Chiral Lagrangian Calculation of Pion-Nucleon Scattering Lengths*

R. D. PECCEI

Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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We have considered two chiral-invariant Lagrangian models of pion-nucleon scattering: Weinberg's model, which contains a direct pion-nucleon scattering term; and an extension of Schwinger's model in which this direct term is replaced by a ρ-mediated term. These models have been extended by including a $\pi NN^*(1236)$ interaction. This has allowed us to calculate the N^* contributions to the S-wave pion-nucleon scattering lengths and to compute the P-wave scattering lengths. The theoretically obtained values for these scattering lengths are found to be in reasonable agreement with experiment, with the possible exception of the $a^-_{P1/2}$ scattering length, for which only the direct model gives agreement with the present experimental data. For the a_S^+ scattering length, agreement was obtained only after the introduction of a contact term. The presence of such a term is made more plausible by the study of the asymptotic behavior of the isospin-even amplitudes. We have also compared our analysis with similar current-algebra calculations and obtain over-all agreement, especially with the ρ-exchange model, although we differ from these calculations in various details.

I. INTRODUCTION

RECENTLY, Weinberg[1] introduced a chiral Lagrangian method which, for soft-pion processes, reproduces the results of current algebra. Since then, much use has been made of chiral Lagrangians in describing low-energy processes.[2] In particular, these ideas have been incorporated by Schwinger in his source theory.[3] Here these effective Lagrangians play a primary role, since they represent the most convenient way of describing the phenomenology.

We will be primarily interested in applying the chiral Lagrangian method to pion-nucleon scattering. The basic Lagrangian for this problem was constructed by Weinberg[1] and subsequently extended by Schwinger[4] to include the spin-1 mesons, ρ and A_1, in the scheme. Both of these Lagrangians give good predictions at threshold, but are not suitable for describing matters much above threshold, since resonance effects become important. We have remedied this situation in part by adding to the pion-nucleon Lagrangians of Weinberg and Schwinger a term which incorporates the interaction of the pion-nucleon system with the first nucleon resonance $N^*(1236)$. This $N^*N\pi$ interaction term has been constructed by making explicit use of both chiral invariance and of a restrictive condition imposed by the spin character of the N^*.

As a first application of this extended Lagrangian we have computed the P-wave pion-nucleon scattering lengths and N^* contributions to the S-wave scattering lengths. The corresponding current-algebra calculations for the P-wave scattering lengths have been done by Schnitzer and by Raman.[5] The latter author also considers, from the current-algebra point of view, the resonance corrections to the S-wave scattering lengths. We will compare the results obtained from the phenomenological Lagrangian method with the current-algebra calculations further on. We remark here only that we obtain general agreement with the work of Schnitzer and Raman, although we differ from them on various points, and of course in the approach.

The plan of this article is as follows. In Sec. II we indicate how to construct a chiral-invariant Lagrangian, and specialize the discussion to the pion-nucleon system. Section III is devoted to the spin-$\frac{3}{2}$ formalism, and the implications of this section are applied in Sec. IV to the construction of a $N^*N\pi$ vertex. In Sec. V we review some kinematical preliminaries for pion-nucleon scattering. In Secs. VI and VII we calculate the isospin-odd and isospin-even scattering lengths, respectively. The asymptotic behavior of the scattering amplitudes, and the implications that this behavior has at threshold, are discussed in Sec. VIII. In Sec. IX we compare our results with the current-algebra calculations and make some final observations.

II. CHIRAL INVARIANCE AND THE PION-NUCLEON SYSTEM

To construct a chiral-invariant Lagrangian for the pion-nucleon system, we follow the nonlinear method discussed by Weinberg.[6] The underlying assumption is that the pion field transforms nonlinearly under the chiral group $SU(2)\times SU(2)$.

If X_a is the generator of chiral transformations, then the most general nonlinear transformation of the pion field under chirality can be shown to be[6]

$$[X_a,\pi_b]=-(i/\lambda)[\tfrac{1}{2}(1-\lambda^2\pi^2)\delta_{ab}+\lambda^2\pi_a\pi_b]. \quad (1)$$

The corresponding transformation for any field ψ is

$$[X_a,\psi]=\lambda(\mathbf{t}\times\boldsymbol{\pi})\psi, \quad (2)$$

where t is the appropriate isospin matrix for the field ψ.

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[2] J. Wess and B. Zumino, Phys. Rev. **163**, 1727 (1967); P. Chang and F. Gürsey, *ibid.* **164**, 1752 (1967); B. W. Lee and H. T. Nieh, *ibid.* **166**, 1507 (1968).

[3] J. Schwinger, Phys. Rev. **158**, 1391 (1967).

[4] J. Schwinger, Phys. Letters **24B**, 473 (1967).

[5] H. Schnitzer, Phys. Rev. **158**, 1471 (1967); K. Raman, *ibid.* **164**, 1736 (1967).

[6] S. Weinberg, Phys. Rev. **166**, 1568 (1968).

Because of the presence of the π field in Eq. (2), it is clear that $\partial_\mu\psi$ does not transform like ψ under chirality. Nevertheless, one can construct a covariant derivative

$$D_\mu\psi=\partial_\mu\psi+2i\lambda^2(1+\lambda^2\pi^2)^{-1}\mathbf{t}\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})\psi \tag{3}$$

such that $D_\mu\psi$ transforms like ψ.

Similarly, one can define a covariant pion derivative

$$D_\mu\boldsymbol{\pi}=(1+\lambda^2\pi^2)^{-1}\partial_\mu\boldsymbol{\pi}, \tag{4}$$

which also obeys the transformation law given by Eq. (2).

