi^0\pi^+K^-), \quad (12a)$$

$$(L_\mu\otimes K_\mu)'_{27;0,2,0}=-(1/\sqrt{3})(m_{K^0}{}^2\sin^2\theta+m_{\pi^-}{}^2)\pi^-\pi^+K_2{}^0+(1/4\sqrt{3})[(3+\cos 2\theta)m_{\pi^0}{}^2-2m_{K^0}{}^2\sin^2\theta]\pi^0\pi^0K_2{}^0 - (1/4\sqrt{3})[2m_{\pi^-}{}^2+(3+\cos 2\theta)m_{\pi^0}{}^2](\pi^0\pi^-K^++\pi^0\pi^+K^-), \quad (12b)$$

$$(L_\mu\otimes J_\mu)'_{27;0,1,0}=+(1/\sqrt{5})(m_{K^0}{}^2-m_{\pi^+}{}^2)\pi^-\pi^+K_1{}^0-(1/4\sqrt{5})(3+5\cos 2\theta)(m_{K^0}{}^2-m_{\pi^0}{}^2)\pi^0\pi^0K_1{}^0 - (i/4\sqrt{5})[2(m_{K^+}{}^2-2m_{\pi^-}{}^2+m_{\pi^0}{}^2)+(3+5\cos 2\theta)(m_{K^+}{}^2-m_{\pi^-}{}^2)](\pi^0\pi^-K^+-\pi^0\pi^+K^-), \quad (12c)$$

$$(L_\mu\otimes K_\mu)'_{27;0,1,0}=+(1/2\sqrt{5})[(1-5\cos 2\theta)m_{K^0}{}^2+m_{\pi^-}{}^2]\pi^-\pi^+K_2{}^0+(1/4\sqrt{5})[(1-5\cos 2\theta)m_{K^0}{}^2 - (3-5\cos 2\theta)m_{\pi^0}{}^2]\pi^0\pi^0K_2{}^0+(1/4\sqrt{5})[2m_{\pi^-}{}^2+(3+5\cos 2\theta)m_{\pi^0}{}^2](\pi^0\pi^-K^++\pi^0\pi^+K^-), \quad (12d)$$

$$(L_\mu\otimes J_\mu)'_{27;0,0,0}=+(2/\sqrt{15})(m_{K^0}{}^2-m_{\pi^+}{}^2)\pi^-\pi^+K_1{}^0-(1/2\sqrt{15})(3-5\cos 2\theta)(m_{K^0}{}^2-m_{\pi^0}{}^2)\pi^0\pi^0K_1{}^0 - (i/2\sqrt{15})[2(m_{K^0}{}^2-2m_{\pi^-}{}^2+m_{\pi^0}{}^2)+(3-5\cos 2\theta)(m_{K^+}{}^2-m_{\pi^-}{}^2)](\pi^0\pi^-K^+-\pi^0\pi^+K^-), \quad (12e)$$

$$(L_\mu\otimes K_\mu)'_{27;0,0,0}=+(1/\sqrt{15})[(1+5\cos 2\theta)m_{K^0}{}^2+2m_{\pi^-}{}^2]\pi^-\pi^+K_2{}^0+(1/2\sqrt{15})[(1+5\cos 2\theta)m_{K^0}{}^2 - (3-5\cos 2\theta)m_{\pi^0}{}^2]\pi^0\pi^0K_2{}^0+(1/2\sqrt{15})[2m_{\pi^-}{}^2+(3-5\cos 2\theta)m_{\pi^0}{}^2](\pi^0\pi^-K^++\pi^0\pi^+K^-), \quad (12f)$$

$$(L_\mu\otimes J_\mu)'_{8;0,0,0}=+(3/\sqrt{10})(m_{K^0}{}^2-m_{\pi^+}{}^2)\pi^-\pi^+K_1{}^0+(3/2\sqrt{10})(m_{K^0}{}^2-m_{\pi^0}{}^2)\pi^0\pi^0K_1{}^0 + (3i/2\sqrt{10})(m_{\pi^-}{}^2-m_{\pi^0}{}^2)(\pi^0\pi^-K^+-\pi^0\pi^+K^-), \quad (12)\mathrm{g}$$

where the factor $\frac{1}{2}\sin 2\theta$ is understood to multiply each term. We note that the mass-breaking modifies the original isospin content; e.g., in the octet the presence of electromagnetic mass differences is responsible for $\Delta I=\frac{3}{2}$ transitions in the charged-K decay.

