Complex $\bar{K}N$ Scattering Lengths and Current Algebra

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We calculate the real and imaginary parts of the s-wave $\bar{K}N$ scattering length in the $I=0$ channel, starting from the predictions of soft-meson current algebra. The multichannel ND^{-1} formalism is used to unitarize the amplitudes obtained from current algebra. We isolate dynamical off-mass-shell corrections in certain residues for one-pole approximations to the numerator functions. These residues, for which we make assumptions of smooth extrapolation, are obtained by comparing the unitarized amplitudes with the results of current algebra. The coupling of the $\bar{K}N$ to the $\pi\Sigma$ channel, which has a lower threshold, generates the imaginary part of the scattering length with the correct sign and reverses the (incorrect) sign of the real part. The magnitudes of the real and imaginary parts agree well with the results of recent experiments and are consistent with the existence of the $Y_0^*(1405)$ virtual bound state below the $\bar{K}N$ threshold.

I. INTRODUCTION

THE recent successes of the hypothesis of partially conserved axial-vector current (PCAC), the algebra of currents, and soft-meson techniques are well known.[1] Following these successes, a number of investigations have considered corrections to the results of current algebra.[2,3] These corrections are necessitated by the various approximations inherent in calculations which make use of PCAC and soft-meson techniques. The corrections may be divided into two broad categories.

(i) There are off-mass-shell corrections which arise if the current-algebra calculation involves soft ($q_\mu \to 0$) or off-mass-shell ($q_\mu^2 \to 0$) mesons. Amplitudes, vertex functions, etc., have to be continued analytically in the masses of the external mesons to estimate corrections arising from off-mass-shell approximations. Techniques based on dispersion theory have been used to study these corrections[4,5]; it is found that, in general, these corrections are small.

(ii) There are also corrections which may be roughly described as arising through "unitarity." By this we mean the following: The use of the algebra of currents in the soft-meson or off-mass-shell limit yields vertex functions, scattering amplitudes, etc., in unphysical kinematic domains. Scattering amplitudes so obtained have no imaginary parts and do not exhibit unitarity. The unitarity corrections arise when one considers the analytic continuation to the physical threshold using analyticity and unitarity.

[1] Y. Tomozawa, Nuovo Cimento **46A**, 707 (1966); S. Weinberg, Phys. Rev. Letters **17**, 616 (1966). See also S. Adler and R. Dashen, *Current Algebras and Applications to Particle Physics* (W. A. Benjamin Inc., New York, 1968), and references therein.
[2] K. Raman Phys. Rev. Letters **17**, 983 (1966); A. P. Balachandran, M. Gundzik, and F. Nicodemi, Nuovo Cimento **44A**, 1257 (1966); K. Raman and E. C. G. Sudarshan, Phys. Rev. **154**, 1479 (1967); S. Okubo, R. E. Marshak, and V. S. Mathur, Phys. Rev. Letters **19**, 407 (1967); K. Raman Phys. Rev. **164**, 1736 (1967); K. Kang and T. Akiba, *ibid.* **164**, 1836 (1967).
[3] S. C. Bhargava, S. N. Biswas, K. C. Gupta, and K. Datta, Phys. Rev. Letters **20**, 558 (1968). See also M. Gundzik, Argonne National Laboratory Report, 1968 (unpublished).
[4] S. Adler, Phys. Rev. **140**, B736 (1965); F. T. Meiere and M. Sugawara, *ibid.* **153**, 1709 (1967).
[5] S. Fubini and G. Furlan, M.I.T. Report, 1968 (unpublished), and references therein.

In the present paper, we consider unitarity corrections to soft-meson calculations for elastic and inelastic s-wave $\bar{K}N$ scattering in the $I=0$ channel. Thus, we consider the processes

$$\begin{aligned}\bar{K}N &\to \bar{K}N\\ &\to \pi\Sigma;\end{aligned}$$

the corrections are considered within a framework developed by us in an earlier paper[3] and applied there to elastic πN scattering. The framework is thus here generalized to include inelastic and multichannel processes.

The investigation of unitarity corrections to soft-meson calculations of $\bar{K}N$ scattering is of interest for the following reasons. It is known that unitarity corrections to threshold parameters in elastic scattering processes are, like off-mass-shell corrections, generally small.[6] This preserves the generally good agreement of the results of soft-meson calculations with experiment. An important exception is elastic $\bar{K}N$ scattering, which is strongly absorptive at the $\bar{K}N$ threshold; hence, the scattering length has a large imaginary part. However, soft-meson calculations in current algebra yield a real scattering length only (and that, too, with an incorrect sign).[7] It has long been noted that unitarity, in this case, couples the elastic $\bar{K}N \to \bar{K}N$ channel with the inelastic processes $\bar{K}N \to \pi\Sigma$, which has a lower threshold. It is therefore of interest to inquire whether the real input from current algebra can generate a large imaginary part for the elastic scattering length if the elastic and inelastic channels are coupled through unitarity. Our present investigation shows that this is indeed possible; in addition to this, we have been able to reproduce the correct sign of the real part of the scattering length, which is negative, with a current-algebra input of the opposite sign. The specific magnitudes of the real and imaginary parts, though somewhat parameter-dependent, agree well with some recent analyses of experimental data.