Because of the structure of Eq. (2), any isotopic spin-invariant Lagrange function constructed out of $D_\mu\pi$, ψ, and $D_\mu\psi$ will then be automatically chiral-invariant. In particular, for the pion-nucleon system we can write the following chiral-invariant Lagrangian[1,4,6]:

$$\mathfrak{L}_{\pi N}=-\bar{N}[\gamma^\mu(D_\mu/i)+M]N-\tfrac{1}{2}D_\mu\boldsymbol{\pi}\cdot D^\mu\boldsymbol{\pi} +(f/m_\pi)\bar{N}i\gamma_\mu\gamma_5\tau\cdot ND^\mu\boldsymbol{\pi}. \tag{5}$$

Because the pion has a finite mass we must add to Eq. (5) a pion-mass term

$$\mathfrak{L}_{\pi N}'=-\tfrac{1}{2}m_\pi^2\pi^2. \tag{5$'$}$$

It is clear that this term breaks chiral invariance; thus we do not have total symmetry of $\mathfrak{L}_{\pi N}$ under chiral transformations, but rather we have a partial symmetry.[4,6]

Since we are primarily interested in pion-nucleon scattering, we abstract from the above the relevant effective Lagrangian that will contribute to this process.

$$\mathfrak{L}_{\pi N}^{\text{eff}}=(f/m_\pi)\bar{N}i\gamma_\mu\gamma_5\tau\cdot N\partial^\mu\boldsymbol{\pi} +(f_0/m_\pi)^2\bar{N}\gamma_\mu\tau\cdot N(\partial^\mu\boldsymbol{\pi}\times\boldsymbol{\pi}). \tag{6}$$

The coupling constant f, which corresponds to the usual derivative-coupling interaction, has the numerical value[7]

$$f=1.01\pm0.01.$$

To determine the value of the other coupling constant f_0 (which is related to the chiral parameter λ by $f_0=m_\pi\lambda$), let us follow Schwinger[8] and construct from the total Lagrangian the current

$$J_\mu=V_\mu-A_\mu=\partial_\mu\boldsymbol{\pi}\times\boldsymbol{\pi}+\bar{N}\gamma_\mu(\tfrac{1}{2}\boldsymbol{\tau})[1-(f/f_0)i\gamma_5]N+\cdots.$$

If we now assume that this is the current associated with the β-decay properties of the pions and the nucleons, it follows that

$$f/f_0=-G_A/G_V=1.18\pm0.02.$$

It is possible, and we think desirable, to extend the idea of chiral symmetry to incorporate the unit spin particles ρ and A_1. This has been done by Schwinger.[4] One starts with two non-Abelian gauge fields ρ^μ and a^μ representing 1^- and 1^+ excitations. Because of the existence of the pion, a^μ is not purely the A_1 field, but rather

$$a^\mu=A_1{}^\mu+\partial^\mu\pi/m_A. \tag{7}$$

When these gauge fields are included, the direct pion-nucleon interaction term which came from the chiral boosting of the nucleon kinetic energy is canceled by a term coming from the nucleon–axial-vector interaction.[4] But it is effectively replaced by a ρ-mediated term which can be characterized by the following Lagrangian:

$$\mathfrak{L}_{\rho\pi N}=g\boldsymbol{\varrho}^\mu\cdot[\partial_\mu\boldsymbol{\pi}\times\boldsymbol{\pi}+\bar{N}\gamma_\mu(\tfrac{1}{2}\boldsymbol{\tau})N]. \tag{8a}$$

This Lagrangian embodies the universal coupling of the ρ to the isotopic current of the pions and nucleons. We note that since this gauge term gives us the same prediction in the forward direction as the direct chiral term, we must have

$$g/\sqrt{2}m_\rho=f_0/m_\pi, \tag{9}$$

which is the Kawarabayashi-Suzuki relation.[9]

These are not all the ρ-mediated terms that contribute to pion-nucleon scattering. From the inclusion of the 1^+ gauge particle, we have an additional $\rho\pi\pi$ interaction,[4]

$$\mathfrak{L}_{\rho\pi\pi}=-(g/4m_\rho^2)\boldsymbol{\varrho}^{\mu\nu}\cdot(\partial_\mu\boldsymbol{\pi}\times\partial_\nu\boldsymbol{\pi}). \tag{8b}$$

We can use Eq. (8b) and the first term of Eq. (8a) to calculate the decay width of the ρ. If we use the value of g implied by the Kawarabayashi-Suzuki relation, we obtain good agreement with the recent value determined for the ρ width in the $e^++e^-\rightarrow\pi^++\pi^-$ experiment, $\Gamma_\rho=93\pm15$ MeV.[10]

Finally, there is also an additional ρ-nucleon interaction term which arises from the coupling of the anomalous magnetic moment of the nucleons to the ρ field strengths.[2]

$$\mathfrak{L}_{\rho NN}=(gK/4M)\boldsymbol{\varrho}^{\mu\nu}\cdot\bar{N}\sigma_{\mu\nu}(\tfrac{1}{2}\boldsymbol{\tau})N, \tag{8c}$$

where $K=\mu_p-\mu_n\simeq3.70$.

The second terms of Eq. (8a) and Eq. (8c) encompass the idea of ρ dominance of the isovector nucleon form factors. This can be seen by noting that the electromagnetic potential A_μ is coupled to the ρ field as follows[11]:

$$\mathfrak{L}_{\rho A}=(e/g)m_\rho^2\rho_3{}^\mu A_\mu.$$

This implies in turn that the nucleon isovector electro-

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[7] J. Hamilton and W. Woolcock, Rev. Mod. Phys. **35**, 737 (1963).

[8] J. Schwinger, Phys. Rev. **167**, 1432 (1968).

[9] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966).

[10] V. L. Auslander *et al.*, Phys. Letters **25B**, 433 (1967).

[11] J. Schwinger, Brandeis Lectures, 1967 (unpublished).

magnetic vertex is given by

$$\mathcal{L}_{ANN}{}^{V}=e\int d^4x'[\bar{N}(x)\gamma_\mu(\tfrac{1}{2}\tau_3)N(x)m_\rho{}^2\Delta_{+\rho}(x-x')A^\mu(x')$$
$$+(K/4M)\bar{N}(x)\sigma_{\mu\nu}(\tfrac{1}{2}\tau_3)N(x)m_\rho{}^2\Delta_{+\rho}(x-x')F^{\mu\nu}(x')],$$

which displays explicitly the ρ-dominance assumption.

We see that the extension of chiral symmetry to include the spin-1 mesons ρ and A_1, together with the idea of ρ dominance of the isovector nucleon form factors, replaces the direct pion-nucleon interaction term in Weinberg's Lagrangian by

$$\mathcal{L}_{\rho\,\mathrm{exch}}=g[\bar{N}\gamma_\mu(\tfrac{1}{2}\boldsymbol{\tau})\cdot N\boldsymbol{\rho}^\mu+(K/4M)\bar{N}\sigma_{\mu\nu}(\tfrac{1}{2}\boldsymbol{\tau})N\cdot\boldsymbol{\rho}^{\mu\nu}]$$
$$+g[(\partial_\mu\boldsymbol{\pi}\times\boldsymbol{\pi})\cdot\boldsymbol{\rho}^\mu-(4m_\rho{}^2)^{-1}(\partial_\mu\boldsymbol{\pi}\times\partial_\nu\boldsymbol{\pi})\cdot\boldsymbol{\rho}^{\mu\nu}]. \quad (10)$$

We then have two basic effective Lagrangians for low-energy pion-nucleon scattering: the direct Lagrangian which is given by Eq. (6), and the ρ-exchange Lagrangian which is given by the first term of Eq. (6) plus Eq. (10). To both of these Lagrangians we must now add contributions that come from the interaction of the nucleons and pions with the nucleon resonances. In particular, we shall consider the $\pi NN^*(1236)$ interaction. Before we can do this, however, we must review certain properties associated with a spin-$\frac{3}{2}$ field.