IV. COMPUTATIONS AND RESULTS

The particular transition matrix elements formed from the expressions listed in (12) do not provide for the $K^0\leftrightarrow\bar{K}^0$ transitions and therefore the proper mixture of $|K_1{}^0\rangle$ and $|K_2{}^0\rangle$ to form the physical states $|K_L\rangle$ and $|K_S\rangle$ must be included in our calculation from experiment.[14–16] In view of this it will be meaningful to demonstrate that the results of the model, when the matrix element is taken between CP eigenstates, does not differ except by a small correction from the amplitudes for physical states.

The parameter α in (7) measures the relative weights of the octet and 27-plet contributions to the weak nonleptonic processes. We choose a value for α such that the ratio of widths[17] of the charged decay $K^+\rightarrow\pi^++\pi^0$ to the mode $K_S\rightarrow\pi^++\pi^-$ will yield the ex-

[13] The convention here is the same as in P. McNamee and F. Chilton, Rev. Mod. Phys. **36**, 1005 (1964).

[14] S. Bennett *et al.*, Phys. Rev. Letters **19**, 997 (1967).
[15] D. Dorfan *et al.*, Phys. Rev. Letters **19**, 987 (1967).
[16] S. Bennett *et al.*, Phys. Letters **27B**, 248 (1968).
[17] α enters here quadratically; thus we consider both roots.

TABLE I. Theoretical results for $K \to 2\pi$ decays. $K_S = K_1^0$, $K_L = K_2^0$ ($|\epsilon| = 0$), $a = \pm 1$.

I'_{27}	a	α	$\lvert\eta_{+-}\rvert = \dfrac{\lvert A(K_L \to \pi^+ + \pi^-)\rvert}{\lvert A(K_S \to \pi^+ + \pi^-)\rvert}$	$\lvert\eta_{00}\rvert = \dfrac{\lvert A(K_L \to \pi^0 + \pi^0)\rvert}{\lvert A(K_S \to \pi^0 + \pi^0)\rvert}$	$\dfrac{\Gamma(K_S \to \pi^+ + \pi^-)}{\Gamma(K_S \to \pi^0 + \pi^0)}$	$\dfrac{\Gamma(K_L \to \pi^+ + \pi^-)}{\Gamma(K_L \to \pi^0 + \pi^0)}$
2	± 1	$+0.0484$	4.77×10^{-3}	2.28×10^{-3}	1.65	7.17
2	± 1	-0.0583	5.38×10^{-3}	3.12×10^{-3}	2.40	7.17
1	± 1	$+0.0467$	38.1×10^{-3}	54.9×10^{-3}	2.90	1.39
1	± 1	-0.0397	33.7×10^{-3}	34.4×10^{-3}	1.45	1.39
0	± 1	$+0.746$	849×10^{-3}	936×10^{-3}	2.38	1.96
0	± 1	-0.374	751×10^{-3}	691×10^{-3}	1.66	1.96

perimental result[18]

$$[\Gamma(K^+ \to \pi^+ + \pi^0)/\Gamma(K_S \to \pi^+ + \pi^-)] = 1/460. \quad (13)$$

It is important to note that this method of determining α, using a process not immediately related to CP violation, will produce the proper magnitude of this effect. Furthermore, we shall first consider no renormalization correction for the K_μ current and therefore set $a = \pm 1$. Thus, for the case $|K_L\rangle = |K_2^0\rangle$, $|K_S\rangle = |K_1^0\rangle$, we combine (12) with prescription (7) and list the results for the various values of isospin in the **27** (I'_{27}) in Table I.

The theoretical results presented in Table I thus indicate that a successful result for the experimentally observed CP violation in the 2π decay of K^0 can only be obtained if the **27** is considered as entering with maximum isospin equal to two.

Although it appears from (8) that the physical $\Delta I = \frac{1}{2}$ contribution from the $I = 2$ term should be small, it actually enters weighted by $m_K^2 \sin^2\theta$ from (12) and contributes nearly the same amount as the $\Delta I = \frac{3}{2}$ term, tending to cancel that term for the process $K_L \to \pi^0 + \pi^0$ when $|\epsilon| = 0$. This explains the large value for $\Gamma(K_L \to \pi^+ + \pi^-)/\Gamma(K_L \to \pi^0 + \pi^0)$ as seen in Table I for $I = 2$.

Considering then only the $I = 2$ contributions in the **27**, we shall introduce the linear combinations of $|K^0\rangle$, $|\bar{K}^0\rangle$, which are indicated by experiment to be the physical states $|K_L\rangle$, $|K_S\rangle$. We employ the standard[19] notation

$$\begin{aligned} |K_L\rangle &= p|K^0\rangle + q|\bar{K}^0\rangle, \\ |K_S\rangle &= p|K^0\rangle - q|\bar{K}^0\rangle, \\ 1 &= |p|^2 + |q|^2, \\ \langle K_L|K_S\rangle &= |p|^2 - |q|^2, \\ \epsilon &= (p-q)/(p+q), \end{aligned}$$

and the experimental value[20] $\mathrm{Re}\epsilon = 1.14 \pm 0.18$, $\mathrm{Im}\epsilon = 1.11 \pm 0.33$. These results are given in Table II for $a = \pm 1$.