[6] See e.g., Bhargava *et al.*, Ref. 3.
[7] Tomozawa, (Ref. 1), obtained the value $+0.70$ for the s-wave $I=0$ $\bar{K}N$ scattering length. The imaginary part of the scattering length is absent and the sign of the real part disagrees with that obtained from most of the recent analyses of experimental data.

2. MODEL

We briefly outline our earlier model[3] for incorporating unitarity corrections to elastic meson-baryon scattering lengths from soft-meson calculations. The off-mass-shell scattering amplitude is written, in each partial wave and isotopic spin channel and for each fixed value of the off-shell mass variable, in the usual N/D form:

$$f_l^I(s,q^2)=\frac{N_l(s,q^2)}{D_l(s,q^2)}, \quad (1)$$

where s is the square of the center-of-mass energy and q^2 is the variable mass of the external mesons. The N function is approximated by a single pole and elastic unitarity is used to obtain $\mathrm{Im}D(s,q^2)$ for evaluating a once-subtracted dispersion relation for D:

$$f_l^I(s,q^2)=\frac{R(q^2)}{s+m_0^2}\bigg/\left[1-\frac{s-s_0}{\pi}\int_{s_{\rm th}}^{\infty}ds'\frac{\rho(s')R(q^2)}{(s'-s)(s'-s_0)(s'+m_0^2)}\right]. \quad (2)$$

If s_0 is now chosen to be the soft-meson threshold s_c, $R(0)$ is evaluated by comparing the amplitude at $q^2=0$ and $s=s_c$ with the value of the scattering length in soft-meson current algebra:

$$f_l^I(s_c,\, q^2=0)\equiv a_c=R(0)/(s_c+m_0^2). \quad (3)$$

The scattering length corrected for unitarity is given by

$$a=a_c\frac{\rho(M_n^2)(m_0^2+M_n^2)}{\rho((M_n+\mu_\pi)^2)[m_0^2+(M_n+\mu_\pi)^2]}[1-a_c\rho(M_n^2)(M_n^2+m_0^2)\,\mathrm{Re}I((M_n+\mu_\pi)^2,\mu_\pi^2)]^{-1}, \quad (4)$$

with

$$I(s)\equiv\left[\int_{s_{\rm th}}^{\infty}\frac{ds'}{\pi}\frac{|\mathbf{q}'|\rho(s')}{(s'-s)(s'-M_n^2)(s'+m_0^2)}\right](s-M_n^2).$$

Only the external mesons are taken off their mass shells; thus, $s_{\rm th}=(s_{\rm th})_{\rm physical}$, and the unitarity corrections are given by Eq. (4).

3. MULTICHANNEL ND^{-1} FORMALISM

To extend our model to multichannel and inelastic processes, we employ the multichannel ND^{-1} formalism developed by Bjorken[8] and applied to meson-baryon scattering by Martin and Wali[9] and Kumar.[10] We use the notation and kinematics of Martin and Wali, modified to allow for off-mass-shell external mesons.

For a general meson-baryon scattering process from channel i to channel j

$$B_i(p_i)+\mu_i(q_i)\rightarrow B_j(p_j)+\mu_j(q_j),$$

we define the Mandelstam kinematic invariants in the usual fashion:

$$s=-(p_i+q_i)^2,\quad p_i^2=-M_i^2,\quad p_j^2=-M_j^2$$
$$t=-(q_i-q_j)^2$$
$$u=-(p_i-q_j)^6$$
$$s+t+u=M_i^2+M_j^2-q_i^2-q_j^2.$$

The external mesons have masses $\sqrt{(-q_i^2)}$, $\sqrt{(-q_j^2)}$. Thus, the initial and final center-of-mass momenta are related to the total center-of-mass energy $W=\sqrt{s}$ through the off-shell variables q_i^2, q_j^2:

$$\begin{aligned}|\mathbf{q}_i^c|&=(1/2W)[(W+M_i)^2+q_i^2]^{1/2}\times[(W-M_i)^2+q_i^2]^{1/2}\\|\mathbf{q}_j^c|&=(1/2W)[(W+M_j)^2+q_j^2]^{1/2}\times[(W-M_j)^2+q_j^2]^{1/2}.\end{aligned} \quad (5)$$