III. SPIN-$\frac{3}{2}$ FORMALISM

The free Lagrangian for a spin-$\frac{3}{2}$ field can be written as[12]

$$\mathcal{L}=-\bar{\psi}^\alpha\{[\gamma^\mu g_{\alpha\beta}+w(\delta_\alpha{}^\mu\gamma_\beta+\delta_\beta{}^\mu\gamma_\alpha)+K\gamma_\alpha\gamma^\mu\gamma_\beta]\partial_\mu/i$$
$$+M(g_{\alpha\beta}+T\gamma_\alpha\gamma_\beta)\}\psi^\beta, \quad (11)$$

where

$$K=-\tfrac{1}{2}(3w^2+2w+1),\quad T=\tfrac{1}{4}[(1+3w)^2+3(1+w)^2],$$

and w is an arbitrary parameter.

If one makes the point transformation

$$\psi_\alpha\rightarrow\psi_\alpha+\tfrac{1}{4}\lambda\gamma_\alpha\gamma_\beta\psi^\beta, \quad (12)$$

then $\mathcal{L}(w)\rightarrow\mathcal{L}(w')$, where $w'=w(1-\lambda)-\frac{1}{2}\lambda$. This transformation does not affect the spin-$\frac{3}{2}$ content of ψ_α, but merely mixes the two classes of spin-$\frac{1}{2}$ components of ψ_α.[12] Thus we see that the particular value of w does not have any physical significance.

By making use of the above Lagrangian, we can write down the equation obeyed by the spin-$\frac{3}{2}$ propagator $G_{\alpha\beta}$. In momentum space it reads

$$[(\gamma\cdot p+M)g_{\alpha\beta}+w(p_\alpha\gamma_\beta+\gamma_\alpha p_\beta)+K\gamma_\alpha\gamma\cdot p\gamma_\beta$$
$$+MT\gamma_\alpha\gamma_\beta]G^\beta{}_\rho(p)=g_{\alpha\rho}. \quad (13)$$

This can be readily solved and yields for the propagator

$$G_{\alpha\beta}(p)=\frac{-\gamma\cdot p+M}{p^2+M^2-i\epsilon}\Bigg[g_{\alpha\beta}+\frac{w}{3(2w+1)^2}\frac{p_\alpha\gamma_\beta}{M}+\frac{3w+2}{3(2w+1)}\frac{\gamma_\alpha p_\beta}{M}+\frac{2w}{3(2w+1)}\frac{p_\alpha p_\beta}{M^2}+\frac{(3w+1)(w+1)}{6(2w+1)^2}\frac{\gamma_\alpha\gamma\cdot p\gamma_\beta}{M}$$
$$+\frac{w(w+1)}{3(2w+1)^2}\frac{p_\alpha\gamma\cdot p\gamma_\beta}{M^2}-\frac{(w+1)}{3(2w+1)}\frac{\gamma_\alpha\gamma\cdot pp_\beta}{M^2}+\left(\frac{1+3w+3w^2}{3(2w+1)^2}+\frac{(w+1)^2}{6(2w+1)}\frac{p^2}{M^2}\right)\gamma_\alpha\gamma_\beta\Bigg]. \quad (14)$$

The choice $w=-1$ simplifies $G_{\alpha\beta}$ considerably, and in fact is what is commonly used in the literature.[5]

Some care must be exercised when using this propagator in the presence of interactions. This can be best illustrated by an example. In view of future applications, let us consider the interaction of ψ_μ with a spin-$\frac{1}{2}$ field ψ and an axial-vector field B_μ. We can write, quite generally,

$$\mathcal{L}_{\mathrm{int}}=h\bar{\psi}^\mu O_{\mu\nu}\psi B^\nu. \quad (15)$$

We note that under the transformation (12) $\mathcal{L}_{\mathrm{int}}$ does not in general remain invariant. But if we are to have a *pure* coupling to the spin-$\frac{3}{2}$ field, it is necessary that $\mathcal{L}_{\mathrm{int}}$ remain invariant under this transformation, since this transformation only mixes the spin-$\frac{1}{2}$ component of ψ_μ. This can be achieved only if we impose a subsidiary condition on the coupling matrix $O_{\mu\nu}$, namely,

$$\gamma_\mu O^{\mu\nu}=0. \quad (16)$$

We remark that only when such subsidiary conditions are imposed is it correct to make a particular choice of w in the propagator $G_{\alpha\beta}(w)$, because only then is the theory truly w-invariant. This is similar to gauge invariance in electromagnetism, where freedom of gauge transformation

$$A_\mu\rightarrow A_\mu+\partial_\mu\phi$$

implies that A_μ must be coupled only to conserved currents. Once the coupling is gauge-invariant one has the freedom to pick any desired gauge. In our case, once we have made the total Lagrangian invariant under (12), and only then, we are free to choose any particularly convenient value of w.

IV. $N^*N\pi$ COUPLING

To construct a chiral-invariant interaction for the $N^*N\pi$ system, we have seen that we must couple the N^*N current to the covariant derivative of the pion field, and not to the pion field itself. The most general such coupling is, neglecting isospin indices,

$$\mathcal{L}_{N^*N\pi}=(h/m_\pi)\bar{N}_\mu{}^*(4g^{\mu\nu}+\gamma^\mu\gamma^\nu)ND_\nu\pi+\mathrm{H.c.} \quad (17)$$

[12] K. Johnson and E. C. G. Sudarshan, Ann. Phys. (N. Y.) 13, 126 (1961).

The combination of $g^{\mu\nu}$ and $\gamma^\mu\gamma^\nu$ taken is such that the restriction (16) is identically satisfied.

It is useful to remark here that the $\gamma^\mu\gamma^\nu$ term, which is necessary for pure spin-$\frac{3}{2}$ coupling, does not contribute to the $N^*N\pi$ interaction when the N^* is on the mass shell, because then effectively $\gamma^\mu N_\mu^*=0$. But this term does contribute off the mass shell, and thus we can view the subsidiary condition, Eq. (16), as a prescription of how to go off the mass shell.