It is especially interesting that the ratio of rates, $\Gamma(K_S \to \pi^+ + \pi^-)/\Gamma(K_S \to \pi^0 + \pi^0)$, for the negative value of α has been increased from the pure $\Delta I = \frac{1}{2}$ value of 2 by means of the $\Delta I = \frac{3}{2}$ contribution in the **27**. Without this contribution, because of electromagnetic mass-breaking and phase-space considerations, the ordinary octet value of 2 is decreased. We note that the value predicted by our model is in reasonable agreement with the experimental value[21] 2.21 ± 0.08.

Finally, the results for several values of the renormalization constant a are given in Table III, where we have used the negative value of α. We note that $a \simeq 0.5$ is in best agreement with the present experimental values[22] $|\eta_{+-}| = (1.90 \pm 0.06) \times 10^{-3}$, $|\eta_{00}| < 3$ (90% confidence). It is significant that this simple model allows, for one value of a, simultaneous agreement with both experimental values of $|\eta_{+-}|$ and $|\eta_{00}|$.

V. CONCLUSIONS

The most striking feature of this model is seen in Table I, where the physical $|K_S\rangle$ and $|K_L\rangle$ are considered the CP eigenstates $|K_1\rangle$ and $|K_2\rangle$. There the current is assumed to be unaffected by the renormalization effect ($J_\mu \pm K_\mu$), and the charged K-decay rate is used to determine one parameter, $\alpha = -0.0583$, for the $I = 2$ portion of the Lagrangian in the Cabibbo frame. These conditions yield $|\eta_{+-}| = 5.38 \times 10^{-3}$ and $|\eta_{00}| = 3.12 \times 10^{-3}$, already quite close to the experimental values.

We emphasize the fact that this model does not provide for the transitions $K^0 \to \bar{K}^0$, which are responsible for making $|K_L\rangle$ and $|K_S\rangle$ non-CP eigenstates. Thus, no prediction can be made about the magnitude and phase of ϵ, which measures the departure of $|K_L\rangle$ and $|K_S\rangle$ from CP eigenstates. If the experimental value of ϵ is introduced into the model (Table II) then changes of the order of 25% occur in $|\eta_{+-}|$ and $|\eta_{00}|$ from those given in Table I, where $\epsilon = 0$. If, in addition, a renormalization constant a is assumed for the nonconserved current K_μ, we see (Table III) that the calculated $|\eta_{+-}|$ and $|\eta_{00}|$ are quite close to the experimental values for $a \simeq +0.5$. Furthermore,

[18] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).

[19] See, e.g., B. R. Martin and E. de Rafael, Phys. Rev. **162**, 1453 (1967).

[20] Reference 14. The values used are in substantial agreement with more recent measurements (Ref. 16). The values given in Ref. 15 would extend the range of our numerical results slightly.

[21] G. H. Trilling, Argonne National Laboratory Report No. ANL-7130, 1965 (unpublished); W. Willis, in *Proceedings of the Heidelberg International Conference on Elementary Particles*, edited by H. Filthuth (Interscience Publishers, Inc., New York, 1968). We understand that at present this ratio is being re-examined experimentally.

[22] Reference 4, and V. Fitch, in *Proceedings of the Second Hawaii Topical Conference in Particle Physics* (University of Hawaii Press, 1968).

TABLE II. Theoretical results for $K \to 2\pi$ decays. (Reϵ = 1.14±0.18; Imϵ = 1.11±0.33.)

I'_{27}	a	α	$10^3\times\|\eta_{+-}\|$	$10^3\times\|\eta_{00}\|$	$\frac{\Gamma(K_S\to\pi^++\pi^-)}{\Gamma(K_S\to\pi^0+\pi^0)}$	$\frac{\Gamma(K_L\to\pi^++\pi^-)}{\Gamma(K_L\to\pi^0+\pi^0)}$
2	+1	+0.0484	5.99±0.36	1.66±0.11	1.65	21.4 ±5.6
2	+1	−0.0583	4.43±0.28	4.39±0.36	2.40	2.45±0.74
2	−1	+0.0484	3.84±0.24	3.59±0.37	1.65	1.87±0.68
2	−1	−0.0583	6.60±0.36	2.33±0.20	2.40	19.3 ±5.6

TABLE III. Theoretical results for $K \to 2\pi$ decays. (Reϵ = 1.14±0.18; Imϵ = 1.11±0.33.)