The scattering amplitude for the transition from channel i to channel j, related to the T matrix through

$$T^{ji}=\frac{4\pi W}{(M_iM_j)^{1/2}}\chi_j^\dagger f^{ji}\chi_i,$$

has the partial-wave expansion

$$f^{ji}(W,x)=\sum[(l+1)f_{l+}^{ji}+lf_{l-}^{ji}]P_l(x)+i\boldsymbol{\sigma}\cdot(\hat{q}_j^c\times\hat{q}_i^c)(f_{l+}^{ji}-f_{l-}^{ji})P_l'(x), \quad (6)$$

where

$$x=\hat{q}_i^c\cdot\hat{q}_j^c. \quad (7)$$

The unitarity relation for $f_{l\pm}$ is as follows:

$$\mathrm{Im}f_{l\pm}=\sum f_{l\pm}^{jn}|\mathbf{q}_n^c|\,\theta(|W|-W_{\rm th}(n))f_{l\pm}^{ni}. \quad (8)$$

Note that $W_{\rm th}(n)$ is a physical threshold; all mesons in the intermediate states are on their mass shells:

$$W_{\rm th}(n)=M_n+\mu_n.$$

Following Martin and Wali,[9] we work in the W plane, and define modified partial-wave amplitudes which ensure correct threshold behavior and are free of kinematic singularities

$$G_{l+}^{ji}=\frac{2W}{q_j^{c\,l}[(W+M_j)^2+q_j^2]^{1/2}}f_{l+}^{ji}\times\frac{2W}{q_i^{c\,l}[(W+M_i)^2+q_i^2]^{1/2}}\equiv h_{l+}^{ji}f_{l+}^{ji}, \quad (9)$$

[8] J. D. Bjorken, Phys. Rev. Letters 4, 473 (1960).
[9] A. W. Martin and K. C. Wali, Phys. Rev. 130, 2455 (1963).
[10] Aditya Kumar, Phys. Rev. 139, B486 (1965).

$$G_{l-}{}^{ji}=\frac{2W}{q_j{}^{c\,l-1}[(W+M_j)^2+q_j{}^2]^{1/2}}f_{l-}{}^{ji}\times\frac{2W}{q_i{}^{c\,l-1}[(W+M_i)^2+q_j{}^2]^{1/2}}\equiv h_{l-}{}^{ji}f_{l-}{}^{ji}. \quad (10)$$

Unitarity now implies

$$\mathrm{Im}G_{l\pm}{}^{-1}=-\rho_{l\pm}. \quad (11)$$

$\rho_{l\pm}$ is a diagonal matrix in channel space

$$\rho_{l\pm}{}^{nn'}=\frac{|\mathbf{q}_n{}^c|}{4W^2}[(W+M_n)^2-\mu_n{}^2]\times\theta(|W|-W_{\mathrm{th}}(n))\delta^{nn'}. \quad (12)$$

Again, note that the physical masses of the mesons in the intermediate state appear in the unitarity relation for $\mathrm{Im}G_{l\pm}$, in contrast to $G_{l\pm}$, which is a function of off-shell variables, both through the off-shell transition amplitude $f_{l\pm}{}^{ji}$ and through the off-shell kinematic factor $h_{l\pm}{}^{ji}$.

Following our model for single-channel elastic meson-baryon scattering, we assume that for each set of fixed values for $q_i{}^2$, $q_j{}^2$ the ND^{-1} decomposition may be performed for the amplitudes $G_{0+}(W,q_i{}^2,q_j{}^2)$. The functions N_{0+} and D_{0+} are, like G_{0+}, 2×2 matrices in channel space ($i=1$ denotes the $\bar{K}N$ channel, $i=2$ the $\pi\Sigma$ channel). We assume once-subtracted dispersion relations for the D functions in the W plane

$$D^{ji}(W_1q_i{}^2,q_j{}^2)=\delta^{ji}+\frac{W-W_0}{\pi}\int dW'\frac{\mathrm{Im}D^{ji}(W',q_i{}^2,q_j{}^2)}{(W'-W_0)(W'-W)}. \quad (13)$$

Henceforth we suppress the subscripts 0+, since we deal with s-wave scattering only.

Elastic unitarity now gives

$$\mathrm{Im}D^{ji}=-\rho^{jK}N^{ki}=-\rho^jN^{ji}\theta(|W|-W_{\mathrm{th}}(j)), \quad (14)$$

where

$$\rho^j=\frac{|\mathbf{q}_j{}^c|}{4W^2}[(W+M_j)^2-\mu_j{}^2]. \quad (15)$$

$W_{\mathrm{th}}(j)$ is the physical threshold for the channel j. We have already noted that the functions $\rho^{nn'}$ depend on the physical masses of the mesons in the intermediate state:

$$D^{ji}(W,q_i{}^2,q_j{}^2)=\delta^{ji}-\frac{W-W_0}{\pi}\left[\int_{-\infty}^{-W_{\mathrm{th}}(j)}+\int_{W_{\mathrm{th}}(j)}^{\infty}\right]\times\frac{dW'\rho^j(W')N^{ji}(W',q_i{}^2,q_j{}^2)}{(W'-W_0)(W'-W)} \quad (16)$$

and

$$G^{ji}(W,q_i{}^2,q_j{}^2)=(ND^{-1})^{ji}=\sum N^{jk}(W,q_j{}^2,q_k{}^2)[D^{-1}(W,q_k{}^2,q_i{}^2)]^{ki}. \quad (17)$$

4. CURRENT ALGEBRA AND THE MULTI-CHANNEL ND^{-1} FORMALISM

We discuss here the results of soft-meson current algebra and their relationship to inputs in a dynamical ND^{-1} theory of multichannel processes.