The coupling constant h can be determined from the known width of the N^*.[13] One has

$$\Gamma_{N^*}=\frac{4}{3\pi}h^2\frac{p^3(E+M)}{M^*m_\pi^2},$$

where p and E are the momentum and energy, respectively, of the nucleon in the N^* rest system. This gives

$$h^2=0.290\pm0.006.$$

It may be remarked that to the Lagrangian given in Eq. (17) it is also possible to add another term in which the N^*N current is coupled to the double derivative of the pion field. In our analysis we have found this term unnecessary.

V. PION-NUCLEON SCATTERING PRELIMINARIES

We now have all the elements needed to calculate pion-nucleon scattering at low energy. The effective Lagrangian to be used is given by Eqs. (17) and (6) (direct model), or, alternatively, by Eqs. (17), (10), and the first term of Eq. (6) (ρ-exchange model).

In what follows, we let p_1 and q_1 represent the four-momenta of the incoming nucleon and pion, respectively; and p_2 and q_2 represent the four-momenta of the outgoing nucleon and pion, respectively. Then in our metric $s=-(p_1+q_1)^2$, $u=-(p_1-q_2)^2$, and $t=-(p_1-p_2)^2$.

It is convenient to decompose the T matrix in invariant amplitudes.[14]

$$T_{\beta\alpha}(s,t)=\bar{u}(p_2)[A_{\beta\alpha}(s,t)+\gamma\cdot QB_{\beta\alpha}(s,t)]u(p_1), \quad (18)$$

where $Q=\frac{1}{2}(q_1+q_2)$.

We note here for reference that the differential cross section in the c.m. system for unpolarized nucleons is given by

$$\frac{d\sigma}{d\Omega}=\frac{M^2}{32\pi^2 s}\sum_{\text{spin}}|T|^2. \quad (19)$$

The isotopic content of the amplitude is

$$A_{\beta\alpha}=\delta_{\beta\alpha}A^+ +\tfrac{1}{2}[\tau_\beta,\tau_\alpha]A^-, \quad (20)$$

where

$$A^+=\tfrac{1}{3}(A_{1/2}+2A_{3/2}),\quad A^-=\tfrac{1}{3}(A_{1/2}-A_{3/2}), \quad (21)$$

with similar relations holding for $B_{\beta\alpha}$.

It is useful to relate the A and B amplitudes directly to partial waves. We write the differential cross section in the c.m. system as

$$d\sigma/d\Omega=|\langle 2|F|1\rangle|^2,$$

where F is taken between Pauli, not Dirac, spinors. Then the scattering amplitude F has the usual partial-wave expansion[14]

$$F=\sum_{l=0}^{\infty}[lf_l^- +(l+1)f_l^+]P_l(\cos\theta) - i\boldsymbol{\sigma}\cdot(\hat{p}_2\times\hat{p}_1)[f_l^+ - f_l^-]P_l'(\cos\theta), \quad (22)$$

where θ is the c.m. angle.

The relation of F to the invariant amplitudes can be seen to be

$$F=\frac{(E+M)}{8\pi\sqrt{s}}[A+(M-\sqrt{s})B] - \frac{(E-M)}{8\pi\sqrt{s}}[A+(M+\sqrt{s})B] \times(\cos\theta+i\boldsymbol{\sigma}\cdot\hat{p}_2\times\hat{p}_1), \quad (23)$$

where E is the nucleon energy in the c.m. system.

The above equations can be inverted to express the partial waves f_l in terms of A and B. One obtains

$$f_l^\pm=\frac{1}{16\pi\sqrt{s}}\int_{-1}^{1}d(\cos\theta) \times\{(E+M)[A+(M-\sqrt{s})B]P_l(\cos\theta) -(E-M)[A+(M+\sqrt{s})B]P_{l\pm1}(\cos\theta)\}. \quad (24)$$

The S- and P-wave scattering lengths can be expressed in terms of the partial waves as follows:

$$a_S=[m_\pi f_0^+]_{q^2=0},\quad a_{P_{1/2}}=[m_\pi^3 f_1^-/q^2]_{q^2=0}, \quad a_{P_{3/2}}=[m_\pi^3 f_1^+/q^2]_{q^2=0}, \quad (25)$$

where $q^2=E^2-M^2$ is the square of the c.m. momentum.

These are all the kinematical relations that we will need. Therefore, in Sec. VI, we will proceed to calculate the isospin-odd scattering lengths.

VI. ISOSPIN-ODD SCATTERING LENGTHS

It is straightforward to work out the contributions to the isospin-odd amplitudes A^- and B^- coming from our effective Lagrangians. We find that the nucleon-exchange term gives

$$A^-=0, \quad (26a)$$

$$B^-=\frac{2f^2}{m_\pi^2}+\frac{4M^2f^2}{m_\pi^2}\left(\frac{1}{s-M^2}+\frac{1}{u-M^2}\right). \quad (26b)$$

[13] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).
[14] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1955).

The ρ-exchange term yields

$$A^{-}=\frac{f_0^2/m_\pi^2}{1-t/m_\rho^2}\frac{(u-s)K}{2M}\left(1-\frac{t}{4m_\rho^2}\right), \qquad (27a)$$

$$B^{-}=\frac{-2f_0^2/m_\pi^2}{1-t/m_\rho^2}(1+K)\left(1-\frac{t}{4m_\rho^2}\right), \qquad (27b)$$

where we have made use of the Kawarabayashi-Suzuki relation, Eq. (9). If we make use of the direct-model Lagrangian, the above would be replaced by

$$A^{-}=0, \qquad (27a')$$

$$B^{-}=-2f_0^2/m_\pi^2. \qquad (27b')$$

Finally the N^* contribution to A^- and B^- is

$$A^{-}=(h^2/3m_\pi^2)\{(8(M^*+M)(t-2m_\pi^2)+(4M^*+6M)(s-M^2)+(4/3M^*)[s^2-s(M^2-4m_\pi^2)-4m_\pi^2(M^2-m_\pi^2)] + (2M/3M^{*2})[3s^2-s(7M^2-8m_\pi^2)+4(M^2-m_\pi^2)^2])/(s-M^{*2})-[(s\to u, t\to t)]\} \qquad (28a)$$

and

$$B^{-}=(h^2/3m_\pi^2)\{(-2(s-M^2)-8(t-m_\pi^2)+M(8M^*+12M)-(8M/3M^*)[s-2M^2+2m_\pi^2] -(2/3M^{*2})[s^2-s(3M^2-4m_\pi^2)+4(M^2-m_\pi^2)^2])/(s-M^{*2})+[(s\to u, t\to t)]\}. \qquad (28b)$$

We should note that, in general for the N^* amplitudes, M^* in the denominator should be replaced by $M^*-\frac{1}{2}i\Gamma$ to account for the N^* width. For our purposes this will not be necessary since we are evaluating our expressions at threshold.