I'_{27}	a	α	$10^3\times\|\eta_{+-}\|$	$10^3\times\|\eta_{00}\|$	$\frac{\Gamma(K_S\to\pi^++\pi^-)}{\Gamma(K_S\to\pi^0+\pi^0)}$	$\frac{\Gamma(K_L\to\pi^++\pi^-)}{\Gamma(K_L\to\pi^0+\pi^0)}$
2	+0.3	−0.0583	1.28±0.02	2.36±0.38	2.40	0.71±0.22
2	−0.3	−0.0583	2.97±0.38	1.19±0.23	2.40	15.0 ±2.0
2	+0.5	−0.0583	1.97±0.16	2.92±0.38	2.40	1.10±0.50
2	−0.5	−0.0583	3.98±0.37	1.26±0.04	2.40	24.0 ±2.8
2	+0.7	−0.0583	2.90±0.24	3.47±0.38	2.40	1.68±0.66
2	−0.7	−0.0583	5.00±0.36	1.59±0.10	2.40	23.7 ±6.4

$\Gamma(K_S\to\pi^++\pi^-)/\Gamma(K_S\to\pi^0+\pi^0)$ is in reasonable agreement with experiment. Thus a combination of the Cabibbo transformation and a CP-violating (27, $I=2$) contribution to the Lagrangian in the Cabibbo frame accounts for the important CP-nonconservation parameters $|\eta_{+-}|$ and $|\eta_{00}|$.

In this model $|\eta_{+-}|$ and $|\eta_{00}|$ are sensitive to the value of the Cabibbo angle.[23] This may indicate a close connection between CP violation and the Cabibbo formalism, even though the Cabibbo transformation does not introduce, per se, CP noninvariance into the Lagrangian.

Further refinements could be made by introducing the $\pi\pi$ final-state interaction phase shifts δ_0 and δ_2, if they were known. This would then allow a calculation of the phases of η_{+-} and η_{00}. The calculation would require projecting out contributions to the amplitudes due to $\Delta I=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ portions of the effective Lagrangian noting particularly that these arise not only from the Clebsch-Gordan reduction but also from the mass breaking. This effect was seen in the $\Delta I=\frac{3}{2}$ operator describing the decay $K^+\to\pi^++\pi^0$ which, due to electromagnetic mass breaking, occurs in the octet $(L_\mu\otimes J_\mu)_8$.

We should also like to draw attention to the fact that if we had assumed that the symmetric octet and 27-plet portion of our effective Lagrangian were given relative weights as specified by the mass operator[24] for mesons then α, the ratio of **27** to symmetric octet could be calculated and is given by $\alpha=a_{27}/a_{8_1}=-0.0928$. This then yields for $a=0.5$

$$|\eta_{+-}|=3.23\times10^{-3},$$
$$|\eta_{00}|=4.16\times10^{-3},$$
$$[\Gamma(K^+\to\pi^++\pi^0)/\Gamma(K_S\to\pi^++\pi^-)]=1/181,$$
$$[\Gamma(K_S\to\pi^++\pi^-)/\Gamma(K_S\to\pi^0+\pi^0)]=2.78,$$
$$[\Gamma(K_L\to\pi^++\pi^-)/\Gamma(K_L\to\pi^0+\pi^0)]=1.85.$$

While this method of fixing α is interesting, it is subject to the criticism that our effective Lagrangian corresponding to the physical process is not purely $I=0$ but also contains $I=2$.

Finally, we note that a possible improvement in this model would result if the coupling $(L_\mu\otimes L_\mu)_{27}$ were included in the effective Lagrangian (7). The unitary transformation induced by[25] $e^{2i\theta(xF_7+yF_6)}$ $(x^2+y^2=1)$ would introduce direct $K^0\leftrightarrow\bar{K}^0$ transitions and CP noninvariance, so allowing the calculation of ϵ. However, the term $(L_\mu\otimes L_\mu)_{27}$ would have to enter with a coupling constant substantially smaller than the weak coupling constant, otherwise it would lead to a mass difference $\Delta m=m_{K_L}-m_{K_S}$ too large; thus this would be classified as a variant on the superweak model, in which, however, the weak-interaction Lagrangian would also be CP-noninvariant.

ACKNOWLEDGMENT

The authors are very grateful to Dr. Eduardo de Rafael for encouragement and for many helpful discussions.

[23] We have used $\theta=\theta_A=0.26$. A change of θ from 0.26 to 0.20 produces a 20% change in $|\eta_{+-}|$ and an 18% change in $|\eta_{00}|$.

[24] First paper of Ref. 11.

[25] R. J. Oakes, Phys. Rev. Letters **20**, 1539 (1968).