In the soft-meson approximation, the amplitudes for the processes $\bar{K}N\to\bar{K}N$, $\bar{K}N\to\pi\Sigma$, and $\pi\Sigma\to\pi\Sigma$ are uncoupled, though related to each other in the chiral algebra of $SU(3)\times SU(3)$. Since the amplitudes are not unitary, it is no longer possible to write the threshold ansatz

$$f(W)=\frac{1}{|\mathbf{q}^c|\cot\delta-i|\mathbf{q}^c|}\equiv\frac{1}{1/a+\frac{1}{2}r_0|\mathbf{q}^c|^2-i|\mathbf{q}^c|}. \quad (18a)$$

Instead, one defines the scattering lengths as boundary values of off-mass-shell partial-wave amplitudes at appropriate thresholds:

$$a_c{}^{ij}\equiv f^{ij}(W,q_i,q_j)\,|_{W=W_c{}^{ij};\,q_i{}^\mu=q_j{}^\mu=0}. \quad (18b)$$

It is easily checked that this definition coincides with the conventional definition (18a) for physical on-mass-shell amplitudes with appropriate changes of thresholds.

For our input from current algebra we thus use three real scattering lengths.[11] This is to be contrasted with the physical situation where the scattering length for $\bar{K}N$ elastic scattering is complex and that for $\pi\Sigma$ elastic scattering is real. These real scattering lengths from current algebra are related to one another by threshold sum rules.[1,12] Thus,

$$a^c(\bar{K}N)=-3a_c{}^{3/2}(\pi N)\frac{\mu_K(1+\mu_\pi/M_n)}{\mu_\pi(1+\mu_K/M_n)}, \quad (19a)$$

$$a_c(\pi\Sigma)=-4a_c{}^{3/2}(\pi N)\frac{(1+\mu_\pi/M_n)}{(1+\mu_\pi/M_\Sigma)}, \quad (19b)$$

$$a_c(\pi\Sigma\to\bar{K}N)=(3/8\sqrt{6})[4a_c{}^{3/2}(\pi N)-a_c(\pi\Sigma)], \quad (19c)$$

$$2a_c{}^{3/2}(\pi N)+a_c{}^{1/2}(\pi N)=0. \quad (19d)$$

It is, however, well known from the analysis of Dalitz and Tuan[13] that s-wave $I=0$ $\bar{K}N$ scattering and reaction processes are described, at threshold, by three

[11] It must be emphasized that the definition of a scattering length for the inelastic channels $\bar{K}N\rightleftarrows\pi\Sigma$ has to be used with caution. Absorptive cross-sections are large at threshold and are not given, in the s wave, by the usual limit of $4\pi a^2$. However, these large values at threshold are provided by appropriate kinematic factors which must be supplied before absorptive cross-sections are computed from the amplitude in the scattering length approximation.

[12] N. G. Deshpande, Phys. Rev. **163**, 1629 (1967).

[13] R. H. Dalitz and S. F. Tuan, Ann. Phys. (N. Y.) **10**, 307 (1960).

real parameters, if use is made of time-reversal invariance and unitarity. The former ensures the equality of transition amplitudes $\bar{K}N \to \pi\Sigma$ and $\pi\Sigma \to \bar{K}N$; the latter enables the evaluation of the absorptive cross section for $\bar{K}N \to \pi\Sigma$ in terms of the scattering lengths for the elastic processes.

This happy circumstance—viz., the equality of the number of real parameters which determine, in soft-meson current algebra and unitarized coupled-channel analyses, the scattering and reaction processes—enables us to use the results of soft-meson current algebra as inputs in a unitarized coupled-channel formalism.

Thus, if we make a one-pole approximation for the N functions and write

$$N^{ij}(W,q_i^2,q_j^2)=R^{ij}(q_i^2,q_j^2)/(W+m^{ij}), \tag{20}$$

the residue matrix

$$R\equiv\begin{pmatrix} R^{11} & R^{12} \\ R^{21} & R^{22}\end{pmatrix} \tag{21}$$

is symmetric with the choice $m^{ij}=m^{ji}$, and the three real elements are determined by comparing the amplitudes $G^{ij}(W,q_i^2,q_j^2)$ with the results of current algebra at the soft-meson thresholds. The pole positions m^{ij} are parameters which are arbitrary, consistent with the requirement that the functions N^{ij} have discontinuities only in the unphysical region of the complex W plane.