The scattering lengths are now computed by making use of Eqs. (24) and (25). In particular, we see that for the S wave these equations reduce to

$$a_S^{-}=\frac{m_\pi}{4\pi(1+m_\pi/M)}(A^{-}-m_\pi B^{-})_{q^2=0}, \qquad (29)$$

which gives

$$a_S^{-}=\tfrac{1}{3}(a_1-a_3)=\frac{1}{4\pi(1+m_\pi/M)}\times\left(\frac{2}{4M^2/m_\pi^2-1}f^2+2f_0^2+\frac{4m_\pi^2}{M^{*2}}h^2\right). \qquad (30)$$

The second term is the result of either the direct model or the ρ-exchange model. This is clear since we have normalized the ρ-exchange model, through the Kawarabayashi-Suzuki relation, to agree with the direct model for S waves.

Putting in the appropriate values for the coupling constants f, f_0, and h, we obtain

$$\tfrac{1}{3}(a_1-a_3)=+0.101,$$

which compares very favorably with the experimental value of Samaranayake and Woolcock,[15]

$$\tfrac{1}{3}(a_1-a_3)_{\text{expt}}=+0.097\pm0.006.$$

As one would expect, the N^* contributes very little ($\sim$1%) to this combination of scattering lengths. In fact, as is well known,[16] the ρ-exchange term alone (or the direct term alone) suffices to obtain agreement within experimental error.

In calculating the P-wave scattering lengths, we are interested in terms of order q^2 only. We can write, using Eq. (24),

$$f_{1^+}{}^{(-)}=\frac{1}{8\pi(1+m_\pi/M)}\times\int_{-1}^{1}[A^{-}(x)-m_\pi B^{-}(x)]x\,dx+O(q^4) \qquad (31)$$

and

$$f_{1^-}{}^{(-)}=f_{1^+}{}^{(-)}-\frac{q^2}{4M^2}f_{0^+}{}^{(-)}-\frac{q^2}{16\pi M}\int_{-1}^{1}B^{-}(x)dx+O(q^4). \qquad (32)$$

A straightforward, albeit lengthy, evaluation of the above gives the following expression for the $P_{3/2}$ scattering lengths for the ρ-exchange model:

$$a_{P_{3/2}}{}^{-}=\tfrac{1}{3}(a_{13}-a_{33})=\frac{1}{36\pi(1+m_\pi/M)}\left\{-\frac{6}{(1-m_\pi/2M)^2}f^2+\left(\frac{9m_\pi^2}{m_\rho^2}-\frac{3m_\pi K}{M}\right)f_0^2+\left[-\frac{16m_\pi}{M^*-(M+m_\pi)} +\frac{m_\pi}{M^{*2}-(M-m_\pi)^2}\left(8M^*+4M-12m_\pi+\frac{8}{3M^*}(-M^2+6Mm_\pi+3m_\pi^2) +\frac{4}{3M^{*2}}(M^3+11M^2m_\pi-9Mm_\pi^2-3m_\pi^3)\right)\right]h^2\right\}. \qquad (33)$$

If we had used the direct model there would have been no f_0 contribution.

[15] V. Samaranayake and W. Woolcock, Phys. Rev. Letters **15**, 936 (1965).
[16] Y. Tomozawa, Nuovo Cimento **46A**, 707 (1966); S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).

Numerically this gives, for the ρ-exchange model,

$$\tfrac{1}{3}(a_{13}-a_{33})=-0.054-0.007-0.026=-0.087\,,$$

while for the direct model the result is

$$\tfrac{1}{3}(a_{13}-a_{33})=-0.080\,.$$

Both of these results agree reasonably well with the experimental value of Hamilton and Woolcock,[7]

$$\tfrac{1}{3}(a_{13}-a_{33})_{\text{expt}}=-0.081\pm0.005\,,$$

and thus do not provide a clear test of which chiral Lagrangian model is preferable.

We see from Eq. (32) that for the $P_{1/2}$ scattering lengths it is only necessary at this stage to compute

$$R^{(-)}=m_\pi^2\int_{-1}^{1}B^{(-)}(x)dx\,.$$

We find

$$\begin{aligned}R^{(-)}=-\frac{4}{(4M^2/m_\pi^2-1)}f^2-4(1+K)f_0^2+\Bigg[&\frac{-\frac{4}{3}(M^{*2}-M^2-m_\pi^2)}{[(M^*+M)^2-m_\pi^2][(M^*-M)^2-m_\pi^2]}\Bigg(6m_\pi+M(8M^*+12M)\\&+\frac{8M}{3M^*}(M^2-3m_\pi^2)-\frac{2}{3M^{*2}}(2M^4-M^2m_\pi^2+9m_\pi^4)\Bigg)\\&+\frac{(32/3)M^2m_\pi^2}{[(M^*+M)^2-m_\pi^2][(M^*-M)^2-m_\pi^2]}\Bigg(1+\frac{4M}{3M^*}-\frac{1}{3M^{*2}}(M^2-6m_\pi^2)\Bigg)\Bigg]h^2\end{aligned}\qquad(34)$$

using the ρ-exchange model. For the direct model, the result is the same except that K is set equal to zero. Then

$$a_{P_{1/2}}{}^-=\tfrac{1}{3}(a_{11}-a_{31})=\tfrac{1}{3}(a_{13}-a_{33})-(m_\pi^2/12M^2)(a_1-a_3)-(m_\pi/16\pi M)R^{(-)}.\qquad(35)$$

Putting in the appropriate numerical values we obtain for the ρ-exchange model

$$\tfrac{1}{3}(a_{11}-a_{31})=0.054+0.033+0.024=+0.003\,,$$

where the first term is the nucleon contribution, the second is the ρ contribution, and the last is the N^* contribution. The direct model replaces the ρ contribution by a much smaller value and gives instead

$$\tfrac{1}{3}(a_{11}-a_{31})=-0.022\,.$$

The quoted experimental values for this combination of scattering lengths differ somewhat (see Table III). If we average these various experimental determinations of $a_{P_{1/2}}{}^-$, we obtain

$$\tfrac{1}{3}(a_{11}-a_{31})_{\text{expt}}=-0.017\,.$$

It appears that the direct chiral Lagrangian model gives a value for $a_{P_{1/2}}{}^-$ which is in better agreement with experiment than the ρ-exchange model. This is somewhat surprising since the ρ-exchange Lagrangian provides a natural extension of the direct chiral Lagrangian, and contains some very interesting predictions in its own right.[4] It is clear that the difference between these two models, as far as this calculation is concerned, resides in the inclusion of the magnetic moment interaction, Eq. (8c). If K were zero, ρ exchange would then give $a_{P_{1/2}}{}^-=-0.020$.