5. DETERMINATION OF THE RESIDUE MATRIX R

The residue matrix R consists of four functions of two off-mass-shell variables $R^{ij}(q_i^2,q_j^2)$. Their determination follows from the requirement that the coupled, unitarized amplitudes agree with the results of current algebra at the appropriate values of the energy and off-mass-shall variables. Thus, we must first obtain expressions for our amplitudes in the appropriate limits.

In our approximation scheme,

$$\begin{aligned} N^{ij}(W,q_i^2,q_j^2)&=R^{ij}(q_i^2,q_j^2)/(W+m^{ij}),\\ D^{ij}(W,q_i^2,q_j^2)&=\delta^{ij}-\pi^{-1}(W-W_0)\\ &\quad\times R^{ij}(q_i^2,q_j^2)I^{ij}(W), \end{aligned} \tag{22}$$

with

$$I^{ij}(W)=\left(\int_{-\infty}^{-(M_i+\mu_i)}+\int_{(M_i+\mu_i)}^{\infty}\right)dW' \times\frac{\rho^i(W')}{(W'-W_0)(W'-W)(W'+m^{ij})}. \tag{23}$$

We note here the separation of the off-mass-shell dependence in the residue matrix R; the "unitarity" integrals I^{ij} start at the physical thresholds and have no off-mass-shell dependance. This is a consequence of the on-mass-shell character of the phase-space functions $\rho^i(W)$ which appear in the unitarity relations.

The unitarized coupled amplitudes are now obtained by inverting the matrix of the D functions:

$$G^{ij}(W,q_i^2,q_j^2) =\sum N^{ik}(W,q_i^2,q_j^2)[D^{-1}(W,q_i^2,q_j^2)]^{kj}. \tag{24}$$

Thus, for example,

$$G^{11}=(\det D)^{-1}(N^{11}D^{22}-N^{12}D^{21}).$$

The explicit expressions for the amplitudes are as follows:

$$G^{11}=R^{11}(W+m^{11})^{-1}(\det D)^{-1}\left[1-\frac{W-W_0}{\pi}R^{22}\left(I^{22}-\frac{R^{12}R^{21}(W+m^{11})}{R^{11}R^{22}(W+m^{12})}I^{21}\right)\right], \tag{25a}$$

$$G^{12}=R^{12}(W+m^{12})^{-1}(\det D)^{-1}\left[1-\frac{W-W_0}{\pi}R^{11}\left(I^{11}-\frac{(W+m^{12})}{(W+m^{11})}I^{12}\right)\right], \tag{25b}$$

$$G^{21}=R^{21}(W+m^{21})^{-1}(\det D)^{-1}\left[1-\frac{W-W_0}{\pi}R^{22}\left(I^{22}-\frac{(W+m^{21})}{(W+m^{22})}I^{21}\right)\right], \tag{25c}$$

$$G^{22}=R^{22}(W+m^{22})^{-1}(\det D)^{-1}\left[1-\frac{W-W_0}{\pi}R^{11}\left(I^{11}-\frac{R^{12}R^{21}(W+m^{22})}{R^{11}R^{22}(W+m^{21})}I^{12}\right)\right]. \tag{25d}$$

The functional dependances have been suppressed:

$$R^{ij}\equiv R^{ij}(q_i^2,q_j^2),\quad I^{ij}\equiv I^{ij}(W),$$

and

$$\det D\equiv\det D(W,q_i^2,q_j^2).$$

The amplitudes f^{ij}, in terms of which the scattering lengths are defined, are given by

$$f^{ij}(W,q_i^2,q_j^2)=h_{ij}^{-1}(W,q_i^2,q_j^2)G^{ij}(W,q_i^2,q_j^2),$$

where $h_{ij}(W,q_i^2,q_j^2)$ are the off-mass-shell kinematic functions defined earlier in Eq. (9).

We now demand that

$$f^{ij}(M_n,0,0)=a_c^{ij}; \tag{26}$$

this, together with the choice $W_0=M_n$ gives [since $\det D(M_n,0,0)=1$]

$$R^{ij}(0,0)=a_c^{ij}(M_n+m^{ij})h^{ij}(M_n,0,0), \tag{27}$$

where we have assumed in determining R^{22} that the soft-meson current-algebra result for the $\pi\Sigma$ scattering length does not vary significantly in extrapolation from $W=M_\Sigma$ to $W=M_n$.[14] The symmetry of the residue matrix follows from Eq. (27) with the choice $m^{ij}=m^{ji}$.

6. REAL AND IMAGINARY PARTS OF THE $\bar{K}N$ SCATTERING LENGTH

Following our earlier model,[3] as well as the work of Fubini and Furlan,[5] we assume smooth extrapolation of all dynamical parameters in the off-mass-shell meson variables q_i^2, q_j^2.[15] The residues $R^{ij}(q_i^2,q_j^2)$ represent, in our formulation, the "strength" of the forces generated by soft-meson current algebra. Thus, our assumption implies

$$R^{ij}(0,0)\approx R^{ij}(-\mu_i^2,\,-\mu_j^2)\equiv R^{ij}.$$

To reduce the parameter dependence of the theory, we choose poles in all the channels at the same position

$$m^{ij}=m.$$

With this choice,

$$I^{11}=I^{12}\equiv I^1\,,$$
$$I^{21}=I^{22}\equiv I^2\,.$$

The $\bar{K}N$ scattering length is evaluated by computing

$$\begin{aligned}f^{11}(W=M_n+\mu_K,\,q_1^2=q_2^2=-\mu_K^2)\\ =h_{11}^{-1}(W=M_n+\mu_K,\,q_1^2=q_2^2=-\mu_K^2)\\ \times G^{11}(W=M_n+\mu_K,\,q_1^2=q_2^2=-\mu_K^2)\,.\end{aligned}$$

Use of Eq. (25a) for G^{11} and of Eq. (27) for the residues leads to the following expression for the scattering length:

$$\begin{aligned}a(\bar{K}N)=\frac{M_n(M_n+m)a_c(\bar{K}N)}{(M_n+\mu_K)(M_n+\mu_K+m)}[\det D(W=M_n+\mu_K)]^{-1}\\ \times\left[1-\frac{\mu_K}{\pi}\frac{4M_n^2(M_n+m)}{(M_n+M_\Sigma)^2}a_c(\pi\Sigma)\left(1-\frac{a_c^2(\bar{K}N\to\pi\Sigma)}{a_c(\bar{K}N)a_c(\pi\Sigma)}\right)I^2\right].\end{aligned}\tag{28}$$

The first factor in this expression represents corrections to the scattering length arising from kinematics alone when the amplitude is evaluated at the physical rather than the soft-meson threshold. This correction does not depend on the coupling of the channels. It depends on $a_c(\bar{K}N)$ only: hence the corrected scattering length is also real. The second factor represents corrections from unitarity and the coupling of the channels; it thus depends on $a_c(\pi\Sigma)$ and $a_c(\bar{K}N\to\pi\Sigma)$ and generates the imaginary part of $a(\bar{K}N)$.

The determination of the real and imaginary parts of the scattering length is now straightforward. We note that

$$\begin{aligned}\det D(W)&=D^{11}(W)D^{22}(W)-D^{12}(W)D^{21}(W)\\ &=(1-\pi^{-1}(W-M_n)R^{11}I^1)(1-\pi^{-1}(W-M_n)R^{22}I^2)-\pi^{-2}(W-M_n)^2R^{12}R^{21}I^1I^2\end{aligned}\tag{29}$$

depends on both the unitarity integrals $I^i(W)$, $i=1$, 2. At the $\bar{K}N$ threshold the integral I^1 is real:

$$\begin{aligned}I^1(W=M_n+\mu_K)\\ =P\left[\int_{M_n+\mu K}^{\infty}dW'\left(\frac{\rho_1(W')}{(W'-M_n)(W'-M_n-\mu_K)(W'+m)}-\frac{\rho_1(-W')}{(W'+M_n)(W'+M_n+\mu_K)(W'-m)}\right)\right].\end{aligned}\tag{30}$$

In contrast, the integral $I^2(W)$, which starts at the $\pi\Sigma$ threshold, is complex at the higher $\bar{K}N$ threshold and generates the imaginary part of $a(\bar{K}N)$:

$$\begin{aligned}I^2(W=M_n+\mu_K+i\epsilon)=P\int_{M_\Sigma+\mu_\pi}^{\infty}dW'\left(\frac{\rho_2(W')}{(W'-M_n)(W'-M_n-\mu_K)(W'+m)}-\frac{\rho_2(-W')}{(W'=M_n)(W'+M_n+\mu_K)(W'-m)}\right)\\ +i\pi\frac{\rho_2(M_n+\mu_K)}{\mu_K(M_n+\mu_K+m)}\\ \equiv \mathrm{Re}I^2+i\,\mathrm{Im}I^2.\end{aligned}\tag{31}$$

[14] The justification for this approximation is as follows. As we have noted in our introductory remarks, unitarity corrections to real scattering lengths are small; they will be even smaller in moving the threshold from one point to another in a domain away from the physical branch point. The corrections will, at best, be of the order of off-mass-shell corrections which we have neglected.