We could assume that the ρ magnetic interaction was not a contact interaction, but rather that it was the result of a vertex modification of the minimal ρNN vertex. If this were the case, then this magnetic interaction, being of higher order, would not contribute in the tree approximation.[2] We have not been able, however, to justify this hypothesis of minimality. Therefore we must continue to include a contact magnetic interaction term, Eq. (8c), so as to preserve ρ dominance of the magnetic isovector form factor.

We should note that the ρ-exchange contributions are to be compared with the "current-commutator" terms in the current-algebra calculations.[5] In fact, our ρ-exchange term is identical to Schnitzer's current-commutator term except that we have an explicit model for his nucleon isovector form factors $F_1(t)$ and $F_2(t)$, namely, ρ dominance. Thus it is not very surprising that both Schnitzer's and Raman's values for $a_{P_{1/2}}{}^-$ should also not agree with the experimental number given above, being too small and positive, respectively (see Table II). The direct model replaces the current-commutator terms in the current-algebra calculations by a much smaller contribution, and therefore gives better agreement with experiment.

Since the experimental determination of this combination of scattering lengths is not that certain, and also in view of the current-algebra results, we believe that before we can make a definitive choice between our chiral Lagrangian models we will need a more accurate determination of $a_{P_{1/2}}{}^{-}$. At the moment, however, the present data favor the direct model.

VII. ISOSPIN-EVEN SCATTERING LENGTHS

The contributions of the nucleon-exchange and the N^*-exchange terms to the isospin-even amplitudes A^+ and B^+ are

$$A^+=\frac{4M}{m_\pi^2}f^2-\frac{2h^2}{3m_\pi^2}\Bigg[\bigg(8(M^*+M)(1-2m_\pi^2)+(4M^*+6M)(s-M^2)+\frac{4}{3M^*}[s^2-s(M^2-4m_\pi^2)-4m_\pi^2(M^2-m_\pi^2)] + \frac{2M}{3M^{*2}}[3s^2-s(7M^2-8m_\pi^2)+4(M^2-m_\pi^2)^2]\bigg)\frac{1}{s-M^{*2}}+[(s\to u,t\to t)]\Bigg] \quad (36)$$

and

$$B^+=\frac{4M^2}{m_\pi^2}\left(\frac{1}{s-M^2}-\frac{1}{u-M^2}\right)f^2+\frac{2}{3}\frac{h^2}{m_\pi^2}\Bigg[\bigg(2(s-M^2)+8(t-m_\pi^2)-M(8M^*+12M)+\frac{8M}{3M^*}(s+2m_\pi^2-2M^2) + \frac{2}{3M^{*2}}[s^2-s(3M^2-4m_\pi^2)+4(M^2-m_\pi^2)^2]\bigg)\frac{1}{s-M^{*2}}-[(s\to u,t\to t)]\Bigg]. \quad (37)$$

Neither ρ-exchange nor the direct-interaction term contributes to these amplitudes, and thus the isospin-even amplitudes do not provide any further test as to which chiral model is preferable.

By using the methods of Sec. VI we can calculate the isospin-even scattering lengths. For the S wave we find

$$\tfrac{1}{3}(2a_3+a_1)=\frac{1}{4\pi(1+m_\pi/M)}\left(-\frac{4Mm_\pi}{4M^2-m_\pi^2}f^2-\frac{8(2M^*+M)m_\pi}{M^{*2}}h^2\right). \quad (38)$$

Numerically, this gives

$$\tfrac{1}{3}(2a_3+a_1)=-0.011-0.050=-0.061.$$

This value for the isospin-even S-wave scattering length does not compare well with the determination of Samaranayake and Woolcock,[15]

$$\tfrac{1}{3}(2a_3+a_1)_{\text{expt}}=-0.013\pm0.003.$$

We remark that there also exists considerable uncertainty about the value of this combination of scattering lengths. For instance, the recent analysis of Hamilton[17] gives

$$\tfrac{1}{3}(2a_3+a_1)_{\text{expt}}=-0.001\pm0.003.$$

However, the consensus is that $a_S{}^+$ is small.

In his current-algebra calculation, Raman[5] has also obtained a value of $a_S{}^+$ which is of the same order of magnitude as ours. He argues that the scalar term which arises from the equal-time commutator of the axial current with the divergence of the axial current provides a correction to the calculated value of $a_S{}^+$, which brings it back more in line with the experimental value. In Sec. VIII we discuss a possible reason for the rather large value of $a_S{}^+$ obtained in our work, and indicate a way to correct it within our approach. We point out here only that this will require looking at the asymptotic behavior of the N^* amplitude.

The $P_{3/2}$ and $P_{1/2}$ scattering lengths are computed by using Eqs. (31) and (32) with A^- and B^- replaced by A^+ and B^+. In this way we find

$$\tfrac{1}{3}(2a_{33}+a_{13})=\frac{1}{36\pi(1+m_\pi/M)}\Bigg\{\frac{6}{(1-m_\pi/2M)^2}f^2+\Bigg[\frac{32m_\pi}{M^*-(M+m_\pi)}+\frac{2m_\pi}{M^{*2}-(M-m_\pi)^2}\bigg(8M^*+4M-12m_\pi+\frac{8}{3M^*}(-M^2+6Mm_\pi+3m_\pi^2)+\frac{4}{3M^{*2}}(M^3+11M^2m_\pi-9Mm_\pi^2-3m_\pi^3)\bigg)\Bigg]h^2\Bigg\}, \quad (39)$$

which gives

$$\tfrac{1}{3}(2a_{33}+a_{13})=0.054+0.072=0.126.$$

This is to be compared with the value of Hamilton and Woolcock,[7]

$$\tfrac{1}{3}(2a_{33}+a_{13})_{\text{expt}}=0.132\pm0.005.$$

[17] J. Hamilton, Phys. Letters **20**, 687 (1966).