[15] We can offer a prescription for continuation of the residues to on-mass-shell points which agrees with the assumption of smooth extrapolation and takes account of some kinematic corrections. Thus, if

$$R^{ij}(0,0)=a_c^{ij}(M_n+m^{ij})h^{ij}(M_n,0,0),$$

continuation to on-mass-shell points is provided by writing

$$R^{ij}(q_i^2,q_j^2)=a_c^{ij}(M_n+m^{ij})h^{ij}(M_n,q_i^2,q_j^2)/h^{ij}(M_n,0,0).$$

Such continuation provides a correction of about 5% to $\bar{K}N$ and of about 1% to $\pi\Sigma$ scattering lengths. However, such continuation is manifestly nonunique and does not incorporate any dynamical corrections. Hence, we use it merely to lend some plausibility to our assumption of smooth extrapolation.

We thus obtain

$$\mathrm{Re}\,\det D(W=M_n+\mu_K)=1-\frac{\mu_K}{\pi}(M_n+m)a_c(\bar{K}N)\left(I^1+\frac{4M_n{}^2a_c(\pi\Sigma)}{(M_n+M_\Sigma)^2a_c(\bar{K}N)}\,\mathrm{Re}I^2\right)+\left(\frac{\mu_K}{\pi}\right)^2\frac{4M_n{}^2(M_n+m)^2}{(M_n+M_\Sigma)^2}a_c(\bar{K}N)a_c(\pi\Sigma)\left(1-\frac{a_c{}^2(\bar{K}N\to\pi\Sigma)}{a_c(\bar{K}N)a_c(\pi\Sigma)}\right)I^1\,\mathrm{Re}I^2,\quad(32)$$

$$\mathrm{Im}\,\det D(W=M_n+\mu_K)=-\frac{\mu_K}{\pi}\frac{4M_n{}^2(M_n+m)}{(M_n+M_\Sigma)^2}a_c(\pi\Sigma)\,\mathrm{Im}I^2+\left(\frac{\mu_K}{\pi}\right)^2\frac{4M_n{}^2(M_n+m)^2}{(M_n+M_\Sigma)^2}a_c(\bar{K}N)a_c(\pi\Sigma)\left[1-\frac{a_c{}^2(\bar{K}N\to\pi\Sigma)}{a_c(\bar{K}N)a_c(\pi\Sigma)}\right]I^1\,\mathrm{Im}I^2.\quad(33)$$

The expressions for the real and imaginary parts of the scattering lengths are

$$\mathrm{Re}a(\bar{K}N)=\frac{M_n(M_n+m)a_c(\bar{K}N)[\mathrm{Re}\,\det D-\Gamma(\mathrm{Re}I^2\,\mathrm{Re}\,\det D+\mathrm{Im}I^2\,\mathrm{Im}\,\det D)]}{(M_n+\mu_K)(M_n+\mu_K+m)[(\mathrm{Re}\,\det D)^2+(\mathrm{Im}\,\det D)^2]},\quad(34)$$

$$\mathrm{Im}a(\bar{K}N)=\frac{M_n(M_n+m)a_c(\bar{K}N)[\Gamma(\mathrm{Re}I^2\,\mathrm{Im}\,\det D-\mathrm{Im}I^2\,\mathrm{Re}\,\det D)-\mathrm{Im}\,\det D]}{(M_n+\mu_K)(M_n+\mu_K+m)[(\mathrm{Re}\,\det D)^2+(\mathrm{Im}\,\det D)^2]},\quad(35)$$

Here

$$\Gamma=\frac{\mu_K}{\pi}\frac{4M_n{}^2(M_n+m)}{(M_n+M_\Sigma)^2}a_c(\pi\Sigma)\left[1-\frac{a_c{}^2(\bar{K}N\to\pi\Sigma)}{a_c(\bar{K}N)a_c(\pi\Sigma)}\right].\quad(36)$$

7. RESULTS AND DISCUSSION

The numerical results for the real and imaginary parts of the $\bar{K}N$ scattering length is shown in Table I. The inputs from current algebra correspond to $a_c{}^{1/2}(\pi N)=0.2m_\pi{}^{-1}$; the other scattering lengths from current algebra are computed using Eqs. (19). The pole position has been varied in the range $0\leqslant W\leqslant M_n$. Our results are compared with the results of previous investigations.[16–18]

The sensitivity of the results to the choice of the pole position is significant and is, we believe, to be expected on general grounds. The region of the real axis in the W plane where $\mathrm{Im}N\neq0$ is small: $-(M_\Sigma+\mu_\pi)\leqslant W\leqslant(M_\Sigma+\mu_\pi)$. Thus, the pole is never far away from the branch points where the unitarity cuts begin, and variations of the pole position thus strongly affect the numerical results. We note nonetheless that the sign of the real part is reversed for all values of m and that the imaginary part generated is always positive. This latter feature is, as is well known, essential to a correct theory, since $\mathrm{Im}a(\bar{K}N)$ is directly proportional to the absorptive cross section for $\bar{K}N\to\pi\Sigma$. We regard these aspects of our results as more significant than the specific numerical magnitudes of the real and imaginary parts of the scattering length.

TABLE I. Real and imaginary parts of s-wave $\bar{K}N$ scattering length ($I=0$) in units of fermis.

Pole position	Present calculation		Experimental information		
m	$\mathrm{Re}a(\bar{K}N)$	$\mathrm{Im}a(\bar{K}N)$	$\mathrm{Re}a(\bar{K}N)$	$\mathrm{Im}a(\bar{K}N)$	Authors
0	−1.84	1.21	−1.85	1.1	Dalitz & Tuan[a]
$\frac{1}{2}M_n$	−1.31	0.85	−1.39	0.9	Fujii[b]
			−1.30	0.84	Akiba & Capps[c]
			−1.57	0.54	Kittel *et al.*[d]
			−1.65	0.73	Kim[d]
M_n	−0.91	0.42	−0.56	0.96	Humphrey & Ross[c]
			±0.49	±0.17	

[a] Ref. 13. [b] Ref. 16. [c] Ref. 17. [d] Ref. 18.

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17 T. Akiba and R. H. Capps, Phys. Rev. Letters **8**, 28 (1962); **8**, 175 (1962); W. E. Humphrey and R. Ross, Phys. Rev. **127**, 1305 (1962).
18 W. Kittel, G. Otter, and I. Wacek, Phys. Letters **21**, 349 (1966); J. K. Kim, Phys. Rev. Letters **19**, 1074 (1967).
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Nonetheless, the close agreement with the results of Fujii[16] for the choice $m=\frac{1}{2}M_n$ suggests an important qualitative observation. It has long been known that the $Y_0^*(1405)$ resonance contributes significantly to $K\bar{N}$ scattering lengths. Thus, von Hippel and Kim,[19] in applying unitarity corrections to $\mathrm{Re}a_c(\bar{K}N)$, specifically note the importance of the Y_0^*, and incorporate its effects in a forward dispersion relation. In our formulation, the Y_0^* must appear as a virtual bound state in the $\bar{K}N$ channel, as a result of our input forces. The close agreement of our results which those of Fujii[16] and of Akiba and Capps,[17] whose analyses include the effects of the Y_0^*, suggests that the existence of this resonance close to threshold is consistent with the results of current algebra when corrected to include the effects of unitarity.

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Relation of Charged-K Decay to CP Violation

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An $SU(3)$ model is described which relates the rate of the charged-K decay $K^+ \to \pi^+ + \pi^0$ to the magnitude of CP violation. A weak-interaction Lagrangian for nonleptonic processes is constructed from octet currents in a Cabibbo frame under the hypothesis that the CP-conserving processes are described predominantly by an octet-dominant interaction Lagrangian, and that the CP-violating processes enter solely through the 27-dimensional representation of $8\otimes 8$. Three types of octet currents are used: L_μ, which is linear in the field operators, and J_μ and K_μ, which are bilinear and possess opposite C-transformation properties. It is shown that if only $I=2$ contributions enter into the 27 portion of the Lagrangian, then, approximating $|K_S\rangle$ and $|K_L\rangle$ by the CP eigenstates $|K_1\rangle$ and $|K_2\rangle$, the observed rate of the charged-K decay and $\theta=0.26$ for the Cabibbo angle fix $|\eta_{+-}|=5.38\times10^{-3}$ and $|\eta_{00}|=3.12\times10^{-3}$, of the proper order of magnitude. Introducing the experimentally observed departure of $|K_S\rangle$ and $|K_L\rangle$ from CP eigenstates and a renormalization constant for the K current yields $|\eta_{+-}|=1.97\times10^{-3}$ and $|\eta_{00}|=2.92\times10^{-3}$, in good agreement with experiment.

I. INTRODUCTION

THERE have been various approaches[1] to an explanation of CP violation since it was first seen in the charged-pion decay of the K^0-$\bar{K}^0$ system by Christenson, Cronin, Fitch, and Turlay.[2] These theories have been phenomenological in nature and in some cases have temporarily been in disagreement with the experimental evidence for the decay rates of the K_L and K_S modes into two charged and two uncharged pions. Specifically, data[3] had indicated that $|\eta_{+-}|\neq|\eta_{00}|$, and thus models which utilized CP-invariant weak-interaction Lagrangians were in apparent conflict with experiment. Recent data[4] indicate a smaller magnitude for $A[K_L\to(\pi\pi)_{I=2}]$. Within the existing experimental error $|\eta_{00}|$ may be equal to $|\eta_{+-}|$, again giving substance to such models as the superweak theory.[5]

It is the purpose of this paper to propose a rather straightforward model within the $SU(3)$ symmetry scheme which relates the rate of the charged K decay, $K^+\to\pi^++\pi^0$, to the magnitude of CP violation. We propose an ordinary symmetric coupled current-current theory within the $SU(3)$ framework. The CP violation is introduced by an octet vector current which, under charge conjugation, transforms into itself and thus

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