Finally, for the $P_{1/2}$ scattering length we find $R^{(+)}$ to be

$$R^{(+)}=\frac{32M^3}{m_\pi(4M^2-m_\pi^2)}f^2+\left[\left(\frac{44}{3}M^2-2M^{*2}+\frac{16MM^*}{3}+4m_\pi^2+\frac{16M}{3M^*}(M^2-m_\pi^2)-\frac{2(M^2-m_\pi^2)^2}{M^{*2}}\right)\frac{(16/3)Mm_\pi}{[(M^*+M)^2-m_\pi^2][(M^*-M)^2-m_\pi^2]}\right]h^2. \quad (40)$$

This gives for the scattering length

$$\tfrac{1}{3}(2a_{31}+a_{11})=-0.106+0.040=-0.066,$$

which again compares favorably with Hamilton and Woolcock's value,

$$\tfrac{1}{3}(2a_{31}+a_{11})_{\text{expt}}=-0.059\pm0.005.$$

Other experimental values are given in Table III, and again the comparison is not unfavorable.

VIII. ASYMPTOTIC BEHAVIOR OF AMPLITUDES

It is convenient to define a new amplitude G which is a linear combination of A and B:

$$G(s,t)=A(s,t)+(M^2+m_\pi^2-s)B(s,t)/2M. \quad (41)$$

By assuming that $G^-(s,0)$ satisfied an unsubtracted dispersion relation, Goldberger, Miyazawa, and Oehme[18] obtained the following sum rule for the isospin-odd S-wave scattering length:

$$\frac{4\pi}{3}\left(1+\frac{m_\pi}{M}\right)(a_1-a_3)=\frac{2f^2}{1-m_\pi^2/4M^2}+\frac{m_\pi^2}{\pi}\int_{m_\pi}^{\infty}\frac{[\sigma^-(w)-\sigma^+(w)]}{(w^2-m_\pi^2)}qdw, \quad (42)$$

where $\sigma^\pm(w)$ are the total cross sections for $\pi\mp+p$ scattering.

Making use of our expression, Eq. (30), for the scattering length combination a_1-a_3 gives us

$$f_0^2=f^2-\frac{2m_\pi^2}{M^{*2}}h^2+\frac{m_\pi^2}{2\pi}\int_{m_\pi}^{\infty}\frac{[\sigma^-(w)-\sigma^+(w)]}{(w^2-m_\pi^2)}qdw. \quad (43)$$

Recalling that $f/f_0=-G_A/G_V$, it is clear that the above is essentially the Adler-Weisberger relation.[19] We should note that this particular way of arriving at the Adler-Weisberger relation was pointed out some time ago by Weinberg.[20]

The essential point in the above derivation was the assumption that G^- satisfies an unsubtracted dispersion relation. What we must examine is whether the chiral Lagrangian method really produces an amplitude which is consistent with the above assumption. Using Eqs. (26), (27), or (27′), and (28), we find that as $s\to\infty$, $G^-(s,0)$ goes like

$$G^-(s,0)\underset{s\to\infty}{\sim}\frac{(s-M^2-m_\pi^2)}{Mm_\pi^2}\times\left[f_0^2-f^2+h^2\left(\frac{8(M+M^*)^2+10m_\pi^2}{9M^{*2}}\right)\right]+O(1/s). \quad (44)$$

The only way that we have to guarantee that G^- behaves properly at infinity is to ask that the square brackets vanish identically. That is,

$$f_0^2-f^2+h^2\left(\frac{8(M+M^*)^2+10m_\pi^2}{9M^{*2}}\right)+\cdots=0. \quad (45)$$

Here the dots stand for contributions of the other resonances of the pion-nucleon system which presumably are also present in G^- for large s.

Comparison with Eq. (43) makes it clear that what we are requiring in the above is just that the Adler-Weisberger relation hold. Thus the h^2 contribution in Eq. (45) is to be identified with the integral over the resonance in Eq. (43). In fact, this is precisely what Schnitzer[5] used to determine the $N^*N\pi$ coupling constant.

If we terminate Eq. (45) with the N^* resonance, we obtain a value of G_A/G_V of around 1.45. This is reasonable since it is well known[19] that the N^* contribution alone in the Adler-Weisberger relation overestimates the value of G_A/G_V.

These remarks indicate that the isospin-odd amplitudes generated from the chiral Lagrangians have the right asymptotic behavior. That this is so increases our confidence in the low-energy results obtained.

We now turn to the isospin-even amplitudes. While G^- corresponds to the difference between the amplitude for π^-+p scattering and the amplitude for π^++p scattering, G^+ corresponds to the sum of these amplitudes. Thus we do not expect that G^+ vanish in the forward direction as $s\to\infty$. Since

$$[d\sigma^+/dt]_{t=0}\propto|G^+(s,0)/s|^2, \quad (46)$$

we must certainly require that as $s\to\infty$ there should be no term in $G^+(s,0)$ which goes as s^2. But we find

[18] M. L. Goldberger, H. Miyazawa, and R. Oehme, Phys. Rev. **99**, 986 (1955).

[19] S. L. Adler, Phys. Rev. Letters **14**, 1051 (1965); W. I. Weisberger, *ibid.* **14**, 1047 (1965).

[20] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).

that our chiral Lagrangian gives us

$$G^+(s,0) \underset{s\to\infty}{\sim} -s^2\left[\frac{4h^2}{9m_\pi^2M^{*2}M}\right]. \tag{47}$$

It has recently been suggested by Weinberg[21] that we should require the sum of all the amplitudes generated by a chiral Lagrangian (in the tree approximation) not to grow faster at high energy than the actual scattering amplitude. If we follow this approach, then we must require that additional terms should be present in our Lagrangian so as to cancel the bad asymptotic behavior of the N^* amplitude above. Clearly various possibilities are now open. We could assume that the higher resonances contribute appropriate terms of order s^2 to $G^+(s,0)$, thus canceling the N^* contribution as $s\to\infty$. This is in effect what happens in the isospin-odd case where the higher resonances provide the tail of the Adler-Weisberger relation. We could equally well assume that the N^* contribution at high energy is canceled by an appropriate contact term. Or we could invoke a mixture of these two mechanisms.

Depending on which of the above approaches we choose, we would naturally also alter the behavior of the amplitude in the low-energy region. The work of Raman[5] shows that the inclusion of resonances higher than the $N^*(1236)$ does not materially affect a_S^+. Hence if the higher resonances alone are enough to cancel the high-energy N^* behavior, we are at a loss to explain the discrepancy between the theoretically calculated and the experimentally obtained value of a_S^+. On the other hand, if we assumed that a contact term was necessary to correct the higher-s behavior of the N^* amplitude, we would alter the value previously calculated for a_S^+ considerably.

As an example of this, we note that if we choose the most obvious contact term, namely,

$$G_{\text{con}}^+(s,t) = +\frac{4}{9}h^2\frac{s^2}{M^{*2}m_\pi^2M}, \tag{48}$$

to cancel the N^* s^2 contribution, then this gives a threshold modification of a_S^+,

$$(a_S^+)_{\text{con}} = \frac{1}{4\pi(1+m_\pi/M)}\left(\frac{4}{9}\frac{(M+m_\pi)^4h^2}{M^{*2}Mm_\pi}\right) = +0.060.$$

In terms of this simple model we would then have

$$\tfrac{1}{3}(2a_3+a_1)_{\text{tot}} = -0.001,$$

which is in reasonable agreement with the experimentally quoted values.

For the $P_{3/2}$ scattering lengths the contact term as chosen has no effect. Its effect on the isospin-even $P_{1/2}$ scattering length cannot be determined without some more specific assumptions, but it should be small.

We would like to emphasize that the above example is merely suggestive, and one should not take very seriously the perhaps fortuitous agreement with experiment.

We present in Tables I–III our final results and compare them with other theoretical analyses and experiment.

TABLE I. S-wave scattering lengths.

Length	Chiral Lagrangian	ρ-exchange alone	Expt.[a]
$\frac{1}{3}(a_1-a_3)$	0.101	0.099	0.097
		Chiral Lagrangian plus contact	
$\frac{1}{3}(2a_3+a_1)$	-0.061	-0.001	-0.013[a] -0.001[b]

[a] Reference 15.
[b] Reference 17.

TABLE II. P-wave scattering lengths—theory.

Length	ρ-exchange model	Direct model	Raman	Schnitzer
$\frac{1}{3}(a_{13}-a_{33})$	-0.087	-0.080	-0.083	-0.075
$\frac{1}{3}(a_{11}-a_{31})$	$+0.003$	-0.022	$+0.012$	-0.005
$\frac{1}{3}(2a_{33}+a_{13})$	$+0.126$	$+0.126$	$+0.133$	$+0.114$
$\frac{1}{3}(2a_{31}+a_{11})$	-0.066	-0.066	-0.064	-0.070

TABLE III. P-wave scattering lengths—experiment.

Length	HW[a]	RWF[b]	SW[c]
$\frac{1}{3}(a_{13}-a_{33})$	-0.081	-0.081	-0.081
$\frac{1}{3}(a_{11}-a_{31})$	-0.021	-0.016	-0.013
$\frac{1}{3}(2a_{33}+a_{13})$	$+0.134$	$+0.137$	$+0.136$
$\frac{1}{3}(2a_{31}+a_{11})$	-0.059	-0.069	-0.055

[a] Reference 7.
[b] We take the 0–350 MeV solution of L. D. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. 138, B190 (1965), with $a_{33}=0.217$.
[c] V. Samaranayake and W. Woolcock, as quoted by K. Raman (Ref. 5).

IX. CONCLUSION

The addition of a $N^*N\pi$ interaction to the chiral Lagrangian models of pion-nucleon scattering has allowed us to calculate P-wave scattering lengths, and N^* contributions to the S-wave scattering lengths. As we can see from Tables I–III, the predicted values and the experimental values are in reasonable agreement, except for a_S^+. For the $a_{P_{1/2}}^-$ scattering length we obtained agreement only with the direct model. As we have emphasized before, we do not fully understand why this should be so, and hope that further experimental analyses will clarify this matter.

The S-wave isospin-even scattering length as calculated directly from the phenomenological Lagrangian was far from in agreement with experiment. A possible

[21] S. Weinberg, Phys. Rev. (to be published).

explanation of this discrepancy is that we have failed to include a contact term for the isospin-even amplitudes. Some supporting evidence for this line of thought comes from Weinberg's hypothesis of the high-energy behavior of chiral Lagrangian amplitudes.[21] Though we cannot show that a contact term is needed purely from asymptotic arguments, we cannot rule it out either. In fact, a simple model constructed for this contact term was able both to fix the asymptotic behavior and to bring the value of a_S^+ within experimental uncertainty. We believe, however, that this point still remains an open and interesting question.

Another example that correct asymptotic behavior can be an important constraint in chiral Lagrangian calculations was provided by the isospin-odd amplitudes. These amplitudes did not have the correct asymptotic behavior unless the coefficient of the most singular term as $s\rightarrow\infty$ vanished. The vanishing of this coefficient was equivalent to requiring that the Adler-Weisberger sum rule hold.

Our results were found to be in general agreement with the current-algebra calculations of the scattering lengths,[5] especially if we used the ρ-exchange model. There were, however, various differences. In particular, our N^* contributions differ somewhat from the ones obtained by Raman and Schnitzer, since we used a different $N^*N\pi$ vertex. Our vertex was constructed such that the pion-nucleon system coupled only to the spin-$\frac{3}{2}$ part of the N^*, irrespective of whether the N^* is on or off the mass shell. To achieve this we had to impose a subsidiary condition on the coupling which forced us to take a particular combination of $g_{\mu\nu}$ and $\gamma_\mu\gamma_\nu$ in the coupling matrix. Raman and Schnitzer in their work retain only the $g_{\mu\nu}$ term.

Clearly, we do not have current-commutator contributions either, since we are using a Lagrangian formalism. These current-commutator terms are replaced in our approach either by the direct pion-nucleon scattering term (direct model) or by the ρ-exchange terms (ρ-exchange model). This last model reproduces the values that Schnitzer obtains from the current-commutator terms, but does not reproduce Raman's values.

In conclusion, it appears that the inclusion of the $N^*(1236)$ in chiral Lagrangian models of pion-nucleon scattering has proven useful, since it provides an alternative way of calculating the low-energy pion-nucleon parameters. We should point out, in this respect, that by using a Lagrangian approach we do not have any ambiguities which may arise from the extrapolation of the amplitudes in a current-algebra calculation. It is in this sense that we believe that the phenomenological Lagrangian method provides, within the limitations of the approach, a more straightforward calculation of the pion-nucleon scattering lengths.

These ideas can also be applied to photoproduction and N^* production. Work is currently being done in these areas and will be reported at a later date.

Note added in revised manuscript. We have recently become aware of the work of T. Pradhan, E. C. G. Sudarshan, and R. P. Saxena, Phys. Rev. Letters **20**, 79 (1968), in which S- and P-wave scattering lengths are calculated using a universal Lagrangian model. The agreement between their values and our values for the scattering lengths is good with two exceptions. Their value for a_{33} is understandably too low since they are using a static approximation. Their value of $a_S^+=0$ is in much better agreement with experiment than ours, but it is not clear to us how they treated the $N^*N\pi$ vertex.

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